Color Instabilities in Quark-Gluon Plasma

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30 years

1) St. Mrówczyński,
Stream instabilities of the quark-gluon plasma,
Physica Letters B **214**, 587 (1988), Erratum B **656**, 273 (2007)

2) St. Mrówczyński,
Plasma Instability at the initial stage of ultrarelativistic heavy-ion collisions,
Physics Letters B **314**, 118 (1993)

⋮

5) St. Mrówczyński and M. Thoma,
Hard loop approach to anisotropic systems,
Physical Review D **62**, 036011 (2000)

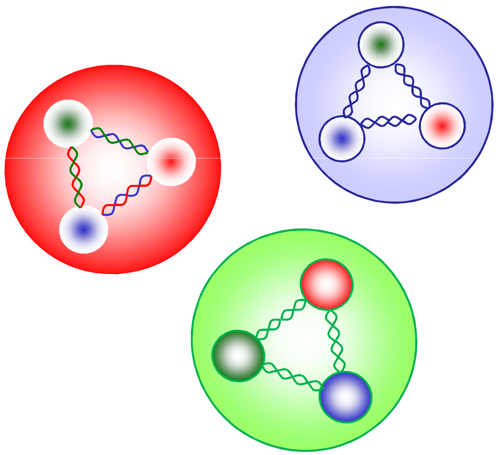
⋮

17) St. Mrówczyński, B. Schenke and M. Strickland,
Color instabilities in the quark-gluon plasma,
Physics Reports **682**, 1 (2017)

Hadrons, Quarks & Gluons

baryons

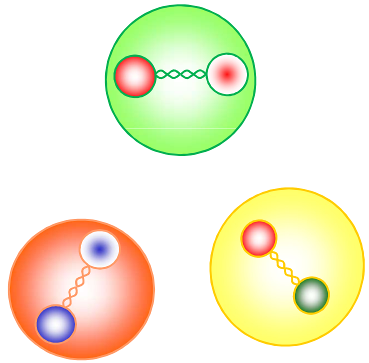
$n, p, \Delta, N^*, \Lambda, \Sigma, \Xi, \Omega, \ldots$



(q, q, q)

mesons

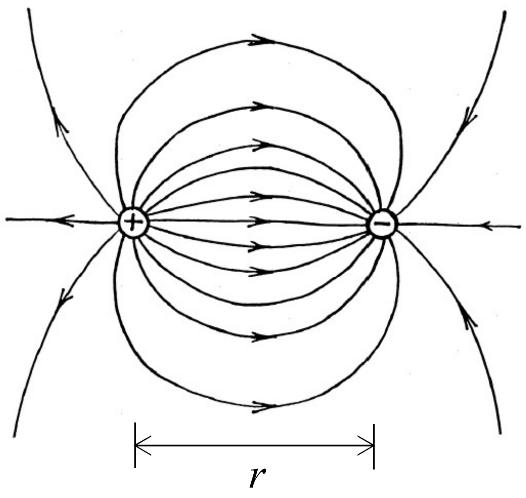
$\pi, K, \rho, \eta, \ldots$



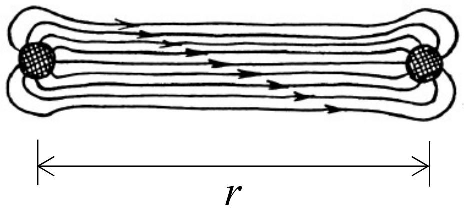
$(q, \overline{q})$

Confinement

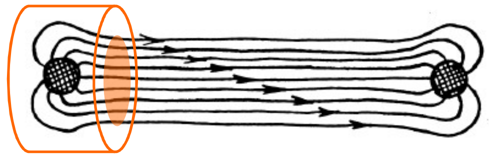
Electrodynamics



Chromodynamics



Gauss law

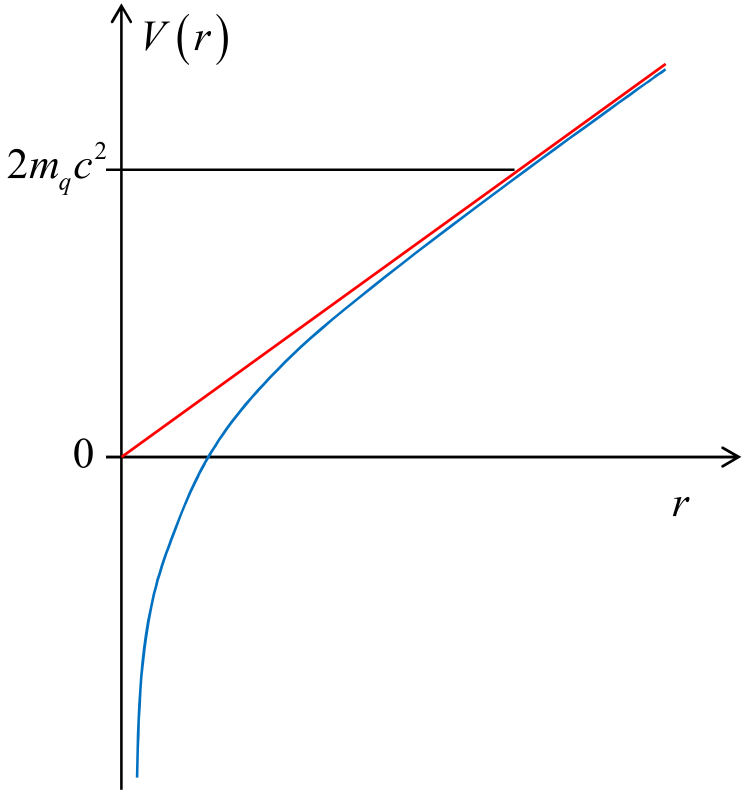


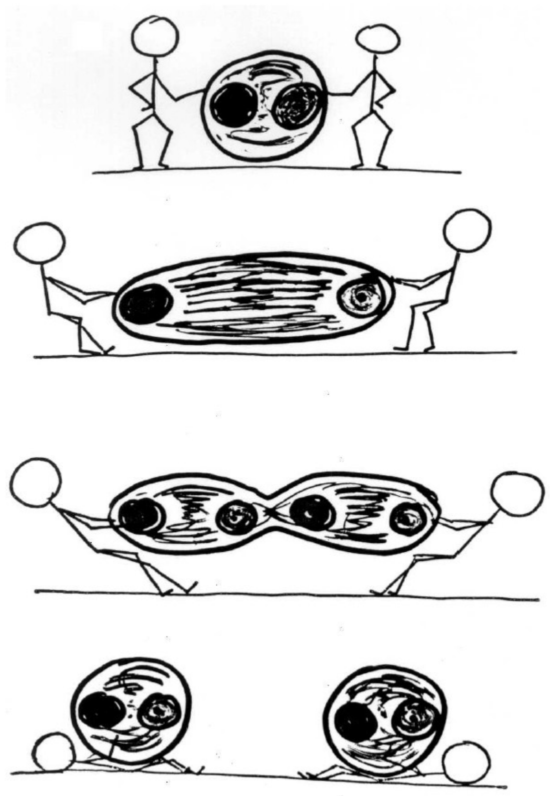
$$\Phi = \sigma E = 4\pi g$$

$$E(r) = \frac{e}{r^2} \Rightarrow V(r) = -\frac{e^2}{r}$$

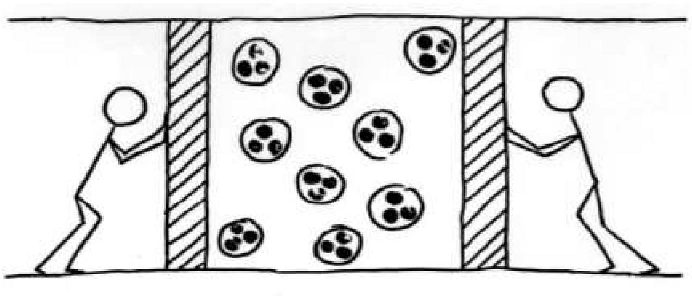
$$E(r) = \frac{4\pi g}{\sigma} \Rightarrow V(r) = \frac{4\pi g}{\sigma} r$$

Confinement cont.

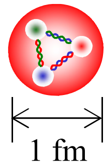




Creation of Quark-Gluon Plasma



compression of nuclear matter



$$\rho_0 = 0.12 \text{ fm}^{-3}$$

nurmal nuclear density

heating up hadron gas

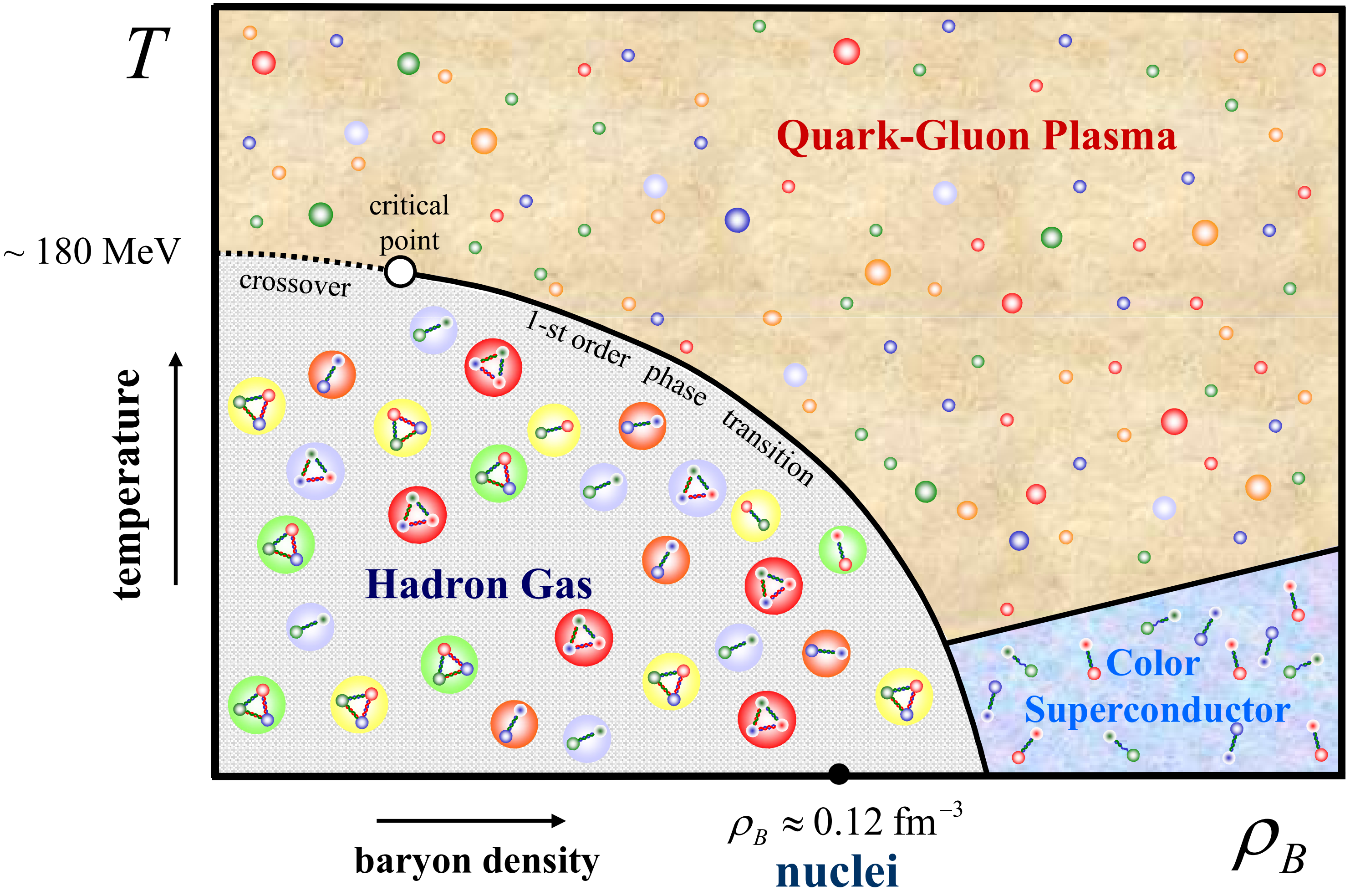
hadron density

$$\rho \sim T^3$$

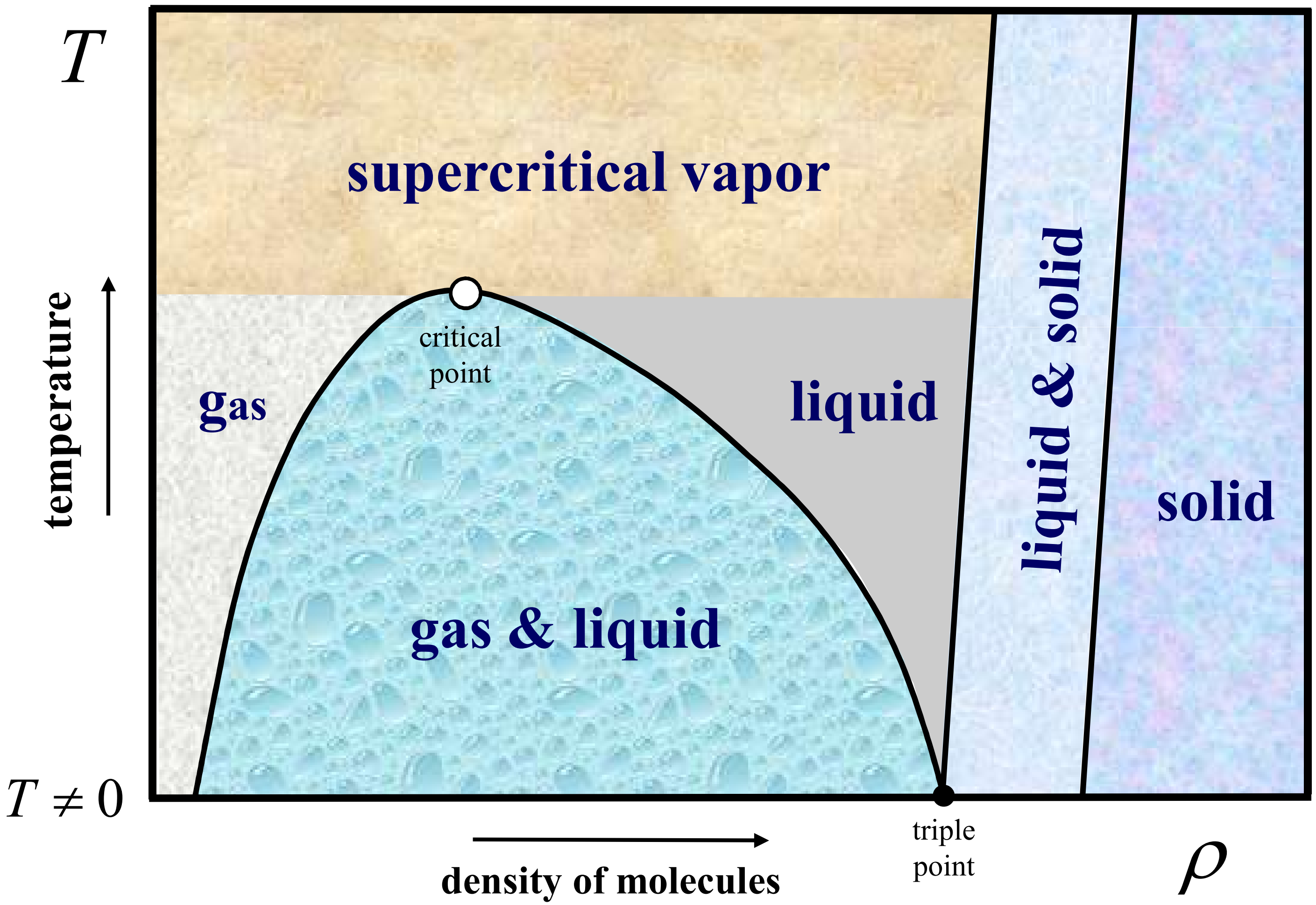
$$m_\pi << T$$

Natural system of units: $\hbar = c = k_B$

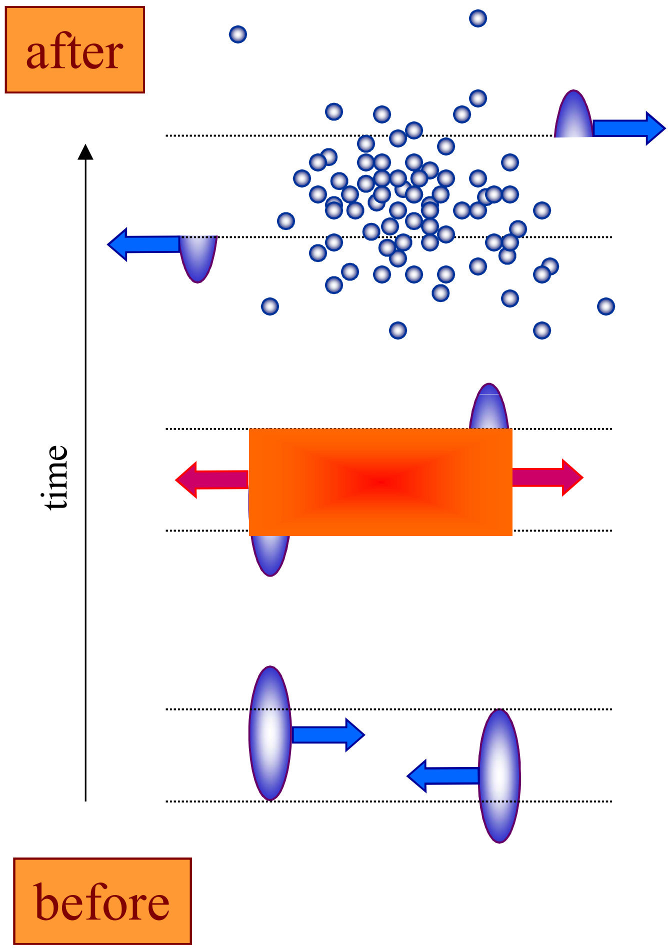
Phase diagram of strongly interacting matter

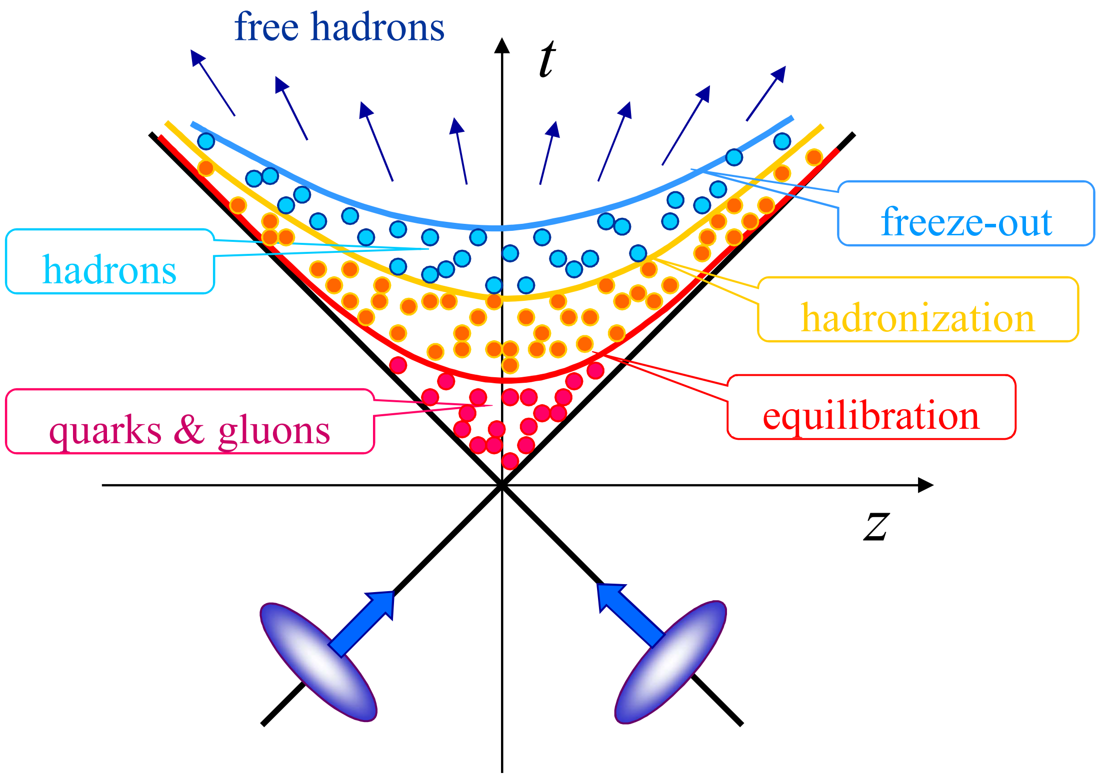


Schematic phase diagram of a simple fluid

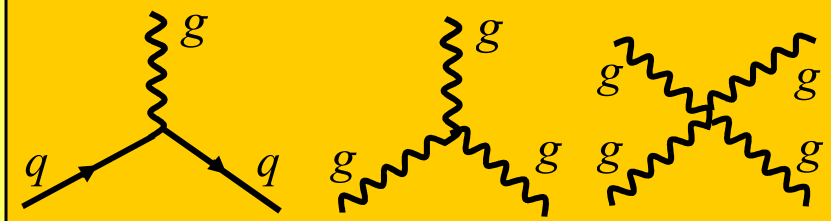
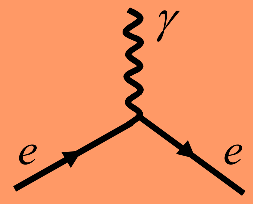


Relativistic heavy-ion collisions





Quark-Gluon Plasma vs. EM Plasma

		Quark-Gluon Plasma	Electromagnetic Plasma
Underlying Microscopic Theory		**QCD**	**QED**
Elementarny Interactions			
Constituents	**Fermions**	quarks, antiquarks	electrons, positrons
	Massless Gauge Bosons	gluons	photons
		-	ions
Coupling		$\alpha(Q^2) = \frac{g^2}{4\pi} \approx 0.1 - 1$	$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$

Ultrarelativistic Quark-Gluon Plasma

Plasma constituents – quarks & gluons – are massless!

$$m_q << T$$

Temperature T is often the only dimensional parameter.

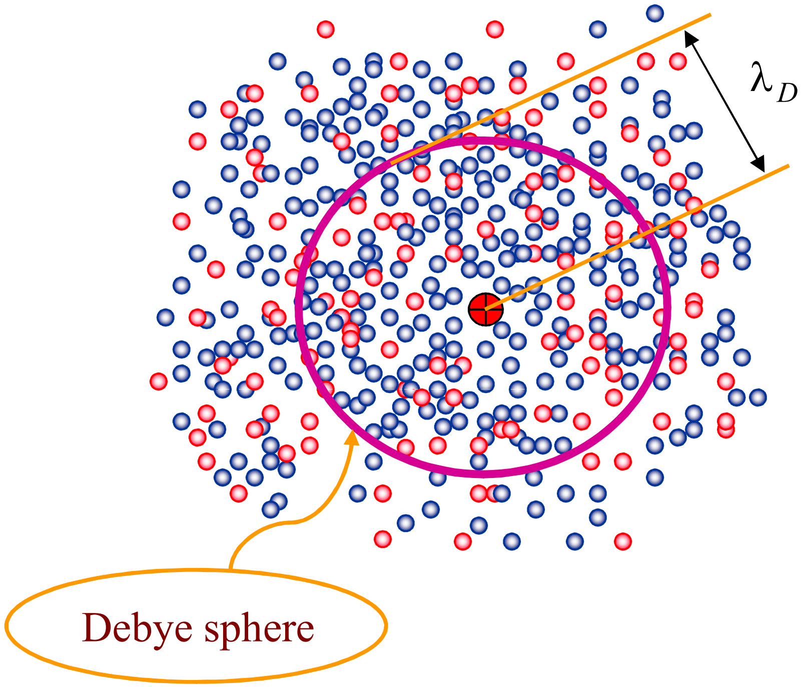
density: $\rho \sim T^3$

inter-particle spacing: $l \sim T^{-1}$

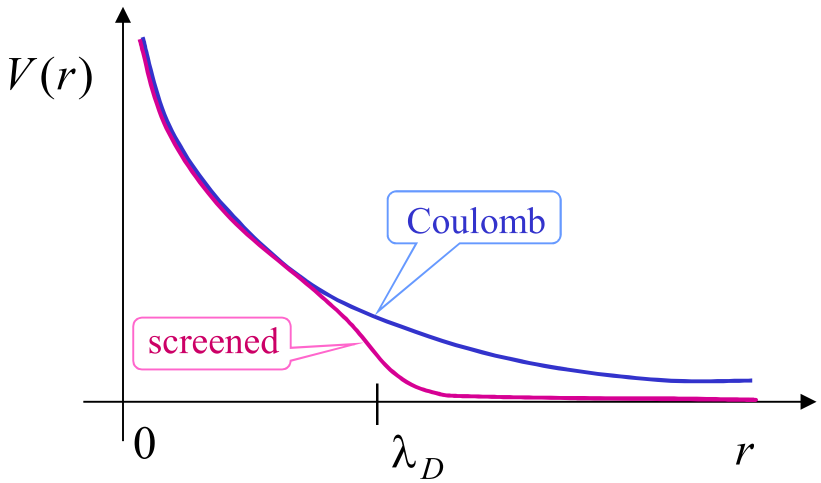
energy density: $\varepsilon \sim T^4$

pressure: $p \sim T^4$

Plasma manifests collective behavior



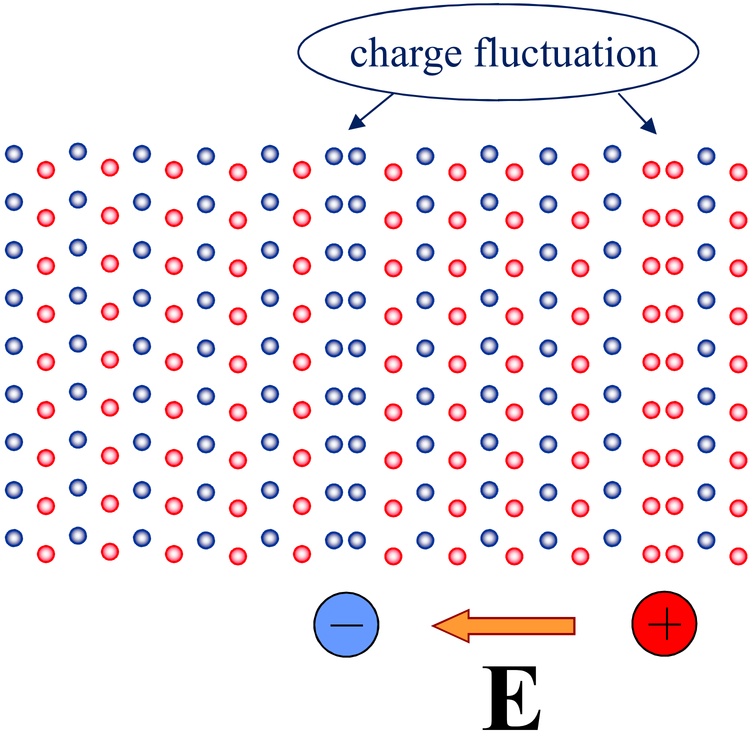
$$\lambda_D = \frac{1}{m_D} \sim \frac{1}{gT} \quad \text{screening length} \qquad V(r) \sim \frac{e^{-\frac{r}{\lambda_D}}}{r}$$



$$V_D = \frac{4}{3}\pi\lambda_D^3 \sim \frac{1}{g^3T^3}, \qquad n \sim T^3, \qquad n\,V_D \sim \frac{1}{g^3} >> 1 \quad \text{if} \quad g << 1$$

In a weakly coupled plasma, there are many particles in a Debye sphere!

Plasma oscillations



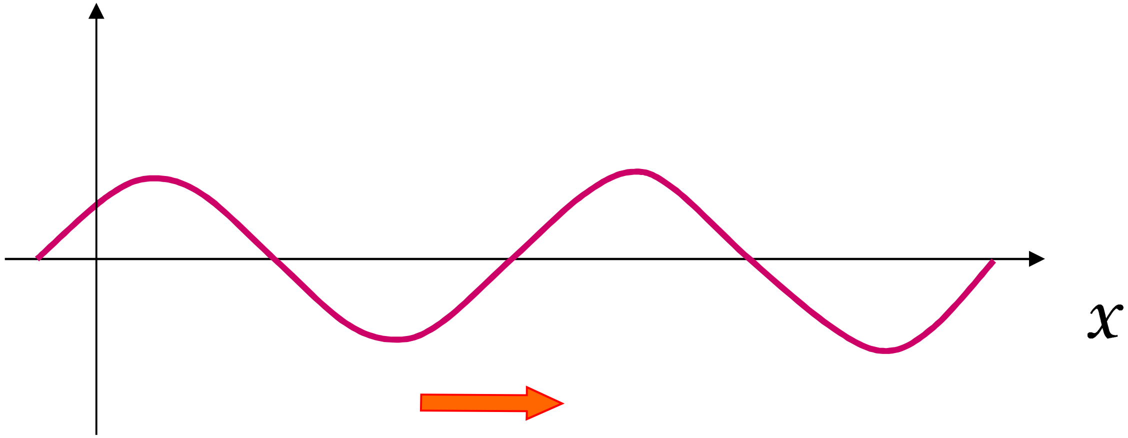
$$\mathbf{E}(t,\mathbf{r}) = \mathbf{E}_0 \cos(\omega(\mathbf{k})t - \mathbf{k}\cdot\mathbf{r} + \varphi)$$

$$\omega(\mathbf{k}) \underset{\mathbf{k}\to 0}{\approx} \omega_0 \sim gT$$

plasma frequency

Landau damping

$$E^x(t,x) = E_0 \cos(\omega_0 t - kx)$$



$$v_\varphi = \frac{\omega_0}{k}$$

Resonance energy transfer from electric field to particles with $v = v_\varphi$

Instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

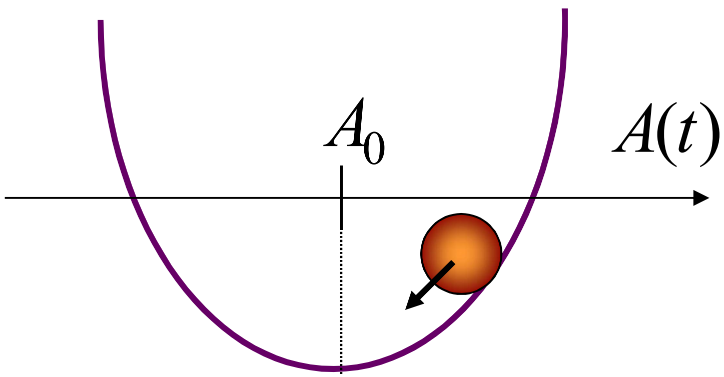
fluctuation

Instability

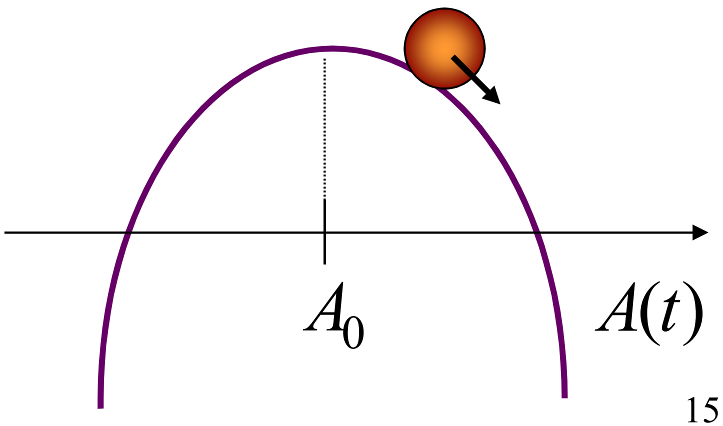
$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma > 0$$

stable configuration



unstable configuration



Plasma instabilities

- **instabilities in configuration space – hydrodynamic instabilities**

- **instabilities in momentum space – kinetic instabilities**

instabilities due to non-equilibrium momentum distribution

$$f(\mathbf{p}) \text{ is not } \sim \exp\left(-\frac{E}{T}\right)$$

Kinetic instabilities

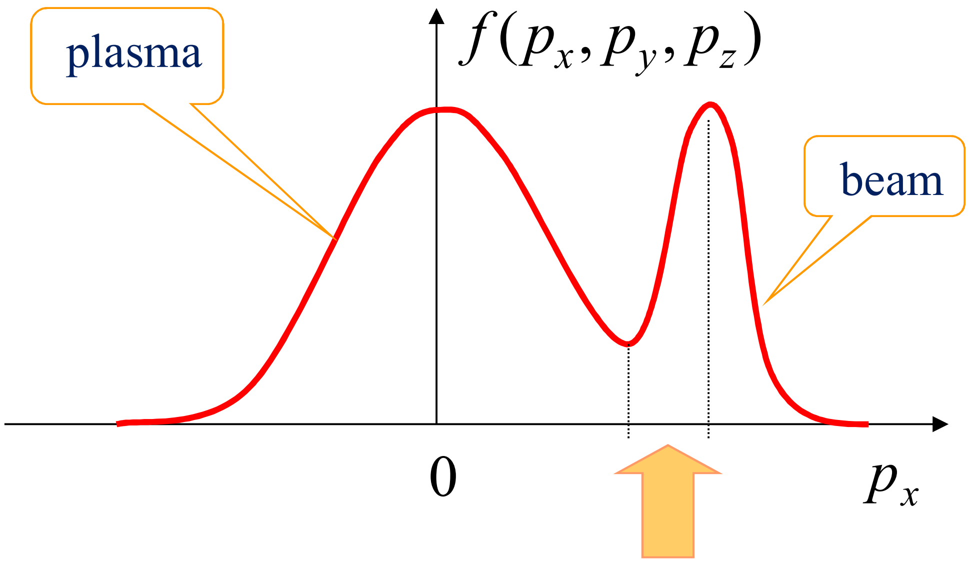
- longitudinal modes – $\mathbf{k} \parallel \mathbf{E}, \;\; \delta\rho \sim e^{-i(\omega t - \mathbf{kr})}$
- transverse modes – $\mathbf{k} \perp \mathbf{E}, \;\; \delta\mathbf{j} \sim e^{-i(\omega t - \mathbf{kr})}$

E – electric field, **k** – wave vector, ρ – charge density, **j** - current

Which modes are relevant for QGP
from relativistic heavy-ion collisions?

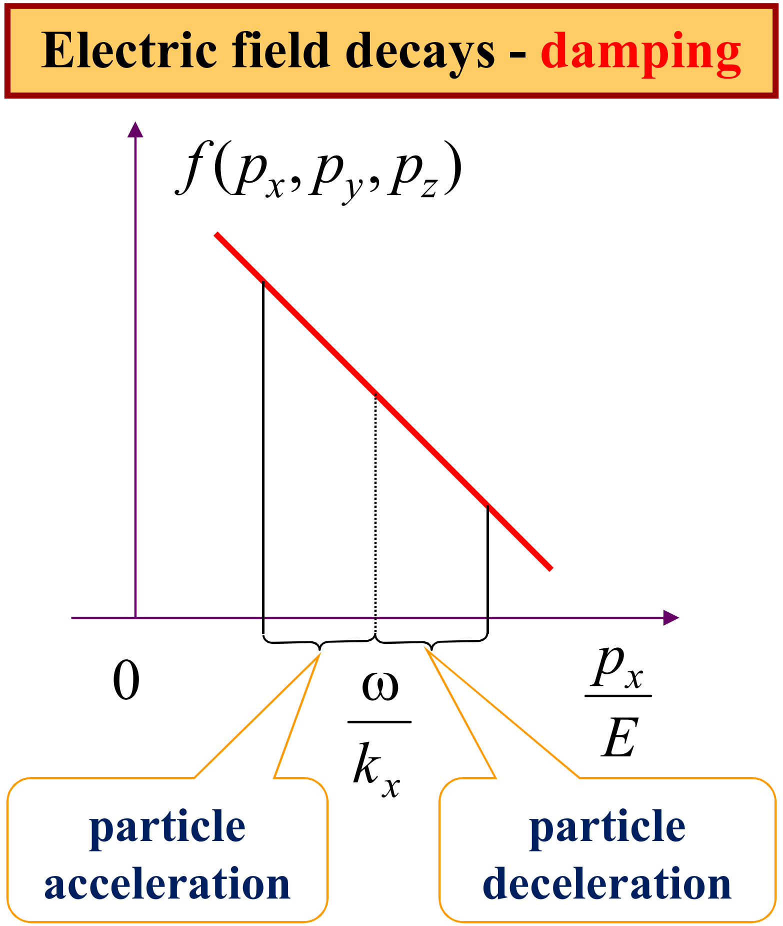
Logitudinal modes

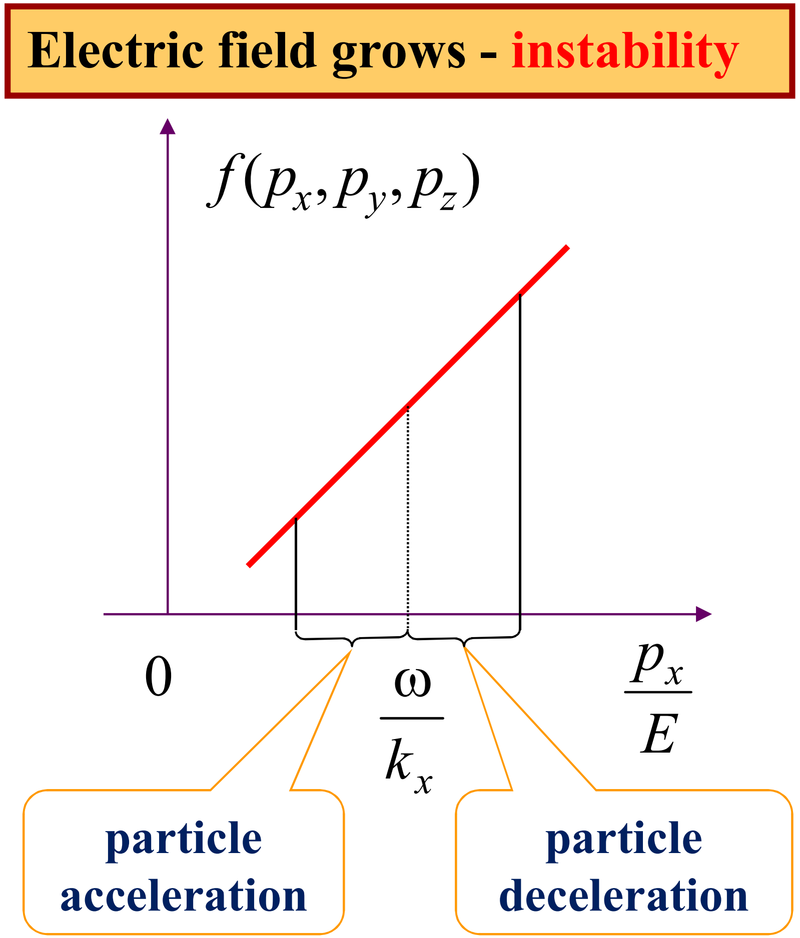
unstable configuration



Energy is transferred from particles to fields.

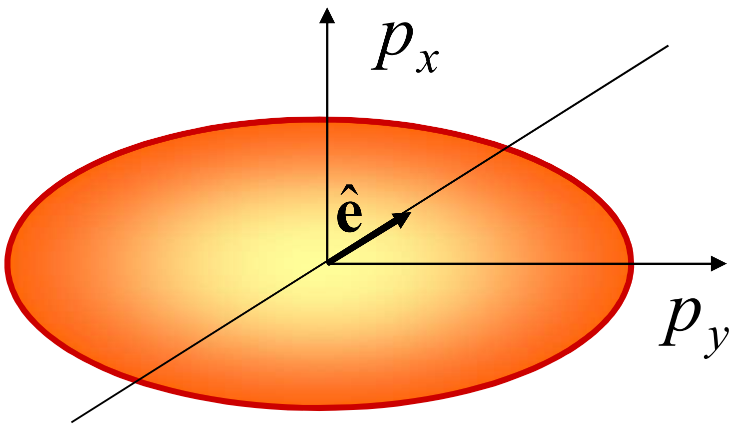
Logitudinal modes

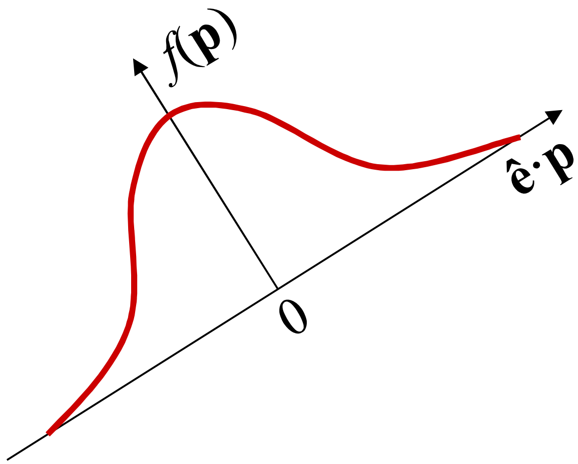




$\frac{\omega}{k_x}$ - phase velocity of the electric field wave, $\frac{p_x}{E}$ - particle's velocity

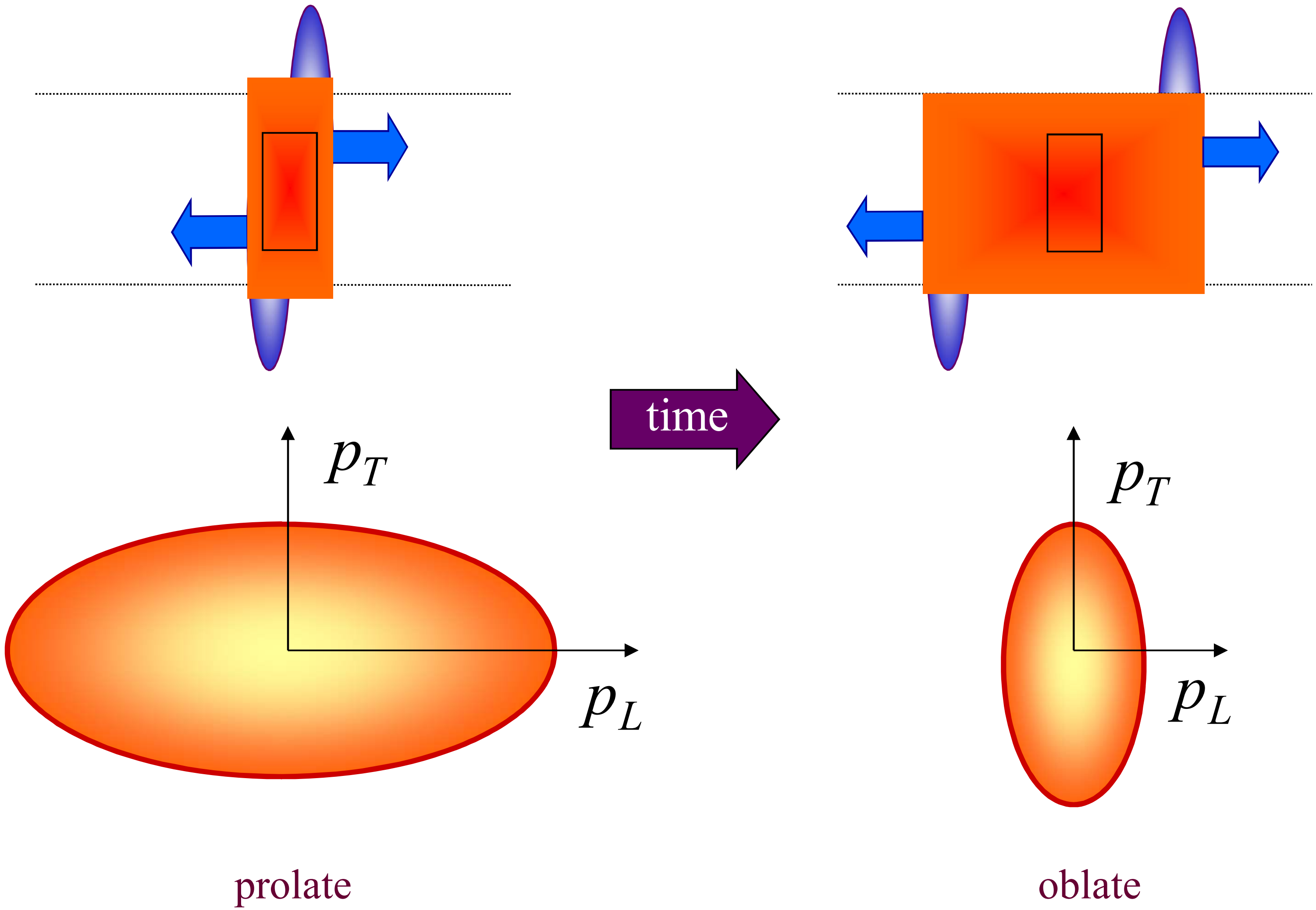
Parton momentum distribution in AA collisions





- Momentum distribution has a single maximum and monotonously decreases in every direction.
- Longitudinal unstable modes are irrelevant for relativistic heavy-ion collisions.
- There are unstable transverse modes.

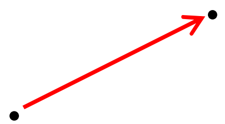
Evolution of Parton Momentum Distribution



Seeds of instability

$\langle j_a^\mu(x) \rangle = 0$ but current fluctuations are finite

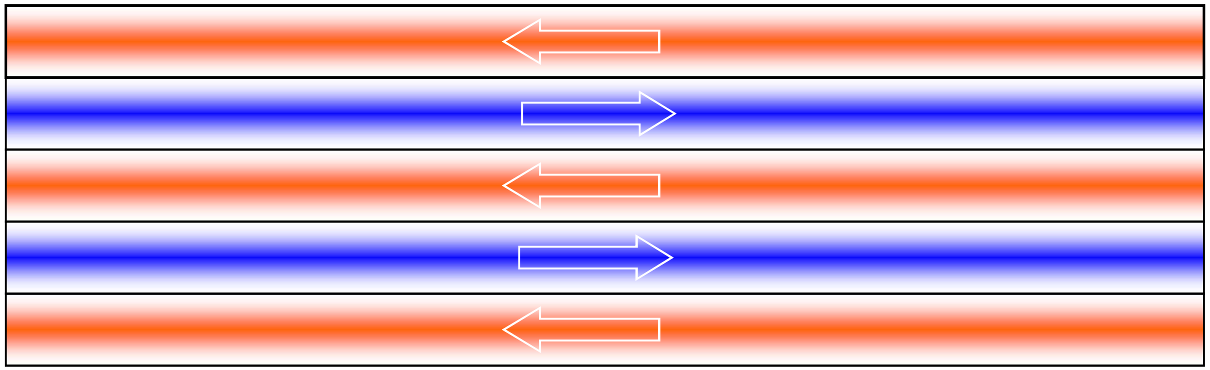
$$\langle j_a^\mu(x_1)\, j_b^\nu(x_2) \rangle = \frac{1}{2}\delta^{ab} \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E_p^2} f(\mathbf{p})\delta^{(3)}(\mathbf{x} - \mathbf{v}t) \neq 0$$



$x_2 = (t_2, \mathbf{x}_2)$

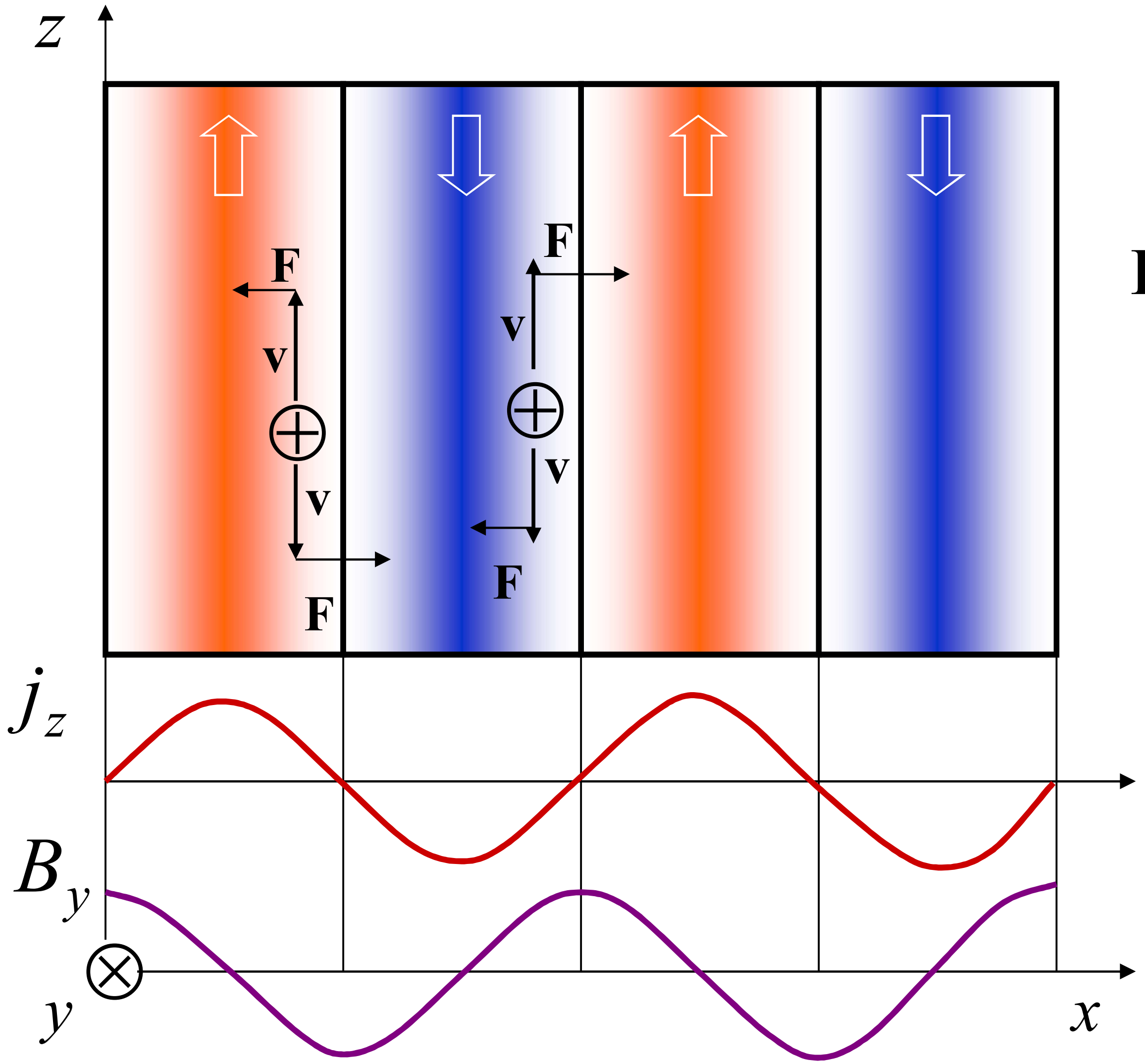
$x_1 = (t_1, \mathbf{x}_1)$

$x \equiv (t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2)$



Direction of the momentum surplus

Mechanism of filamentation



Lorentz force

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

Ampere's law

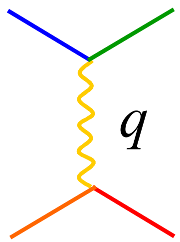
$$\nabla \times \mathbf{B} = \mathbf{j}$$

Time scale & collisional damping

Time scale of collective phenomena

$$t_{\text{collec}} \sim \frac{1}{gT} \;\Rightarrow\; \nu_{\text{collec}} \sim \frac{1}{t_{\text{collec}}} \sim gT$$

Parton-parton scattering



hard scattering: $q \sim T$

soft scattering: $q \sim gT$

Frequency of collisions

$$\nu_{\text{hard}} \sim g^4 \ln(1/g)\, T$$

$$\nu_{\text{soft}} \sim g^2 \ln(1/g)\, T$$

$$g^2 << 1 \;\Rightarrow\; \nu_{\text{hard}} << \nu_{\text{soft}} << \nu_{\text{collec}}$$

The instabilities are fast!

Growth of instabilities – 1+1 numerical simulations

SU(2) Hard Loop Dynamics

1+1 dimensions

$$A_a^\mu = A_a^\mu(t,z)$$

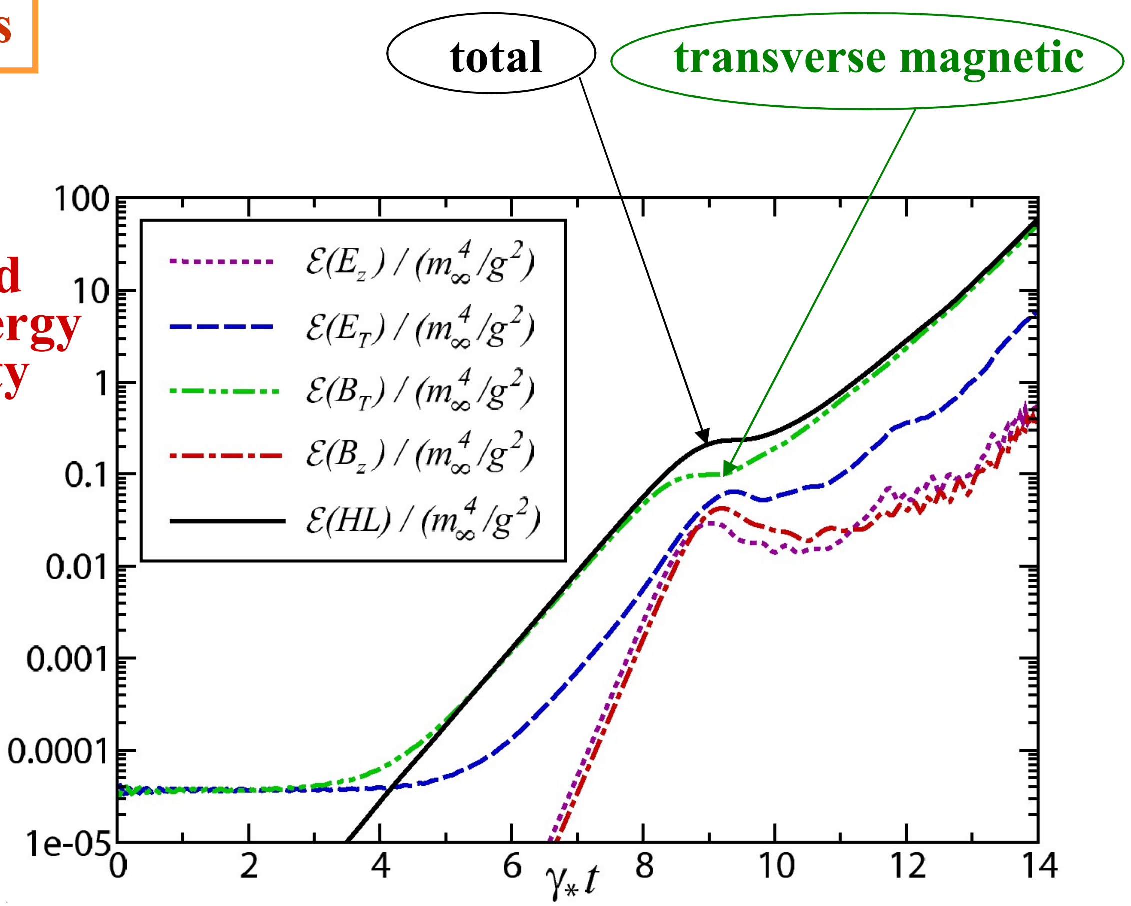
Anisotropic particle's momentum distribution

$$f(\mathbf{p}) = f_{\text{iso}}(|\mathbf{p}| + \zeta p_z)$$

$$m_D^2 = -\frac{\alpha_s}{\pi}\int_0^\infty dp\,p^2 \frac{df_{\text{iso}}(p)}{dp}$$

$$(m_D, \zeta)$$

Scaled field energy density



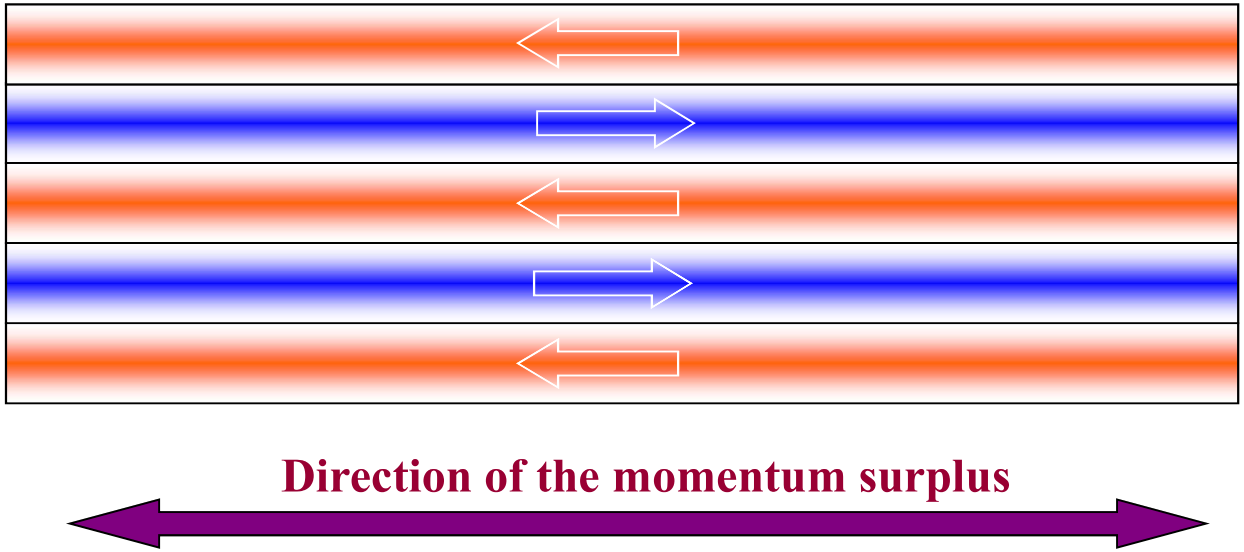
Strong anisotropy $\zeta = 10$ γ_* - maximal growth rate

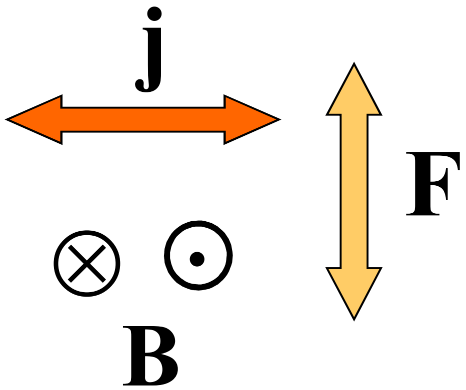
A. Rebhan, P. Romatschke & M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005)

What is the role of instabilities in nuclear collisions?

Instabilities speed up equilibration of quark-gluon plasma

Isotropization - particles





$$\Delta \mathbf{p} = \int dt\, \mathbf{F}$$

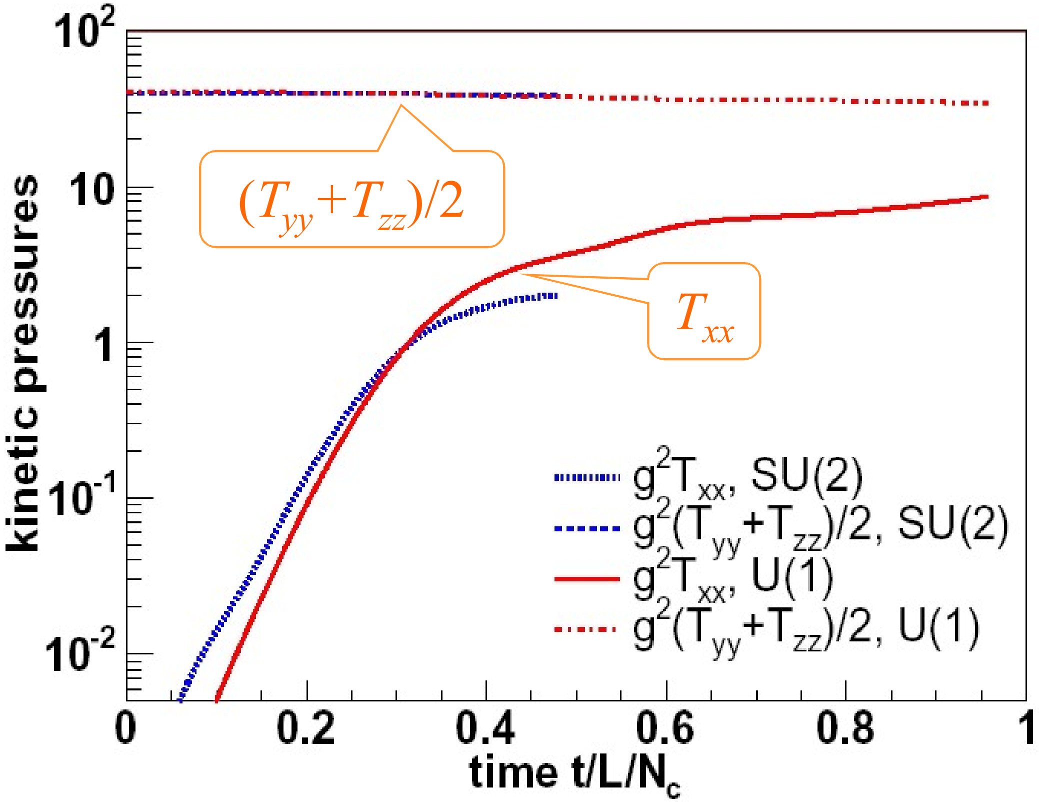
Isotropization – numerical simulation

Classical system of colored particles & fields

$$T_{ij} = \int \frac{d^3p}{(2\pi)^3} \frac{p_i p_j}{E} f(\mathbf{p})$$

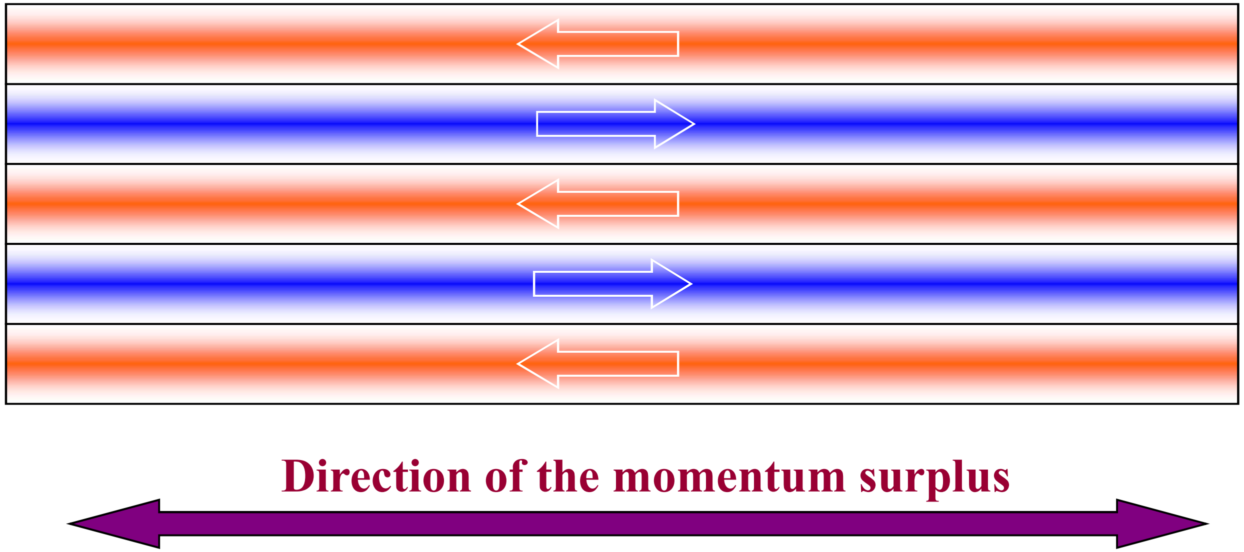
Isotropy:

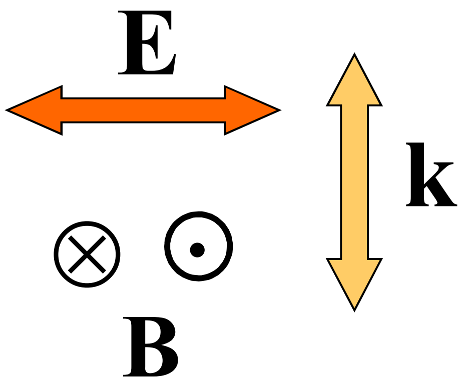
$$T_{xx} = (T_{yy} + T_{zz})/2$$



A. Dumitru & Y. Nara, Phys. Lett. **B621**, 89 (2005).

Isotropization - fields





$$\mathbf{P}_{\text{fields}} \sim \mathbf{B}^a \times \mathbf{E}^a \sim \mathbf{k}$$

Conclusions

- QGP is similar to EM plasmas
- Non-equilibrium QGP can be unstable
- Unstable transverse modes are relevant for AA collisions
- Instabilities drive equilibration