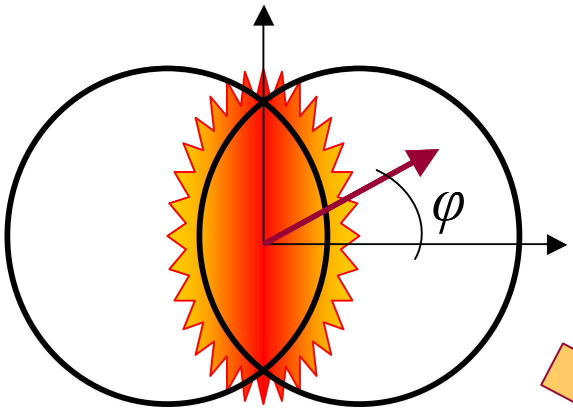
Instabilities Driven Equilibration of the Quark-Gluon Plasma

Stanisław Mrówczyński

Świętokrzyska Academy, Kielce, Poland
& Institute for Nuclear Studies, Warsaw, Poland

Evidence of the early stage equilibration

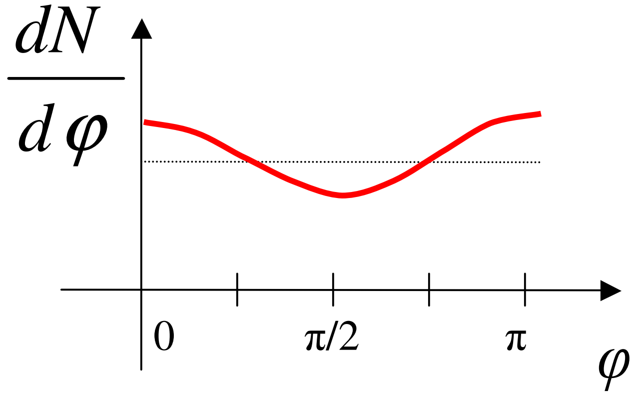
Success of hydrodynamic models in describing elliptic flow



Hydrodynamics

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\nabla\right)\mathbf{v} = -\frac{\nabla p}{\rho}$$

Hydrodynamic requires local thermodynamical equilibrium!



Equilibration is fast

$$v_2 \sim \varepsilon = \left\langle \frac{x^2 - y^2}{x^2 + y^2} \right\rangle$$

Eccentricity decays due to the free streaming!

$\varepsilon \searrow \;\Rightarrow\; v_2 \searrow$ ➡ $t_{eq} \leq 0.6\ \text{fm}/c$

time of equilibration

U. Heinz, AIP Conf. Proc.**739,** 163 (2004)

Collisions are too slow

Time scale of hard parton-parton scattering

hard scattering ~ momentum transfer of order of T

$$t_{\text{hard}} \sim \frac{1}{g^4 \ln(1/g)\, T}$$

either single hard scattering or multiple soft scatterings

$$t_{\text{eq}} \approx t_{\text{hard}} \geq 2.6 \ \text{fm}/c$$

R. Baier, A.H. Mueller, D. Schiff & D.T. Son, Phys. Lett. **B539**, 46 (2002)

Instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

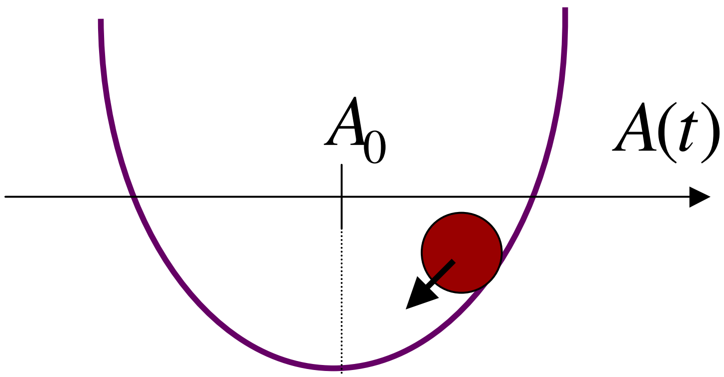
fluctuation

Instability

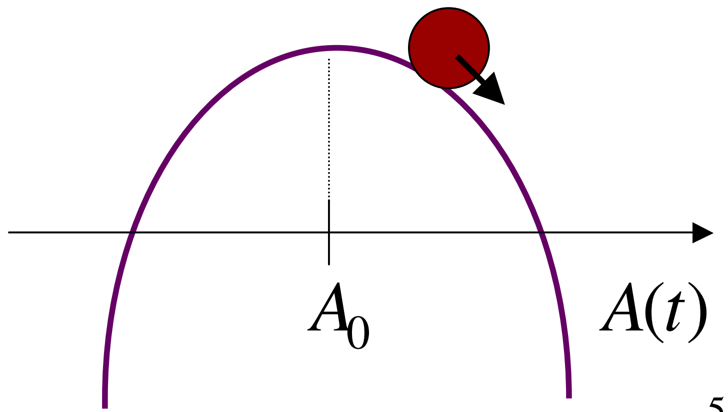
$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma > 0$$

stable configuration



unstable configuration



Plasma instabilities

- **instabilities in configuration space – hydrodynamic instabilities**

- **instabilities in momentum space – kinetic instabilities**

instabilities due to non-equilibrium momentum distribution

$$f(\mathbf{p}) \;\; \text{is} \;\; \text{not} \;\; \sim \exp\left(-\frac{E}{T}\right)$$

Kinetic instabilities

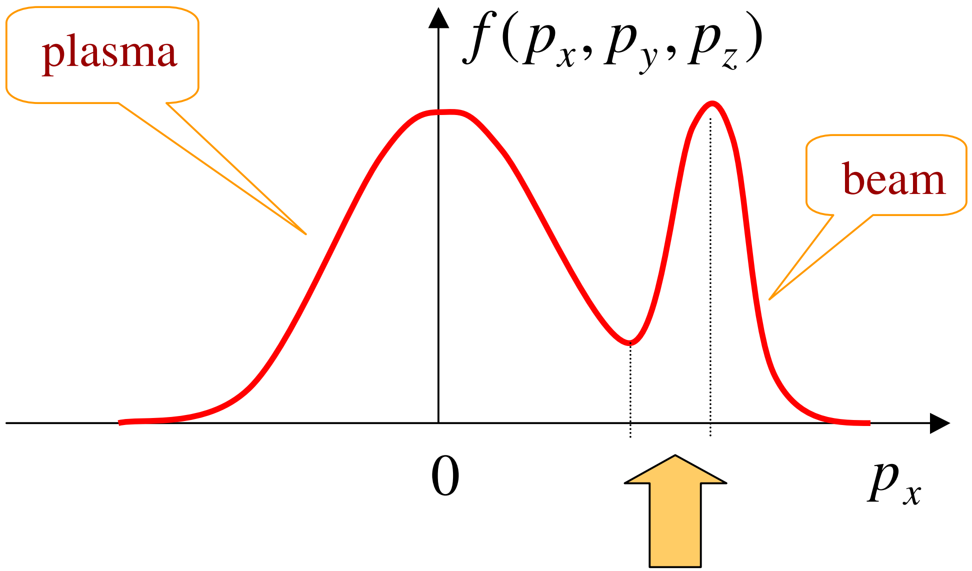
- **longitudinal modes** – $\mathbf{k} \parallel \mathbf{E}, \;\; \delta\rho \sim e^{-i(\omega t - \mathbf{kr})}$

- **transverse modes** – $\mathbf{k} \perp \mathbf{E}, \;\; \delta\mathbf{j} \sim e^{-i(\omega t - \mathbf{kr})}$

E – electric field, **k** – wave vector, ρ – charge density, **j** - current

Logitudinal modes

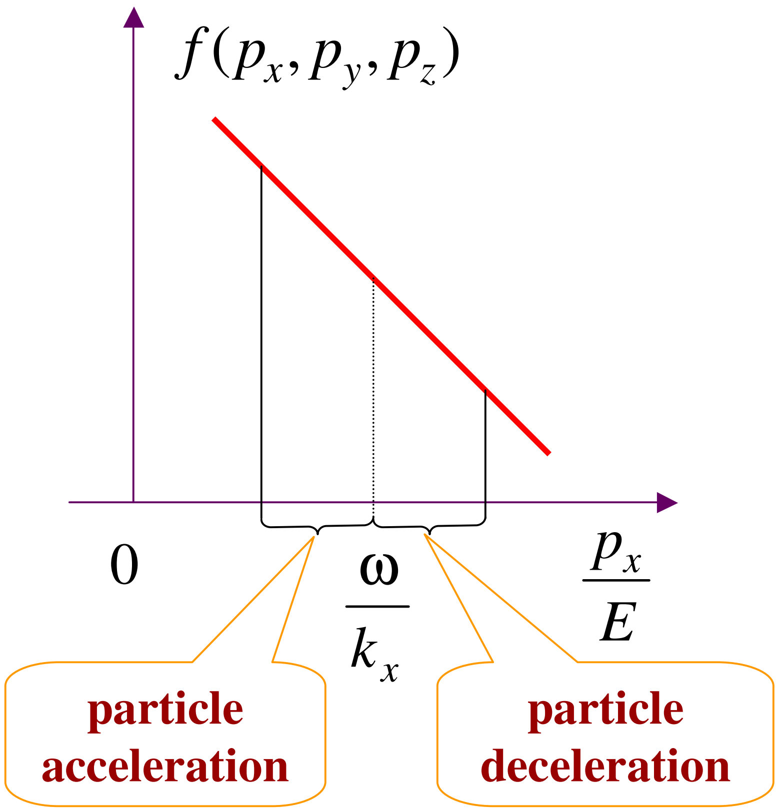
unstable configuration



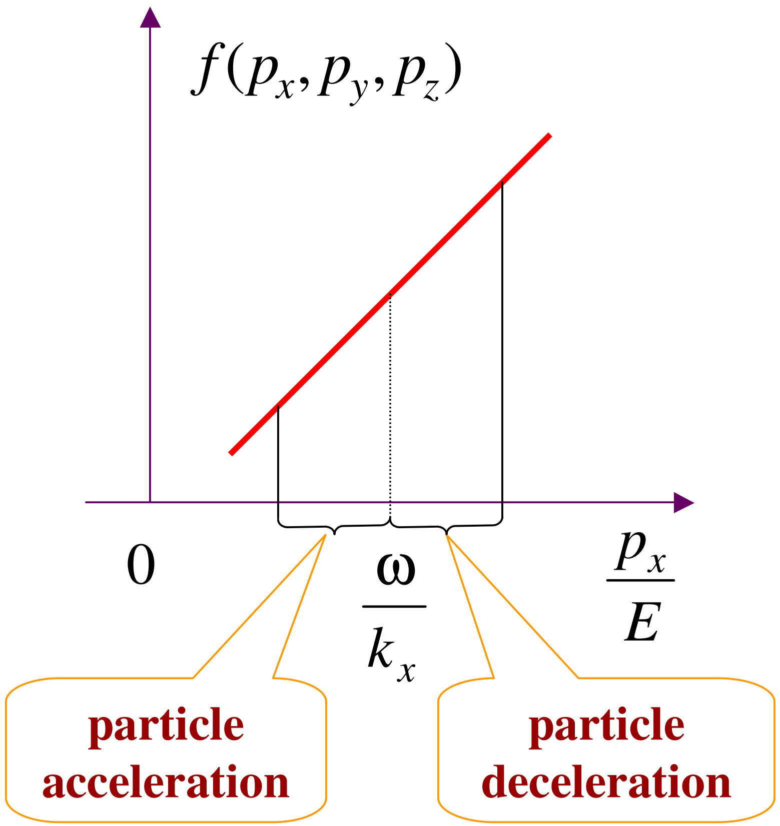
Energy is transferred from particles to fields

Logitudinal modes

Electric field decays - damping



Electric field grows - instability

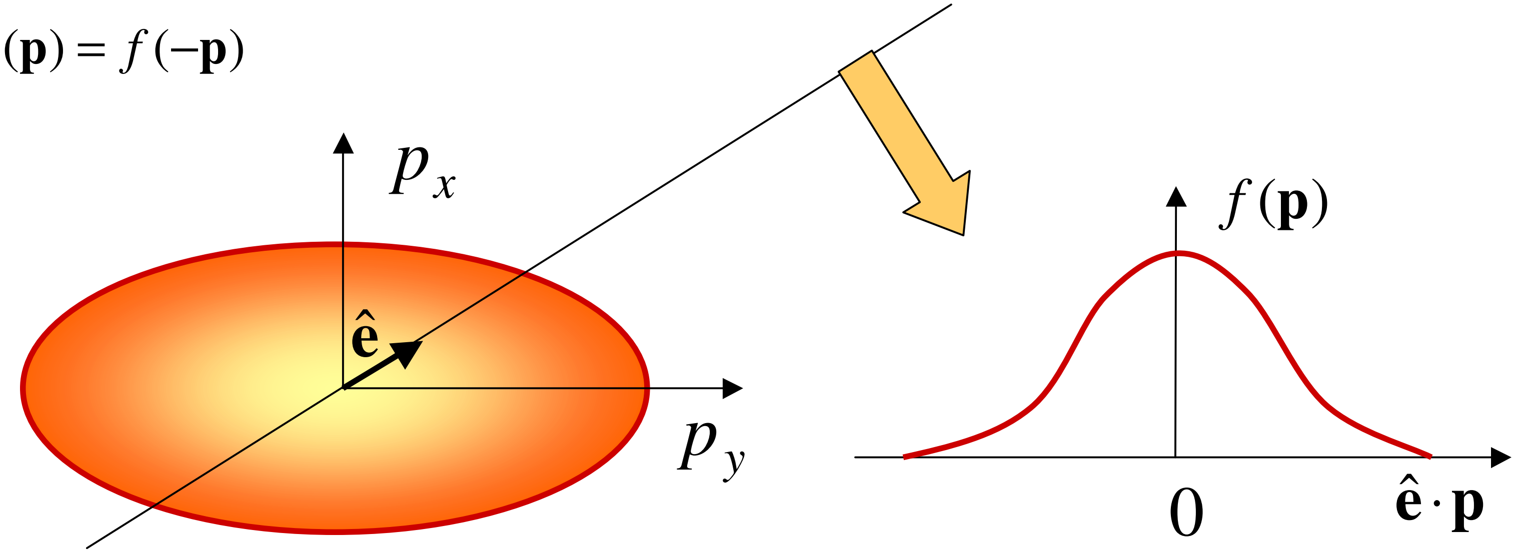


$\frac{\omega}{k_x}$ - phase velocity of the electric field wave, $\frac{p_x}{E}$ - particle's velocity

Transverse modes

Unstable modes occur due to anisotropy of the momentum distribution

$f(\mathbf{p}) = f(-\mathbf{p})$

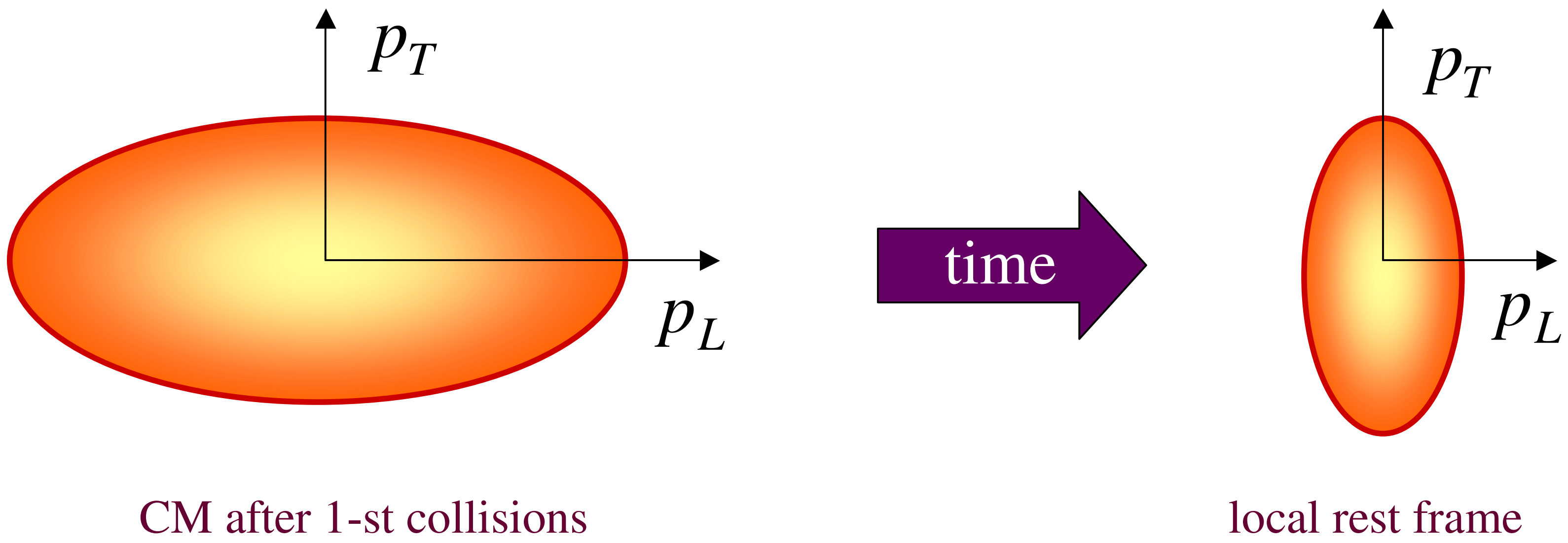


Momentum distribution distribution can monotonously decrease in every direction

Transverse modes are relevant for relativistic nuclear collisions!

Momentum Space Anisotropy in Nuclear Collisions

Parton momentum distribution is initially strongly anisotropic

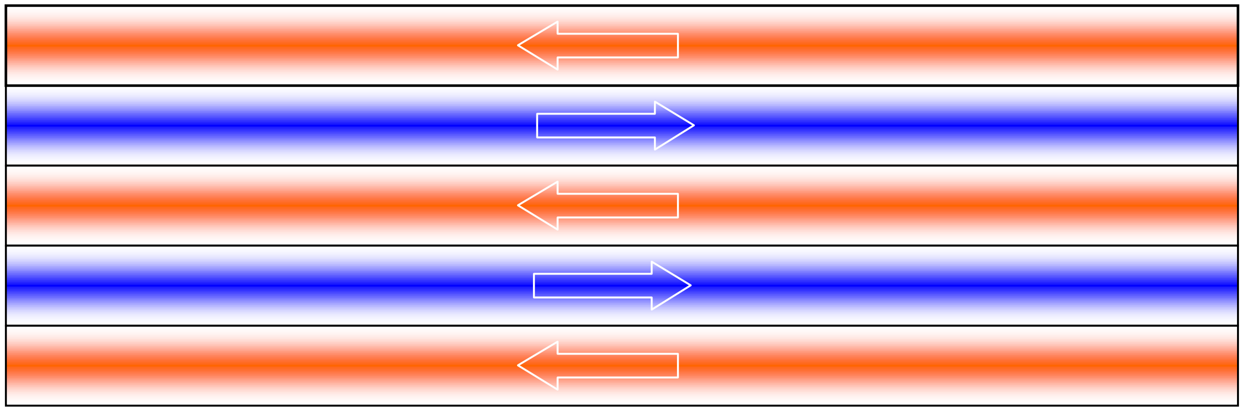


Seeds of instability

$\left\langle j_a^{\mu}(x) \right\rangle = 0$ **but current fluctuations are finite**

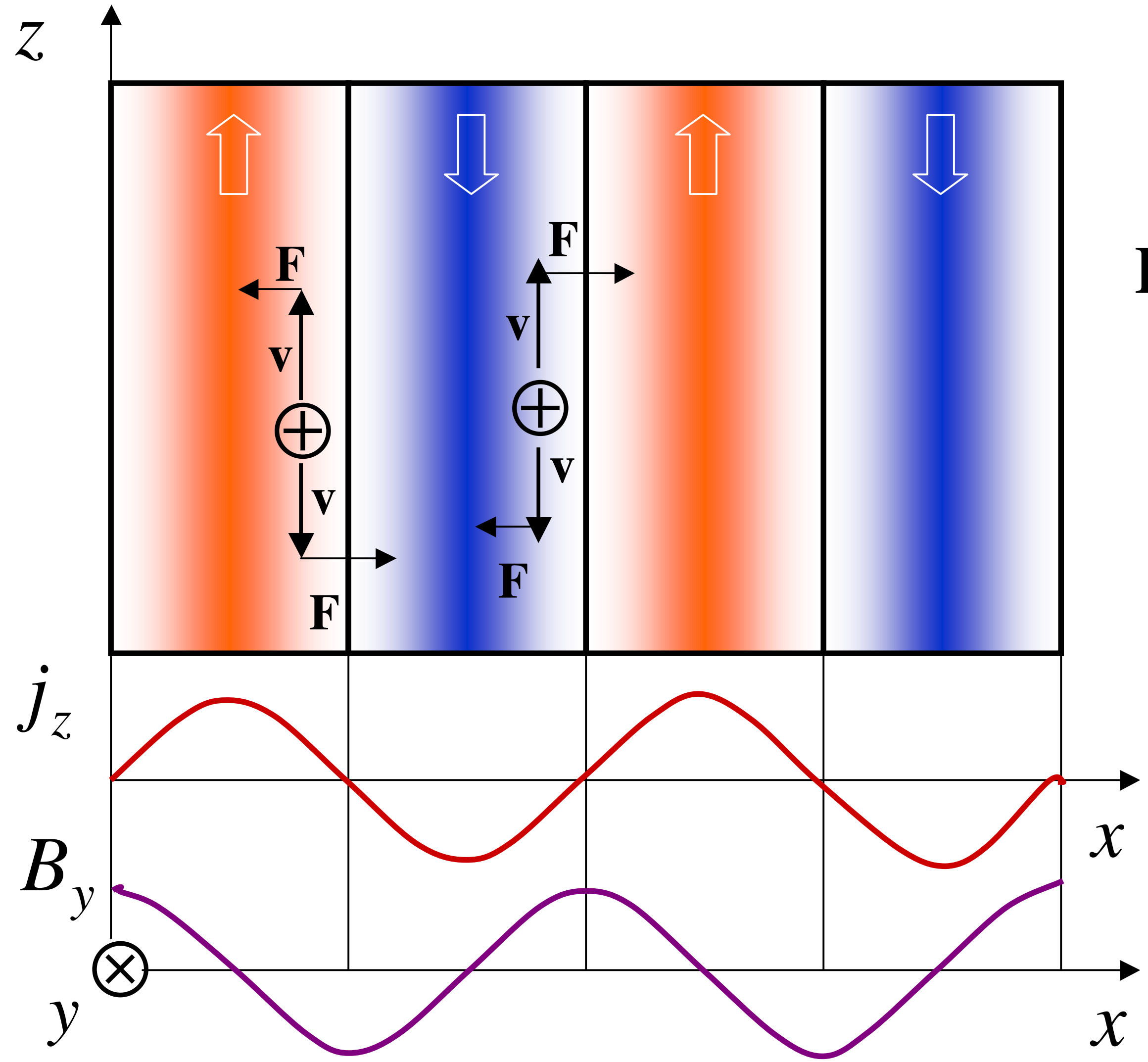
$$\left\langle j_a^{\mu}(x_1)\, j_b^{\nu}(x_2) \right\rangle = \frac{1}{2}\delta^{ab} \int \frac{d^3p}{(2\pi)^3} \frac{p^{\mu}p^{\nu}}{E_p^2} f(\mathbf{p}) \delta^{(3)}(\mathbf{x} - \mathbf{v}t) \neq 0$$

$$x_1 = (t_1, \mathbf{x}_1), \quad x_2 = (t_2, \mathbf{x}_2), \quad x = (t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2)$$



Direction of the momentum surplus

Mechanism of filamentation



Lorentz force

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

Ampere's law

$$\nabla \times \mathbf{B} = \mathbf{j}$$

Dispersion equation

Equation of motion of chromodynamic field A^μ in momentum space

$$[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)]A_\nu(k) = 0$$

gluon self-energy

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

$$k^\mu \equiv (\omega, \mathbf{k})$$

Instabilities – solutions with Im$\omega > 0$ $\Rightarrow$ $A^\mu(x) \sim \mathrm{e}^{\mathrm{Im}\,\omega t}$

Dynamical information is hidden in $\Pi^{\mu\nu}(k)$. How to get it?

Transport theory – distribution functions

Distribution functions of quarks $Q(p,x)$ and antiquarks $\bar{Q}(p,x)$

are gauge dependent $N_c \times N_c$ matrices

$$Q(p,x) \to U(x)\,Q(p,x)\,U^{-1}(x)$$

Distribution function of gluons $G(p,x)$ is $(N_c^2-1)\times(N_c^2-1)$ matrix

Transport theory – transport equations

fundamental: quarks, antiquarks; adjoint: gluons

$$p_\mu D^\mu Q - \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial^\nu_p Q\} = C \quad \text{quarks}$$

$$p_\mu D^\mu \overline{Q} + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial^\nu_p \overline{Q}\} = \overline{C} \quad \text{antiquarks}$$

$$p_\mu \mathcal{D}^\mu G - \frac{g}{2} p^\mu \{\mathcal{F}_{\mu\nu}, (x)\partial^\nu_p G\} = C_g \quad \text{gluons}$$

free streaming — mean-field force — collisions

$$D^\mu \equiv \partial^\mu - ig[A^\mu, \ldots.], \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

$$D_\mu F^{\mu\nu} = j^\nu[Q, \overline{Q}, G]$$

mean-field generation

collisionless limit: $C = \overline{C} = C_g = 0$

Transport theory - linearization

fluctuation

$$Q(p,x) = Q_0(p) + \delta Q(p,x)$$

stationary colorless state $Q_0^{ij}(p) = \delta^{ij} n(p)$

$$|Q_0(p)| >> |\delta Q(p,x)|, \quad |\partial_p^\mu Q_0(p)| >> |\partial_p^\mu \delta Q(p,x)|$$

Linearized transport equations

$$p_\mu D^\mu \delta Q(p,x) - g p^\mu F_{\mu\nu}(x) \partial_p^\nu Q_0(p) = 0$$

$$p_\mu D^\mu \delta \overline{Q}(p,x) + g p^\mu F_{\mu\nu}(x) \partial_p^\nu \overline{Q}_0(p) = 0$$

$$p_\mu \mathcal{D}^\mu \delta G(p,x) - g p^\mu \mathcal{F}_{\mu\nu}(x) \partial_p^\nu G_0(p) = 0$$

Transport theory – polarization tensor

$$\delta Q(p,x) = g\int d^4x'\,\Delta_p(x-x')\,p^\mu F_{\mu\nu}(x)\partial_p^\nu Q_0(p)$$

$$p_\mu D^\mu \Delta_p(x) = \delta^{(4)}(x)$$

$$j^\mu[\delta Q, \delta\overline{Q}, \delta G]$$

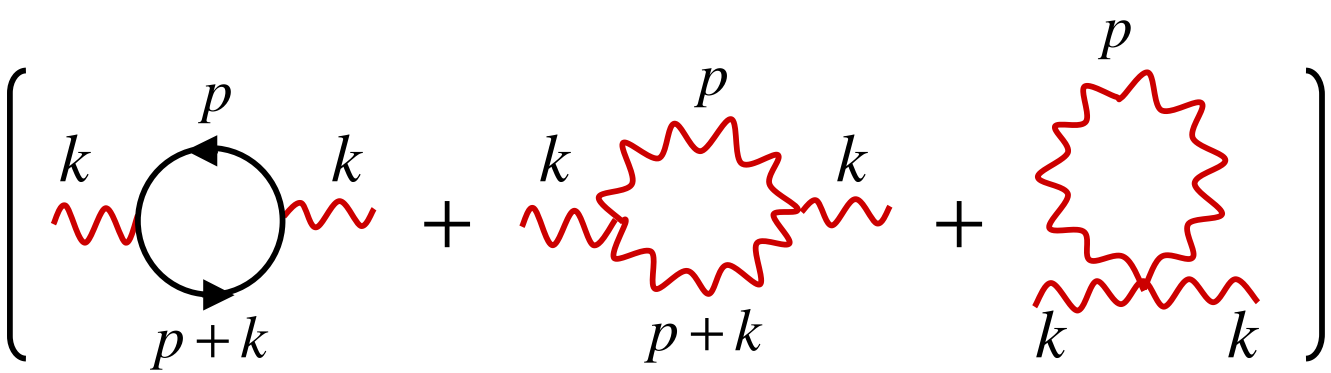
$$j^\mu(k) = \Pi^{\mu\nu}(k)A_\nu(k)$$

$$f(\mathbf{p}) \equiv n(\mathbf{p}) + \overline{n}(\mathbf{p}) + 2n_g(\mathbf{p})$$

$$\Pi^{\mu\nu}(k) = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E}\left[g^{\nu\lambda} - \frac{p^\nu k^\lambda}{p^\sigma k_\sigma + i0^+}\right]\frac{\partial f(\mathbf{p})}{\partial p^\lambda}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_\mu \Pi^{\mu\nu}(k) = 0$$

Diagrammatic Hard Loop approach

$\Pi^{\mu\nu}(k) =$ 

Hard loop approximation: $k^\mu << p^\mu$

$$\Pi^{\mu\nu}(k) = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E}\left[g^{\nu\lambda} - \frac{p^\nu k^\lambda}{p^\sigma k_\sigma + i0^+}\right]\frac{\partial f(\mathbf{p})}{\partial p^\lambda}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_\mu \Pi^{\mu\nu}(k) = 0$$

St. M. & M. Thoma, Phys. Rev. C **62**, 036011 (2000)

Dispersion equation

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

$$k_\mu \Pi^{\mu\nu}(k) = 0$$

$$\varepsilon^{ij}(k) = \delta^{ij} - \frac{1}{\omega^2}\Pi^{ij}(k)$$ **chromodielectric tensor**

$$k^\mu \equiv (\omega, \mathbf{k})$$

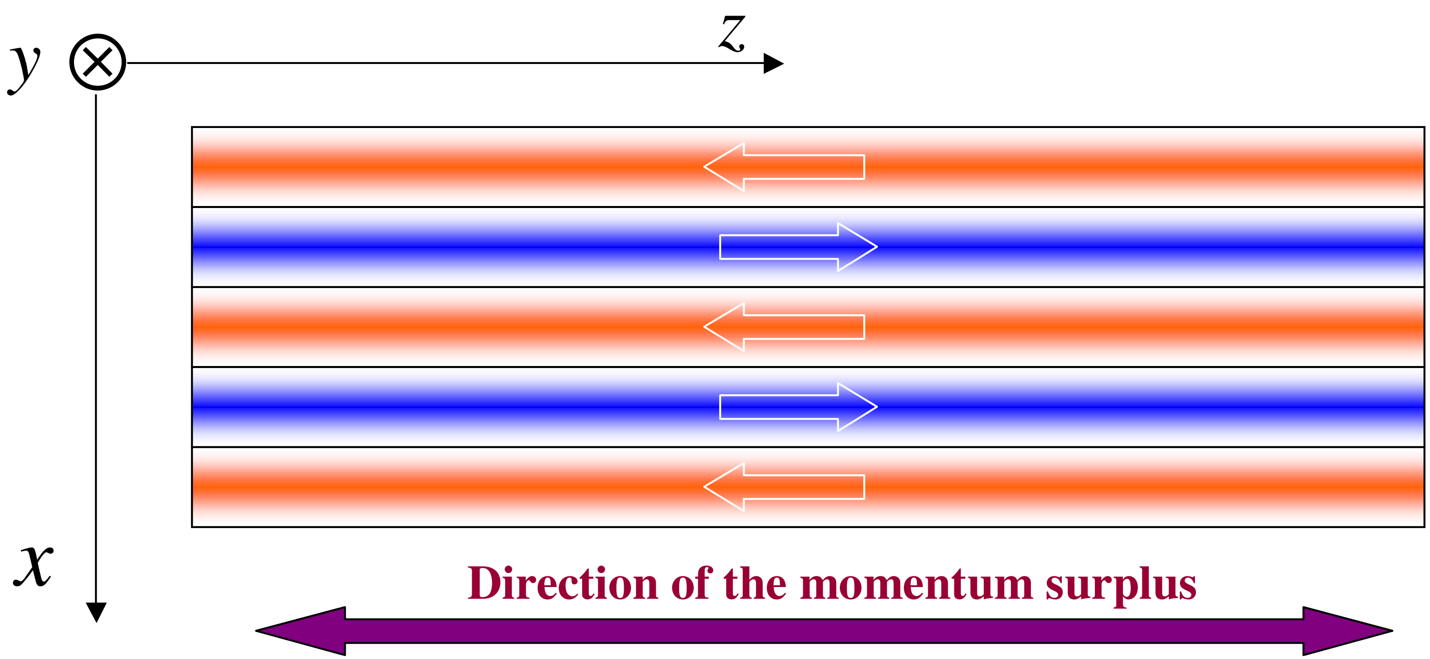
Dispersion equation

$$\det[\mathbf{k}^2 \delta^{ij} - k^i k^j - \omega^2 \varepsilon^{ij}(k)] = 0$$

$$\varepsilon^{ij}(k) = \delta^{ij} + \frac{g^2}{2\omega}\int \frac{d^3p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{kv} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^l}\left[\left(1 - \frac{\mathbf{kv}}{\omega}\right)\delta^{lj} + \frac{k^l v^j}{\omega}\right]$$

$$\mathbf{v} \equiv \mathbf{p}/E$$

Dispersion equation – configuration of interest



$$\mathbf{j} = (0,0,j), \quad \mathbf{E} = (0,0,E), \quad \mathbf{k} = (k,0,0)$$

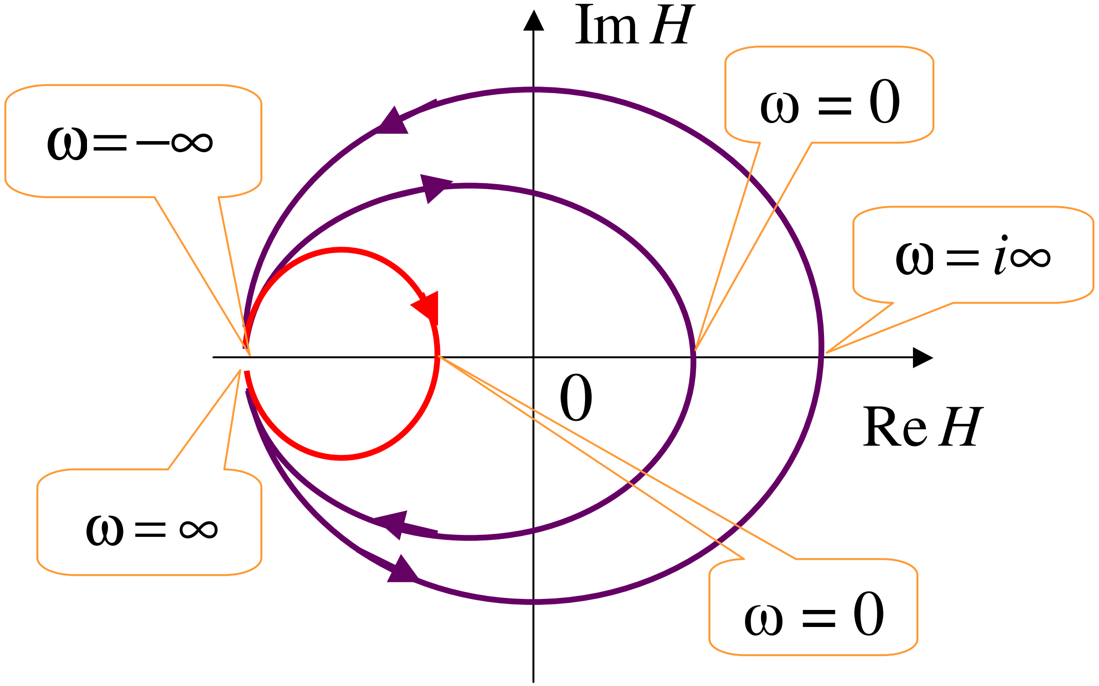
Dispersion equation

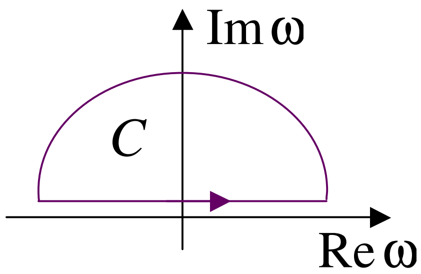
$$k^2 - \omega^2 \varepsilon^{zz}(\omega, k) = 0$$

Existence of unstable modes – Penrose criterion

$$H(\omega) \equiv k^2 - \omega^2 \varepsilon^{zz}(\omega, k)$$

$$\oint_C \frac{d\omega}{2\pi i} \frac{1}{H(\omega)} \frac{dH(\omega)}{d\omega} = \begin{cases} \oint_C \frac{d\omega}{2\pi i} \frac{d \ln H(\omega)}{d\omega} = \ln H(\omega)\Big|_{\phi=\pi^+}^{\phi=\pi^-} \\ \text{number of zeros of } H(\omega) \text{ in } C \end{cases}$$





There are unstable modes if

$$H(\omega = 0) < 0$$

Anisotropy!

Unstable solutions

$$f(\mathbf{p}) = \frac{2^{1/2}}{\pi^{3/2}} \frac{\rho \sigma_\perp^4}{\sigma_\parallel} \frac{1}{(p_\perp^2 + \sigma_\perp^2)^3} e^{-\frac{p_\parallel^2}{2\sigma_\parallel^2}}$$

$$\rho = 6 \text{ fm}^{-3}$$

$$\alpha_s = g^2 / 4\pi = 0.3$$

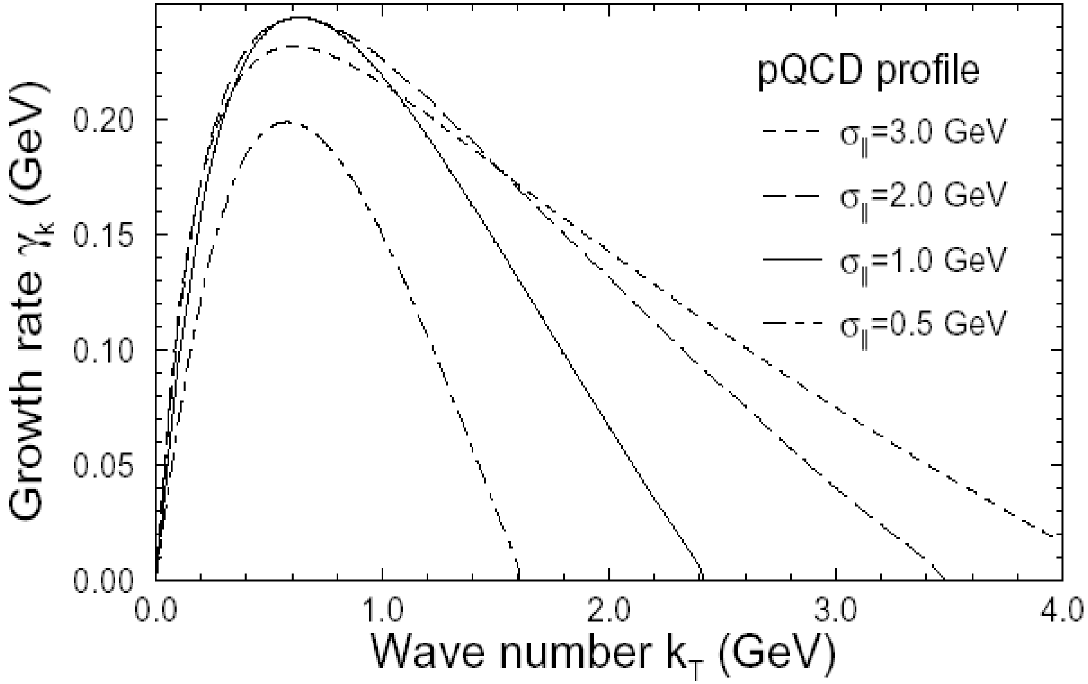
$$\sigma_\perp = 0.3 \text{ GeV}$$

$$k^2 - \omega^2 \varepsilon^{zz}(\omega, k) = 0$$

solution

$$\omega(k) = \pm i \gamma_k$$

$$0 < \gamma_k \in \Re$$



J. Randrup & St. M., Phys. Rev. C **68**, 034909 (2003)

Growth of instabilities – 1+1 numerical simulations

SU(2) Hard Loop Dynamics

1+1 dimensions

$$A_a^\mu = A_a^\mu(t, z)$$

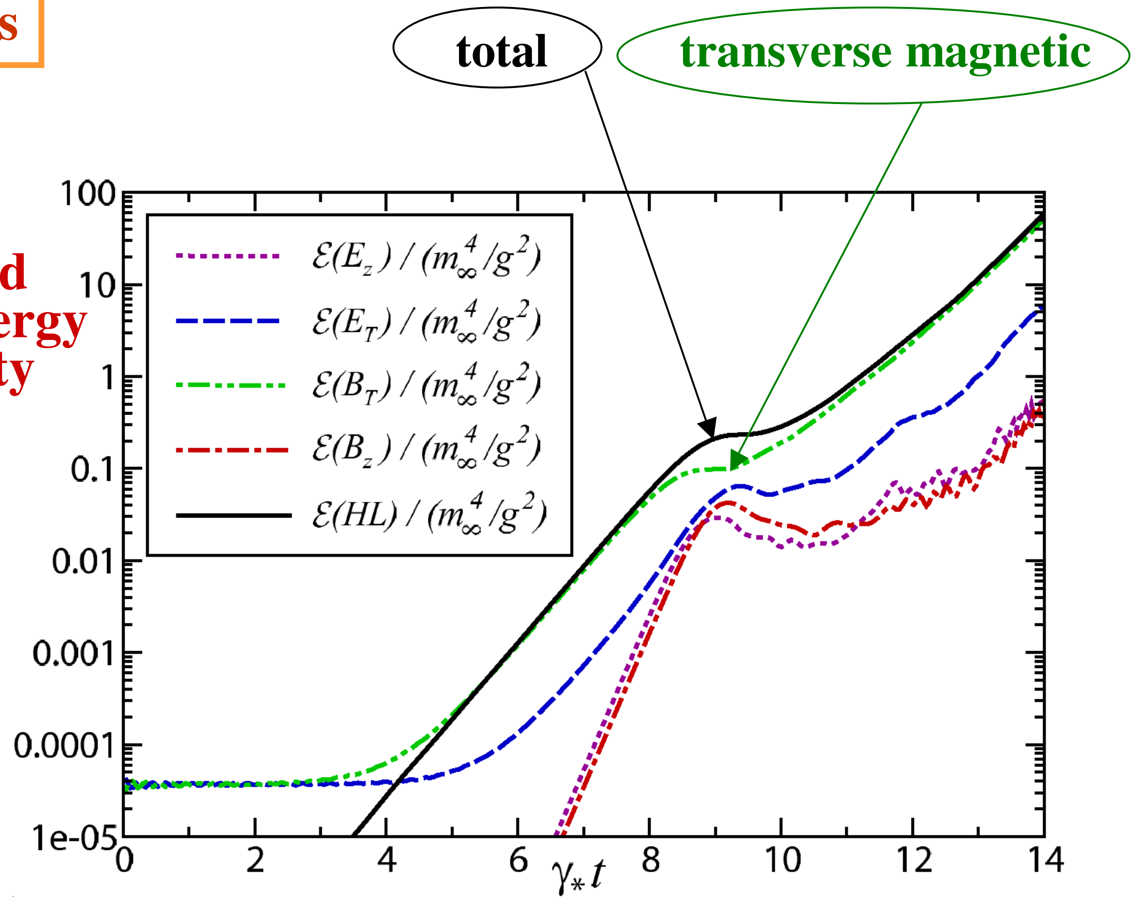
Anisotropic particle's momentum distribution

$$f(\mathbf{p}) = f_{\rm iso}(|\mathbf{p}| + \zeta p_z)$$

$$m_D^2 = -\frac{\alpha_s}{\pi}\int_0^\infty dp\, p^2 \frac{df_{\rm iso}(p)}{dp}$$

$$(m_D, \zeta)$$

Scaled field energy density



Strong anisotropy $\zeta = 10$ γ_* - maximal growth rate

A. Rebhan, P. Romatschke & M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005)

Growth of instabilities – 1+1 numerical simulations

Classical system of colored particles & fields

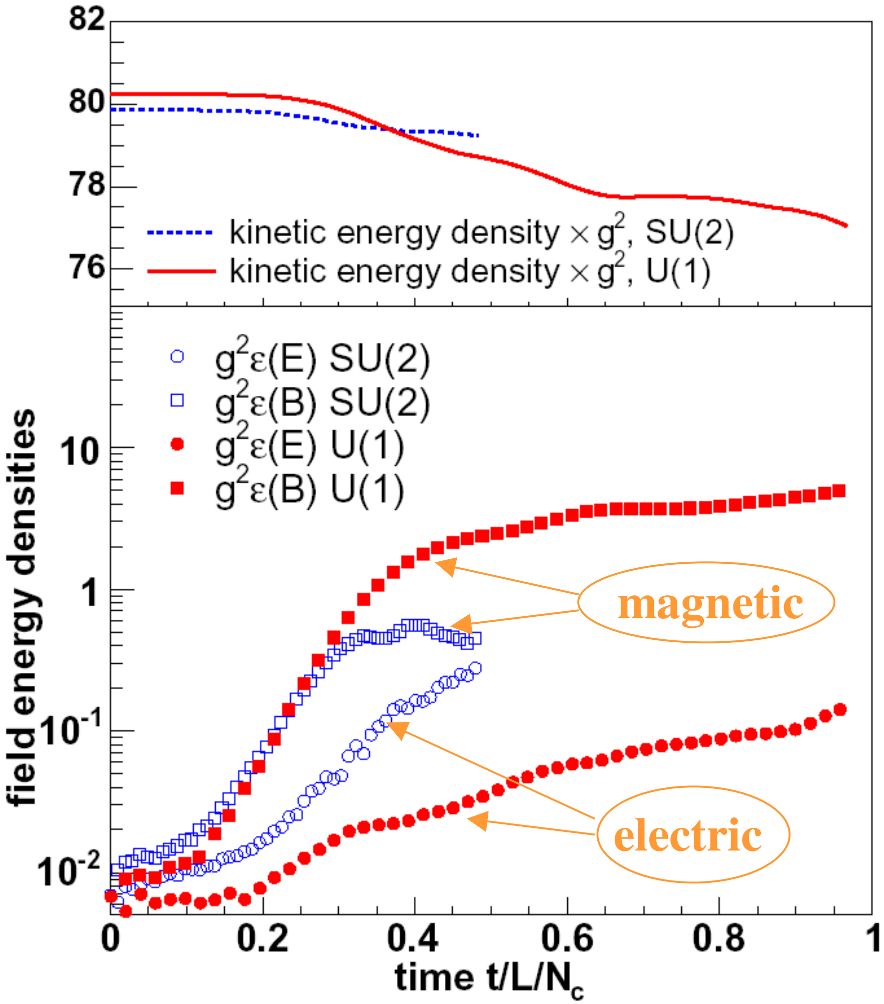
initial fields: **Gaussian noise as in Color Glass Condensate**

initial anisotropic particle distribution:

$$f_0(\mathbf{p},\mathbf{x}) \sim \delta(p_x)\, e^{-\frac{\sqrt{p_y^2+p_z^2}}{p_{\text{hard}}}}$$

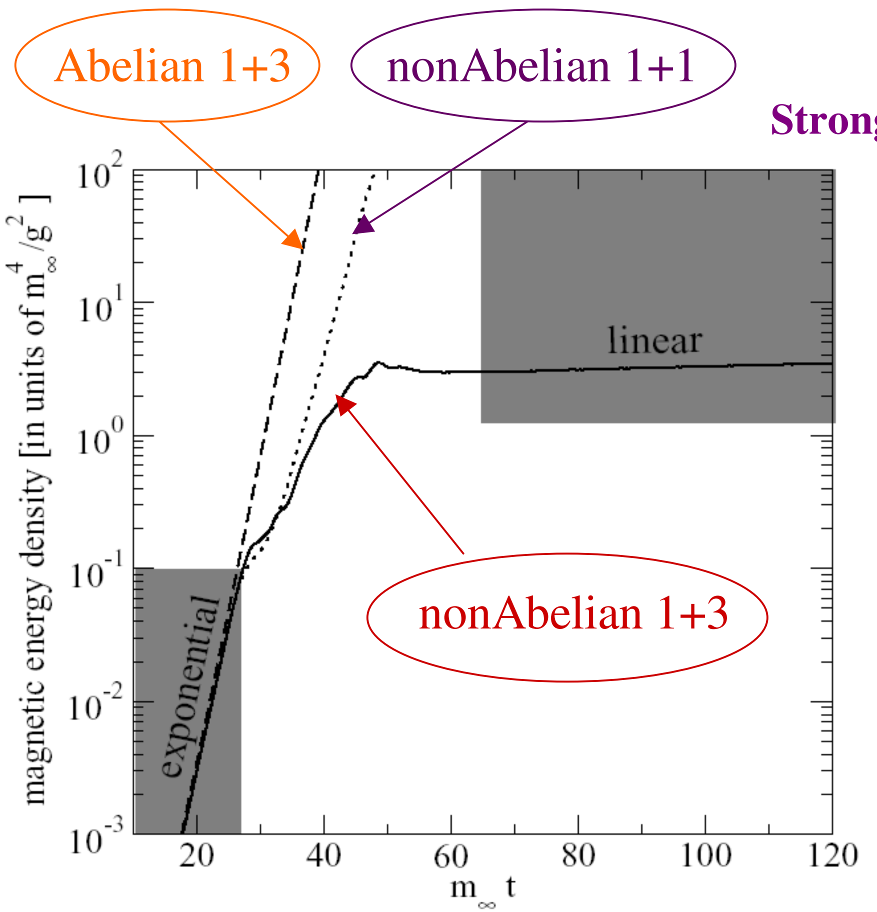
$$p_{\text{hard}} = 10\ \text{GeV}$$

$$L = 40\ \text{fm} \qquad \rho = 10\ \text{fm}^{-3}$$



A. Dumitru & Y. Nara, Phys. Lett. **B621**, 89 (2005).

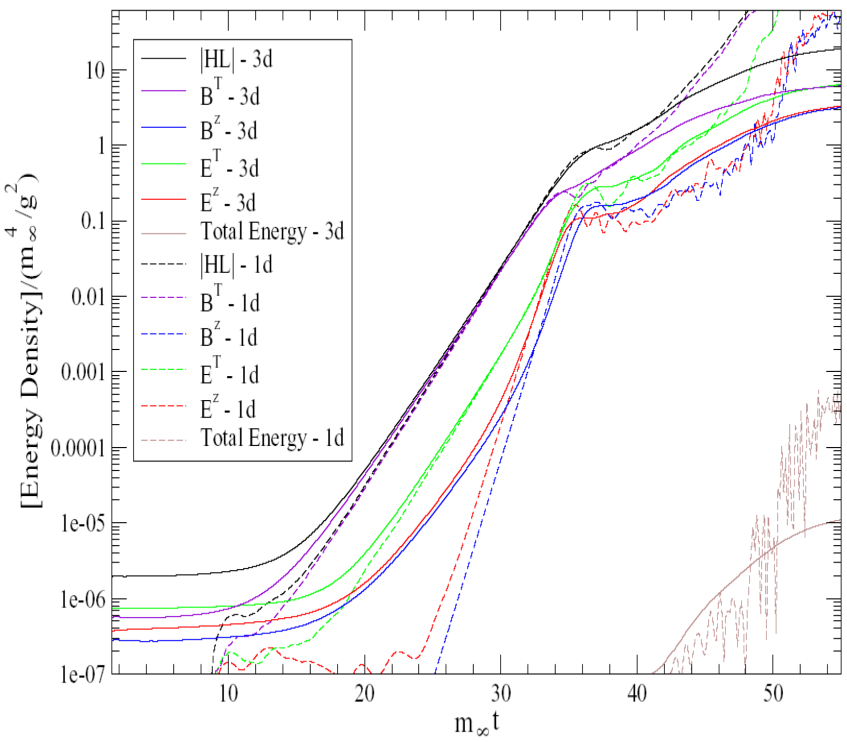
Growth of instabilities – 1+3 numerical simulations



P. Arnold, G.D. Moore & L.G. Yaffe,
Phys. Rev. **D72**, 054003 (2005)

SU(2) Hard Loop Dynamics

Strongly anisotropic particle's momentum distribution



A. Rebhan, P. Romatschke & M.Strickland,
JHEP **0509**, 041 (2005)

Abelanization

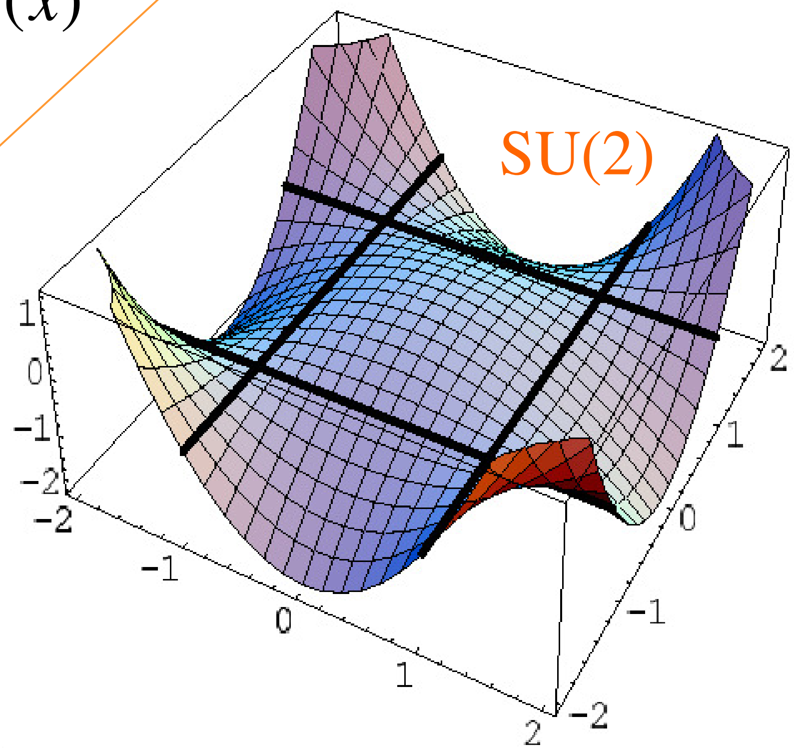
$$V_{\text{eff}}[\mathbf{A}^a] = -\mu^2 \mathbf{A}^a \cdot \mathbf{A}^a + \frac{1}{4} g^2 f_{abc} f_{ade} (\mathbf{A}^b \cdot \mathbf{A}^d)(\mathbf{A}^c \cdot \mathbf{A}^e)$$

the gauge $A_0^a = 0, \quad A_i^a(t, x, y, z) = A_i^a(x)$

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} = -\frac{1}{2} \mathbf{B}^a \mathbf{B}^a$$

$$= -\frac{1}{4} g^2 f_{abc} f_{ade} (\mathbf{A}^b \cdot \mathbf{A}^d)(\mathbf{A}^c \cdot \mathbf{A}^e)$$

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a + \frac{g}{2} f_{abc} \mathbf{A}^b \times \mathbf{A}^c$$



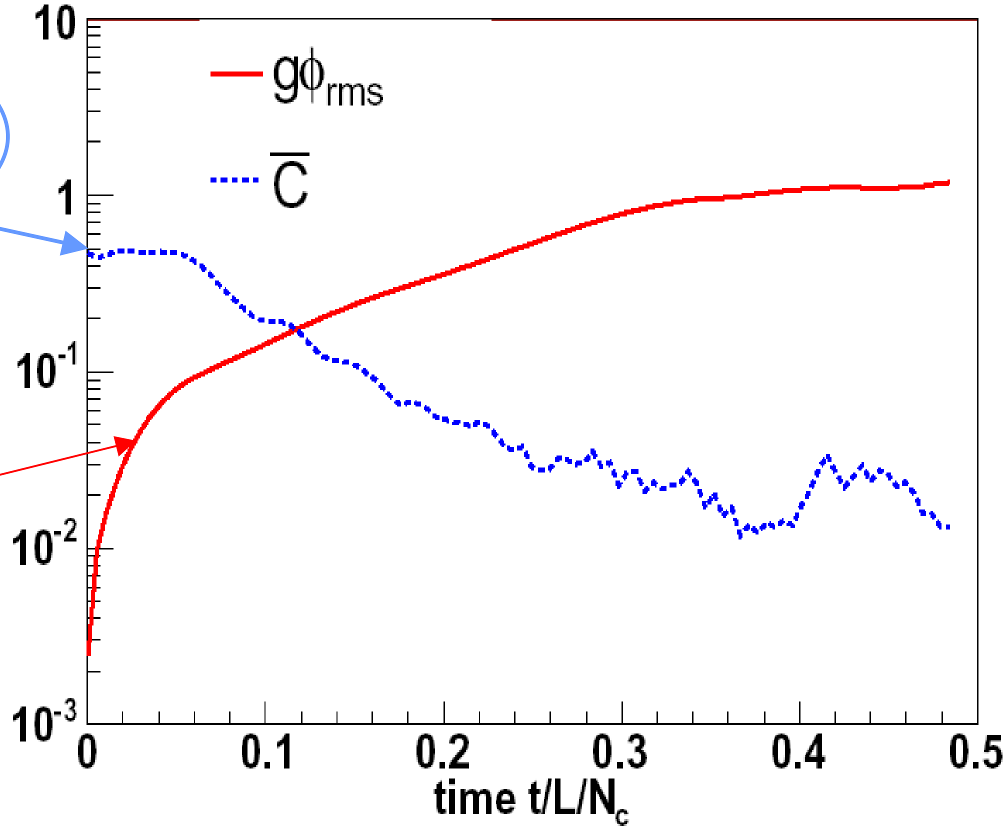
P. Arnold & J. Lenaghan, Phys. Rev. D **70**, 114007 (2004)

Abelanization – 1+1 numerical simulations

Classical system of colored particles & fields

$$\overline{C} \equiv \int_0^L \frac{dx}{L} \frac{\sqrt{\mathrm{Tr}((i[A_y, A_z])^2)}}{\mathrm{Tr}[\mathbf{A}^2]}$$

$$\phi_{\mathrm{rms}} \equiv \sqrt{\int_0^L \frac{dx}{2L} \mathrm{Tr}[\mathbf{A}^2]}$$



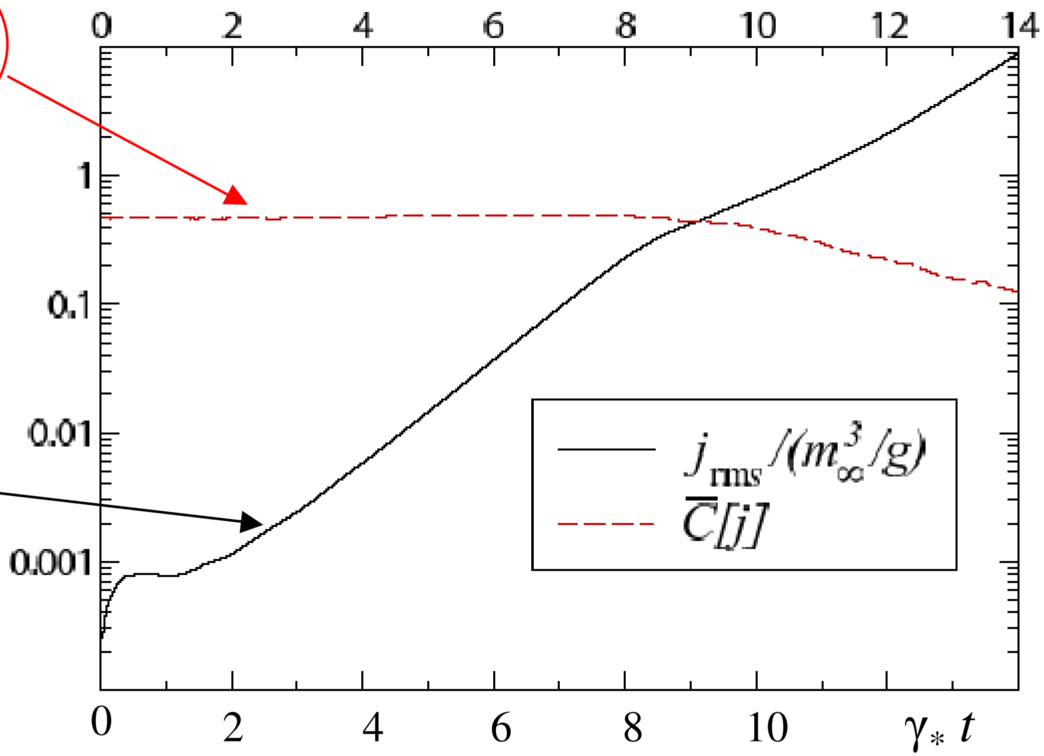
A. Dumitru & Y. Nara, Phys. Lett. **B621**, 89 (2005).

Abelanization – 1+1 numerical simulations

SU(2) Hard Loop Dynamics

$$\overline{C} \equiv \int_0^L \frac{dz}{L} \frac{\sqrt{\mathrm{Tr}((i[j_x, j_y])^2)}}{\mathrm{Tr}[\mathbf{j}^2]}$$

$$j_{\mathrm{rms}} \equiv \sqrt{\int_0^L \frac{dz}{L} 2\,\mathrm{Tr}[\mathbf{j}^2]}$$



A. Rebhan, P. Romatschke & M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005)

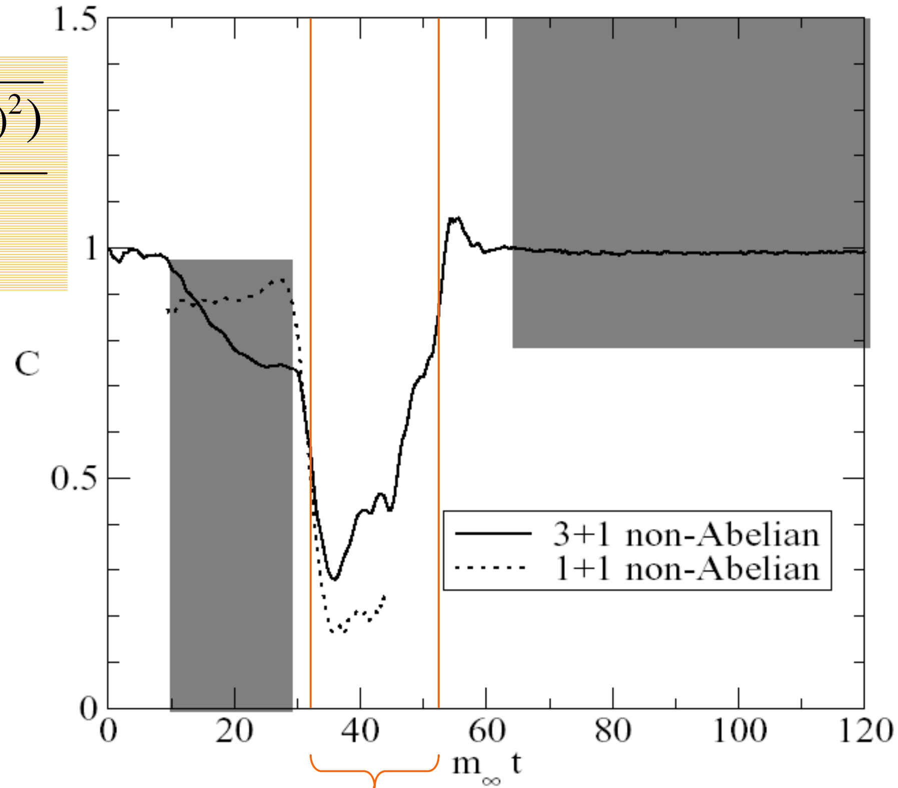
Abelanization – 1+3 numerical simulations

SU(2) Hard Loop Dynamics

$$C \equiv \frac{3}{\sqrt{2}} \frac{\int \frac{d^3x}{V} \sqrt{\mathrm{Tr}((i[j_x, j_y])^2 + (i[j_y, j_z])^2 + (i[j_z, j_x])^2)}}{\int \frac{d^3x}{V} \mathrm{Tr}(\mathbf{j}^2)}$$

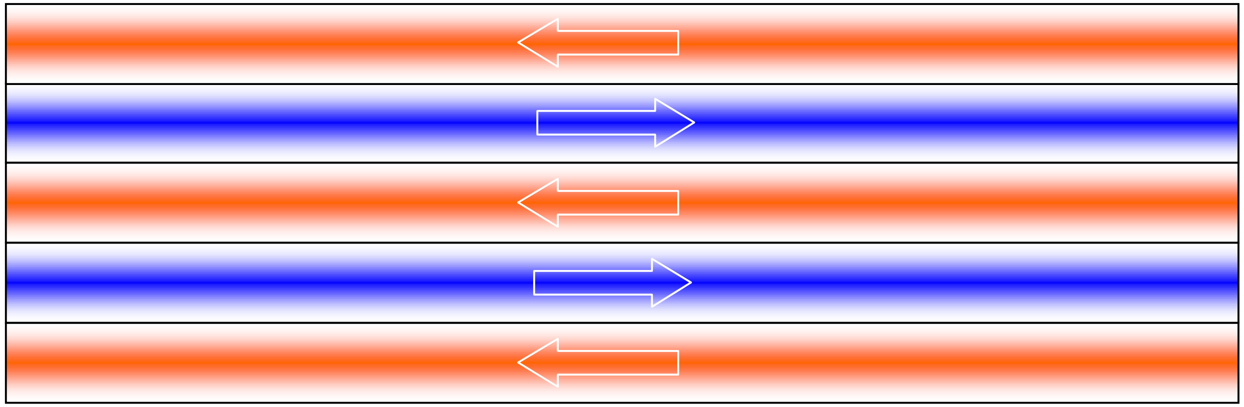
$$A_i^a \sim e^{\gamma t}$$

$$A_i^a \sim \frac{k_{\mathrm{field}}}{g} << \frac{p_{\mathrm{hard}}}{g}$$

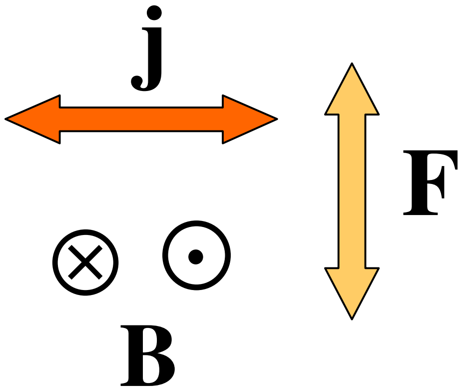


P. Arnold, G.D. Moore & L.G. Yaffe, Phys. Rev. **D72**, 054003 (2005)

Isotropization - particles

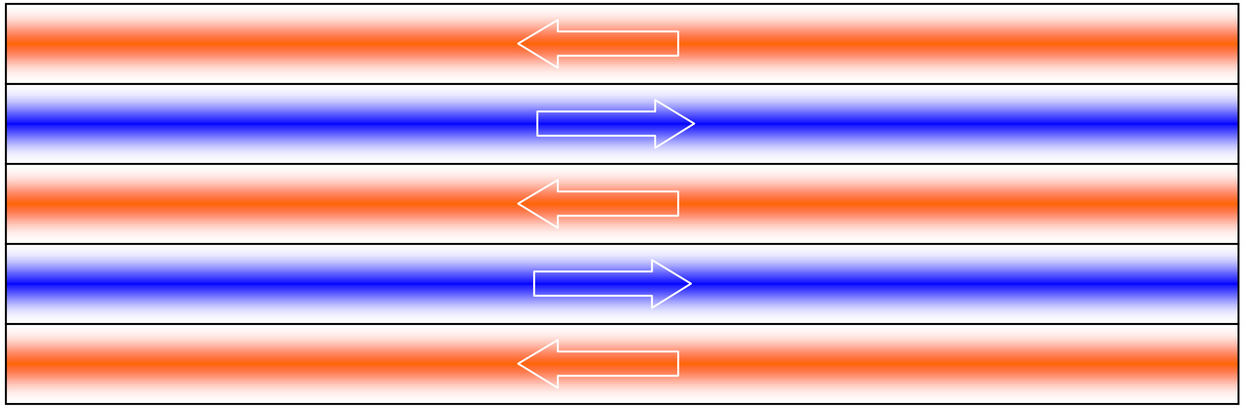


Direction of the momentum surplus

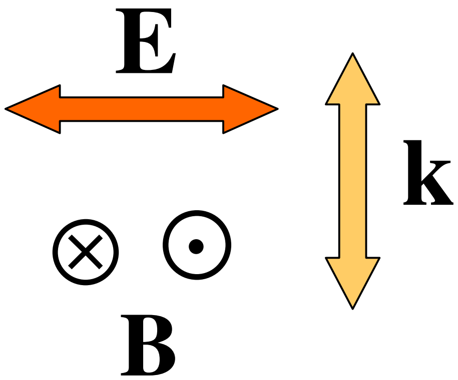


$$\Delta \mathbf{p} = \int dt\, \mathbf{F}$$

Isotropization - fields



Direction of the momentum surplus



$$\mathbf{P}_{\text{fields}} \sim \mathbf{B}^{a} \times \mathbf{E}^{a} \sim \mathbf{k}$$

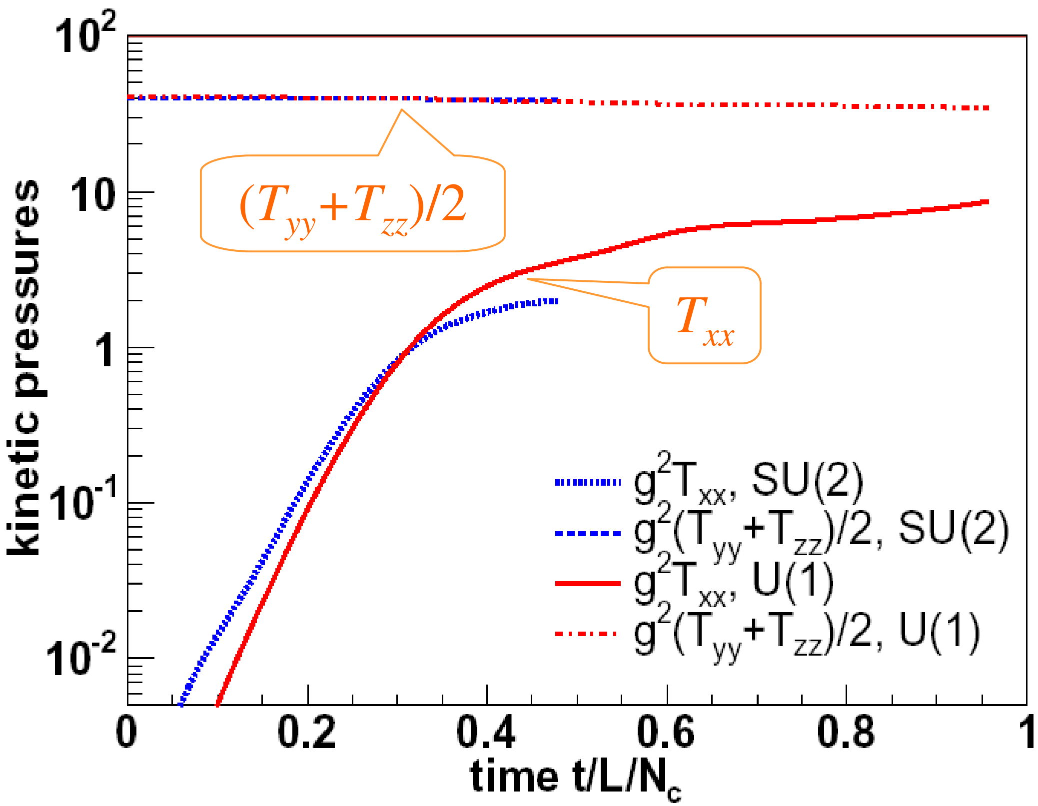
Isotropization – numerical simulation

Classical system of colored particles & fields

$$T_{ij} = \int \frac{d^3 p}{(2\pi)^3} \frac{p_i p_j}{E} f(\mathbf{p})$$

Isotropy:

$$T_{xx} = (T_{yy} + T_{zz})/2$$



A. Dumitru & Y. Nara, Phys. Lett. **B621**, 89 (2005).

Conclusion

The scenario of instabilities driven equilibration is a plausible solution of the fast equilibration problem

St. Mrówczyński, Acta Physica Polonica **B372**, 427 (2006)