QCD vs. $\mathcal{N}$=4 super Yang-Mills Plasma

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Outline

1. Motivation

2. QCD vs. $\mathcal{N}$=4 super Yang-Mills

3. Basic plasma characteristics

4. Collective modes
    1. Dispersion equations
    2. Perturbative computation of self-energies
    3. Effective action

5. Collisional characteristics
    1. Elementary processes
    2. Transport coefficients
    3. Energy loss & $\hat{q}$

6. Conclusions

Motivation

AdS/CFT duality

weakly coupled
gravity in AdS

⟺

strongly coupled
$\mathcal{N} = 4$ super Yang-Mills

QCD vs. super Yang-Mills ?

Motivation cont.

Does rudimentary SUSY induce instabilities in fermionic sector?

QED PLASMA

There are unstable photon modes

⟺ SUSY

SUSY QED PLASMA

Are there unstable photino modes?

Bigger project

A systematic comparison
of supersymmetric plasma systems
to their non-supersymmetric counterparts
in a weak coupling domain

A. Czajka & St. Mrówczyński, arXiv: 1203.1856 [hep-th]
Physical Review **D** in print — $\mathcal{N}$=4 super Yang-Mills

A. Czajka & St. Mrówczyński, Physical Review **D83** (2011) 065021 — SUSY
A. Czajka & St. Mrówczyński, Physical Review **D84** (2011) 105020 — QED

Lagrangian of 𝒩=4 super Yang-Mills

$$\mathcal{L} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \frac{i}{2} \overline{\Psi}_i^a (\not{D}\Psi_i)^a + \frac{1}{2} (D_\mu \Phi_A)_a (D^\mu \Phi_A)_a$$
$$-\frac{1}{4} g^2 f^{abe} f^{cde} \Phi_A^a \Phi_B^b \Phi_A^c \Phi_B^d$$
$$-i \frac{g}{2} f^{abc} (\overline{\Psi}_i^a \alpha_{ij}^p X_p^b \Psi_j^c + i \overline{\Psi}_i^a \beta_{ij}^p \gamma_5 Y_p^b \Psi_j^c)$$

Type of the field		Range of the field's index	Spin	Number of degrees of freedom
A^μ	- vector	$\mu, \nu = 0, 1, 2, 3$	1	$2 \times (N_c^2 - 1)$
Φ_A	- real (pseudo-)scalar	$A, B = 1, 2, \ldots 6$	0	$6 \times (N_c^2 - 1)$
λ_i	- Majorana spinor	$i, j = 1, 2, 3, 4$	$\frac{1}{2}$	$8 \times (N_c^2 - 1)$

Basic plasma characteristics

	QGP	$\mathcal{N}$=4 SYMP
energy density - ε	$\frac{\pi^2 T^4}{60}\left[4(N_c^2-1)+7N_f N_c\right]$	$\frac{\pi^2 T^4}{2}(N_c^2-1)$
particle density - n	$\frac{2\zeta(3)T^3}{\pi^2}\left[2(N_c^2-1)+3N_f N_c\right]$	$\frac{14\zeta(3)T^3}{\pi^2}(N_c^2-1)$
Debye mass - m_D^2	$\frac{g^2 T^2}{6}\left(2N_c+N_f\right)$	$2g^2T^2N_c$
plasma parameter - λ $\left(\lambda \equiv \frac{1}{\frac{4}{3}\pi r_D^3 n}\right)$	$0.042g^3$	$0.257g^3$

All chemical potentials are assumed to vanish in both QGP and $\mathcal{N}$=4 SYMP

Collective modes

Gluon dispersion equation

Equation of motion of gluon field $A^\mu(k)$

$$[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)]A_\nu(k) = 0$$

$$k^\mu \equiv (\omega, \mathbf{k})$$

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

Collective modes - solutions: $\omega(\mathbf{k})$

$\Pi^{\mu\nu}$ – retarded polarization tensor encodes gluon interaction with surrounding plasma

Fermion & scalar dispersion equations

Fermion field

$$\det[k_\mu \gamma^\mu - \Sigma(k)] = 0$$

Scalar field

$$k^2 + P(k) = 0$$

Keldysh–Schwinger formalism

Description of non-equilibrium many-body systems

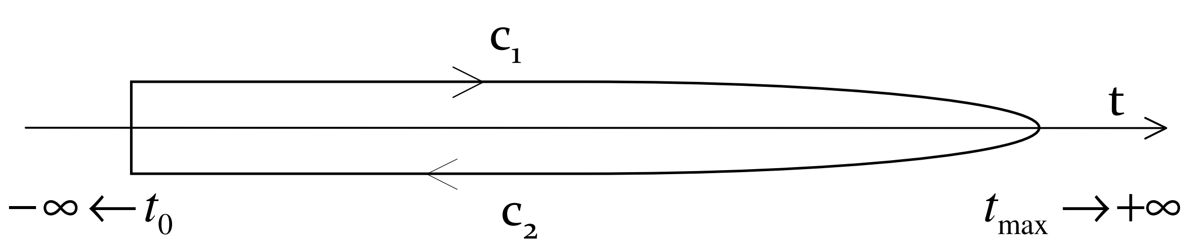
Contour Green function of scalar field

$$iG(x,y) \stackrel{\text{def}}{=} \left\langle \tilde{T}\phi(x)\phi(y) \right\rangle$$

$$\langle \ldots \rangle = \mathrm{Tr}[\hat{\rho}(t)\ldots]$$

$\tilde{T}$ - ordering along the contour

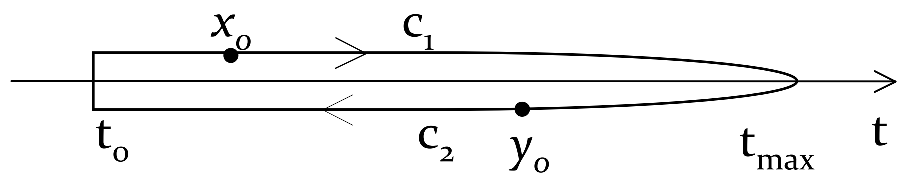
$$\tilde{T}A(x)B(y) = \Theta(x_0,y_0)A(x)B(y) \pm \Theta(y_0,x_0)B(y)A(x)$$



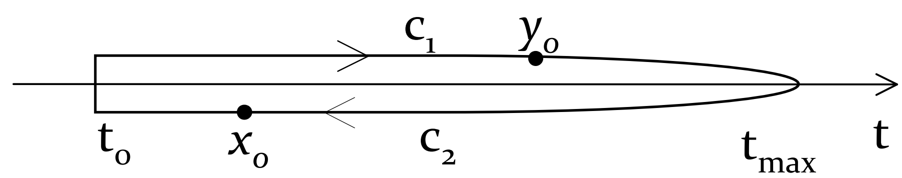
Keldysh–Schwinger Green functions

Unordered functions – phase-space densities

$$G^{>}(x,y)$$

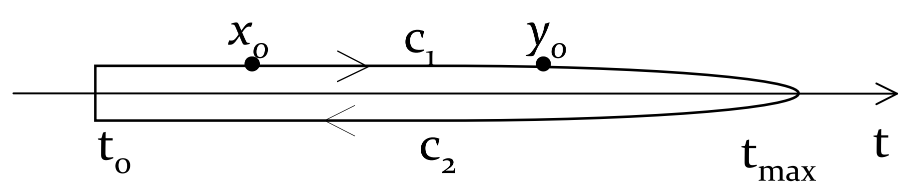


$$G^{<}(x,y)$$

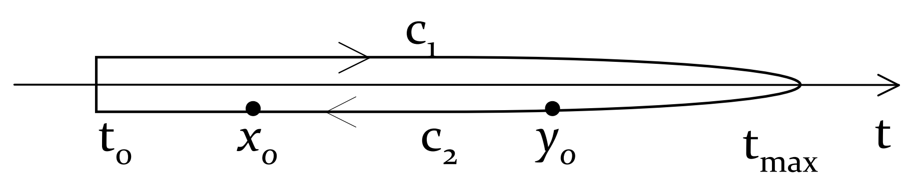


Ordered functions – propagators

$$G^{c}(x,y)$$



$$G^{a}(x,y)$$



Polarization tensor

Dyson–Schwinger equation

$$D = D_0 - D_0 \Pi D$$

Full contour propagator

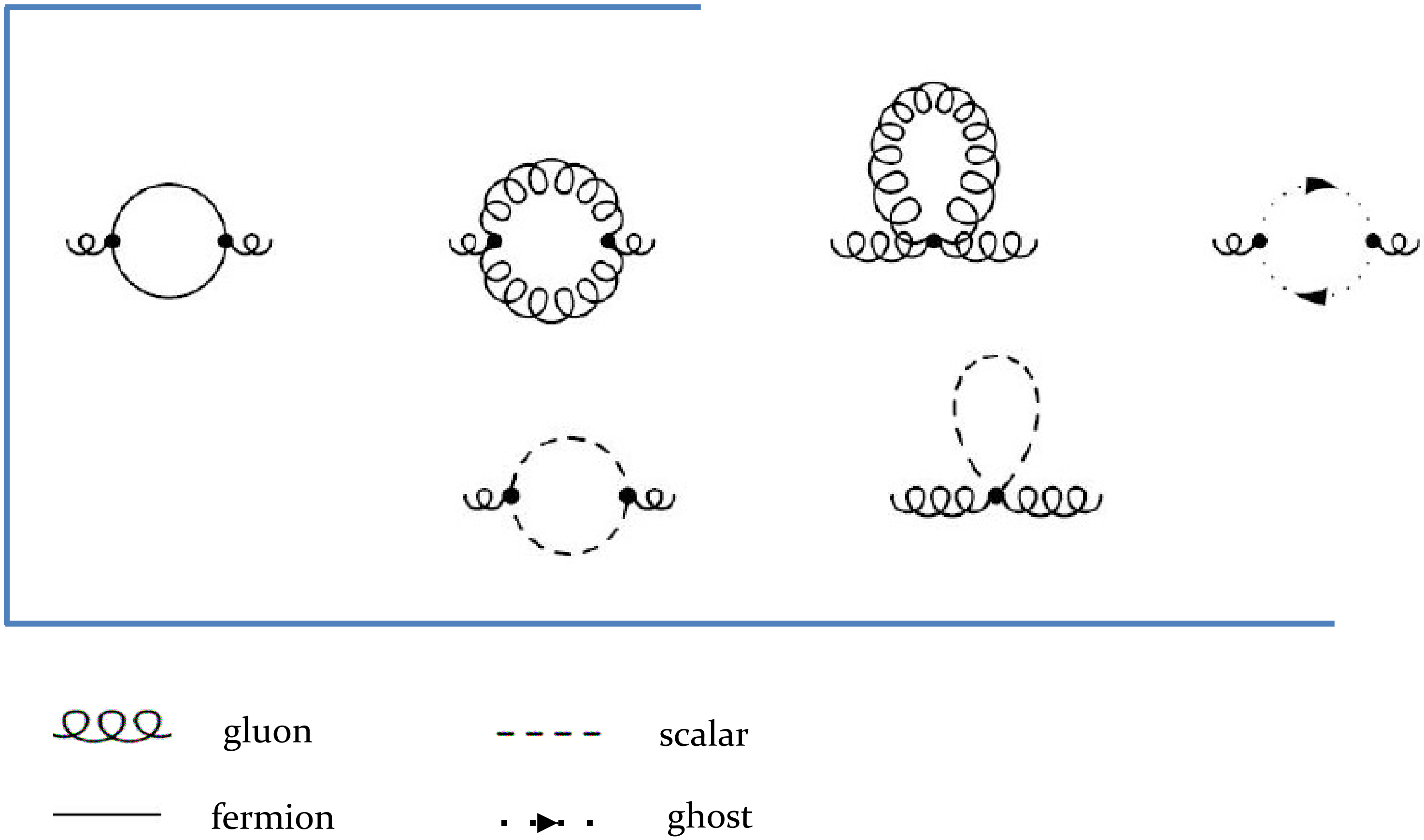
Free contour propagator

Contour polarization tensor

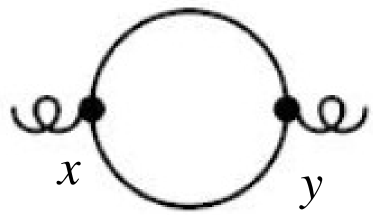
$D(x, y) = D(x - y)$ **homogeneity, translational invariance**

Lowest order contributions to Π

Contour–ordered Green functions have perturbative expansion similar to that of time-ordered Green functions



Fermion-loop contribution to Π



Contour polarization tensor

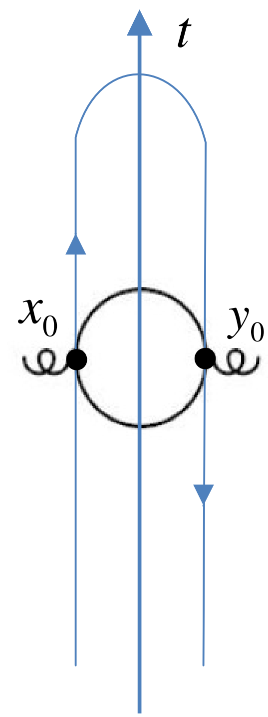
$$\Pi_{ab}^{\mu\nu}(x,y) = -ig^2 N_c \delta_{ab} \, \mathrm{Tr}[\gamma^\mu S(x,y)\gamma^\nu S(y,x)]$$

From contour to retarded Π

$$\Pi^{+}(x,y)=\Theta(x_0-y_0)\left(\Pi^{>}(x,y)-\Pi^{<}(x,y)\right)$$

Contour polarization tensor

$$\left(\Pi(x,y)\right)^{\mu\nu}_{ab}=-ig^2N_c\delta_{ab}\,\mathrm{Tr}[\gamma^\mu S(x,y)\gamma^\nu S(y,x)]$$



Unordered polarization tensor

$$\left(\Pi^{>}(x,y)\right)^{\mu\nu}_{ab}=-ig^2N_c\delta_{ab}\,\mathrm{Tr}[\gamma^\mu S^{>}(x,y)\gamma^\nu S^{<}(y,x)]$$

Fermion-loop contribution to Π⁺

$$\left(\Pi^{+}(k)\right)^{\mu\nu}_{ab} = -\frac{ig^{2}}{2} N_c \delta_{ab} \int \frac{d^4 p}{(2\pi)^4} \times$$

$$\times \mathrm{Tr}[\gamma^{\mu} S^{+}(p+k)\gamma^{\nu} S^{\mathrm{sym}}(p) + \gamma^{\mu} S^{\mathrm{sym}}(p)\gamma^{\nu} S^{-}(p-k)]$$

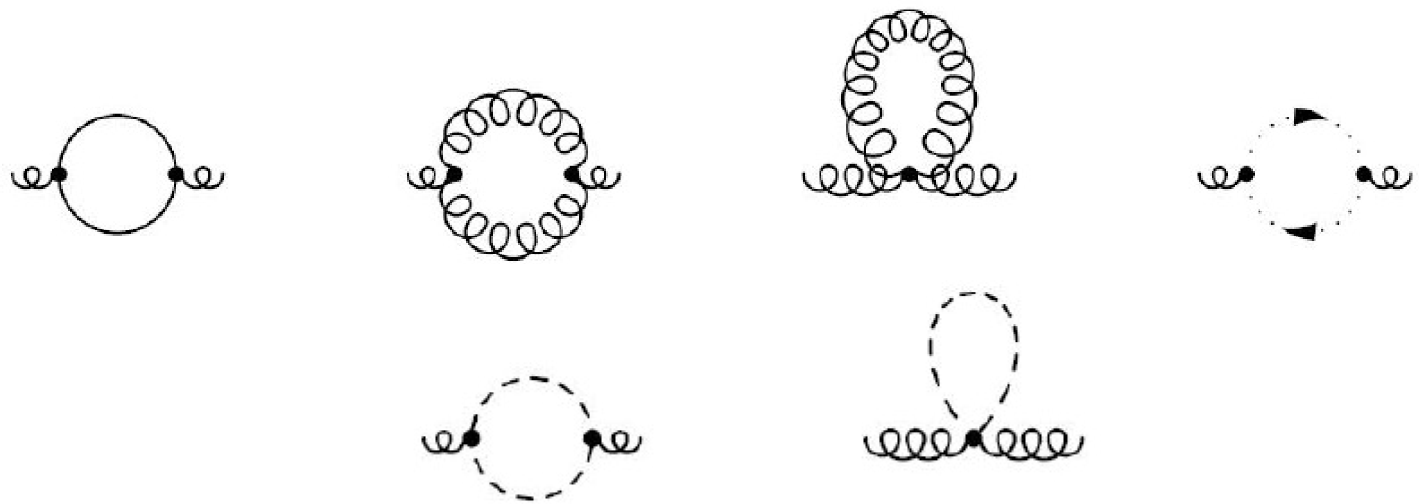
Free Green functions

$$S^{\pm}(p) = \frac{p^{\mu}\gamma_{\mu}}{p^2 \pm i p_0 0^{+}} \qquad S^{\mathrm{sym}}(p) = S^{>}(p) + S^{<}(p)$$

$$S^{>}(p) = \frac{i\pi}{E_p} p^{\mu}\gamma_{\mu}\left[\delta(E_p - p_0)[n_f(\mathbf{p}) - 1] + \delta(E_p + p_0) n_f(-\mathbf{p})\right]$$

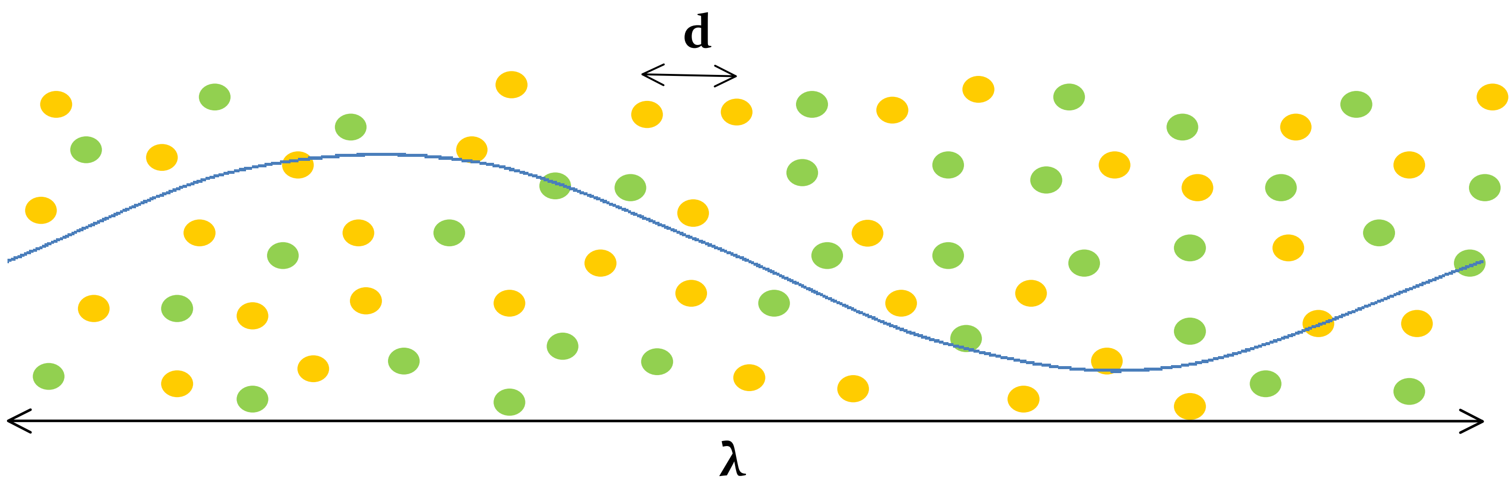
$$S^{<}(p) = \frac{i\pi}{E_p} p^{\mu}\gamma_{\mu}\left[\delta(E_p - p_0) n_f(\mathbf{p}) + \delta(E_p + p_0)[n_f(-\mathbf{p}) - 1]\right]$$

Contributions to Π in $\mathcal{N}$=4 SYMP



$$\left(\Pi^{+}(k)\right)_{ab}^{\mu\nu} = g^{2} N_{c} \delta_{ab} \int \frac{d^{4}p}{(2\pi)^{4}} \;\ldots\ldots$$

Hard Loop Approximation



Wavelength of a quasi-particle is much bigger than inter-particle distance in the plasma

$$\lambda >> d$$

Hard Loop Approximation cont.

The only dimensional parameter in free ultrarelativistic equilibrium plasma is temperature *T*.

particle density $\rho \sim \frac{1}{d^3} \sim T^3 \sim |\mathbf{p}|^3$

$\frac{1}{d} \sim |\mathbf{p}|$ momentum of plasma constituent

$\frac{1}{\lambda} \sim |\mathbf{k}|$ wave vector of collective mode

collective modes $k^\mu << p^\mu$

HL polarization tensor

$k^\mu << p^\mu$

$$\Pi^{\mu\nu}_{ab}(k) = g^2 N_c \delta_{ab} \int \frac{d^3 p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{k^2 p^\mu p^\nu - [p^\mu k^\nu + k^\mu p^\nu - g^{\mu\nu}(k\cdot p)](k\cdot p)}{(k\cdot p + i0^+)^2}$$

$$f(\mathbf{p}) \equiv 2n_g(\mathbf{p}) + 8n_f(\mathbf{p}) + 6n_s(\mathbf{p})$$

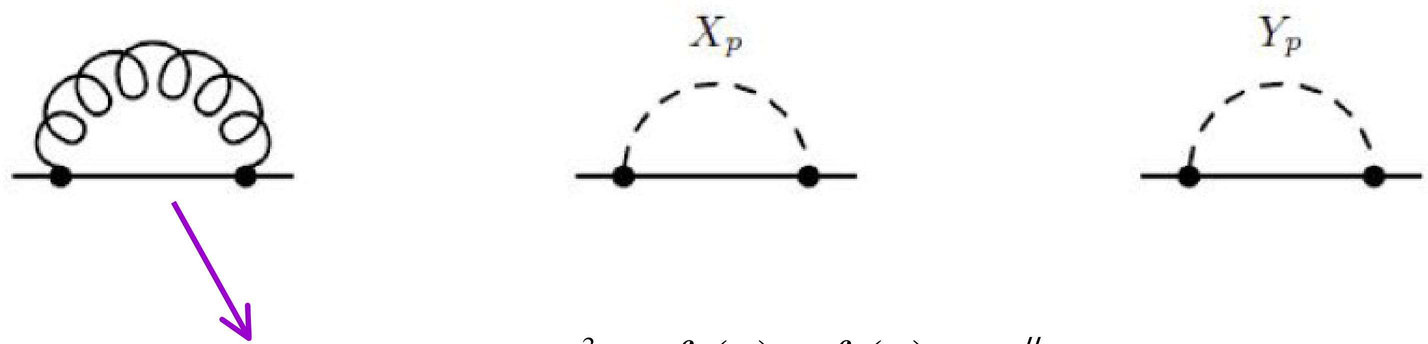
- **the same structure as in QED and QCD**
- **symmetric** $\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k)$
- **transversal** $k_\mu \Pi^{\mu\nu}(k) = 0$ **Gauge independence!**

Effects of SUSY

- vacuum contribution vanishes $(\Pi(k) = 0$ for $f(\mathbf{p}) = 0)$
- the coefficients in front of the distribution functions are the numbers of dof

$$f_{QGP}(\mathbf{p}) \equiv 2n_g(\mathbf{p}) + \frac{N_f}{N_c}\left(n_q(\mathbf{p}) + n_{\bar{q}}(\mathbf{p})\right)$$

Fermion self-energy



$$\Sigma_{QED}(k) = e^2 \int \frac{d^3p}{(2\pi)^3} \frac{f_\gamma(\mathbf{p}) + f_e(\mathbf{p})}{E_p} \frac{p^\mu \gamma_\mu}{k \cdot p + i0^+}$$

$$\Sigma_{ab}^{ij}(k) = \frac{g^2}{2} N_c \delta_{ab} \delta^{ij} \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{p^\mu \gamma_\mu}{k \cdot p + i0^+}$$

$$f(\mathbf{p}) \equiv 2n_g(\mathbf{p}) + 8n_f(\mathbf{p}) + 6n_s(\mathbf{p})$$

The fermion self-energy has **the same structure** for the $\mathcal{N}$=4 SYM, SUSY QED and usual QED plasma

Scalar self-energy

$$P_{ab}^{AB}(k) = -2g^2 N_c \delta_{ab} \delta^{AB} \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p}$$

$$f(\mathbf{p}) \equiv 2n_g(\mathbf{p}) + 8n_f(\mathbf{p}) + 6n_s(\mathbf{p})$$

Scalar self-energy:

- **independent of *k***
- **vanishes in the vacuum limit**

Hard loop effective action

Self-energy constrains the form of effective action

$$\mathcal{L}_2^{(\Psi)}(x) = \int d^4y\, \overline{\Psi}(x)\Sigma(x-y)\Psi(y)$$

$$\Sigma(x,y) = \frac{\delta^2 S[\Psi,\overline{\Psi}]}{\delta\overline{\Psi}(x)\delta\Psi(y)}$$

$$\mathcal{L}_{\mathrm{HL}} = -\frac{1}{4}F_a^{\mu\nu}F_{\mu\nu}^a + \frac{i}{2}\overline{\Psi}_i^a(\not{D}\Psi_i)^a + \frac{1}{2}(D_\mu\Phi_A)_a(D^\mu\Phi_A)_a + \mathcal{L}_{HL}^{(A)} + \mathcal{L}_{HL}^{(\Psi)} + \mathcal{L}_{HL}^{(\Phi)}$$

From effective action to self-energies

$$\mathcal{L}_{\rm HL}^{(A)}(x) = g^2 N_c \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} F_{\mu\nu}^{a}(x) \left(\frac{p^\nu p^\rho}{(p\cdot D)^2} \right)_{ab} F_{\rho}^{b\mu}(x)$$

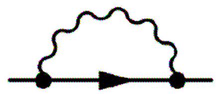
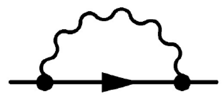
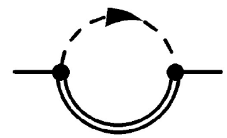
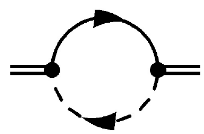
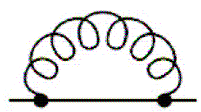
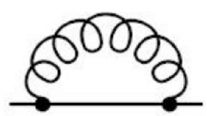
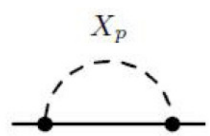
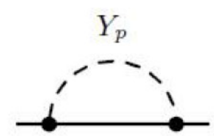
$$\mathcal{L}_{\rm HL}^{(\Psi)}(x) = g^2 N_c \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \overline{\Psi}_i^{a}(x) \left(\frac{p\cdot \gamma}{p\cdot D} \right)_{ab} \Psi_i^{b}(x)$$

$$\mathcal{L}_{\rm HL}^{(\Phi)}(x) = -2 g^2 N_c \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \Phi_A^a(x) \Phi_A^a(x)$$

The structure of each term of the effective action appears to be unique

Structure of each self-energy is unique

Fermionic HL self-energies

electron in QED			
electron in SUSY QED			
photino in SUSY QED			
quark in QCD			
fermion in $\mathcal{N}=4$ super Yang-Mills			

$$\Sigma(k) = \# \, g^2 \int \frac{d^3 p}{(2\pi)^3} \frac{f(\mathbf{p})}{E_p} \frac{p^\mu \gamma_\mu}{k \cdot p + i0^+}$$

$k^\mu << p^\mu$

Gauge bosons collective modes

Dispersion equation

$$\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$$

solutions: $\omega(\mathbf{k})$ $\qquad k^\mu \equiv (\omega, \mathbf{k})$

The structure of $\Pi^{\mu\nu}(k)$

- coincides with the gluon polarization tensor of QCD plasma
- such as of QED and SUSY QED plasma

The spectrum of collective excitations of gauge bosons
in $\mathcal{N}$=4 super Yang-Mills, QCD, QED
and SUSY QED plasma is the same

There is a whole variety of possible collective excitations,
there are unstable modes

Fermion collective modes

The structure of fermion self-energy is

- such as of quark self-energy in QCD plasma
- such as of QED and SUSY QED plasma

There are identical spectra of collective excitations of fermions in all systems

No unstable modes found!

Supersymmetry does not change anything

Scalar collective modes

The self-energy is independent of momentum, negative and real

$$P(k) = -m_{\text{eff}}^2$$

m_{eff} is the effective scalar mass

The solutions of dispersion equation

$$E_p = \pm\sqrt{m_{\text{eff}}^2 + \mathbf{p}^2}$$

Collisional characteristics

Elementary processes

Process	$\frac{1}{g^4}\frac{1}{N_c(N_c^2-1)}\sum \lvert M\rvert^2$
$GG \leftrightarrow GG$	$2\left(\frac{s^2+u^2}{t^2}+\frac{u^2+t^2}{s^2}+\frac{t^2+s^2}{u^2}+3\right)$
$GF \leftrightarrow GF$	$32\left(\frac{s^2+u^2}{t^2}-\frac{u}{s}-\frac{s}{u}\right)$
$GG \leftrightarrow FF$	$32\left(\frac{t^2+u^2}{s^2}-\frac{u}{t}-\frac{t}{u}\right)$
$GS \leftrightarrow GS$	$24\left(\frac{s^2+u^2}{t^2}+1\right)$
$GG \leftrightarrow SS$	$24\left(\frac{t^2+u^2}{s^2}+1\right)$
$GF \leftrightarrow SF$	$-96\left(\frac{u}{s}+\frac{s}{u}+1\right)$
$GS \leftrightarrow FF$	$-96\left(\frac{u}{t}+\frac{t}{u}+1\right)$
$FS \leftrightarrow FS$	$-96\left[\frac{2us}{t^2}+3\left(\frac{u}{s}+\frac{s}{u}\right)+1\right]$
$SS \leftrightarrow FF$	$-96\left[\frac{2ut}{s^2}+3\left(\frac{u}{t}+\frac{t}{u}\right)+1\right]$
$SS \leftrightarrow SS$	$72\left(\frac{s^2+u^2}{t^2}+\frac{u^2+t^2}{s^2}+\frac{t^2+s^2}{u^2}+3\right)$
$FF \leftrightarrow FF$	$128\left(\frac{s^2+u^2}{t^2}+\frac{u^2+t^2}{s^2}+\frac{t^2+s^2}{u^2}+3\right)$

Coulomb-like scattering

S. C. Huot, S. Jeon, and G. D. Moore, Phys. Rev. Lett. **98**, 172303 (2007)

Transport coefficients

Collisional processes

⇩

transport properties of ultrarelativistic plasma

- ✓ Temperature T is the only dimensional parameter
- ✓ Coulomb-like scatterings dominate the interaction

shear viscosity
$$\eta \sim \frac{T^3}{g^4 \ln g^{-1}}$$

S. C. Huot, S. Jeon, and G. D. Moore, Phys. Rev. Lett. **98**, 172303 (2007)

Energy loss & momentum broadening

- **are not constrained by dimensional arguments**

$$\frac{dE}{dx} \sim T^2, ET, E^2, \ldots$$

E – energy of test particle

$$\hat{q} \sim T^3, ET^2, E^2T, E^3, \ldots$$

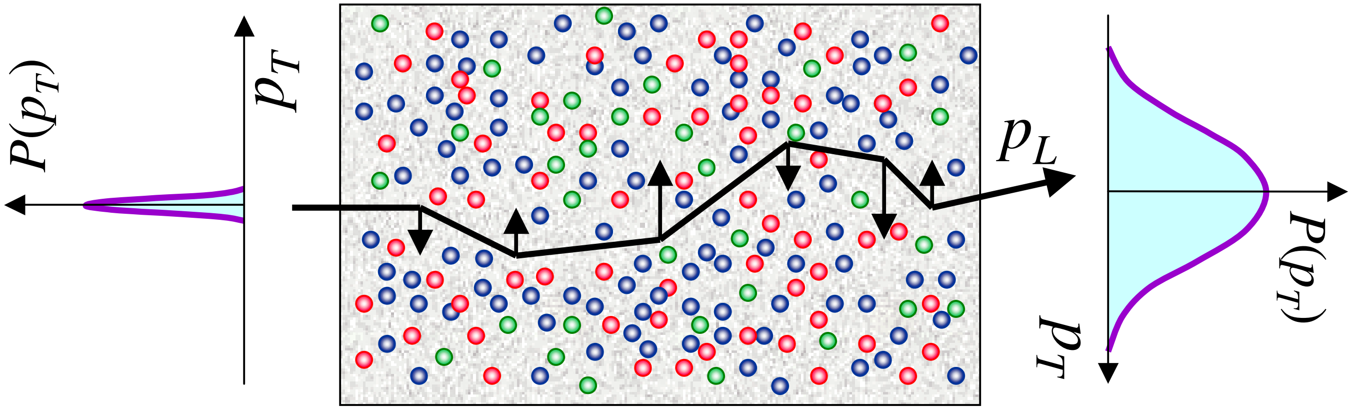
- **depend on a specific scattering process under consideration**

Momentum broadening

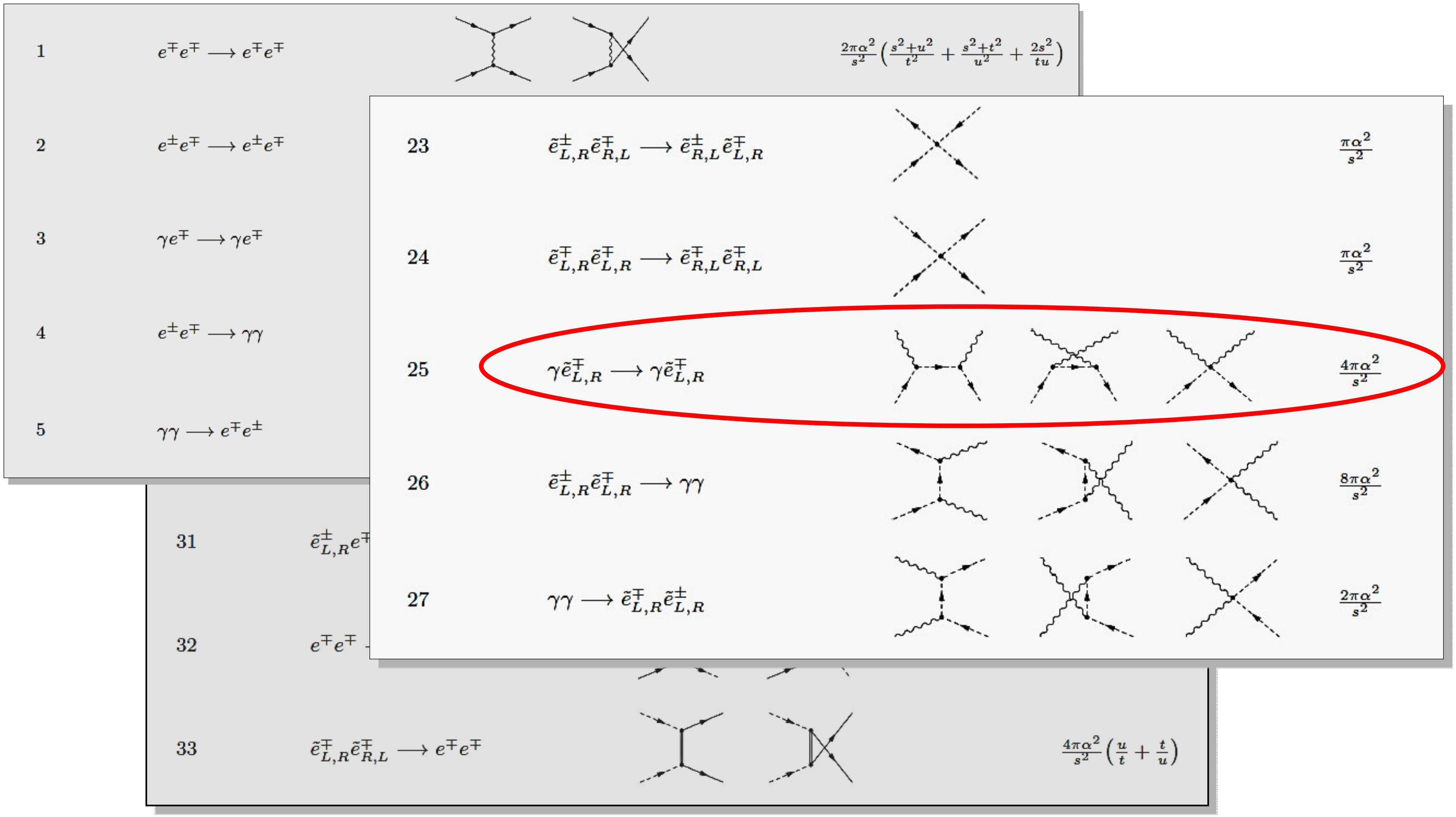
Radiative energy loss of a fast parton is controlled by

$$\hat{q} \equiv \frac{d\left\langle \Delta p_T^2(t) \right\rangle}{dt}$$

Baier, Dokshitzer, Mueller, Peigne & Schiff 1996



Elementary processes in SUSY QED



1	$e^\mp e^\mp \longrightarrow e^\mp e^\mp$	$\frac{2\pi\alpha^2}{s^2}\left(\frac{s^2+u^2}{t^2}+\frac{s^2+t^2}{u^2}+\frac{2s^2}{tu}\right)$
2	$e^\pm e^\mp \longrightarrow e^\pm e^\mp$	
3	$\gamma e^\mp \longrightarrow \gamma e^\mp$	
4	$e^\pm e^\mp \longrightarrow \gamma\gamma$	
5	$\gamma\gamma \longrightarrow e^\mp e^\pm$	

23	$\tilde e^\pm_{L,R}\tilde e^\mp_{R,L} \longrightarrow \tilde e^\pm_{R,L}\tilde e^\mp_{L,R}$	$\frac{\pi\alpha^2}{s^2}$
24	$\tilde e^\mp_{L,R}\tilde e^\mp_{L,R} \longrightarrow \tilde e^\mp_{R,L}\tilde e^\mp_{R,L}$	$\frac{\pi\alpha^2}{s^2}$
25	$\gamma\tilde e^\mp_{L,R} \longrightarrow \gamma\tilde e^\mp_{L,R}$	$\frac{4\pi\alpha^2}{s^2}$
26	$\tilde e^\pm_{L,R}\tilde e^\mp_{L,R} \longrightarrow \gamma\gamma$	$\frac{8\pi\alpha^2}{s^2}$
27	$\gamma\gamma \longrightarrow \tilde e^\mp_{L,R}\tilde e^\pm_{L,R}$	$\frac{2\pi\alpha^2}{s^2}$

31	$\tilde e^\pm_{L,R} e^\mp$	
32	$e^\mp e^\mp$	
33	$\tilde e^\mp_{L,R}\tilde e^\mp_{R,L} \longrightarrow e^\mp e^\mp$	$\frac{4\pi\alpha^2}{s^2}\left(\frac{u}{t}+\frac{t}{u}\right)$

Energy loss and momentum broadening in SUSY QED

A selectron is traversing an equilibrium photon gas.

$$\left|\mathcal{M}\right|^2 = 4e^4$$

$$\frac{dE}{dx} = -\frac{e^4}{2^5 3\pi} T^2 \left[1 - \frac{12\zeta(3)}{\pi^2}\frac{T}{E}\right] \underset{E>>T}{\approx} -\frac{e^4}{2^5 3\pi} T^2$$

$$\hat{q} = \frac{e^4}{12\pi^3} T^3 \left[\zeta(3) + \frac{\pi^4}{45}\frac{T}{E}\right] \underset{E>>T}{\approx} \frac{e^4 \zeta(3)}{12\pi^3} T^3$$

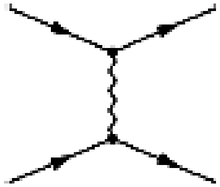
Comparison with Coulomb-like interaction

Energy loss for contact interaction

$$\frac{dE}{dx} = -\frac{e^4}{2^5 3\pi} T^2$$

Energy loss for Coulomb-like interaction

$$\frac{dE}{dx} = -\frac{e^4}{48\pi^3} T^2 \left(\ln \frac{E}{eT} + 2.031 \right)$$



E. Braaten and M. H. Thoma, Phys. Rev. D **44**, 1298 (1991)

Energy loss

		Contact $\lvert\mathcal{M}\rvert^2 \sim e^4$	Coulomb $\lvert\mathcal{M}\rvert^2 \sim e^4 \frac{s^2}{t^2}$
energy change in single collision	ΔE	$\sim E$	$\sim e^2 T$
cross section	σ	$\sim \frac{e^4}{ET}$	$\sim \frac{e^2}{T^2}$
density	ρ	$\sim T^3$	$\sim T^3$
inverse mean path	$\lambda^{-1} = \sigma\rho$	$\sim \frac{e^4 T^2}{E}$	$\sim e^2 T$
energy loss	$\frac{dE}{dx} \sim \frac{\Delta E}{\lambda}$	$\sim e^4 T^2$	$\sim e^4 T^2$

Different interactions lead to the same energy loss!

Conclusions

- The collective modes of $\mathcal{N}$=4 super Yang-Mills plasma are the same as those of QGP
  - The structures of self-energies appear to be unique
  - There are no unstable fermion modes

- The transport characteristics of SUSY plasma are similar to those of QGP

Both systems are very similar to each other in the weak coupling regime!