Chromodynamic Fluctuations in the Quark-Gluon Plasma

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- $\left\langle E_a^i(t,\mathbf{r}) \right\rangle = 0, \quad \left\langle E_a^i(t,\mathbf{r}) E_b^j(t',\mathbf{r}') \right\rangle = ?$
- $\left\langle B_a^i(t,\mathbf{r}) \right\rangle = 0, \quad \left\langle B_a^i(t,\mathbf{r}) B_b^j(t',\mathbf{r}') \right\rangle = ?$

Motivation

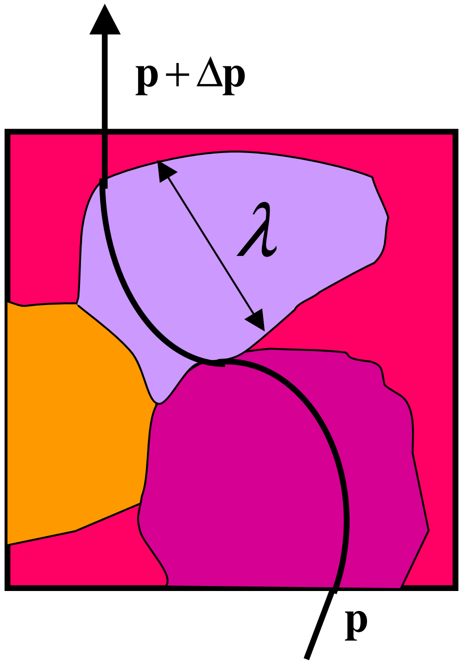
- Color fluctuations in equilibrium (white) QGP are small but the fluctuations can be large in non-equilibrium unstable QGP.
- QGP from the early stage of relativistic heavy-ion collisions is unstable with respect to magnetic modes.
- QGP becomes spontaneously chromomagnetized.
- What is the structure of chromomagnetic field in the plasma?

Motivation cont.

Q: Why the chromomagnetic field in the plasma matters?

A1: It controls plasma transport properties.

A2: Weakly coupled magnetized plasma can behave as strongly coupled.



Example: Viscosity of magnetized plasma

anomalous viscosity: $$\eta_A \sim \frac{1}{g^2 \langle \mathbf{B}^2 \rangle \lambda}$$

$$\frac{1}{\eta} = \frac{1}{\eta_A} + \frac{1}{\eta_C}$$

λ - size of magnetic domain

M. Asakawa, S.A. Bass and B. Müller, Prog. Theor. Phys. **116**, 725 (2007) [arXiv:hep-ph/0608270].

How to compute fluctuations of B & E?

- Equilibrium methods are not applicable.
- We deal with the initial value problem.

The kinetic theory method by Klimontovich & Silin, Rostoker, Tsytovich, see E.M. Lifshitz and L.P. Pitaevskii, *Physical Kinetics*

St. Mrówczynski, arXiv:0711.2003 [physics] Electromagnetic Fluctuations
St. Mrówczynski, arXiv:0801.0536 [hep-ph] Chromodynamic Fluctuations

Transport theory – distribution functions

QGP is assumed to be weakly coupled, QGP = pQGP, $g^2 << 1$

$Q(x,p)$, $\overline{Q}(x,p)$ - distribution functions of quarks and antiquarks,
gauge dependent $N_c \times N_c$ matrices, $x \equiv (t, \mathbf{r})$, $p \equiv (E_p, \mathbf{p})$

on-mass-shell

The gauge transformation:

$$Q(x,p) \rightarrow U(x)\, Q(x,p)\, U^{-1}(x)$$

$G(p,x)$ - distribution function of gluons, $(N_c^2 - 1) \times (N_c^2 - 1)$ matrix

Transport theory – transport equations

fundamental:

$$p_\mu D^\mu Q - \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu Q\} = C[Q, \overline{Q}, G]$$ quarks

$$p_\mu D^\mu \overline{Q} + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu \overline{Q}\} = \overline{C}[Q, \overline{Q}, G]$$ antiquarks

adjoint:

$$p_\mu \mathcal{D}^\mu G - \frac{g}{2} p^\mu \{\mathcal{F}_{\mu\nu}, (x) \partial_p^\nu G\} = C_g[Q, \overline{Q}, G]$$ gluons

free streaming

mean-field force

collisions

$$D^\mu \equiv \partial^\mu - ig[A^\mu, \ldots.], \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

$$D_\mu F^{\mu\nu} = j^\nu[Q, \overline{Q}, G]$$ mean-field generation

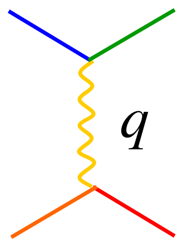
collisionless limit: $C = \overline{C} = C_g = 0$

Time scale of collisional processes

Time scale of processes driven by parton-parton scattering

$$t_{\text{hard}} \sim \frac{1}{g^4 \ln(1/g)\, T}$$

$$t_{\text{soft}} \sim \frac{1}{g^2 \ln(1/g)\, T}$$



hard scattering: $q \sim T$

soft scattering: $q \sim gT$

Time scale of collective phenomena

$$t_{\text{collec}} \sim \frac{1}{g\, T}$$

$$g^2 << 1 \quad \Rightarrow \quad t_{\text{hard}} >> t_{\text{soft}} >> t_{\text{collec}}$$

The instabilities are fast!

Equations to be solved, quarks only

Transport equation

$$\left(D^0 + \mathbf{v}\cdot\mathbf{D}\right)Q(t,\mathbf{r},\mathbf{p}) - \frac{g}{2}\{\mathbf{E}(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{r}), \nabla_p Q(t,\mathbf{r},\mathbf{p})\} = 0$$

$$\mathbf{v} \equiv \frac{\mathbf{p}}{E_p}$$

Yang-Mills (Maxwell) equations

$$\mathbf{D}\cdot\mathbf{E}(t,\mathbf{r}) = \rho(t,\mathbf{r}), \qquad \mathbf{D}\cdot\mathbf{B}(t,\mathbf{r}) = 0,$$

$$\mathbf{D}\times\mathbf{E}(t,\mathbf{r}) = -D^0\mathbf{B}(t,\mathbf{r}), \quad \mathbf{D}\times\mathbf{B}(t,\mathbf{r}) = \mathbf{j}(t,\mathbf{r}) + D^0\mathbf{E}(t,\mathbf{r})$$

$$\begin{cases} \rho_a(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\mathrm{Tr}\left[\tau^a Q(t,\mathbf{r},\mathbf{p})\right] \\ \mathbf{j}_a(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\mathbf{v}\,\mathrm{Tr}\left[\tau^a Q(t,\mathbf{r},\mathbf{p})\right] \end{cases}$$

Small fluctuations

fluctuation

$$Q(t,\mathbf{r},\mathbf{p}) = Q_0(\mathbf{p}) + \delta Q\,(t,\mathbf{r},\mathbf{p})$$

stationary colorless state $Q_0^{ij}(\mathbf{p}) = \delta^{ij} n(\mathbf{p})$

$$|Q_0(\mathbf{p})| >> |\delta Q(t,\mathbf{r},\mathbf{p})|, \quad |\nabla_p Q_0(\mathbf{p})| >> |\nabla_p \delta Q(t,\mathbf{r},\mathbf{p})|$$

$$\mathbf{E}(t,\mathbf{r}), \mathbf{B}(t,\mathbf{r}), A^0(t,\mathbf{r}), \mathbf{A}(t,\mathbf{r}) \sim \delta Q(t,\mathbf{r},\mathbf{p})$$

Linearized equations

Transport equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\delta Q(t,\mathbf{r},\mathbf{p}) - g\left(\mathbf{E}(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{r})\right)\nabla_p n(\mathbf{p}) = 0$$

Yang-Mills (Maxwell) equations

$$\nabla\cdot\mathbf{E}(t,\mathbf{r}) = \rho(t,\mathbf{r}), \qquad \nabla\cdot\mathbf{B}(t,\mathbf{r}) = 0,$$

$$\nabla\times\mathbf{E}(t,\mathbf{r}) = -\frac{\partial\mathbf{B}(t,\mathbf{r})}{\partial t}, \quad \nabla\times\mathbf{B}(t,\mathbf{r}) = \mathbf{j}(t,\mathbf{r}) + \frac{\partial\mathbf{E}(t,\mathbf{r})}{\partial t}$$

$$\begin{cases} \rho_a(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\,\mathrm{Tr}\left[\tau^a\,\delta Q(t,\mathbf{r},\mathbf{p})\right], \\ \mathbf{j}_a(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\,\mathbf{v}\,\mathrm{Tr}\left[\tau^a\,\delta Q(t,\mathbf{r},\mathbf{p})\right], \end{cases}$$

Fully Abelian problem!

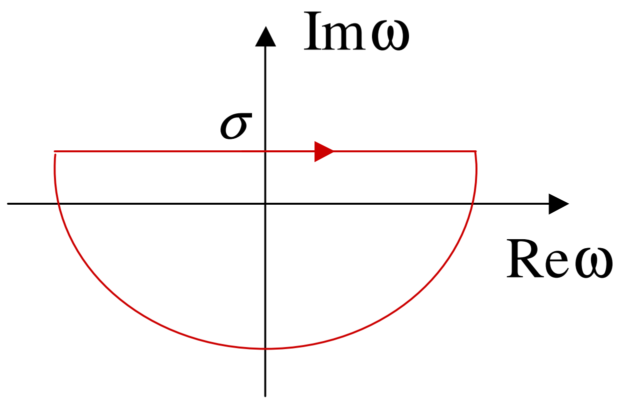
Gauge dependence discussed a posteriori

Initial value problem

$$\delta Q(t=0,\mathbf{r},\mathbf{p}) = \delta Q_0(\mathbf{r},\mathbf{p}),$$
$$\mathbf{E}(t=0,\mathbf{r},\mathbf{p}) = \mathbf{E}_0(\mathbf{r},\mathbf{p}), \quad \mathbf{B}(t=0,\mathbf{r},\mathbf{p}) = \mathbf{B}_0(\mathbf{r},\mathbf{p})$$

$$\begin{cases} f(\omega,\mathbf{k}) = \int_0^{\infty} dt \int d^3r \, e^{i(\omega t - \mathbf{kr})} f(t,\mathbf{r}) \\ f(t,\mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} \, e^{-i(\omega t - \mathbf{kr})} f(\omega,\mathbf{k}) \end{cases}$$

$$0 < \sigma \in R$$



Transformed linear equations

Transport equation

$$-i(\omega - \mathbf{v}\cdot\mathbf{k})\delta Q(\omega,\mathbf{k},\mathbf{p}) - g\left(\mathbf{E}(\omega,\mathbf{k}) + \mathbf{v}\times\mathbf{B}(\omega,\mathbf{k})\right)\nabla_p n(\mathbf{p}) = \delta Q_0(\mathbf{k},\mathbf{p})$$

Yang-Mills (Maxwell) equations

$$i\mathbf{k}\cdot\mathbf{E}(\omega,\mathbf{k}) = \rho(\omega,\mathbf{k}), \qquad i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0,$$

$$i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = i\omega\mathbf{B}(\omega,\mathbf{k}) + \mathbf{B}_0(\mathbf{k}),$$

$$i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = \mathbf{j}(\omega,\mathbf{k}) - i\omega\mathbf{E}(\omega,\mathbf{k}) - \mathbf{E}_0(\mathbf{k})$$

$$\begin{cases} \rho_a(\omega,\mathbf{k}) = -g\int \dfrac{d^3p}{(2\pi)^3}\,\mathrm{Tr}\left[\tau^a\,\delta Q(\omega,\mathbf{k},\mathbf{p})\right], \\[2ex] \mathbf{j}_a(\omega,\mathbf{k}) = -g\int \dfrac{d^3p}{(2\pi)^3}\,\mathbf{v}\,\mathrm{Tr}\left[\tau^a\,\delta Q(\omega,\mathbf{k},\mathbf{p})\right], \end{cases}$$

Solutions

From transport equation

$$\delta Q(\omega,\mathbf{k},\mathbf{p}) = i\,\frac{g\big(\mathbf{E}(\omega,\mathbf{k}) + \mathbf{v}\times\mathbf{B}(\omega,\mathbf{k})\big)\nabla_p n(\mathbf{p}) + \delta Q_0(\mathbf{k},\mathbf{p})}{\omega - \mathbf{v}\cdot\mathbf{k}}$$

$$\mathbf{j}_a(\omega,\mathbf{k}) = -g\int\frac{d^3p}{(2\pi)^3}\,\mathbf{v}\,\mathrm{Tr}\left[\tau^a\,\delta Q(\omega,\mathbf{k},\mathbf{p})\right]$$

$$= \cdots = -i\omega\big(\hat{\varepsilon}(\omega,\mathbf{k}) - \mathbf{1}\big)\mathbf{E}(\omega,\mathbf{k})$$

$$+\frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3}\frac{\mathbf{v}}{\omega-\mathbf{v}\cdot\mathbf{k}}\frac{\mathbf{v}\times\mathbf{B}_0(\mathbf{k})}{\omega}\cdot\nabla_p n(\mathbf{p})$$

$$-ig\int\frac{d^3p}{(2\pi)^3}\frac{\mathbf{v}}{\omega-\mathbf{v}\cdot\mathbf{k}}\,\delta Q_0(\mathbf{k},\mathbf{p})$$

$$\mathbf{B}(\omega,\mathbf{k}) = \frac{\mathbf{k}}{\omega}\times\mathbf{E}(\omega,\mathbf{k}) + \frac{i}{\omega}\mathbf{B}_0(\mathbf{k})$$

$$\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} + \frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{v}\cdot\mathbf{k}}\left(\left(1-\frac{\mathbf{v}\cdot\mathbf{k}}{\omega}\right)\delta^{jl} + \frac{v^j k^l}{\omega}\right)\nabla_p^l n(\mathbf{p})$$

Solutions cont.

From Maxwell equations

$$\left[\left(\omega^2-\mathbf{k}^2\right)\delta^{ij}+k^ik^j\right]E^j(\omega,\mathbf{k})=-i\omega\, j^i(\omega,\mathbf{k})+i\omega E_0^i(\mathbf{k})+i\left(\mathbf{k}\times\mathbf{B}_0(\mathbf{k})\right)^i$$

From transport equation

$$j^i(\omega,\mathbf{k})=-i\omega\left(\varepsilon^{ij}(\omega,\mathbf{k})-\delta^{ij}\right)E^j(\omega,\mathbf{k})$$
$$+\frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{v}\cdot\mathbf{k}}\frac{\mathbf{v}\times\mathbf{B}_0(\mathbf{k})}{\omega}\cdot\nabla_p n(\mathbf{p})-ig\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{v}\cdot\mathbf{k}}\delta Q_0(\mathbf{k},\mathbf{p})$$

$$\left[-\mathbf{k}^2\delta^{ij}+k^ik^j+\omega^2\varepsilon^{ij}(\omega,\mathbf{k})\right]E^j(\omega,\mathbf{k})=-g\,\omega\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{v}\cdot\mathbf{k}}\delta Q_0(\mathbf{k},\mathbf{p})$$
$$-i\frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{v}\cdot\mathbf{k}}\frac{\mathbf{v}\times\mathbf{B}_0(\mathbf{k})}{\omega}\cdot\nabla_p n(\mathbf{p})+i\omega E_0^i(\mathbf{k})-i\left(\mathbf{k}\times\mathbf{B}_0(\mathbf{k})\right)^i$$

Inverting of Σ

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

$$\left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k}) = ?$$

Isotropic system

$$\varepsilon^{ij}(\omega,\mathbf{k}) \equiv \varepsilon_L(\omega,\mathbf{k})\frac{k^i k^j}{\mathbf{k}^2} + \varepsilon_L(\omega,\mathbf{k})\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

$$\left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k}) = \frac{1}{\omega^2\varepsilon_L(\omega,\mathbf{k})}\frac{k^i k^j}{\mathbf{k}^2} + \frac{1}{\omega^2\varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

Fluctuations of E field

The solution

$$E^i(\omega,\mathbf{k}) = \left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k})\left[\ldots\ldots\ \delta Q_0(\mathbf{k},\mathbf{p}) + \ldots\ldots\ \mathbf{E}_0(\mathbf{k}) + \ldots\ldots\ \mathbf{B}_0(\mathbf{k})\right]^j$$

The correlation function

$$\begin{aligned}\left\langle E^i(\omega,\mathbf{k})E^j(\omega',\mathbf{k}')\right\rangle = \left(\Sigma^{-1}\right)^{ik}(\omega,\mathbf{k})\left(\Sigma^{-1}\right)^{jl}(\omega',\mathbf{k}')\Big[\ldots\ldots\ &\left\langle \delta Q_0(\mathbf{k},\mathbf{p})\delta Q_0(\mathbf{k}',\mathbf{p}')\right\rangle \\ + \ldots\ldots\ &\left\langle \delta Q_0(\mathbf{k},\mathbf{p})E_0^m(\mathbf{k}')\right\rangle + \ldots\ldots\ \left\langle \delta Q_0(\mathbf{k},\mathbf{p})B_0^m(\mathbf{k}')\right\rangle \\ + \ldots\ldots\ &\left\langle E_0^m(\mathbf{k})E_0^n(\mathbf{k}')\right\rangle + \ldots\ldots\ \left\langle E_0^m(\mathbf{k})B_0^n(\mathbf{k}')\right\rangle \\ + \ldots\ldots\ &\left\langle B_0^m(\mathbf{k})B_0^n(\mathbf{k}')\right\rangle\Big]^{kl}\end{aligned}$$

$\langle \cdots \rangle$ - statistical ensemble average

B, ρ, j are given by E

From Maxwell equations

$$\begin{cases} \mathbf{B}(\omega,\mathbf{k}) = \dfrac{\mathbf{k}}{\omega}\times\mathbf{E}(\omega,\mathbf{k}) + \dfrac{i}{\omega}\mathbf{B}_0(\mathbf{k}) \\ \rho(\omega,\mathbf{k}) = i\mathbf{k}\cdot\mathbf{E}(\omega,\mathbf{k}) \\ \mathbf{j}(\omega,\mathbf{k}) = i\omega\mathbf{E}(\omega,\mathbf{k}) - i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) + \mathbf{E}_0(\mathbf{k}) \end{cases}$$

Initial fluctuations

color indices $i, j, k, l = 1, 2, \ldots, N_c$

$$\left\langle \delta Q_0^{ij}(\mathbf{r},\mathbf{p})\; \delta Q_0^{kl}(\mathbf{r}',\mathbf{p}') \right\rangle = ?$$

Assumption

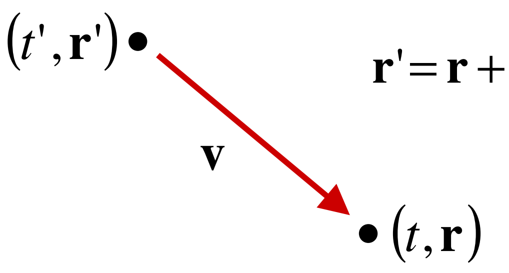
The initial fluctuations are given by $\left\langle \delta Q^{ij}(t=0,\mathbf{r},\mathbf{p})\; \delta Q^{kl}(t'=0,\mathbf{r}',\mathbf{p}') \right\rangle_{\text{free}}$

colorless state

$$\delta Q^{ij}(t,\mathbf{r},\mathbf{p}) \equiv Q^{ij}(t,\mathbf{r},\mathbf{p}) - \left\langle Q^{ij}(t,\mathbf{r},\mathbf{p}) \right\rangle = Q^{ij}(t,\mathbf{r},\mathbf{p}) - \delta^{ij} n(\mathbf{p})$$

Classical limit

$$\left\langle \delta Q^{ij}(t,\mathbf{r},\mathbf{p})\; \delta Q^{kl}(t',\mathbf{r}',\mathbf{p}') \right\rangle_{\text{free}} = \delta^{il}\delta^{jk}(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')(2\pi)^3\delta^{(3)}\big(\mathbf{r}'-\mathbf{r}-\mathbf{v}(t'-t)\big)n(\mathbf{p})$$



$$\mathbf{r}' = \mathbf{r} + \mathbf{v}(t'-t)$$

Fluctuations of free distribution functions

Keldysh-Schwinger formalism

$x \equiv (t, \mathbf{r})$

$$\begin{cases} i\Delta^{>}_{ij}(x_1, x_2) \equiv i\left\langle \varphi_i(x_1)\, \varphi^*_j(x_2) \right\rangle \\ i\Delta^{<}_{ij}(x_1, x_2) \equiv i\left\langle \varphi^*_j(x_2)\, \varphi_i(x_1) \right\rangle \end{cases}$$

$$\Delta(x, p) \equiv \int d^4u\, e^{ipu} \Delta(x + u/2, x - u/2)$$

averaged distribution function

$$\begin{cases} \Theta(p^0) i\Delta^{<}_{ij}(x, p) \equiv \dfrac{\pi}{E_p} \delta(E_p - p^0) \left\langle Q^{ij}(x, \mathbf{p}) \right\rangle \\ \Theta(-p^0) i\Delta^{>}_{ij}(x, p) \equiv \dfrac{\pi}{E_p} \delta(E_p + p^0) \left\langle \overline{Q}^{ij}(x, -\mathbf{p}) \right\rangle \end{cases}$$

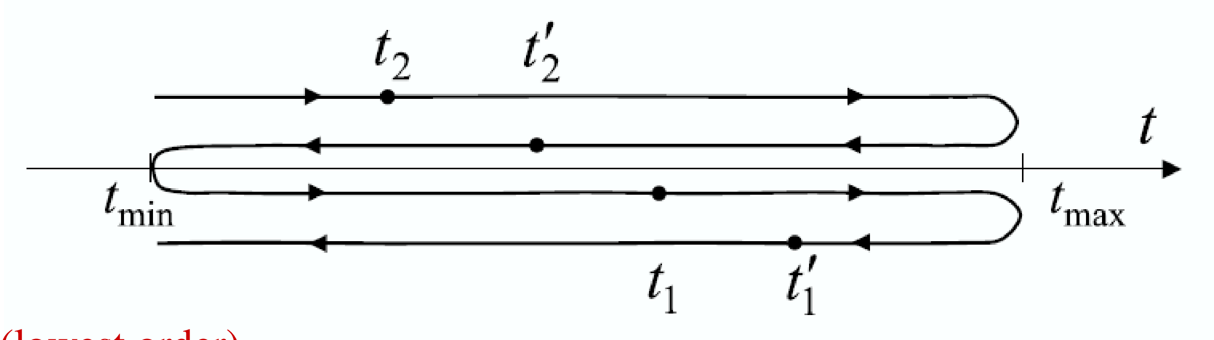
$$\delta Q^{ij}(x, \mathbf{p}) \equiv Q^{ij}(x, \mathbf{p}) - \left\langle Q^{ij}(x, \mathbf{p}) \right\rangle$$

$$\left\langle \delta Q^{ij}(x_1, \mathbf{p}_1) \delta Q^{kl}(x_2, \mathbf{p}_2) \right\rangle = 4 E_{p_1} E_{p_2} \int \frac{dp_1^0}{2\pi} \Theta(p_1^0) \int \frac{dp_1^0}{2\pi} \Theta(p_1^0) \int d^4u_1 \int d^4u_2\, e^{i(p_1u_1 + p_2u_2)}$$
$$\times W_{ijkl}(x_1 + u_1/2, x_1 - u_1/2, x_2 + u_2/2, x_2 - u_2/2)$$

$$W_{ijkl}(x_1, x'_1, x_2, x'_2) \equiv \left\langle \varphi^*_j(x'_1) \varphi_i(x_1)\, \varphi^*_l(x'_2) \varphi_k(x_2) \right\rangle - \left\langle \varphi^*_j(x'_1) \varphi_i(x_1) \right\rangle \left\langle \varphi^*_l(x'_2) \varphi_k(x_2) \right\rangle$$

Fluctuations of free distribution functions cont.

$$\left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle = \left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle$$



Wick theorem (lowest order)

$$\left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle = \left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\right)\right\rangle\left\langle T_c\left(\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle + \left\langle T_c\left(\varphi_j^*(x'_1)\varphi_k(x_2)\right)\right\rangle\left\langle T_c\left(\varphi_l^*(x'_2)\varphi_i(x_1)\right)\right\rangle$$

$$\left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle = \left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\right\rangle\left\langle \varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle + \left\langle \varphi_j^*(x'_1)\varphi_k(x_2)\right\rangle\left\langle \varphi_i(x_1)\varphi_l^*(x'_2)\right\rangle$$

Fluctuations of free distribution functions cont.

$$\left\langle Q^{ij}(x,\mathbf{p})\right\rangle = \delta^{ij} n(\mathbf{p})$$ fluctuations around colorless state

$$\left\langle \delta Q^{ij}(x_1,\mathbf{p}_1)\delta Q^{kl}(x_2,\mathbf{p}_2)\right\rangle_{\text{free}} = \delta^{il}\delta^{jk}(2\pi)^3\delta^{(3)}(\mathbf{p}_1-\mathbf{p}_2)\int\frac{d^3q}{(2\pi)^3}\frac{E_{p_1}E_{p_2}}{E_{p_1-q/2}E_{p_2+q/2}}e^{iq(x_1-x_2)}$$
$$\times n(\mathbf{p}_1+\mathbf{q}/2)\left[1+n(\mathbf{p}_1-\mathbf{q}/2)\right]$$

Classical limit: 1) $|\mathbf{x}_1-\mathbf{x}_2|>>1/|\mathbf{p}| \quad\Rightarrow\quad |\mathbf{p}_1|>>|\mathbf{q}|,$ 2) $n(\mathbf{p})<<1$

$$\left\langle \delta Q^{ij}(x,\mathbf{p})\,\delta Q^{kl}(x',\mathbf{p}')\right\rangle_{\text{free}} = \delta^{il}\delta^{jk}(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')(2\pi)^3\delta^{(3)}(\mathbf{r}'-\mathbf{r}-\mathbf{v}(t'-t))n(\mathbf{p})$$

$x\equiv(t,\mathbf{r}),\quad x'\equiv(t',\mathbf{r}')$

Initial fluctuations cont.

$$\left\langle \delta Q_0^{ij}(\mathbf{k},\mathbf{p})\, \delta Q_0^{kl}(\mathbf{k}',\mathbf{p}') \right\rangle = \delta^{il}\delta^{jk}(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')n(\mathbf{p})$$

$$\left\langle \mathbf{E}_0^a(\mathbf{k})\tau_{ij}^b \delta Q_0^{ji}(\mathbf{k}',\mathbf{p}') \right\rangle = i\frac{g}{2}\delta^{ab}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\frac{(\mathbf{k}\cdot\mathbf{v}')\mathbf{v}'-\mathbf{k}}{(\mathbf{k}\cdot\mathbf{v}')^2-\mathbf{k}^2}\, n(\mathbf{p})$$

$$\left\langle \mathbf{B}_0^a(\mathbf{k})\tau_{ij}^b \delta Q_0^{ji}(\mathbf{k}',\mathbf{p}') \right\rangle = i\frac{g}{2}\delta^{ab}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\frac{\mathbf{k}\times\mathbf{v}'}{(\mathbf{k}\cdot\mathbf{v}')^2-\mathbf{k}^2}\, n(\mathbf{p}')$$

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i,j,k – color indices of fundamental representation; *a,b,c* of adjoint representation

Fluctuations of E field

$$\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle = \left(\Sigma^{-1}\right)^{ik}(\omega,\mathbf{k})\left(\Sigma^{-1}\right)^{jl}(\omega',\mathbf{k}')\Big[\ldots\ldots\left\langle \delta Q_0(\mathbf{k},\mathbf{p})\delta Q_0(\mathbf{k}',\mathbf{p}')\right\rangle$$
$$+\ldots\ldots\left\langle \delta Q_0(\mathbf{k},\mathbf{p})E_0^m(\mathbf{k}')\right\rangle + \ldots\ldots\left\langle \delta Q_0(\mathbf{k},\mathbf{p})B_0^m(\mathbf{k}')\right\rangle$$
$$+\ldots\ldots\left\langle E_0^m(\mathbf{k})E_0^n(\mathbf{k}')\right\rangle + \ldots\ldots\left\langle E_0^m(\mathbf{k})B_0^n(\mathbf{k}')\right\rangle$$
$$+\ldots\ldots\left\langle B_0^m(\mathbf{k})B_0^n(\mathbf{k}')\right\rangle\Big]^{kl}$$

i,j,k – coordinate space indices; *a,b,c* – color indices of adjoint representation

Isotropic system

$$\left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k}) = \frac{1}{\omega^2\varepsilon_L(\omega,\mathbf{k})}\frac{k^i k^j}{\mathbf{k}^2} + \frac{1}{\omega^2\varepsilon_T(\omega,\mathbf{k})-\mathbf{k}^2}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

Fluctuations in isotropic (stable) system

$$\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle = \frac{g^2}{2}\delta^{ab}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\int\frac{d^3p}{(2\pi)^3}n(\mathbf{p})F(\omega,\mathbf{k},\omega',\mathbf{k}',\mathbf{p})$$

$F(\omega,\mathbf{k},\omega',\mathbf{k}',\mathbf{p})$ has poles at:

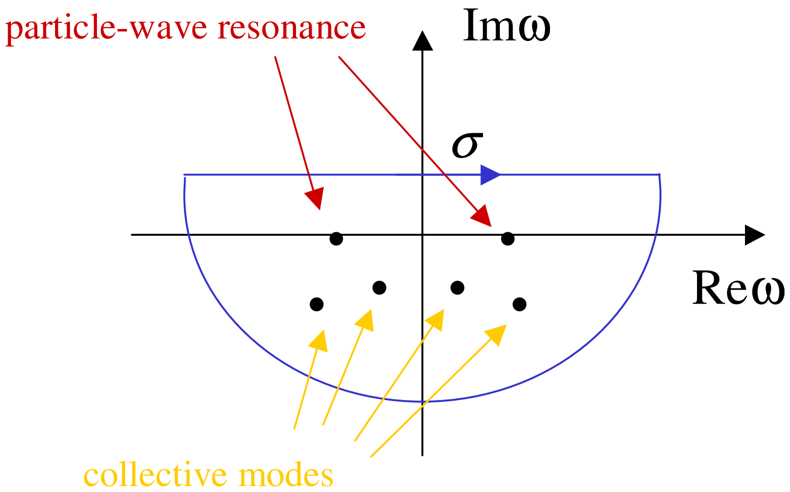
particle-wave resonance
$$\begin{cases}\omega-\mathbf{v}\cdot\mathbf{k}=0\\ \omega'-\mathbf{v}'\cdot\mathbf{k}'=0\end{cases}$$

collective longitudinal modes
$$\begin{cases}\varepsilon_L(\omega,\mathbf{k})=0\\ \varepsilon_L(\omega',\mathbf{k}')=0\end{cases}$$

collective transverse modes
$$\begin{cases}\omega^2\varepsilon_T(\omega,\mathbf{k})-\mathbf{k}^2=0\\ \omega'^2\,\varepsilon_T(\omega',\mathbf{k}')-\mathbf{k}'^2=0\end{cases}$$

Fluctuations in isotropic (stable) system

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle = \int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi}\int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega'}{2\pi}\int\frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\,e^{-i(\omega t+\omega' t'-\mathbf{kr}-\mathbf{k}'\mathbf{r}')}\times\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle$$



$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle \sim f(\mathbf{r}-\mathbf{r}')$$

$$\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle \sim \delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle = \left(\begin{matrix}\text{collective}\\ \text{modes}\end{matrix}\right)\left(e^{-\gamma t}\ \text{or}\ e^{-\gamma t'}\right) + \left(\begin{matrix}\text{particle-wave}\\ \text{resonance}\end{matrix}\right) f(t-t')$$

$$\gamma \equiv \mathrm{Im}\,\omega > 0$$

Fluctuations in isotropic (stable) system cont.

Long time limit $t, t' \to \infty \quad \left\langle E_a^i(t,\mathbf{r}) E_b^j(t',\mathbf{r}')\right\rangle_\infty = f(t'-t, \mathbf{r}'-\mathbf{r})$

$$\left\langle E_a^i(t,\mathbf{r}) E_b^j(t',\mathbf{r}')\right\rangle_\infty = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i\left(\omega(t-t') - \mathbf{k}(\mathbf{r}-\mathbf{r}')\right)} \left\langle E_a^i E_b^j \right\rangle_{\omega,\mathbf{k}}$$

Fluctuation spectrum

$$\left\langle E_a^i E_b^j \right\rangle_{\omega,\mathbf{k}} = \frac{g^2}{2} \delta^{ab} \int \frac{d^3p}{(2\pi)^3} n(\mathbf{p})\, 2\pi\delta(\omega - \mathbf{kv}) \frac{\omega^2}{\mathbf{k}^4}$$
$$\times \left[\frac{\omega^2 k^i k^j}{|\,\omega^2 \varepsilon_L(\omega,\mathbf{k})\,|^2} + \frac{(\mathbf{kv}k^i - \mathbf{k}^2 v^i)(\mathbf{kv}k^j - \mathbf{k}^2 v^j)}{|\,\omega^2 \varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2\,|^2} \right]$$

Fluctuations in equilibrium system

Long time limit $\quad t, t' \to \infty \quad \left\langle E_a^i(t,\mathbf{r}) E_b^j(t',\mathbf{r}')\right\rangle_\infty = f(t'-t, \mathbf{r}'-\mathbf{r})$

$$\left\langle E_a^i(t,\mathbf{r}) E_b^j(t',\mathbf{r}')\right\rangle_\infty = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i\left(\omega(t-t')-\mathbf{k}(\mathbf{r}-\mathbf{r}')\right)} \left\langle E_a^i E_b^j \right\rangle_{\omega,\mathbf{k}}$$

$$\operatorname{Im} \varepsilon_L(\omega,\mathbf{k}) = \frac{\pi g^2 \omega}{2T\mathbf{k}} \int \frac{d^3p}{(2\pi)^3} n(\mathbf{p})\, 2\pi\delta(\omega - \mathbf{k}\mathbf{v}), \quad \operatorname{Im} \varepsilon_T(\omega,\mathbf{k}) = ...$$

Fluctuation dissipation relation

$$\left\langle E_a^i E_b^j \right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab} T \omega^3 \left[\frac{k^i k^j}{\mathbf{k}^2} \frac{\operatorname{Im} \varepsilon_L(\omega,\mathbf{k})}{|\,\omega^2 \varepsilon_L(\omega,\mathbf{k})\,|^2} + \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) \frac{\operatorname{Im} \varepsilon_T(\omega,\mathbf{k})}{|\,\omega^2 \varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2\,|^2} \right]$$

B fluctuations in equilibrium system

Fluctuation dissipation relation

$$\left\langle B_a^i B_b^j \right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab} T\omega\left(\mathbf{k}^2\delta^{ij} - k^i k^j\right)\frac{\operatorname{Im}\varepsilon_T(\omega,\mathbf{k})}{|\,\omega^2\varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2\,|^2}$$

Fluctuations in unstable systems

Two-stream system $n(\mathbf{p}) = (2\pi)^3 n\left[\delta^{(3)}(\mathbf{p}-\mathbf{q}) + \delta^{(3)}(\mathbf{p}+\mathbf{q})\right]$

Longitudinal electric field: $\omega_+(\mathbf{k})$ - stable mode, $\omega_-(\mathbf{k})$ - unstable mode

$$\left\langle E_a^i(\omega,\mathbf{k})E_b^i(\omega',\mathbf{k}')\right\rangle = \frac{g^2}{2}\delta^{ab}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\frac{\mathbf{k}\cdot\mathbf{k}'}{\mathbf{k}^2\mathbf{k}'^2}$$
$$\times\frac{1}{\varepsilon_L(\omega,\mathbf{k})}\frac{1}{\varepsilon_L(\omega',\mathbf{k}')}\int\frac{d^3p}{(2\pi)^3}\frac{n(\mathbf{p})}{(\omega-\mathbf{v}\cdot\mathbf{k})(\omega'-\mathbf{v}'\cdot\mathbf{k}')}$$

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle_{\text{unstable}} = \frac{g^2}{2}\delta^{ab}\, n\int\frac{d^3k}{(2\pi)^3}\frac{e^{-i\mathbf{k}(\mathbf{r}-\mathbf{r}')}}{\mathbf{k}^2}\frac{1}{\left(\omega_+^2-\omega_+^2\right)^2}\frac{\left(\gamma_k^2+(\mathbf{k}\mathbf{u})^2\right)^2}{\gamma_k^2}$$
$$\times\left[\left(\gamma_k^2+(\mathbf{k}\mathbf{u})^2\right)\cosh\left(\gamma_k(t+t')\right)+\left(\gamma_k^2-(\mathbf{k}\mathbf{u})^2\right)\cosh\left(\gamma_k(t-t')\right)\right]$$

$$\mathbf{u}\equiv\frac{\mathbf{q}}{E_q},\quad \gamma_k\equiv\operatorname{Im}\omega_-(\mathbf{k})$$

Inclusion of antiquarks & gluons

for massless quarks

$$n(\mathbf{p}) \to n(\mathbf{p}) + \overline{n}(\mathbf{p}) + 2N_c n_g(\mathbf{p})$$

Gauge dependence

Generic correlation function: $L_{ab}(x,x') \equiv \langle H_a(x) K_b(x') \rangle$

Infinitesimal gauge transformation

$$H_a(x) \to H_a(x) + f_{abc}\lambda_b(x) H_c(x)$$

$$L_{ab}(x,x') \to L_{ab}(x,x') + f_{acd}\lambda_c(x) L_{db}(x,x') + f_{bcd}\lambda_c(x') L_{ad}(x,x')$$

colorless background

Actual correlation function: $L_{ab}(x,x') \equiv \delta^{ab} L(x,x')$

$$L_{ab}(x,x') \to \left(\delta^{ab} + f_{acb}\lambda_c(x) + f_{bca}\lambda_c(x')\right) L(x,x')$$

$L_{aa}(x,x') = \left(N_c^2 - 1\right) L(x,x')$ **- gauge invariant!**

Conclusion & outlook

- The method allows one to compute fluctuation spectra of chromodynamic fields in stable and unstable pQGP.
- Computation of $\left\langle B_a^i(t,\mathbf{r}) B_b^j(t',\mathbf{r}') \right\rangle$ in a phenomenologically interesting, anisotropic (unstable) configuration is under way.