

# Energy loss in Unstable Quark-Gluon Plasma

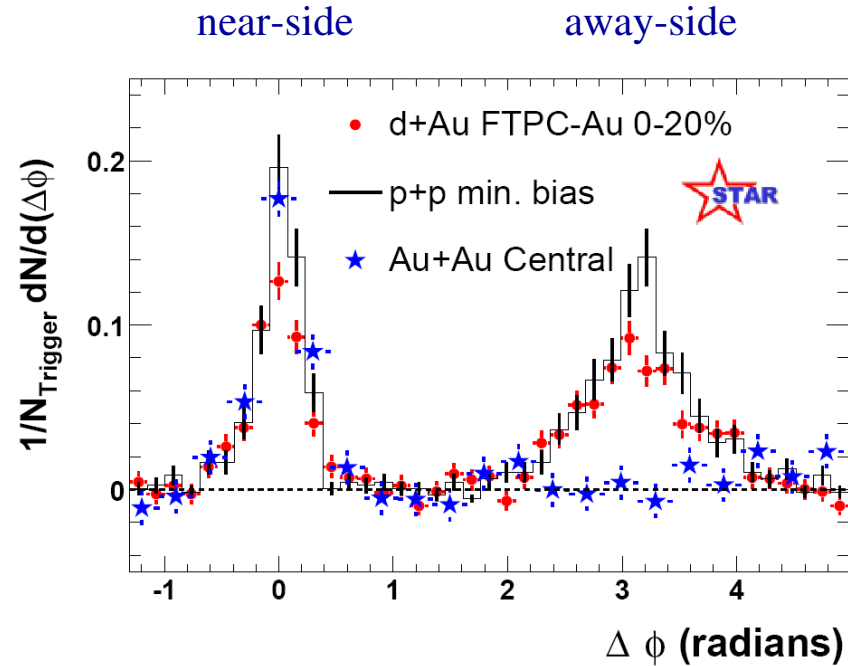
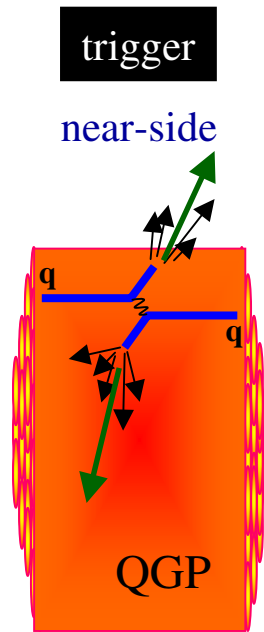
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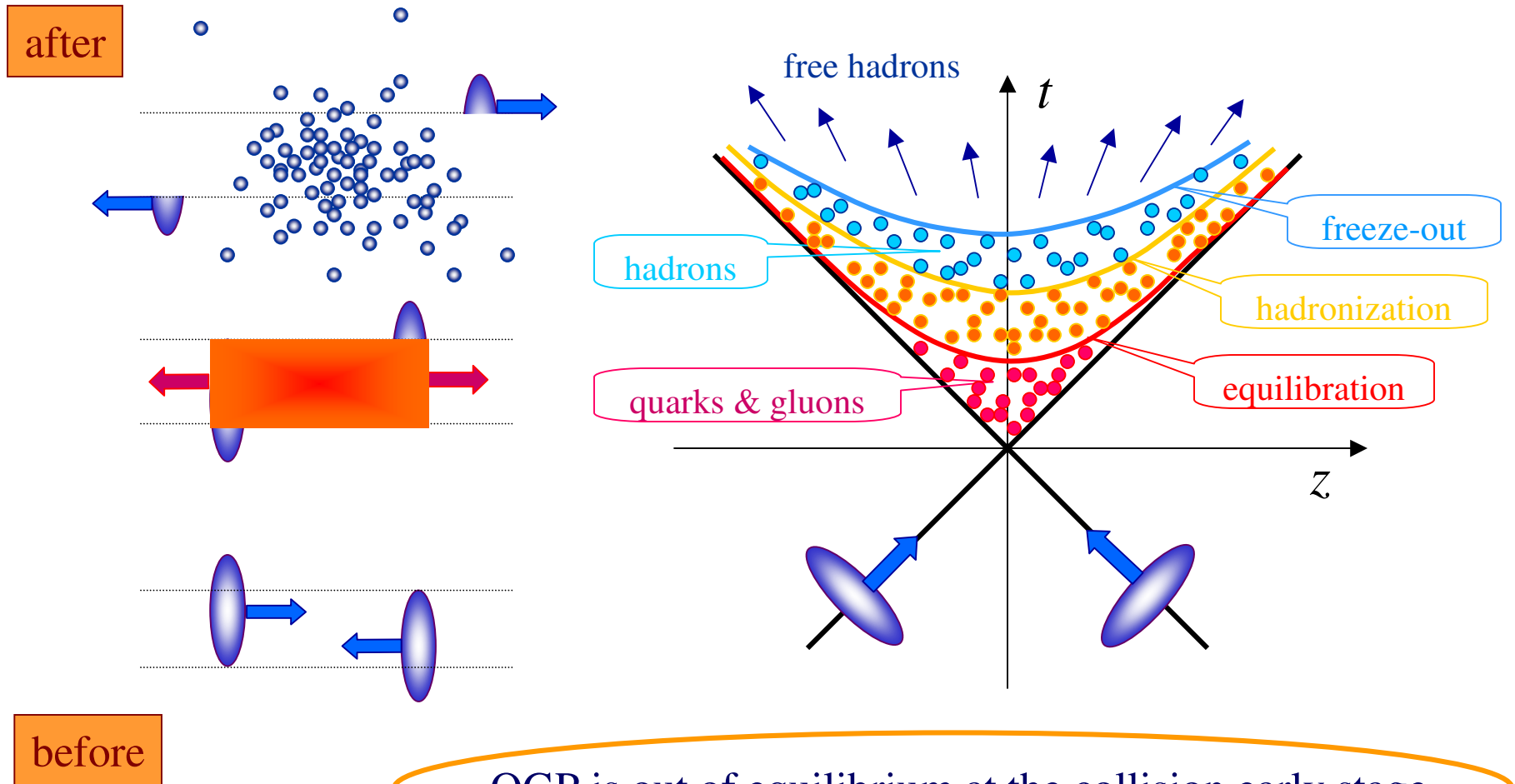
**Margaret Carrington & Katarzyna Deja**

# Motivation - jet quenching



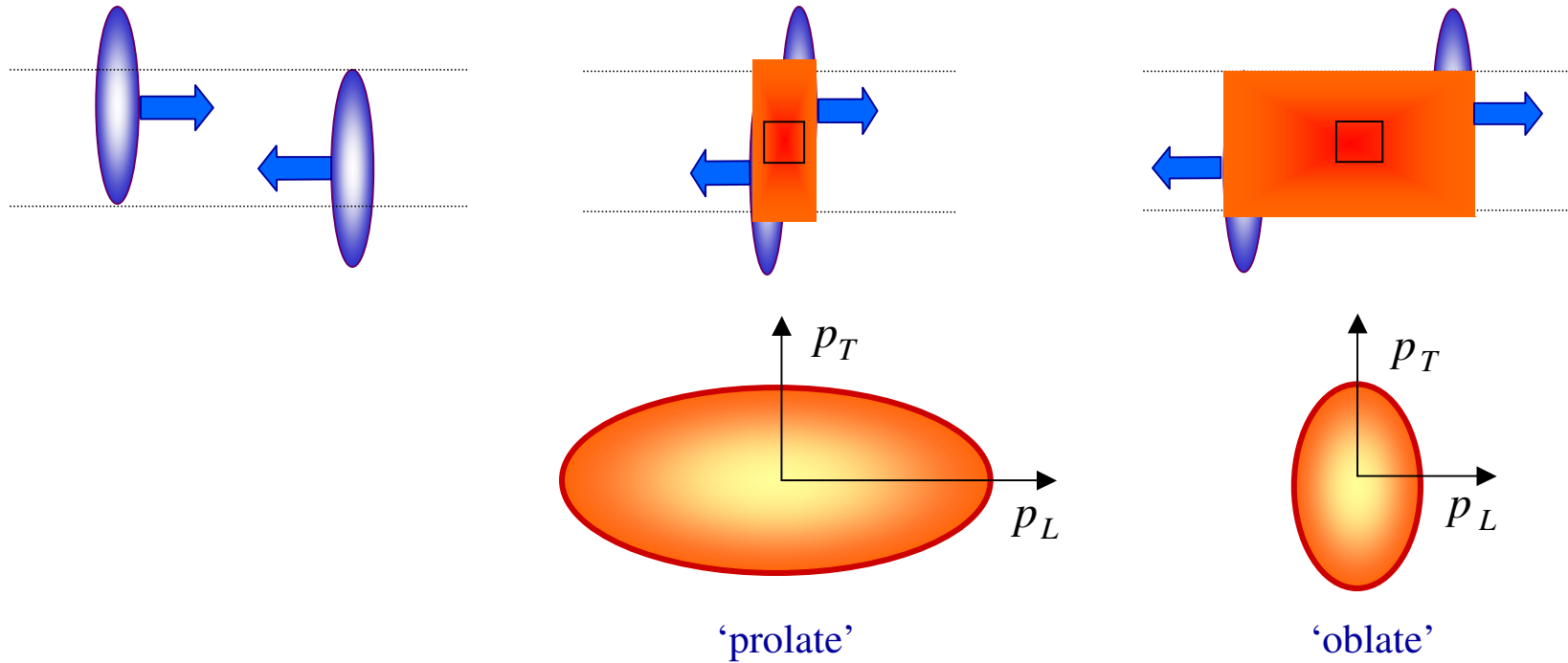
Away-side jet is suppressed  
in central collisions

# Scenario of relativistic heavy-ion collisions



QGP is out of equilibrium at the collision early stage

# Anisotropic QGP



Anisotropic QGP is unstable due to magnetic plasma modes

## Energy loss in unstable QGP

► Is  $dE/dx$  in unstable QGP sizeable?

Yes!

► How to compute  $dE/dx$  in unstable QGP?

Solve initial value problem!

# A test parton in QGP

Wong's equation of motion (Hard Loop Approximation)

$$\left\{ \begin{array}{l} \frac{dx^\mu(\tau)}{d\tau} = u^\mu(\tau) \\ \frac{dp^\mu(\tau)}{d\tau} = gQ_a(\tau) F_a^{\mu\nu}(x(\tau)) u_\nu(\tau) \\ \frac{dQ_a(\tau)}{d\tau} = -gf^{abc} p_\mu(\tau) A_b^\mu(x(\tau)) Q_c(\tau) \end{array} \right.$$

## Simplifications

Gauge condition:  $p_\mu(\tau) A_b^\mu(x(\tau)) = 0 \Rightarrow Q_a(\tau) = \text{const}$

Parton travels with constant velocity:  $u^\mu = (\gamma, \gamma \mathbf{v}) = \text{const}$

## Parton's energy loss

$$\frac{dE(t)}{dt} = gQ_a \mathbf{E}_a(t, \mathbf{r}(t)) \cdot \mathbf{v}$$

induced & spontaneously  
generated chromoelectric field

parton's current:  $\mathbf{j}_a(t, \mathbf{r}) = gQ_a \mathbf{v} \delta^{(3)}(\mathbf{r} - \mathbf{v}t)$

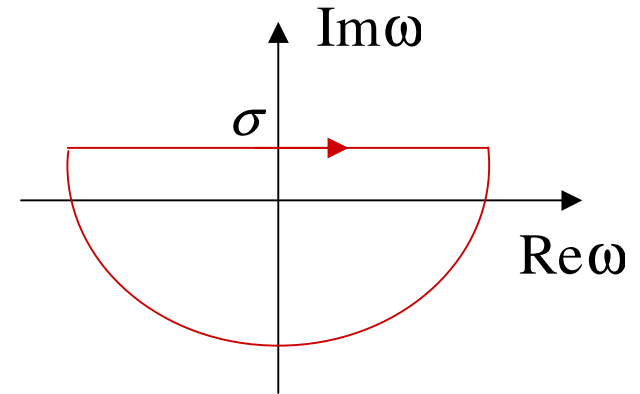
$$\frac{dE(t)}{dt} = \int d^3r \mathbf{E}_a(t, \mathbf{r}) \cdot \mathbf{j}_a(t, \mathbf{r})$$

# Initial value problem

## One-sided Fourier transformation

$$\left\{ \begin{aligned} f(\omega, \mathbf{k}) &= \int_0^{\infty} dt \int d^3 r e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} f(t, \mathbf{r}) \\ f(t, \mathbf{r}) &= \int_{-\infty + i\sigma}^{\infty + i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3 k}{(2\pi)^3} e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})} f(\omega, \mathbf{k}) \end{aligned} \right.$$

$$0 < \sigma \in \mathbb{R}$$



$$\mathbf{j}_a(t, \mathbf{r}) = gQ_a \mathbf{v} \delta^{(3)}(\mathbf{r} - \mathbf{v}t) \quad \Rightarrow \quad \mathbf{j}_a(\omega, \mathbf{k}) = \frac{igQ_a \mathbf{v}}{\omega - \mathbf{k} \cdot \mathbf{v}}$$

$$\frac{dE(t)}{dt} = gQ_a \int_{-\infty + i\sigma}^{\infty + i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3 k}{(2\pi)^3} e^{-i(\omega - \mathbf{k} \cdot \mathbf{v})t} \mathbf{E}_a(\omega, \mathbf{k}) \cdot \mathbf{v}$$



## Induced Electric Field

Linearized Yang-Mills (Maxwell) equations (Hard Loop Approximation)

$$\begin{aligned}
 i\mathbf{k} \cdot \mathbf{D}(\omega, \mathbf{k}) &= \rho(\omega, \mathbf{k}), & i\mathbf{k} \cdot \mathbf{B}(\omega, \mathbf{k}) &= 0, \\
 i\mathbf{k} \times \mathbf{E}(\omega, \mathbf{k}) &= i\omega\mathbf{B}(\omega, \mathbf{k}) + \mathbf{B}_0(\mathbf{k}), \\
 i\mathbf{k} \times \mathbf{B}(\omega, \mathbf{k}) &= \mathbf{j}(\omega, \mathbf{k}) - i\omega\mathbf{E}(\omega, \mathbf{k}) - \mathbf{D}_0(\mathbf{k})
 \end{aligned}$$

$$D^i(\omega, \mathbf{k}) = \varepsilon^{ij}(\omega, \mathbf{k}) E^j(\omega, \mathbf{k})$$

Chromodielectric tensor

$$\varepsilon^{ij}(\omega, \mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega} \int \frac{d^3 p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{k}\mathbf{v} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^l} \left[ \left(1 - \frac{\mathbf{k}\mathbf{v}}{\omega}\right) \delta^{lj} + \frac{k^l v^j}{\omega} \right] \quad \text{dynamical information}$$

$$E^i(\omega, \mathbf{k}) = -i(\Sigma^{-1})^{ij}(\omega, \mathbf{k}) [\omega\mathbf{j}(\omega, \mathbf{k}) + \mathbf{k} \times \mathbf{B}_0(\mathbf{k}) - \omega\mathbf{D}_0(\mathbf{k})]^j$$

$$\Sigma^{ij}(\omega, \mathbf{k}) \equiv -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega, \mathbf{k})$$

## Energy-Loss formula

$$\frac{dE(t)}{dt} = gQ_a v^i \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\bar{\omega})t}$$

$$\times (\Sigma^{-1})^{ij}(\omega, \mathbf{k}) \left[ \frac{igQ_a \omega \mathbf{v}}{\omega - \bar{\omega}} + \mathbf{k} \times \mathbf{B}_0(\mathbf{k}) - \omega \mathbf{D}_0(\mathbf{k}) \right]^j$$

$$\bar{\omega} \equiv \mathbf{k} \cdot \mathbf{v}$$

$$\Sigma^{ij}(\omega, \mathbf{k}) \equiv -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega, \mathbf{k})$$

**Dispersion equation**

$$\det[\Sigma(\omega, \mathbf{k})] = 0$$

## Initial values of the fields

$$\text{Maxwell equations \& } \mathbf{j}_a(t, \mathbf{r}) = gQ_a \mathbf{v} \delta^{(3)}(\mathbf{r} - \mathbf{v}t)$$



**Initial values:**

$$D_0^i(\mathbf{k}) = -igQ_a \bar{\omega} \varepsilon^{ij}(\bar{\omega}, \mathbf{k}) (\Sigma^{-1})^{jk}(\bar{\omega}, \mathbf{k}) v^k$$

$$B_0^i(\mathbf{k}) = -igQ_a \varepsilon^{ijk} k^j (\Sigma^{-1})^{kl}(\bar{\omega}, \mathbf{k}) v^l$$

When the test paron enters the plasma at  $t = 0$ , the instabilities are initiated.

## Energy-Loss formula

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3 k}{(2\pi)^3} e^{-i(\omega-\bar{\omega})t} (\Sigma^{-1})^{ij}(\omega, \mathbf{k})$$

$$\times \left[ \underbrace{\frac{\omega \delta^{jl}}{\omega - \bar{\omega}}}_{\mathbf{j}(\omega, \mathbf{k})} - \underbrace{(k^j k^k - \mathbf{k}^2 \delta^{jk}) (\Sigma^{-1})^{kl}(\bar{\omega}, \mathbf{k})}_{\mathbf{B}_0(\mathbf{k})} + \underbrace{\omega \bar{\omega} \varepsilon^{jk}(\bar{\omega}, \mathbf{k}) (\Sigma^{-1})^{kl}(\bar{\omega}, \mathbf{k})}_{\mathbf{D}_0(\mathbf{k})} \right]$$

$\mathbf{j}(\omega, \mathbf{k})$

$\mathbf{B}_0(\mathbf{k})$

$\mathbf{D}_0(\mathbf{k})$

Averaging over parton's colors:  $\int dQ Q_a Q_b = C_2 \delta^{ab}$ ,  $C_2 \equiv \begin{cases} \frac{1}{2} & \text{for quark} \\ N_c & \text{for gluon} \end{cases}$

$$C_R = \begin{cases} C_2 \frac{N_c^2 - 1}{N_c} = \frac{N_c^2 - 1}{2N_c} & \text{for quark (R = F)} \\ C_2 = N_c & \text{for gluon (R = G)} \end{cases}$$

## Stable isotropic plasma

$$\triangleright \quad \varepsilon^{ij}(\omega, \mathbf{k}) = \varepsilon_L(\omega, \mathbf{k}) \frac{k^i k^j}{\mathbf{k}^2} + \varepsilon_T(\omega, \mathbf{k}) \left( \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right)$$

$$\triangleright \quad (\Sigma^{-1})^{ij}(\omega, \mathbf{k}) = \frac{1}{\omega^2 \varepsilon_L(\omega, \mathbf{k})} \frac{k^i k^j}{\mathbf{k}^2} + \frac{1}{\omega^2 \varepsilon_T(\omega, \mathbf{k}) - \mathbf{k}^2} \left( \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right)$$

$$\begin{aligned} \frac{dE(t)}{dt} = & ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3 k}{(2\pi)^3} e^{-i(\omega-\bar{\omega})t} (\Sigma^{-1})^{ij}(\omega, \mathbf{k}) \\ & \times \left[ \frac{\omega \delta^{jl}}{\omega - \bar{\omega}} - (k^j k^k - \mathbf{k}^2 \delta^{jk})(\Sigma^{-1})^{kl}(\bar{\omega}, \mathbf{k}) + \omega \bar{\omega} \varepsilon^{jk}(\bar{\omega}, \mathbf{k})(\Sigma^{-1})^{kl}(\bar{\omega}, \mathbf{k}) \right] \end{aligned}$$

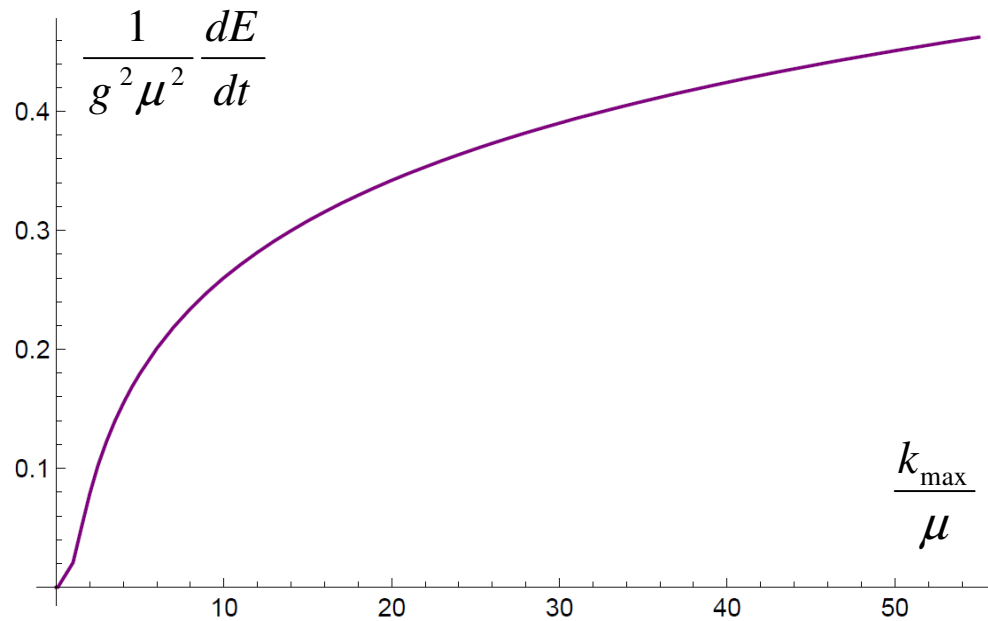
The only stationary contribution:  $\omega = \bar{\omega} \equiv \mathbf{k} \cdot \mathbf{v}$

$$\frac{dE(t)}{dt} = ig^2 C_R \int \frac{d^3 k}{(2\pi)^3} \frac{\bar{\omega}}{\mathbf{k}^2} \left[ \frac{1}{\varepsilon_L(\bar{\omega}, \mathbf{k})} + \frac{\mathbf{k}^2 \mathbf{v}^2 - \bar{\omega}^2}{\bar{\omega}^2 \varepsilon_T(\bar{\omega}, \mathbf{k}) - \mathbf{k}^2} \right]$$

equivalent to the standard result by Braaten & Thoma 13

# Energy loss in equilibrium QGP

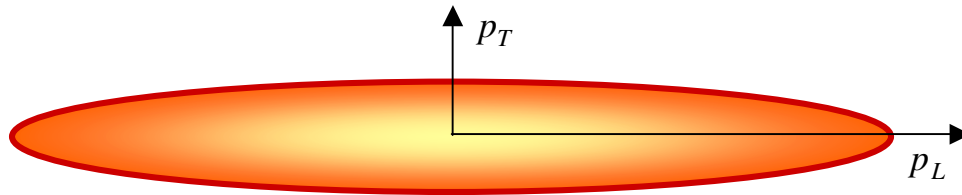
$$\frac{dE(t)}{dt} = ig^2 C_R \int \frac{d^3 k}{(2\pi)^3} \frac{\bar{\omega}}{\mathbf{k}^2} \left[ \frac{1}{\varepsilon_L(\bar{\omega}, \mathbf{k})} + \frac{\mathbf{k}^2 \mathbf{v}^2 - \bar{\omega}^2}{\bar{\omega}^2 \varepsilon_T(\bar{\omega}, \mathbf{k}) - \mathbf{k}^2} \right]$$



Debye mass

$$\mu^2 \equiv g^2 \int \frac{d^3 p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

# Extremely prolate system



$$f(\mathbf{p}) \sim \delta(p_x) \delta(p_y)$$

## Collective modes

$$\det[\Sigma^{ij}(\omega, \mathbf{k})] = 0$$

$$\Sigma^{ij}(\omega, \mathbf{k}) \equiv -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega, \mathbf{k})$$

$$\varepsilon^{ij}(\omega, \mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega} \int \frac{d^3 p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{k}\mathbf{v} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^j} \left[ \left(1 - \frac{\mathbf{k}\mathbf{v}}{\omega}\right) \delta^{ij} + \frac{k^i v^j}{\omega} \right]$$

## Spectrum of collective modes

$$\left\{ \begin{array}{l} \omega_1(\mathbf{k}) = \mu^2 + \mathbf{k}^2 \\ \omega_2(\mathbf{k}) = \mu^2 + (\mathbf{k} \cdot \mathbf{n})^2 \\ \omega_{\pm}(\mathbf{k}) = \frac{1}{2} \left( \mathbf{k}^2 + (\mathbf{k} \cdot \mathbf{n})^2 \pm \sqrt{\mathbf{k}^4 + (\mathbf{k} \cdot \mathbf{n})^4 + 4\mu^2 \mathbf{k}^2 - 4\mu^2 (\mathbf{k} \cdot \mathbf{n})^2 - 2\mathbf{k}^2 (\mathbf{k} \cdot \mathbf{n})^2} \right) \end{array} \right. \quad \mathbf{n} \equiv (0,0,1)$$

# Unstable chromomagnetic mode

stationary state

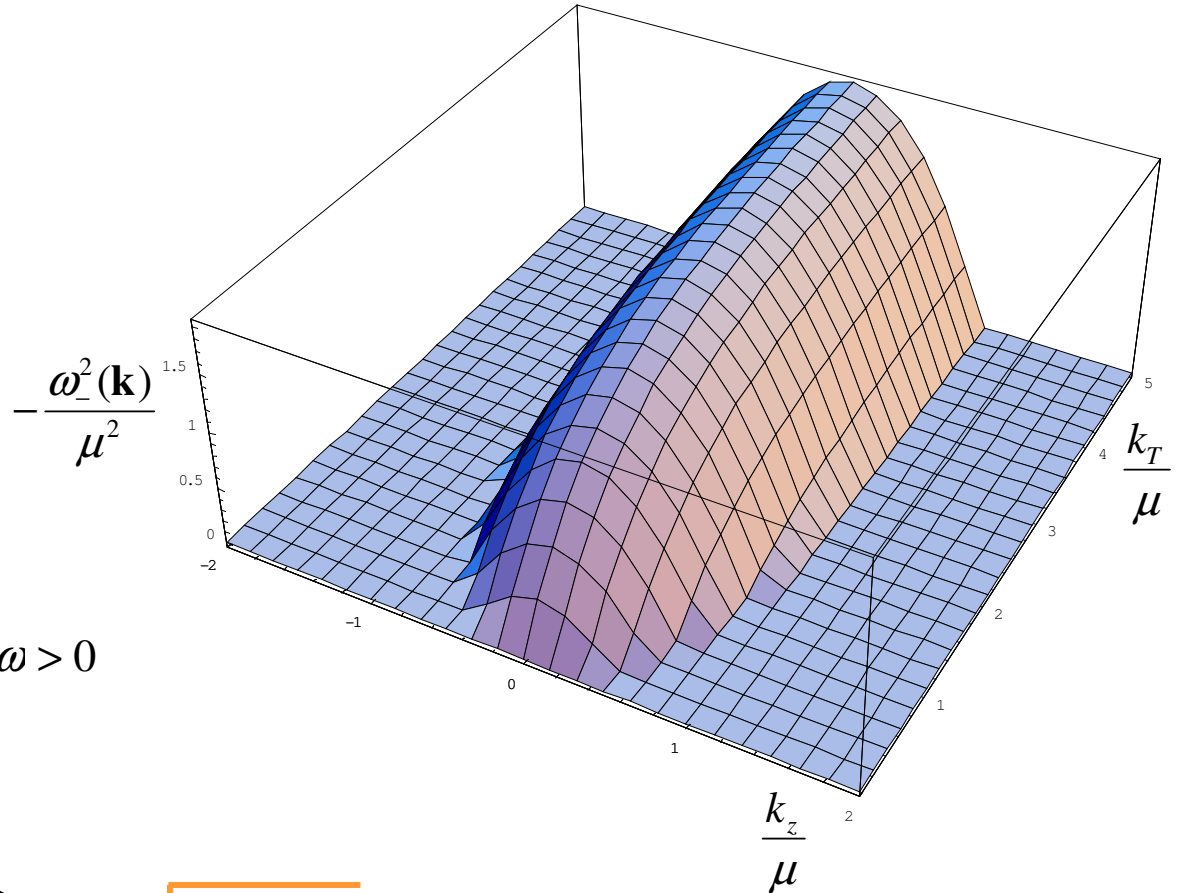
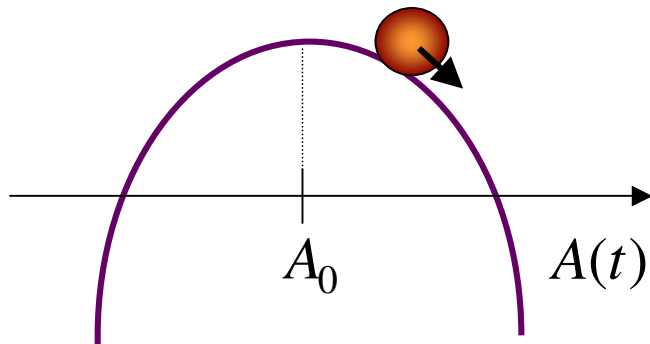
$$A(t) = A_0 + \delta A(t)$$

fluctuation

**Instability**

$$\delta A(t) \propto e^{\text{Im } \omega t}$$

$$\text{Im } \omega > 0$$



$$\frac{dE(t)}{dt} \sim \int \frac{d^3k}{(2\pi)^3} e^{\text{Im } \omega t} \dots$$



## Energy loss in extremely prolate system

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\bar{\omega})t} (\Sigma^{-1})^{ij}(\omega, \mathbf{k})$$

$$\times \left[ \frac{\omega \delta^{jl}}{\omega - \bar{\omega}} - (k^j k^k - \mathbf{k}^2 \delta^{jk})(\Sigma^{-1})^{kl}(\bar{\omega}, \mathbf{k}) + \omega \bar{\omega} \varepsilon^{jk}(\bar{\omega}, \mathbf{k})(\Sigma^{-1})^{kl}(\bar{\omega}, \mathbf{k}) \right]$$

► Inversion of matrix  $\Sigma$  depending on  $\mathbf{k}$  and  $\mathbf{n}$

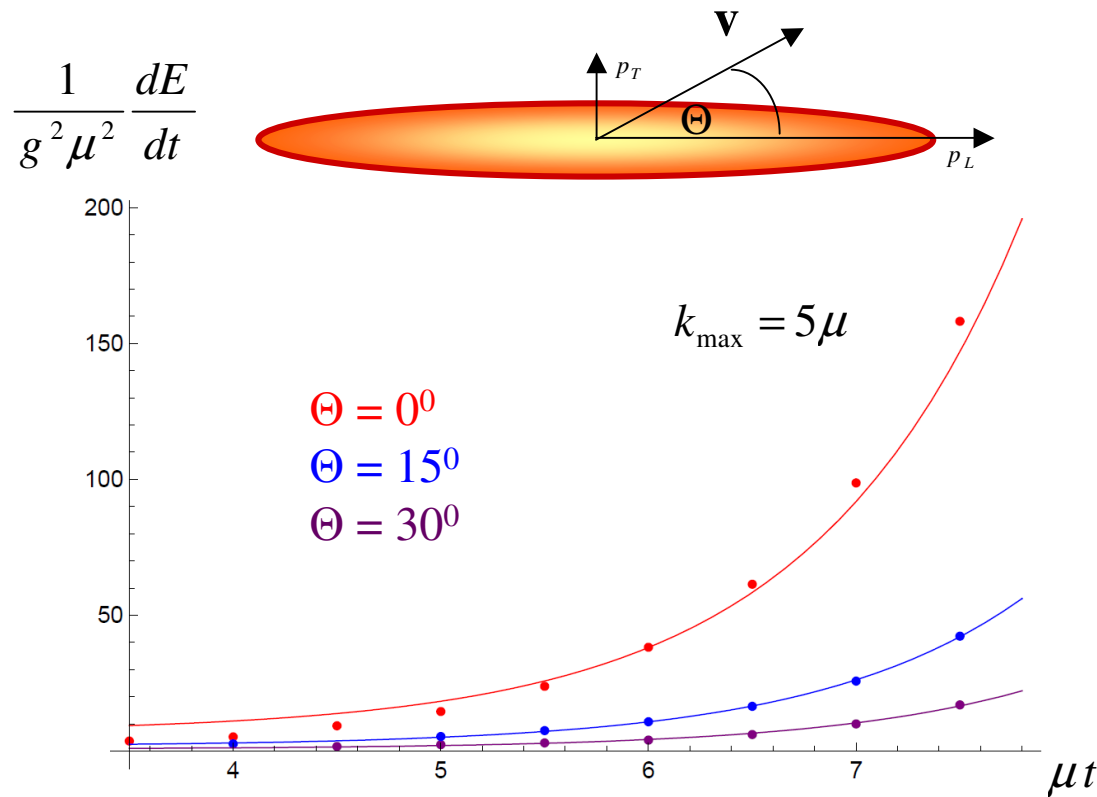
$$\Sigma = aA + bB + cC + dD$$

basis of matrices

$$\left\{ \begin{array}{l} A^{ij} = \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}, \quad B^{ij} = \frac{k^i k^j}{\mathbf{k}^2}, \\ C^{ij} = \frac{n_T^i n_T^j}{\mathbf{n}_T^2}, \quad D^{ij} = n_T^i k^j + k^i n_T^j \end{array} \right. \quad n_T^i \equiv \left( \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) n^j$$

$$\Sigma^{-1} = \alpha A + \beta B + \gamma C + \delta D$$

# Energy loss in extremely prolate system cont.



Debye mass

$$\mu^2 \equiv g^2 \int \frac{d^3 p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

Equilibrium value

$$\frac{1}{g^2 \mu^2} \frac{dE}{dt} \approx 0.2$$

## Conclusions

- ▶ Energy loss is found as a solution of initial value problem
- ▶ Extremely prolate QGP is discussed as an example
- ▶ Strong time and directional dependence of  $dE/dx$  is demonstrated
- ▶  $dE/dx$  in unstable QGP is much bigger than in equilibrium one

for more see:

M. Carrington, K. Deja and St. Mrówczyński,  
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and poster presented by Katarzyna Deja