Energy Loss in Unstable Quark-Gluon Plasma

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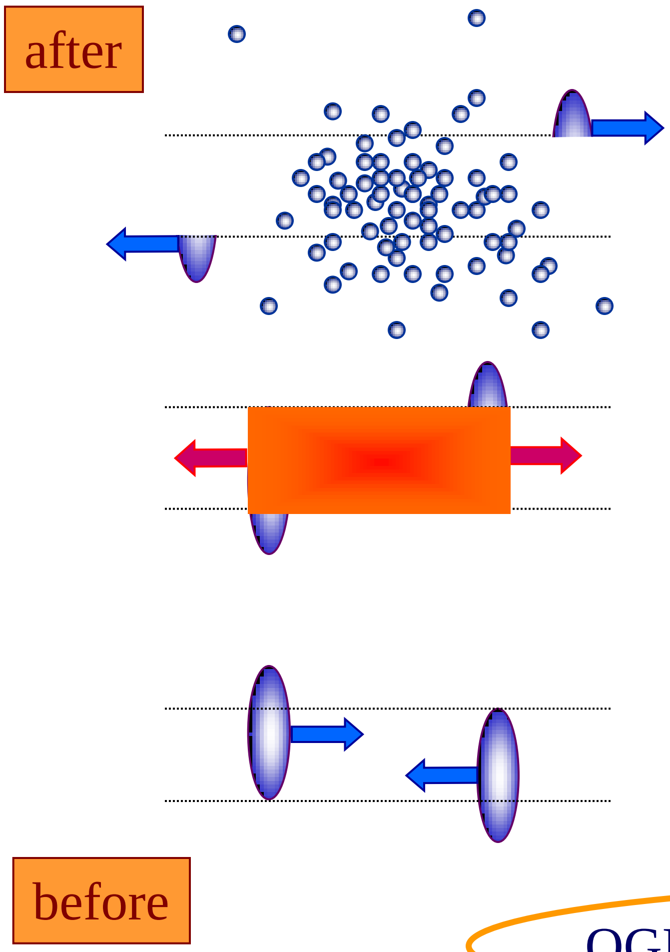
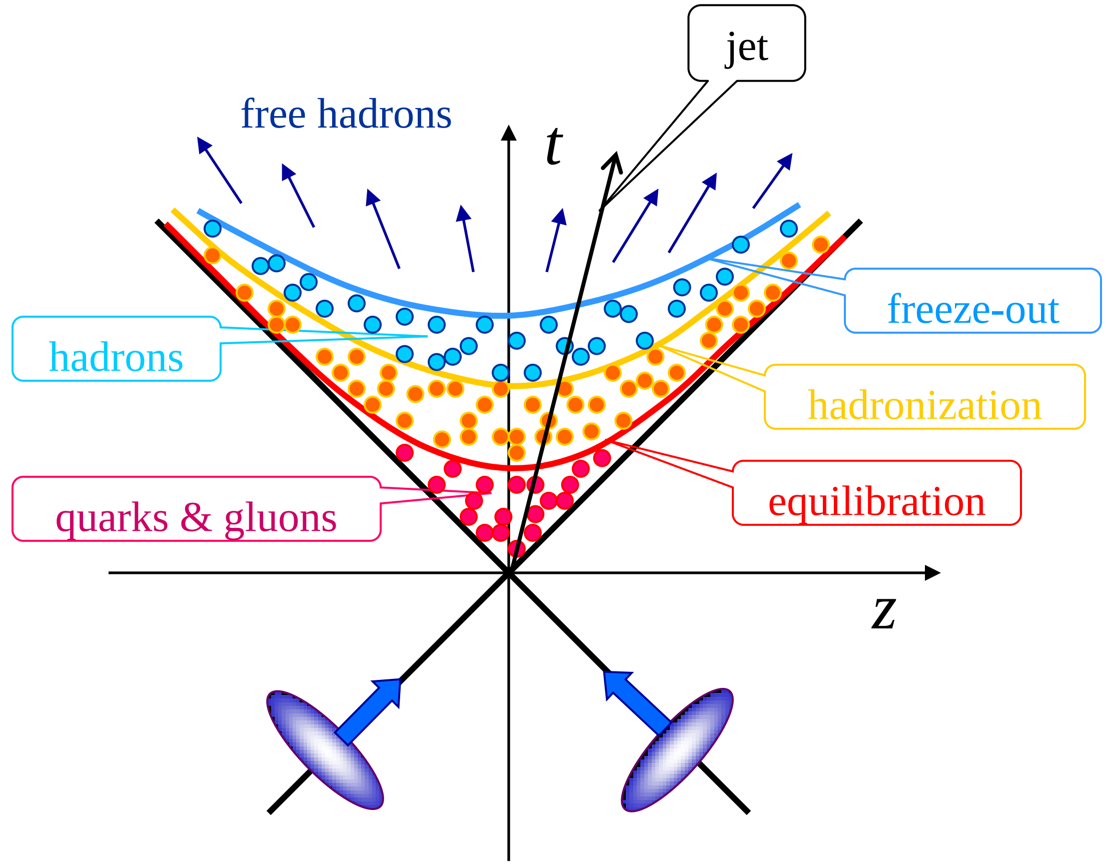
based on arXiv:1506.09082

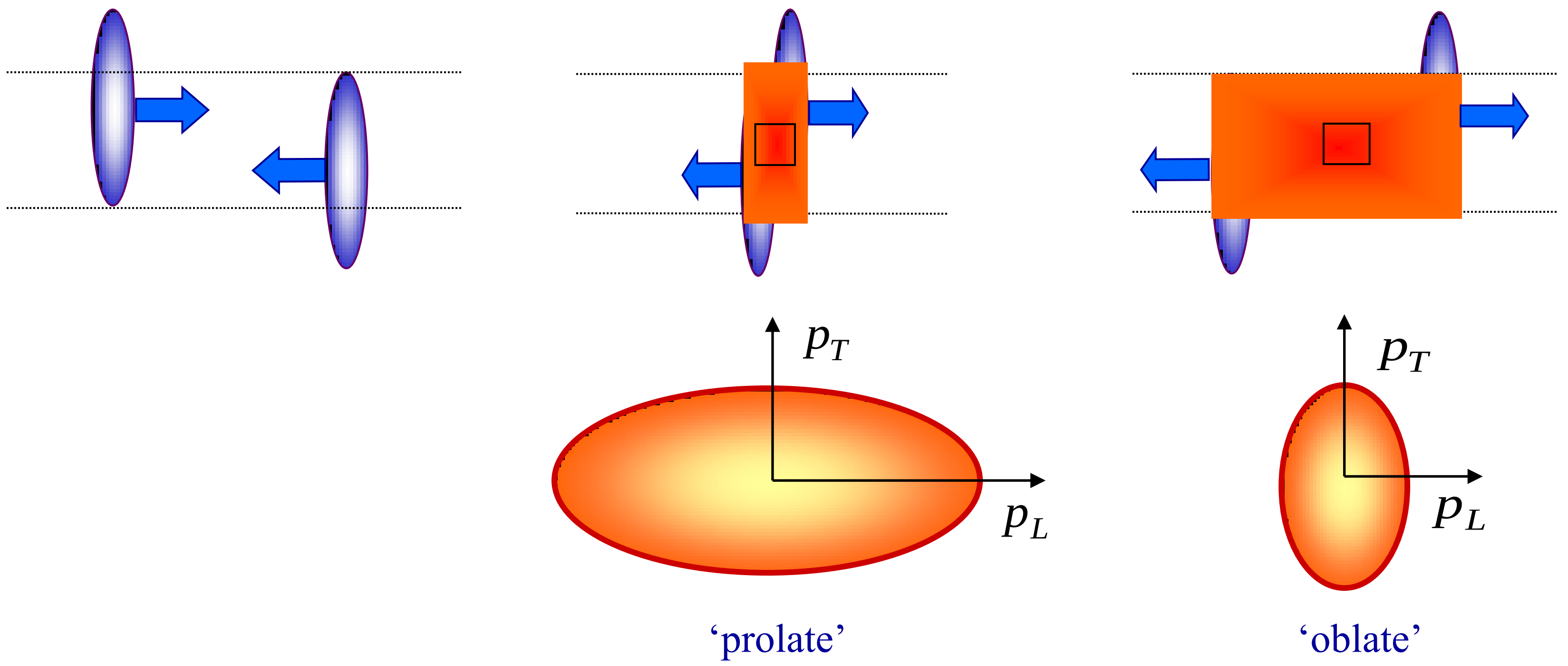
Scenario of relativistic heavy-ion collisions





QGP is out of equilibrium at the collision early stage

Anisotropic QGP



Anisotropic QGP is unstable due to magnetic plasma modes

Questions

What happens to a high-energy parton when it is traversing an unstable QGP?

- Does the parton loose or gains an energy?
- What is a magnitude of energy transfer?

A test parton in QGP

Wong's equation of motion (Hard Loop Approximation)

$$\begin{cases} \dfrac{dx^\mu(\tau)}{d\tau} = u^\mu(\tau) \\ \dfrac{dp^\mu(\tau)}{d\tau} = gQ_a(\tau)\, F_a^{\mu\nu}(x(\tau))\, u_\nu(\tau) \\ \dfrac{dQ_a(\tau)}{d\tau} = -gf^{abc} p_\mu(\tau)\, A_b^\mu(x(\tau))\, Q_c(\tau) \end{cases}$$

Simplifications

Gauge condition: $p_\mu(\tau)\, A_b^\mu\big(x(\tau)\big) = 0 \quad \Rightarrow \quad Q_a(\tau) = \text{const}$

Parton travels with constant velocity: $u^\mu = (\gamma, \gamma \mathbf{v}) = \text{const}$

Evolution of parton's energy

$$\frac{dE(t)}{dt} = gQ_a \, \mathbf{E}_a(t, \mathbf{r}(t)) \cdot \mathbf{v}$$

induced & spontaneously generated chromoelectric field

parton's current: $\mathbf{j}_a(t, \mathbf{r}) = gQ_a \mathbf{v} \delta^{(3)}(\mathbf{r} - \mathbf{v}t)$

$$\frac{dE(t)}{dt} = \int d^3r \, \mathbf{E}_a(t, \mathbf{r}) \cdot \mathbf{j}_a(t, \mathbf{r})$$

Analog of collisional energy loss

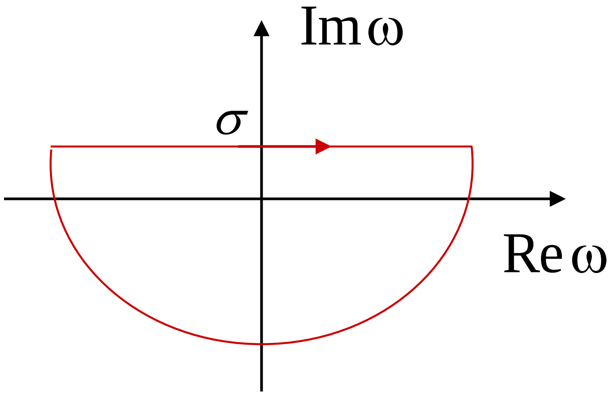
Initial value problem

One-sided Fourier transformation

$$f(\omega,\mathbf{k}) = \int_0^\infty dt \int d^3r\, e^{i(\omega t - \mathbf{kr})} f(t,\mathbf{r})$$

$$f(t,\mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega t - \mathbf{kr})} f(\omega,\mathbf{k})$$

$$0 < \sigma \in R$$



$$\mathbf{j}_a(t,\mathbf{r}) = gQ_a \mathbf{v}\delta^{(3)}(\mathbf{r} - \mathbf{v}t) \quad \Rightarrow \quad \mathbf{j}_a(\omega,\mathbf{k}) = \frac{igQ_a\mathbf{v}}{\omega - \mathbf{k}\cdot\mathbf{v}}$$

$$\frac{dE(t)}{dt} = gQ_a \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega - \mathbf{kv})t} \mathbf{E}_a(\omega,\mathbf{k}) \cdot \mathbf{v}$$

Induced Electric Field

Linearized Yang-Mills (Maxwell) equations (Hard Loop Approximation)

$$i\mathbf{k}\cdot\mathbf{D}(\omega,\mathbf{k}) = \rho(\omega,\mathbf{k}), \qquad i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0,$$
$$i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = i\omega\mathbf{B}(\omega,\mathbf{k}) + \mathbf{B}_0(\mathbf{k}),$$
$$i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = \mathbf{j}(\omega,\mathbf{k}) - i\omega\mathbf{E}(\omega,\mathbf{k}) - \mathbf{D}_0(\mathbf{k})$$

$$D^i(\omega,\mathbf{k}) = \varepsilon^{ij}(\omega,\mathbf{k})\,E^j(\omega,\mathbf{k})$$

Chromodielectric tensor

$$\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{kv}+i0^+}\frac{\partial f(\mathbf{p})}{\partial p^l}\left[\left(1-\frac{\mathbf{kv}}{\omega}\right)\delta^{lj} + \frac{k^l v^j}{\omega}\right]$$

dynamical information about medium

$$E^i(\omega,\mathbf{k}) = -i(\Sigma^{-1})^{ij}(\omega,\mathbf{k})\left[\omega\mathbf{j}(\omega,\mathbf{k}) + \mathbf{k}\times\mathbf{B}_0(\mathbf{k}) - \omega\mathbf{D}_0(\mathbf{k})\right]^j$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

Formula of evolution of parton's energy

$$\frac{dE(t)}{dt} = gQ_a v^i \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t}$$
$$\times (\Sigma^{-1})^{ij}(\omega,\mathbf{k}) \left[\frac{igQ_a \omega \mathbf{v}}{\omega - \overline{\omega}} + \mathbf{k} \times \mathbf{B}_0(\mathbf{k}) - \omega \mathbf{D}_0(\mathbf{k}) \right]^j$$

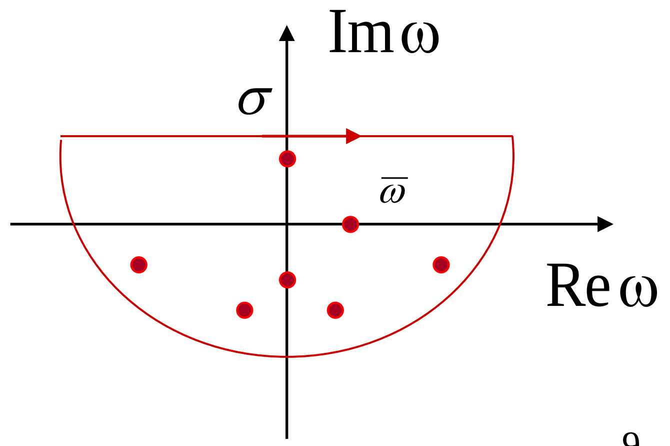
$\overline{\omega} \equiv \mathbf{k} \cdot \mathbf{v}$

initial values of the fields

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega,\mathbf{k})$$

Dispersion equation

$$\det[\Sigma(\omega,\mathbf{k})] = 0$$



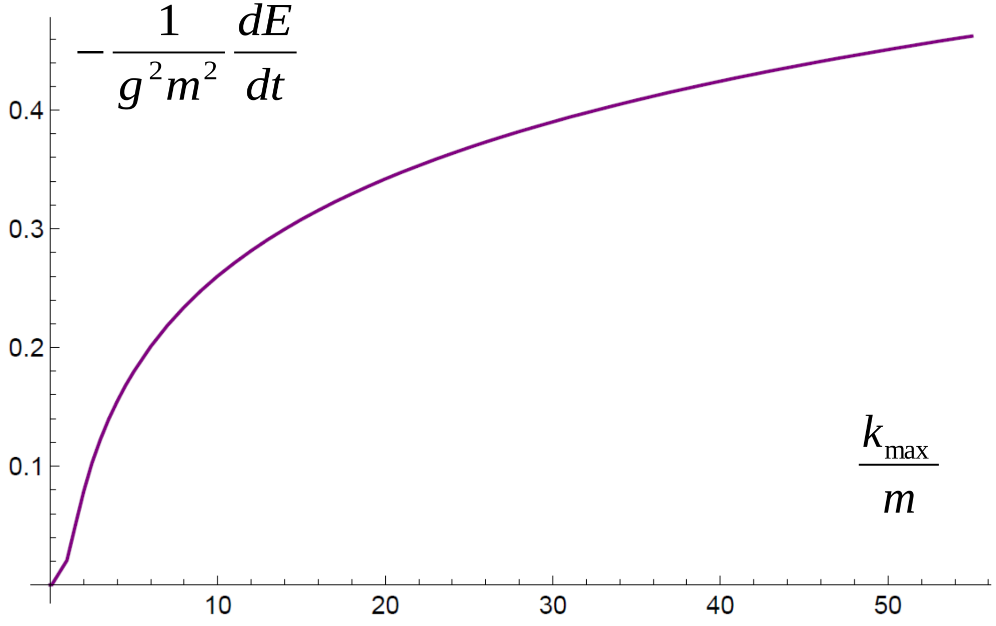
Energy loss in equilibrium QGP

The initial conditions are *forgotten*

$$\frac{dE(t)}{dt} = ig^2 C_R \int \frac{d^3k}{(2\pi)^3} \frac{\overline{\omega}}{\mathbf{k}^2} \left[\frac{1}{\varepsilon_L(\overline{\omega},\mathbf{k})} + \frac{\mathbf{k}^2\mathbf{v}^2 - \overline{\omega}^2}{\overline{\omega}^2 \varepsilon_T(\overline{\omega},\mathbf{k}) - \mathbf{k}^2} \right]$$

$\overline{\omega} \equiv \mathbf{k}\cdot\mathbf{v}$

equivalent to the standard result by Braaten & Thoma



Debye mass

$$m^2 \equiv g^2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

Energy loss in vacuum or self-interaction

vacuum dielectric functions: $\varepsilon_L(\omega,\mathbf{k}) = \varepsilon_T(\omega,\mathbf{k}) = 1$

$$\frac{dE(t)}{dt} = ig^2 C_R \int \frac{d^3k}{(2\pi)^3} \frac{\overline{\omega}}{\mathbf{k}^2} \left[\frac{1}{\varepsilon_L(\overline{\omega},\mathbf{k})} + \frac{\mathbf{k}^2\mathbf{v}^2 - \overline{\omega}^2}{\overline{\omega}^2 \varepsilon_T(\overline{\omega},\mathbf{k}) - \mathbf{k}^2} \right] = 0$$

$\overline{\omega} \equiv \mathbf{k}\cdot\mathbf{v}$

No effect of self-interaction!

How to choose the field initial values?

1) The initial fields vanish: $\mathbf{D}_0(\mathbf{k}) = \mathbf{B}_0(\mathbf{k}) = 0$

2) The initial fields are independent of the parton's current.

1) is equivalent to 2)

The effect of the initial fields cancels out after an averaging over parton's colors.

$$\int dQ\, Q_a = 0, \qquad \int dQ\, Q_a\, Q_b = C_2 \delta^{ab}, \qquad C_2 \equiv \begin{cases} \frac{1}{2} & \text{for quark} \\ N_c & \text{for gluon} \end{cases}$$

Energy loss with *uncorrelated* initial condition

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^j \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t} \frac{\omega}{\omega-\overline{\omega}} (\Sigma^{-1})^{ij}(\omega,\mathbf{k})$$

$$\Sigma^{ij}(\omega,\mathbf{k}) = -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\delta^{ij}$$ in vacuum

$$\mathrm{Si}(z) \equiv \int_0^z \frac{dx \sin x}{x}$$

- $$\left.\frac{dE(t)}{dt}\right|_{\text{vacuum}} = -\frac{g^2 C_R}{4\pi^2 t^2}\left[2\left(\mathrm{Si}(k_{\max}t) - \sin(k_{\max}t)\right) + \left(2k_{\max}t - \mathrm{Si}(2k_{\max}t)\right)\right] \neq 0$$

- $$\frac{dE(t)}{dt} \in R$$

Energy loss is real but includes self-interaction!

How to choose the field initial values?

State of the test parton is, in general, correlated with state of the plasma.

Maximal correlation: the initial fields are induced by the parton's current.

$$\mathbf{j}_a(t,\mathbf{r}) = gQ_a\mathbf{v}\delta^{(3)}(\mathbf{r}-\mathbf{v}t), \quad t\in(-\infty,\infty)$$

Maxwell equations ⇓ Two-side Fourier transformation

Initial values:

$$D_0^i(\mathbf{k}) = -igQ_a\overline{\omega}\varepsilon^{ij}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{jk}(\overline{\omega},\mathbf{k})v^k$$

$$B_0^i(\mathbf{k}) = -igQ_a\varepsilon^{ijk}k^j(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k})v^l$$

Energy loss with *correlated* initial condition

$$\frac{dE(t)}{dt} = ig^2 v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\varpi)t}$$

$$\times (\Sigma^{-1})^{ij}(\omega,\mathbf{k}) \left\{ \frac{\omega\delta^{jl}}{\omega-\varpi} + \right.$$

$$\left. + \cos\varphi \left[\underbrace{(k^j k^k - \mathbf{k}^2)(\Sigma^{-1})^{jk}(\varpi,\mathbf{k})}_{\mathbf{k}\times\mathbf{B}_0(\mathbf{k})} - \underbrace{\omega\varpi\varepsilon^{ij}(\varpi,\mathbf{k})(\Sigma^{-1})^{jk}(\varpi,\mathbf{k})}_{\omega\mathbf{D}_0(\mathbf{k})} \right] \right\}$$

$-1 \le \cos\varphi \le 1$ - arbitrary phase factor

$$\cos\varphi = \begin{cases} +1 & \text{maximal correlation} \\ -1 & \text{maximal anticorrelation} \end{cases}$$

Energy loss with *correlated* initial condition cont.

$$\Sigma^{ij}(\omega,\mathbf{k}) = -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\delta^{ij}$$ in vacuum

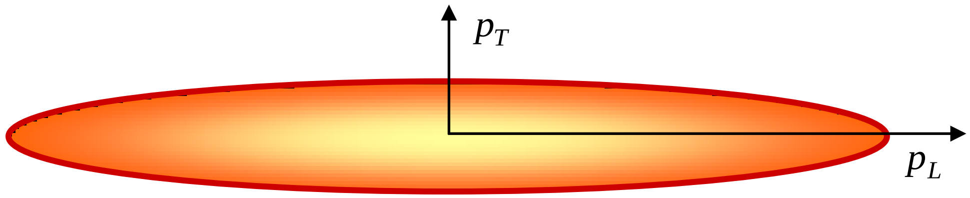
$$\left.\frac{dE(t)}{dt}\right|_{\text{vacuum}} = -\frac{(1-\cos\varphi)g^2 C_R}{4\pi^2 t^2}$$
$$\times\left[2\left(\mathrm{Si}(k_{\max}t) - \sin(k_{\max}t)\right) + \left(2k_{\max}t - \mathrm{Si}(2k_{\max}t)\right)\right] \neq 0$$

$$\frac{dE(t)}{dt} \in R$$

$$\mathrm{Si}(z) \equiv \int_0^z \frac{dx \sin x}{x}$$

Energy loss is real but includes self-interaction!

Extremely prolate QGP



$$f(\mathbf{p}) \sim \delta(p_T)$$

Collective modes

$$\det[\Sigma^{ij}(\omega,\mathbf{k})] = 0$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega,\mathbf{k})$$

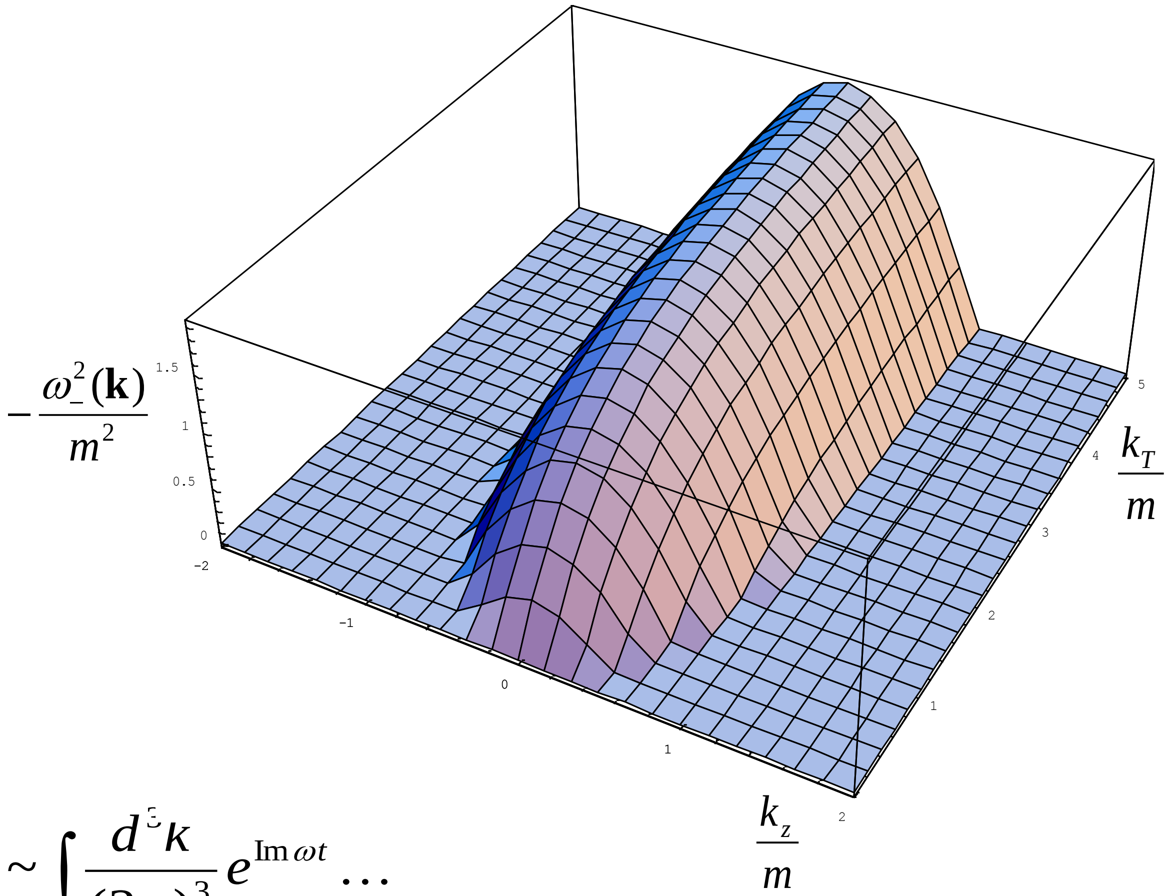
$$\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega}\int \frac{d^3p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{kv} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^l}\left[\left(1 - \frac{\mathbf{kv}}{\omega}\right)\delta^{lj} + \frac{k^l v^j}{\omega}\right]$$

Spectrum of collective modes

$$\begin{cases} \omega_1(\mathbf{k}) = \mu^2 + \mathbf{k}^2 \\ \omega_2(\mathbf{k}) = \mu^2 + (\mathbf{k}\cdot\mathbf{n})^2 \\ \omega_\pm(\mathbf{k}) = \frac{1}{2}\left(\mathbf{k}^2 + (\mathbf{k}\cdot\mathbf{n})^2 \pm \sqrt{\mathbf{k}^4 + (\mathbf{k}\cdot\mathbf{n})^4 + 4\mu^2\mathbf{k}^2 - 4\mu^2(\mathbf{k}\cdot\mathbf{n})^2 - 2\mathbf{k}^2(\mathbf{k}\cdot\mathbf{n})^2}\right) \end{cases}$$

$$\mu^2 \equiv m^2/2 \qquad \mathbf{n} \equiv (0,0,1)$$

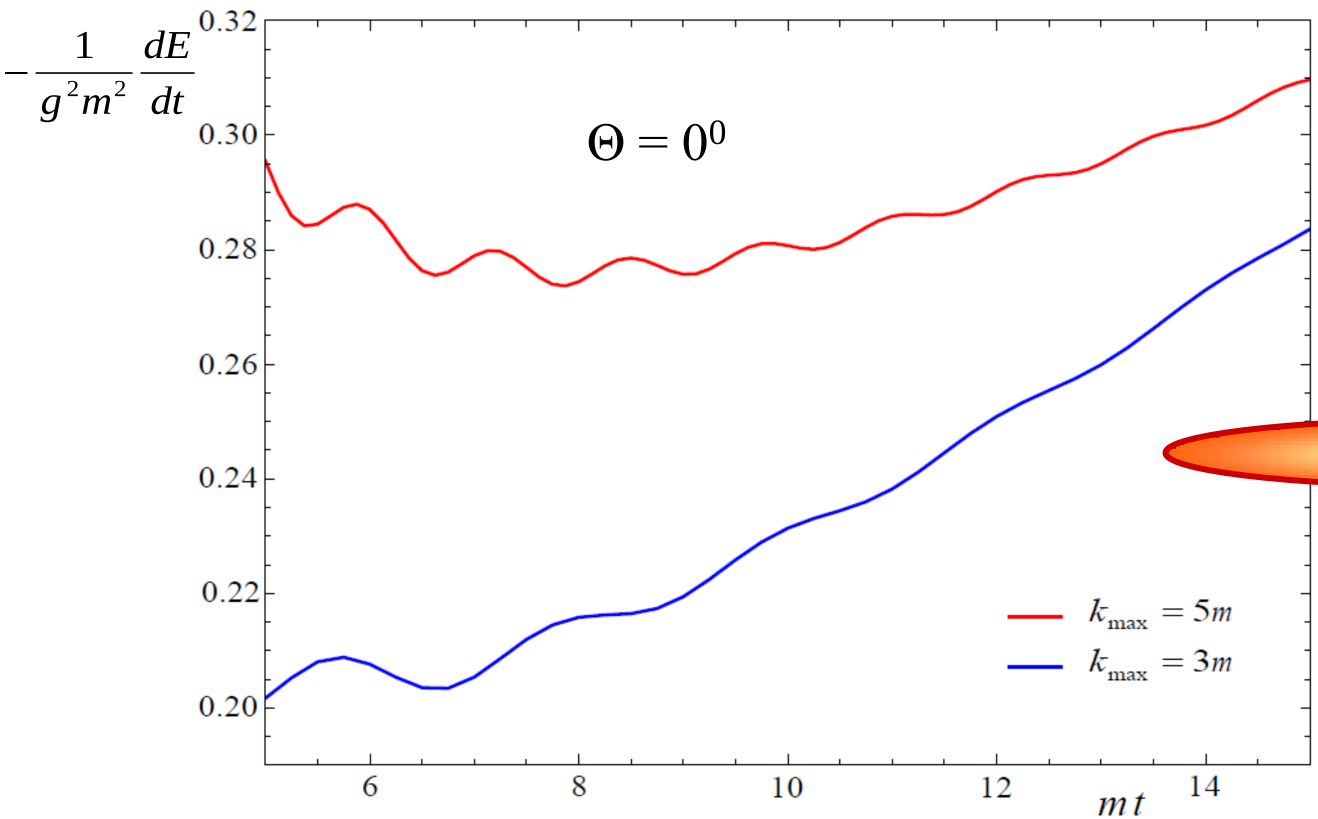
Unstable chromomagnetic mode

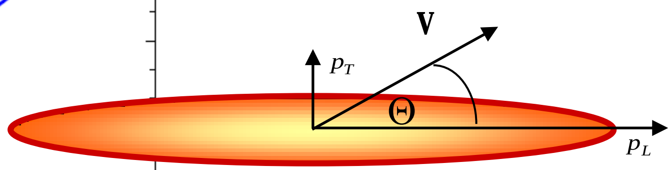


$$\frac{dE(t)}{dt} \sim \int \frac{d^3k}{(2\pi)^3} e^{\mathrm{Im}\,\omega t} \ldots$$

Energy loss in extremely prolate QGP

Uncorrelated initial condition

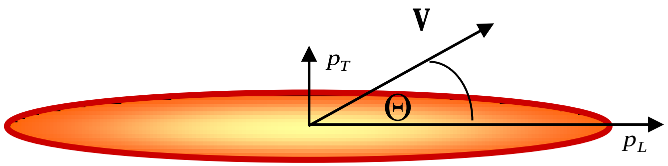


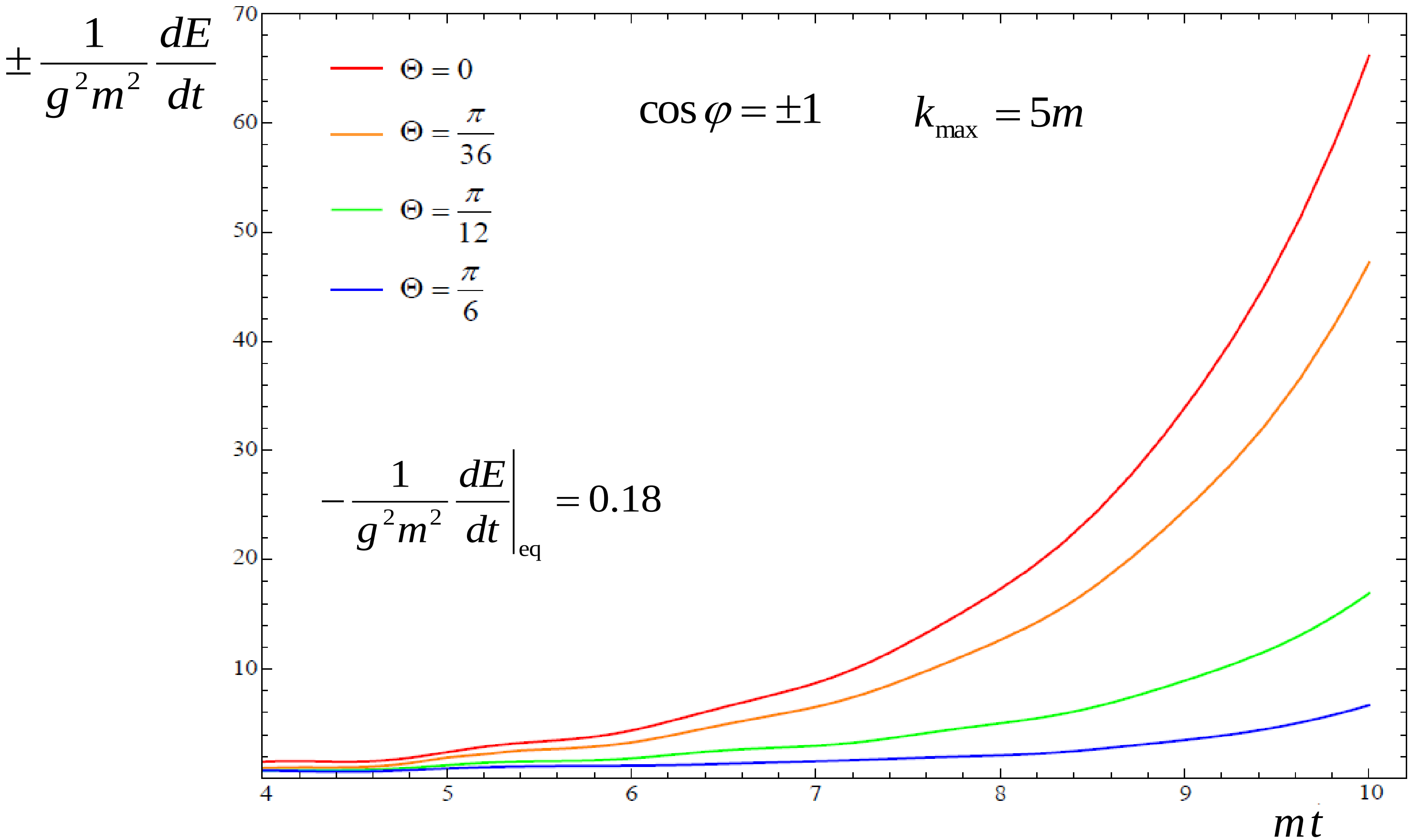


$$-\frac{1}{g^2 m^2}\frac{dE}{dt}\bigg|_{\mathrm{eq}} = \begin{cases} 0.12, & k_{\max} = 3m \\ 0.18, & k_{\max} = 5m \end{cases}$$

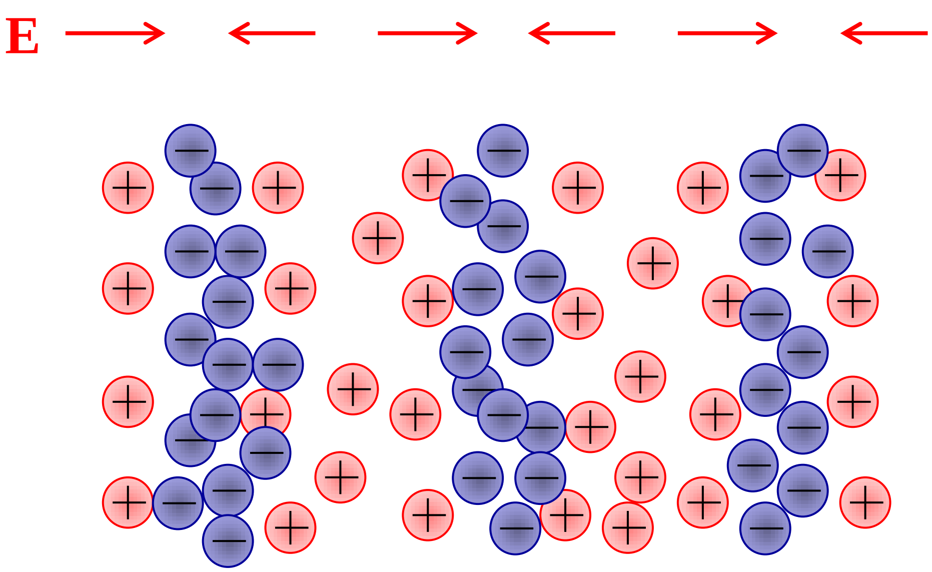
Energy loss in extremely prolate QGP cont.

Correlated initial condition: energy gain for $\cos\varphi < 0$; energy loss for $\cos\varphi > 0$





Plasma accelerator



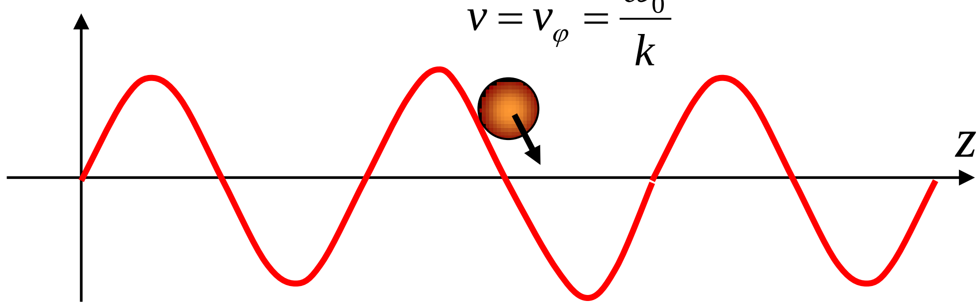
T. Tajima & J. M. Dawson,
Phys. Rev. Lett. **43**, 267 (1979)

$$E_e = 1\,\mathrm{GeV}\ @\ 3.3\,\mathrm{cm}$$

W. P. Leemans *et al.*,
Nature Phys. **2**, 696 (2006).

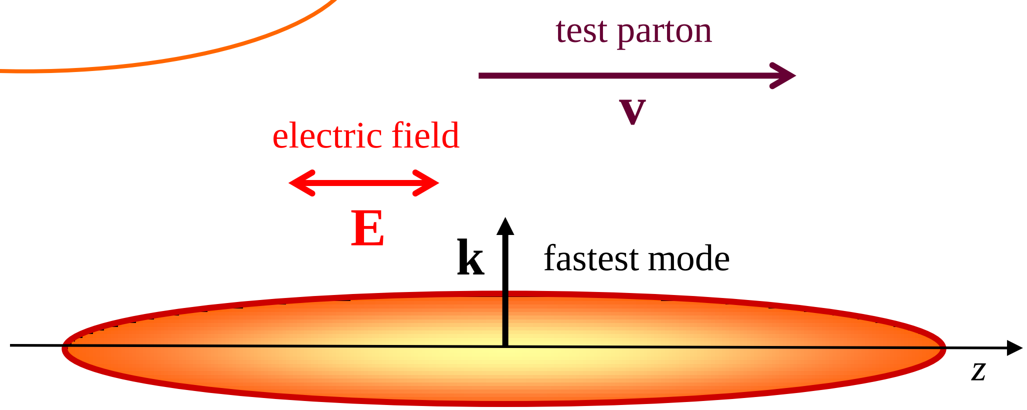
$$E^z(t,x) = E_\mathbf{0}\cos(\omega_0 t - kz)$$

$$v = v_\varphi = \frac{\omega_0}{k}$$



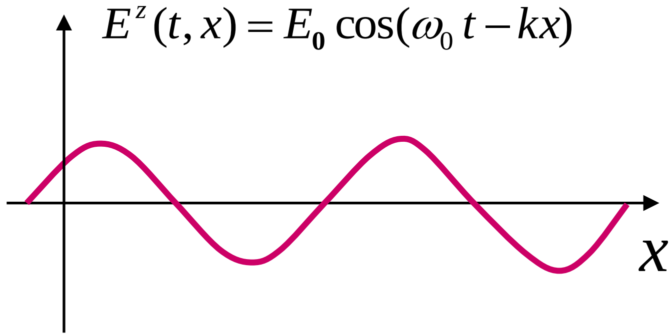
Angular dependence

$$\frac{dE}{dt} \sim \mathbf{E} \cdot \mathbf{j} \sim \mathbf{E} \cdot \mathbf{v}$$

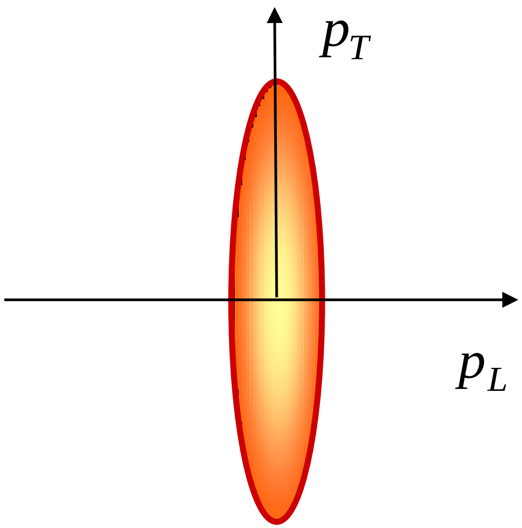


The largest *dE/dt* for **v** along axis *z* !

$$E^z(t,x) = E_0 \cos(\omega_0 t - kx)$$



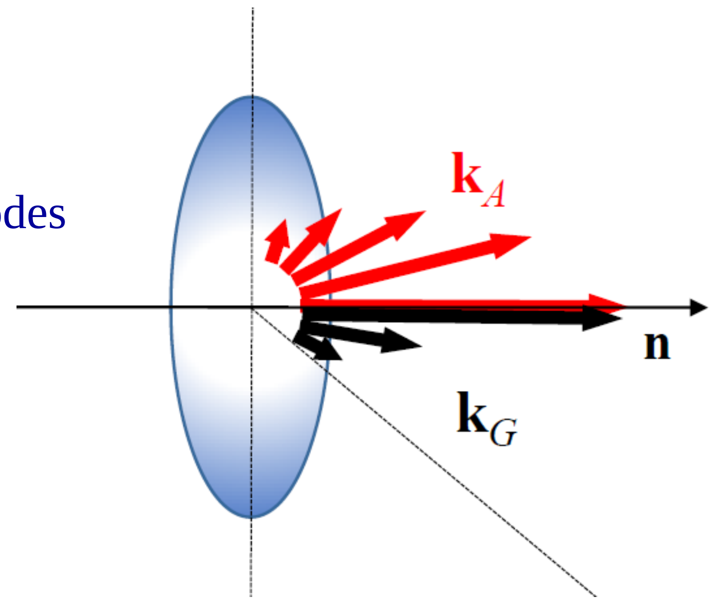
Extremely oblate QGP



Extremely oblate

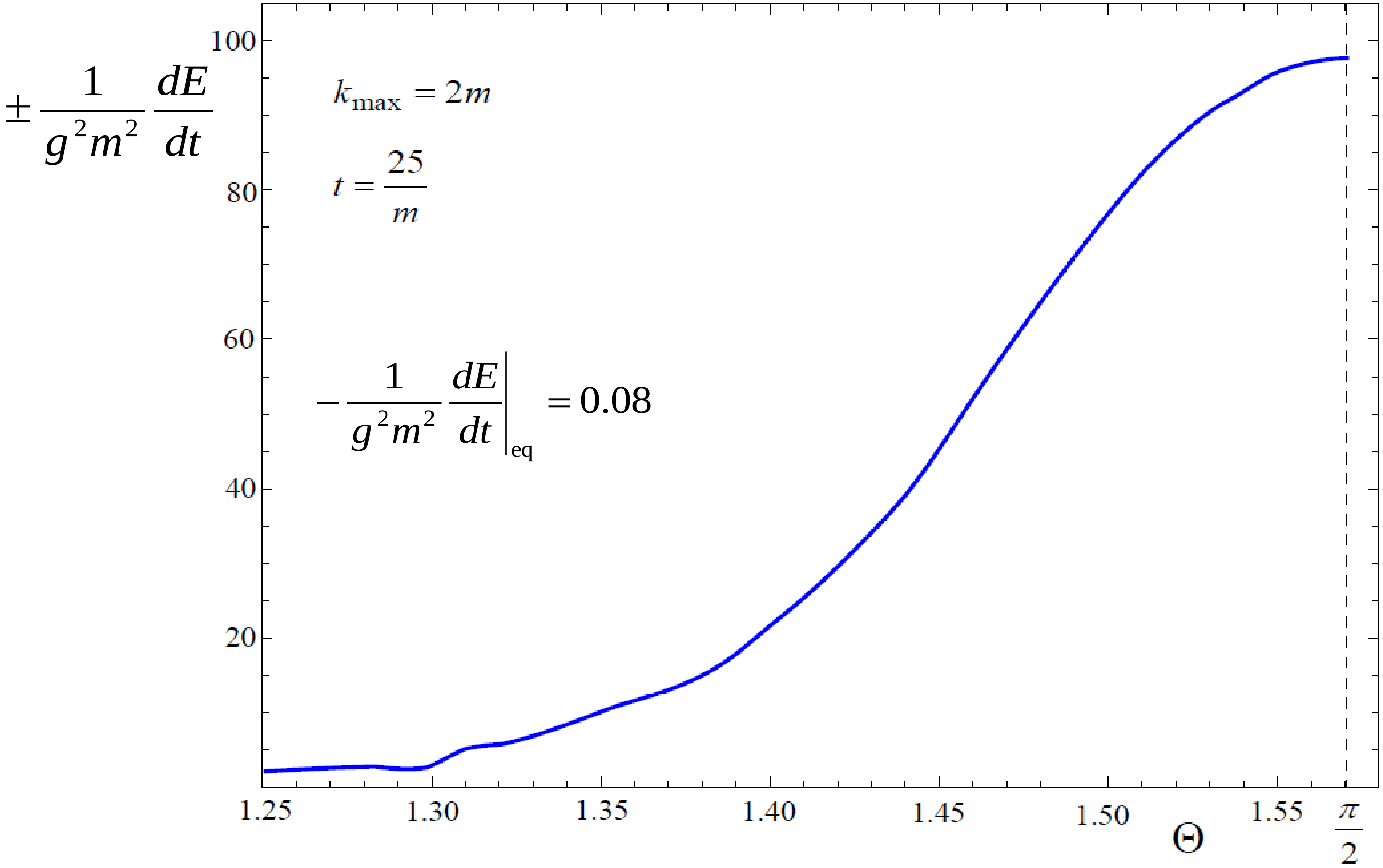
$$f(\mathbf{p}) \sim \delta(p_L)$$

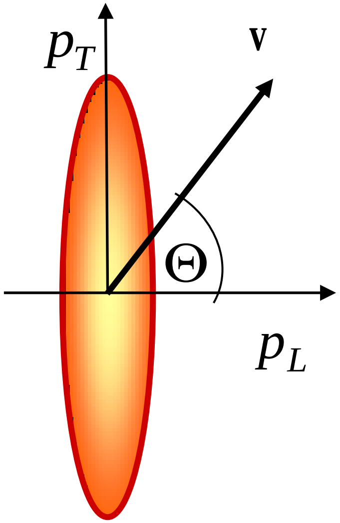
Two unstable modes



Energy loss in extremely oblate QGP

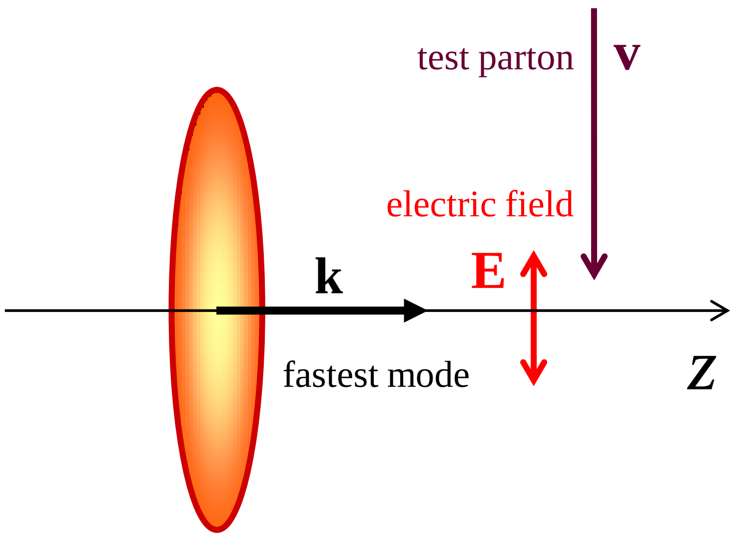
Correlated initial condition
- energy gain for $\cos\varphi < 0$
- energy loss for $\cos\varphi > 0$



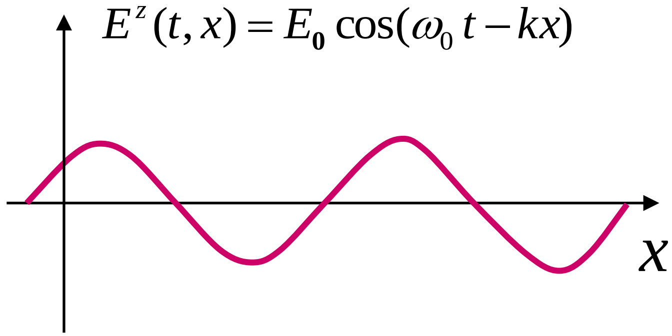


Angular dependence

$$\frac{dE}{dt} \sim \mathbf{E}\cdot\mathbf{j} \sim \mathbf{E}\cdot\mathbf{v}$$



$$E^z(t,x) = E_0 \cos(\omega_0 t - kx)$$



The largest *dE/dt* for **v** transverse to *z* !

Conclusions

- dE/dt crucially depends on initial conditions.
- $dE/dt > 0$ & $dE/dx < 0$
- dE/dt strongly varies with time and direction.
- $|dE/dt|$ can be much bigger than in equilibrium QGP.