The earliest phase of relativistic heavy-ion collisions

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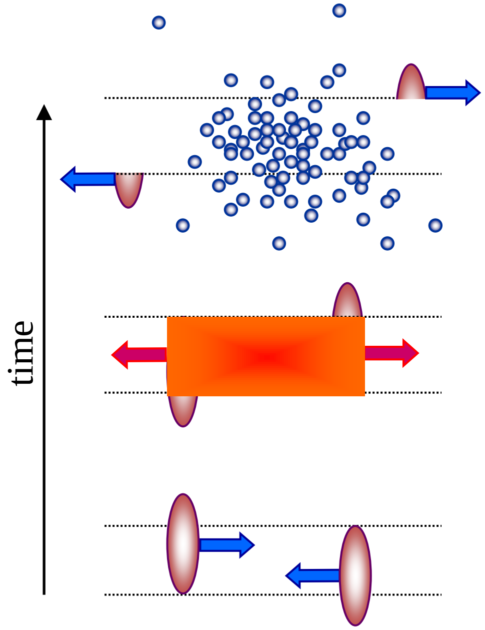
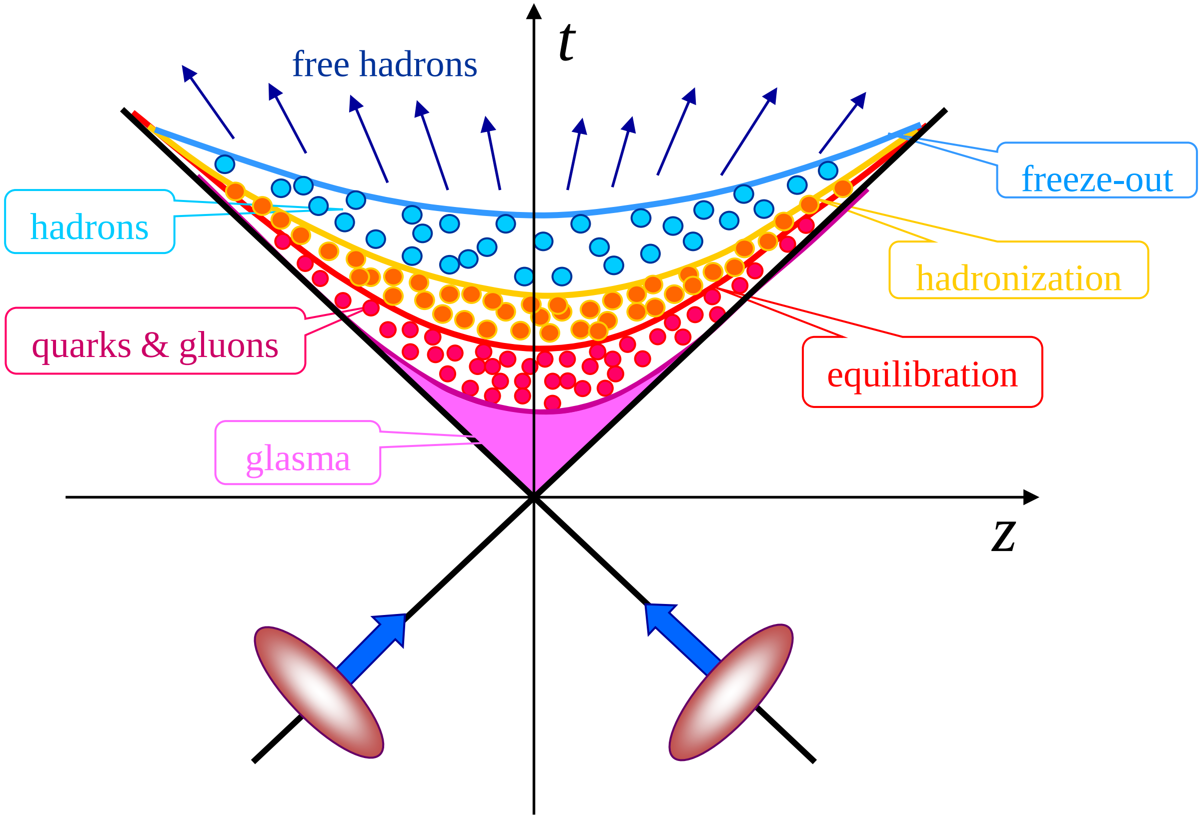
Happy Birthday!

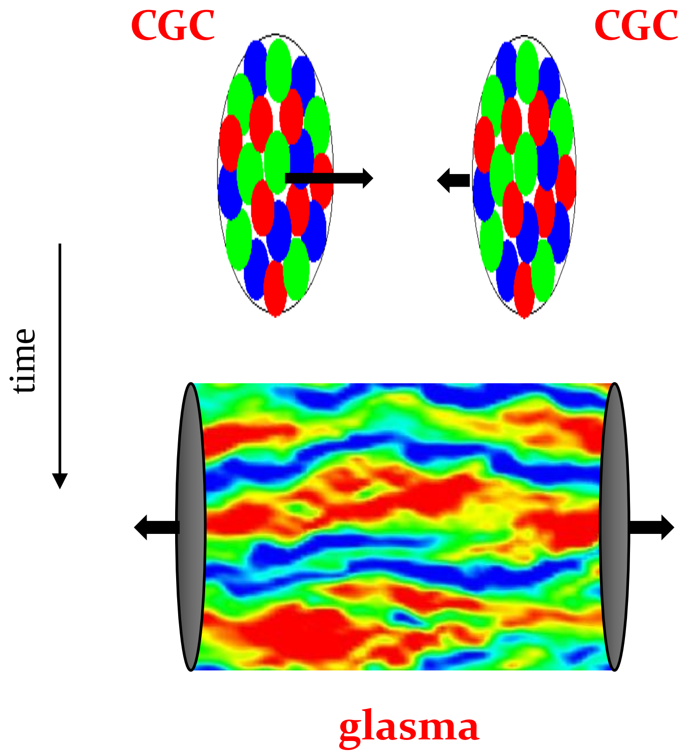
Relativistic heavy-ion collisions





Color Glass Condensate & glasma

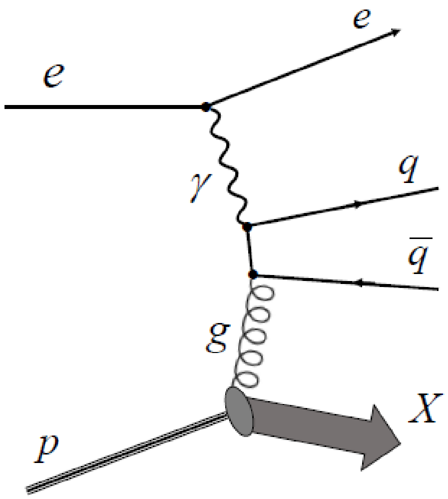
Color charges confined in the colliding nuclei generate **CGC**
which in turn produce **glasma** – the system of strong mostly classical
chromodynamic fields which evolves towards equilibrium.

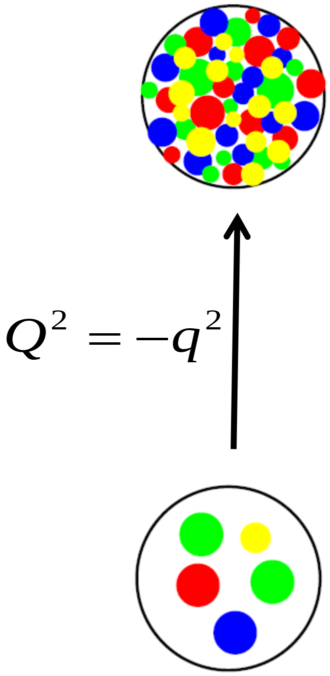


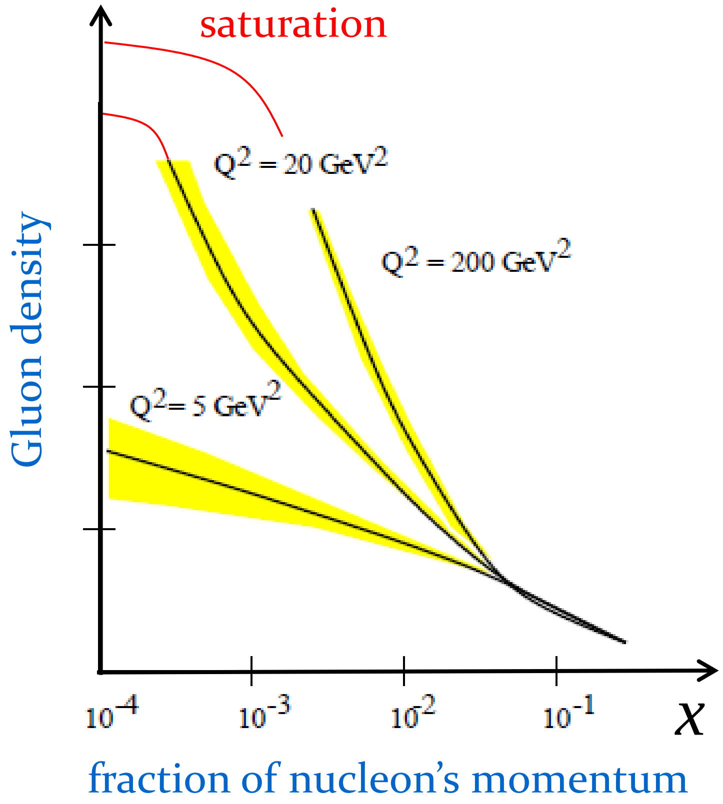
E. Iancu, R. Venugopalan, in *Quark-Gluon Plasma 3*,
ed. by R.C. Hwa, X.-N. Wang (World Scientific, Singapore, 2004

Saturation

Deep Inelastic Scattering

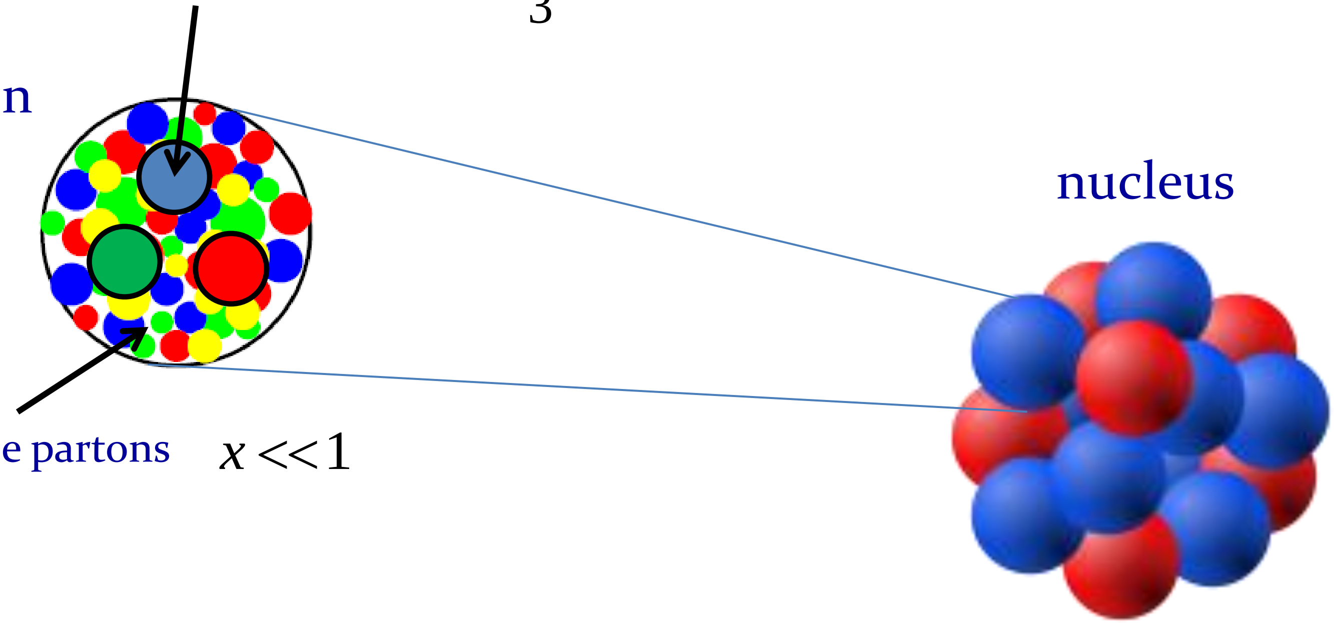






Saturated gluon system can be described in terms of classical chromodynamic fields.

Wee partons & valence quarks



valence quark $x \approx \frac{1}{3}$

nucleon

nucleus

sea or wee partons $x << 1$

In relativistic heavy-ion collsions

- Saturated wee partons – classical chromodynamic fields
- Valence quarks – classical sources of chromodynamic fields

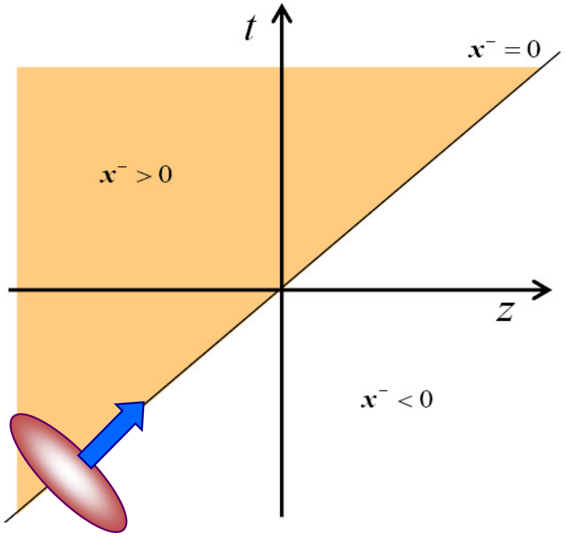
Ultrarelativistic heavy-ion collisions in light-cone variables

natural units $c = 1$

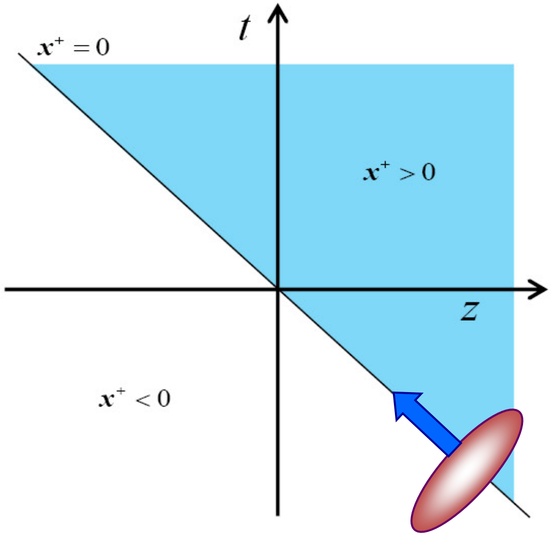
$$x^{\pm} \equiv \frac{t \pm z}{\sqrt{2}} \qquad x_{\mu} y^{\mu} = x_{+} y^{+} + x_{-} y^{-} - \vec{x}\cdot\vec{y} = x^{-} y^{+} + x^{+} y^{-} - \vec{x}\cdot\vec{y}$$

$$x^{\pm} = 0 \quad \Rightarrow \quad z = \pm t$$

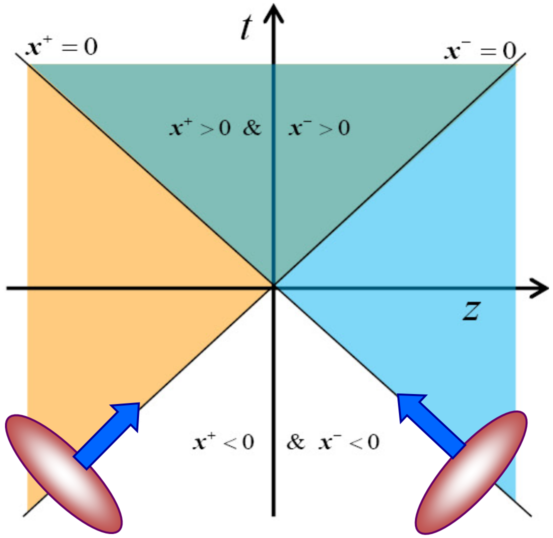
$\Theta(x^{-}) > 0$



$\Theta(x^{+}) > 0$



$\Theta(x^{-})\Theta(x^{+}) > 0$



$x^{-} = 0$

$x^{+} = 0$

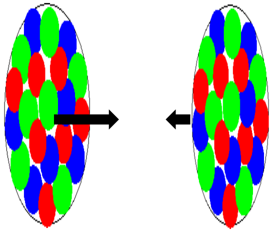
Color Glass Condensate & glasma

Classical Yang-Mills equation

$$\boxed{D_\mu F^{\mu\nu}(x) = j^\nu(x)}$$

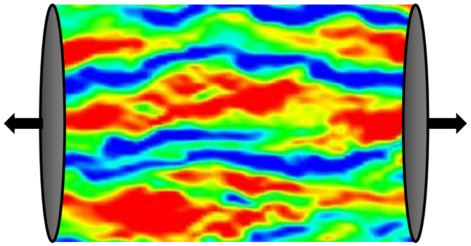
$$j^\mu(x) = j_1^\mu(x) + j_2^\mu(x)$$

$$j_{1,2}^\mu(x) = \pm\delta^{\mu\mp}\delta(x^\pm)\rho_{1,2}(\mathbf{x}_\perp)$$



β_1^i β_2^i

pre-collision potentials

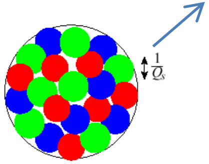


$\alpha, \alpha_\perp^i$

post-collision potentials

Ansatz of gauge potentials

$$\begin{cases} A^+(x) = \Theta(x^+)\Theta(x^-)x^+\alpha(\tau,\mathbf{x}_\perp) \\ A^-(x) = -\Theta(x^+)\Theta(x^-)x^-\alpha(\tau,\mathbf{x}_\perp) \\ A^i(x) = \Theta(x^+)\Theta(x^-)\alpha_\perp^i(\tau,\mathbf{x}_\perp) \\ \qquad +\Theta(-x^+)\Theta(x^-)\beta_1^i(\mathbf{x}_\perp) + \Theta(x^+)\Theta(-x^-)\beta_2^i(\mathbf{x}_\perp) \end{cases}$$



Boundary condition

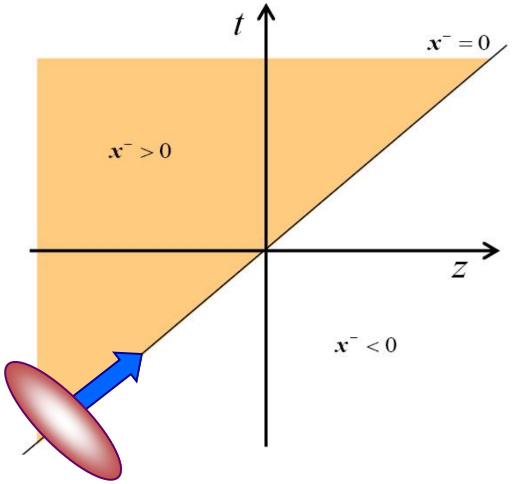
$$\begin{cases} \alpha(0,\mathbf{x}_\perp) = \beta_1^i(\mathbf{x}_\perp) + \beta_2^i(\mathbf{x}_\perp) \\ \alpha_\perp^i(0,\mathbf{x}_\perp) = -\dfrac{ig}{2}[\beta_1^i(\mathbf{x}_\perp), \beta_2^i(\mathbf{x}_\perp)] \end{cases}$$

Gauge condition

$$x^+A^- + x^-A^+ = 0$$

A. Kovner, L.D. McLerran & H. Weigert, Phys. Rev. D **52**, 3809 (1995).

Pre-collision potentials



$$\boxed{j^\mu(x^-,\mathbf{x}_\perp) = \delta^{\mu+}\delta(x^-)\rho(\mathbf{x}_\perp)}$$

Gauge condition: $A^i(x^-,\mathbf{x}_\perp) = 0$

$$D_\mu F^{\mu\nu} = j^\nu \quad \Rightarrow \begin{cases} A^-(x^-,\mathbf{x}_\perp) = 0 \\ A^+(x^-,\mathbf{x}_\perp) = \delta(x^-)\Lambda(\mathbf{x}_\perp) \end{cases}$$

Poisson equation

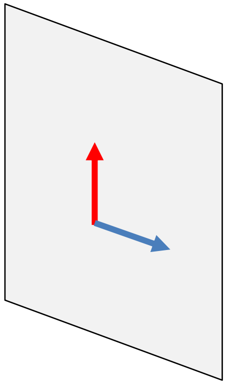
$$\boxed{\nabla_\perp^2 \Lambda(\mathbf{x}_\perp) = -\rho(\mathbf{x}_\perp)}$$

$$\Lambda(\mathbf{x}_\perp) = \int d^2x'_\perp\, G(\mathbf{x}_\perp - \mathbf{x}'_\perp)\rho(\mathbf{x}'_\perp)$$

$$G(\mathbf{x}_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}\frac{e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}}{\mathbf{k}_\perp^2 + m^2} = \frac{1}{2\pi}K_0(m|\mathbf{x}_\perp|)$$

IR regulator $m = \Lambda_{\text{QCD}}$

$$E^z = B^z = 0$$
$$\mathbf{E}_\perp, \mathbf{B}_\perp \sim \delta(x^-)$$



$x^- = 0$

Pre-collision potentials cont.

Gauge transformation: $A^{\mu}(x^{-},\mathbf{x}_{\perp}) \quad \rightarrow \quad \beta^{\mu}(x^{-},\mathbf{x}_{\perp})$

gauge condition

$$A^{i}(x^{-},\mathbf{x}_{\perp})=0$$

light-come gauge

$$\beta^{+}(x^{-},\mathbf{x}_{\perp})=0$$

$$\beta^{\mu}(x)=U(x)A^{\mu}(x)U^{\dagger}(x)+\frac{i}{g}U(x)\partial^{\mu}U^{\dagger}(x)$$

$$U(x^{-},\mathbf{x}_{\perp})A^{+}(x^{-},\mathbf{x}_{\perp})U^{\dagger}(x^{-},\mathbf{x}_{\perp})+\frac{i}{g}U(x^{-},\mathbf{x}_{\perp})\partial^{+}U^{\dagger}(x^{-},\mathbf{x}_{\perp})=0$$

$$\begin{cases} U(x^{-},\mathbf{x}_{\perp})=P\exp\left(ig\int\limits_{-\infty}^{x^{-}}dz^{-}A^{+}(z^{-},\mathbf{x}_{\perp})\right) \\ \beta^{i}(x^{-},\mathbf{x}_{\perp})=\frac{i}{g}U(x^{-},\mathbf{x}_{\perp})\partial^{i}U^{\dagger}(x^{-},\mathbf{x}_{\perp}) \end{cases}$$

pure gauge except $x^{-} = 0$

J. Jalilian-Marian, A. Kovner, L.D. McLerran & H. Weigert, Phys. Rev. D **55**, 5414 (1997).

Proper time expansion

$$\alpha(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha_{(n)}(\mathbf{x}_\perp),\qquad \alpha_\perp^i(\tau,\mathbf{x}_\perp)=\sum_{n=0}^{\infty}\tau^n\alpha^i_{\perp(n)}(\mathbf{x}_\perp)$$

Proper time τ is treated as a small parameter $\tau << Q_s^{-1}$

Yang-Mills equations for the expanded potentials are solved recursively

$$\alpha_{(n)}=\alpha^i_{\perp(n)}=0 \quad \text{for} \quad n=1,3,5,\dots$$

0th order - oboundary conditions

$$\begin{cases}\alpha_{(0)}=-\dfrac{ig}{2}[\beta_1^i,\beta_2^i]\\ \alpha^i_{\perp(0)}=\beta_1^i+\beta_2^i\end{cases}$$

Post-collision potentials are expressed through pre-collision potentials

2nd order

$$\begin{cases}\alpha_{(2)}=-\dfrac{ig}{16}[D^j,[D^j,[\beta_1^i,\beta_2^i]]]\\ \alpha^i_{\perp(2)}=\dfrac{ig}{4}\varepsilon^{zij}\varepsilon^{zkl}[D^j,[\beta_1^k,\beta_2^l]]\end{cases}$$

$$D^i\equiv\partial^i-ig(\beta_1^i+\beta_2^i)$$

Fully analytic approach!

R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Proper time expansion cont.

Chromoelectric and chromomagnetic fields

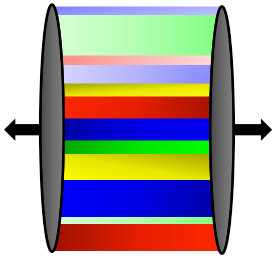
$$E^i = F^{i0}, \quad B^i = \frac{1}{2}\varepsilon^{ijk} F^{kj}$$

0th order

$$\mathbf{E}_{(0)} = (0, 0, E), \qquad \mathbf{B}_{(0)} = (0, 0, B)$$

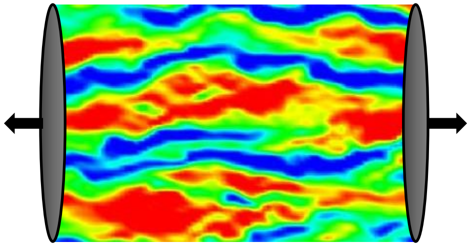
$$E^z_{(0)}(\mathbf{x}_\perp) = -ig[\beta^i_1(\mathbf{x}_\perp), \beta^i_2(\mathbf{x}_\perp)]$$

$$B^z_{(0)}(\mathbf{x}_\perp) = -ig\varepsilon^{zij}[\beta^i_1(\mathbf{x}_\perp), \beta^j_2(\mathbf{x}_\perp)]$$



E & *B* fields along the axis *z*

At higher orders transverse fields show up

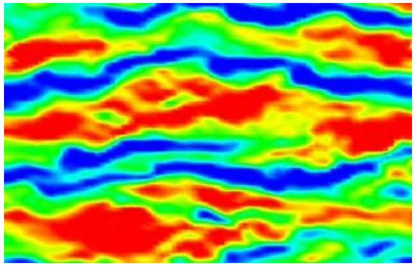


R. J. Fries, J. I. Kapusta, and Y. Li, arXiv:nucl-th/0604054
G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Observables - energy-momentum tensor

$$\boxed{T^{\mu\nu}(x) = 2\mathrm{Tr}[F^{\mu\rho}(x)F_{\rho}{}^{\nu}(x) + \frac{1}{4}g^{\mu\nu}F^{\rho\sigma}(x)F_{\rho\sigma}(x)]}$$

$$F^{\mu\nu}(x) = \partial^{\mu}A^{\nu}(x) - \partial^{\nu}A^{\mu}(x) - ig[A^{\mu}(x), A^{\nu}(x)]$$



Physical quantities

$\langle T^{00}(x)\rangle$ - energy density

$\langle T^{0i}(x)\rangle$ - energy flux, Poynting vector

$\langle T^{xx}(x)\rangle, \langle T^{yy}(x)\rangle, \langle T^{zz}(x)\rangle$ - pressures

$\langle T^{ij}(x)\rangle$ - momentum flux

$\langle \ldots \rangle$ averaging over colour configuration of initial nuclei

one-point correlators

Averaging over colour configuration of initial nuclei

- $$Observable = \int \sum \partial_x^i \partial_z^j \left\langle \beta^k(\mathbf{x}_\perp)\beta^l(\mathbf{y}_\perp)\dots\beta^m(\mathbf{z}_\perp)\right\rangle$$

- $$\beta^i(\mathbf{x}_\perp) \sim \rho(\mathbf{x}_\perp)$$

Wick theorem

- $$\left\langle \rho_a^k(\mathbf{x}_\perp)\rho_b^l(\mathbf{y}_\perp)\dots\rho_c^m(\mathbf{z}_\perp)\right\rangle = \sum\prod\left\langle \rho_a^i(\mathbf{x}_\perp)\rho_b^j(\mathbf{y}_\perp)\right\rangle$$

Glasma graph approximation

- $$\left\langle \beta_a^k(\mathbf{x}_\perp)\beta_b^l(\mathbf{y}_\perp)\dots\beta_c^m(\mathbf{z}_\perp)\right\rangle = \sum\prod\left\langle \beta_a^i(\mathbf{x}_\perp)\beta_b^j(\mathbf{y}_\perp)\right\rangle = \sum\prod B_{ab}^{ij}(\mathbf{x}_\perp,\mathbf{y}_\perp)$$

Basic two-point correlator in transversaly uniform system

$$B^{ij}_{ab}(\mathbf{x}_\perp - \mathbf{y}_\perp) \equiv \left\langle \beta^i_a(\mathbf{x}_\perp)\beta^j_b(\mathbf{y}_\perp)\right\rangle = \int d^2x'_\perp\, d^2y'_\perp \cdots\cdots \left\langle \rho^i_a(\mathbf{x}'_\perp)\rho^j_b(\mathbf{y}'_\perp)\right\rangle$$

$$\left\langle \rho^i_a(\mathbf{x}_\perp)\rho^j_b(\mathbf{y}_\perp)\right\rangle = g^2\mu\,\delta^{ab}\delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

color charge surface density

$$\mu = g^{-4}Q_s^2$$

$$\boxed{B^{ij}_{ab}(\mathbf{x}_\perp - \mathbf{y}_\perp) \equiv \delta^{ab}\left(\delta^{ij}C_1(r) - \hat{r}^i\hat{r}^jC_2(r)\right)}$$

$$\mathbf{r} \equiv \mathbf{x}_\perp - \mathbf{y}_\perp, \quad r \equiv |\mathbf{r}|, \quad \hat{r}^i \equiv \frac{r^i}{r}$$

$$\begin{cases} C_1(r) \equiv \dfrac{m^2K_0(mr)}{g^2N_c\left(mrK_1(mr)-1\right)}\left\{\exp\left[\dfrac{g^4N_c\mu\left(mrK_1(mr)-1\right)}{4\pi m^2}\right]-1\right\} \\[2ex] C_2(r) \equiv \dfrac{m^3rK_1(mr)}{g^2N_c\left(mrK_1(mr)-1\right)}\left\{\exp\left[\dfrac{g^4N_c\mu\left(mrK_1(mr)-1\right)}{4\pi m^2}\right]-1\right\} \end{cases}$$

$$\underset{r<<m^{-1}}{\approx} \quad \#\log(mr)$$

UV regularization required

$$r > Q_s^{-1}$$

J. Jalilian-Marian, A. Kovner, L. D. McLerran and H. Weigert, Physical Review D **55**, 5414 (1997)

Basic correlator of real nucleus

System nonuniform in the transverse plane

Projected Woods-Saxon distribution

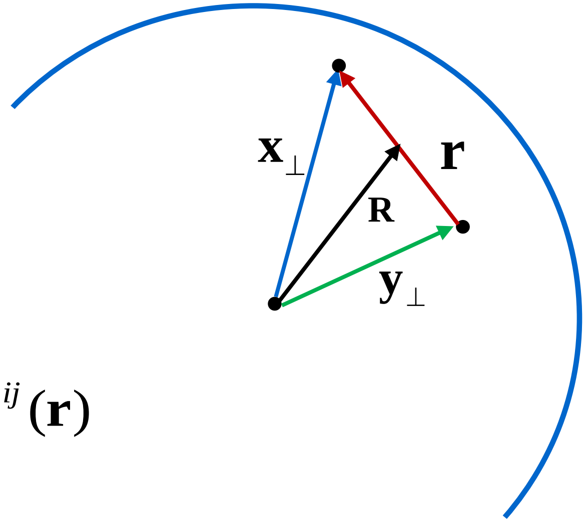
$$\mu(\mathbf{x}_\perp) = \frac{\bar{\mu}}{\ln(1+e^{R_A/a})}\int_{-\infty}^{\infty}\frac{dz}{1+\exp\left[\left(\sqrt{\mathbf{x}_\perp{}^2+z^2}-R_A\right)/a\right]}$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{x}_\perp + \mathbf{y}_\perp), \qquad \mathbf{r} \equiv \mathbf{x}_\perp - \mathbf{y}_\perp$$

$$B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{R}, \mathbf{r}) \approx \text{``}\textit{gradient expansion in } \mathbf{R}\text{''}$$

strong dependence on **r**

weak dependence of **R**



System uniform in the transverse plane

$$B_{ab}^{ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp) = \delta^{ab} f^{ij}(\mathbf{r})$$

G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Numerical results

Pb-Pb collisions at LHC

$N_c = 3$

$g = 1$

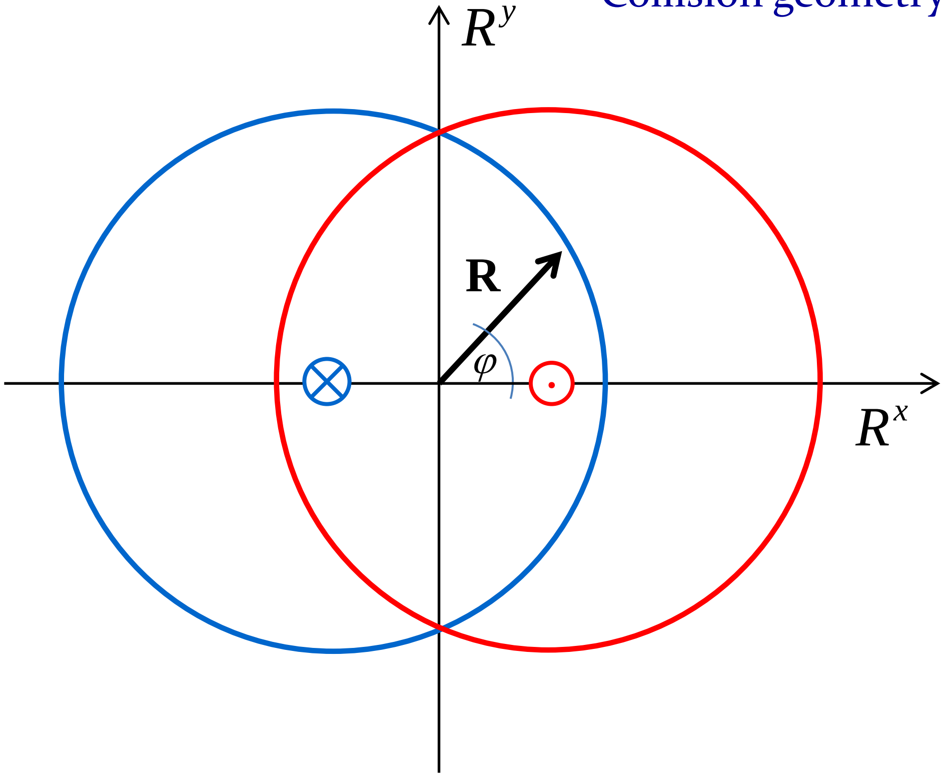
$Q_s = 2\ \text{GeV}$

$m = 0.2\ \ \text{GeV}$

$R_A = 7.4\ \text{fm}$

$a = 0.5\ \text{fm}$

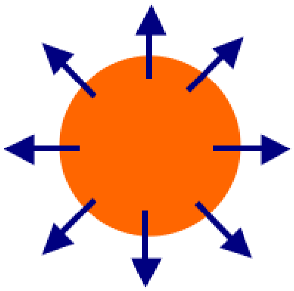
Collision geometry

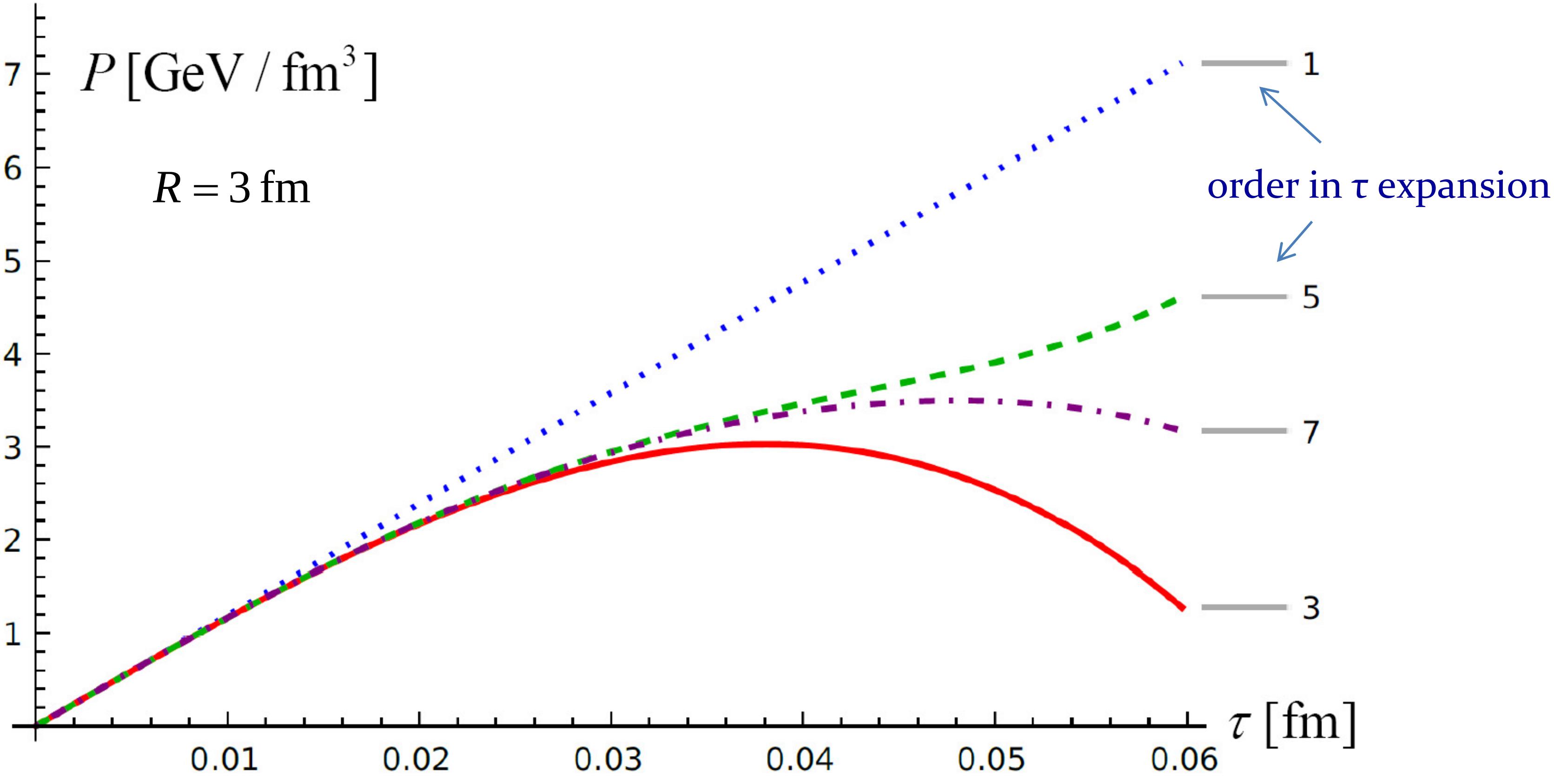


Radial flow

Pb-Pb collisions at b = 6 fm

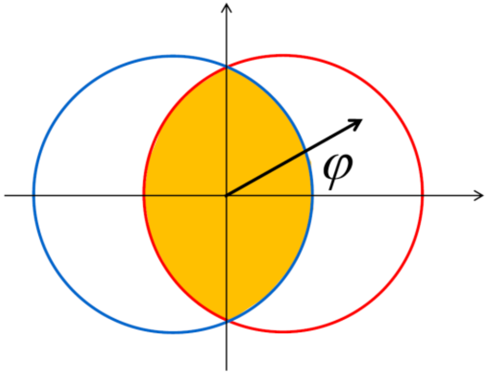
$$P \equiv R^i T^{0i}$$





M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Radial flow cont.



Pb-Pb collisions at b = 6 fm

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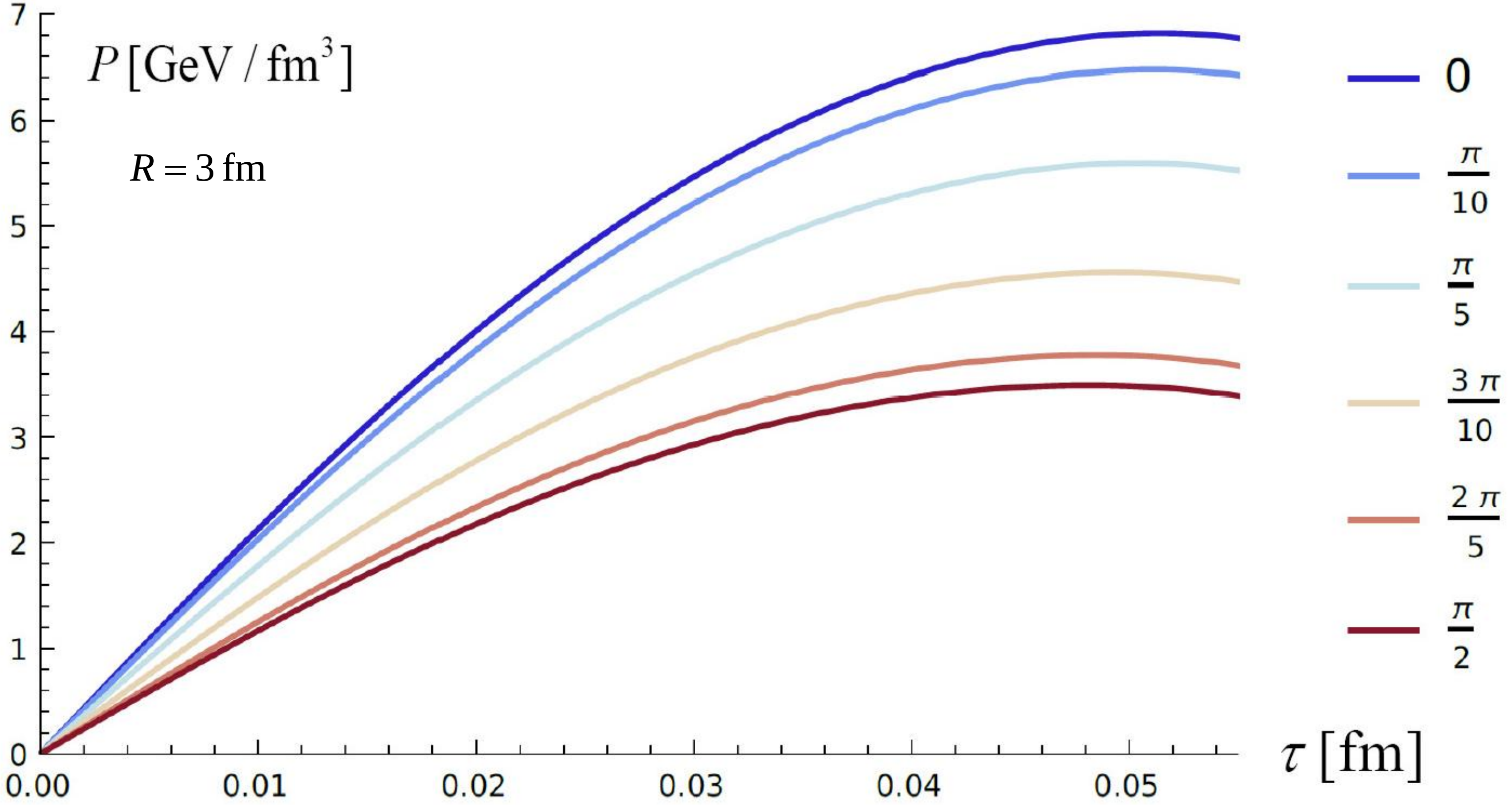
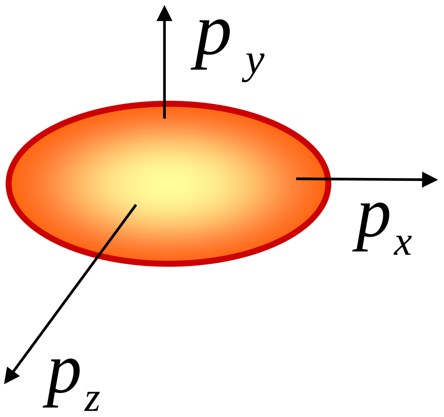


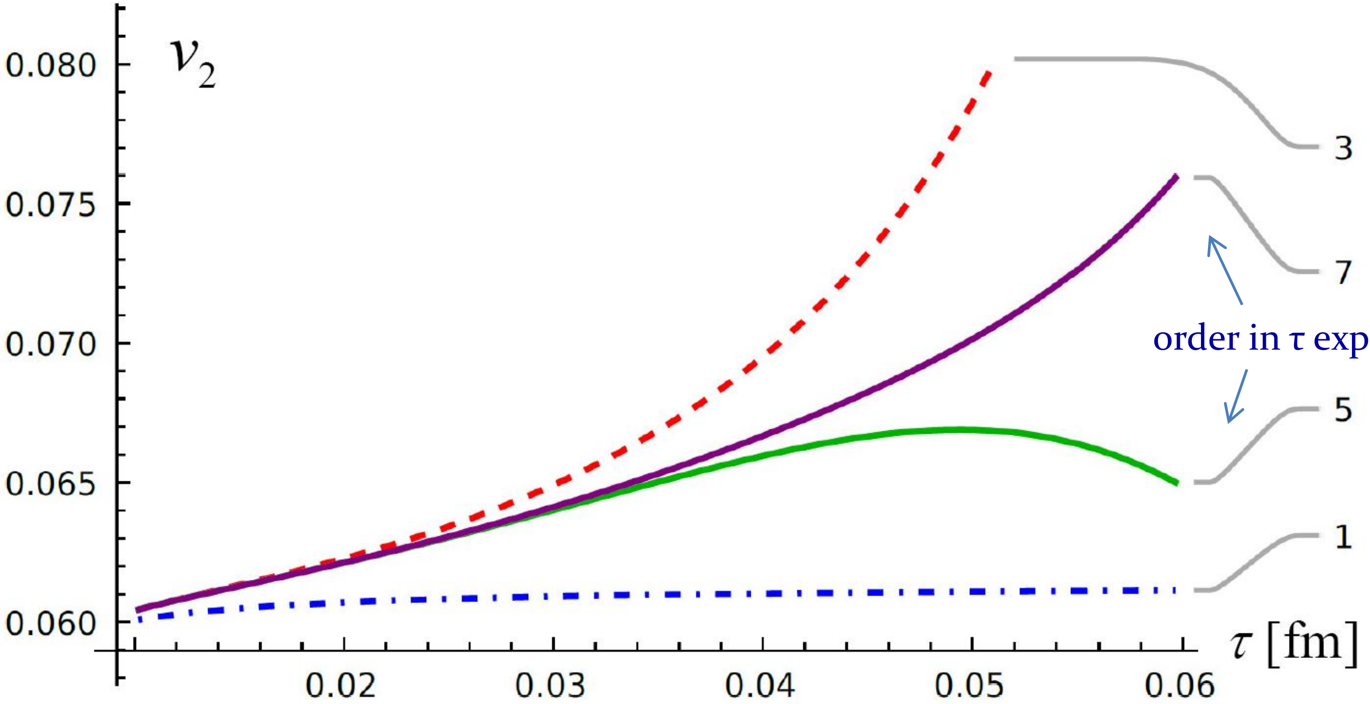
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Elliptic flow

Pb-Pb collisions at *b* = 2 fm

$$v_2 = \frac{\int d^2R \, \frac{T_{0x}^2 - T_{0y}^2}{\sqrt{T_{0x}^2 + T_{0y}^2}}}{\int d^2R \sqrt{T_{0x}^2 + T_{0y}^2}}$$

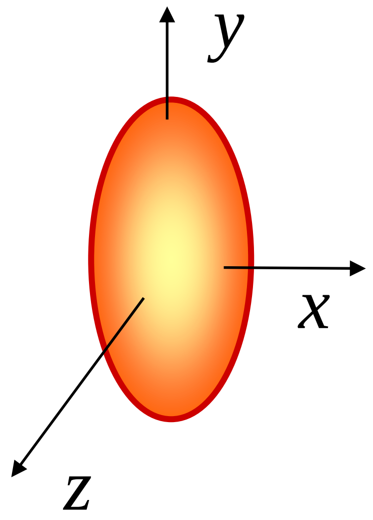




order in τ expansion

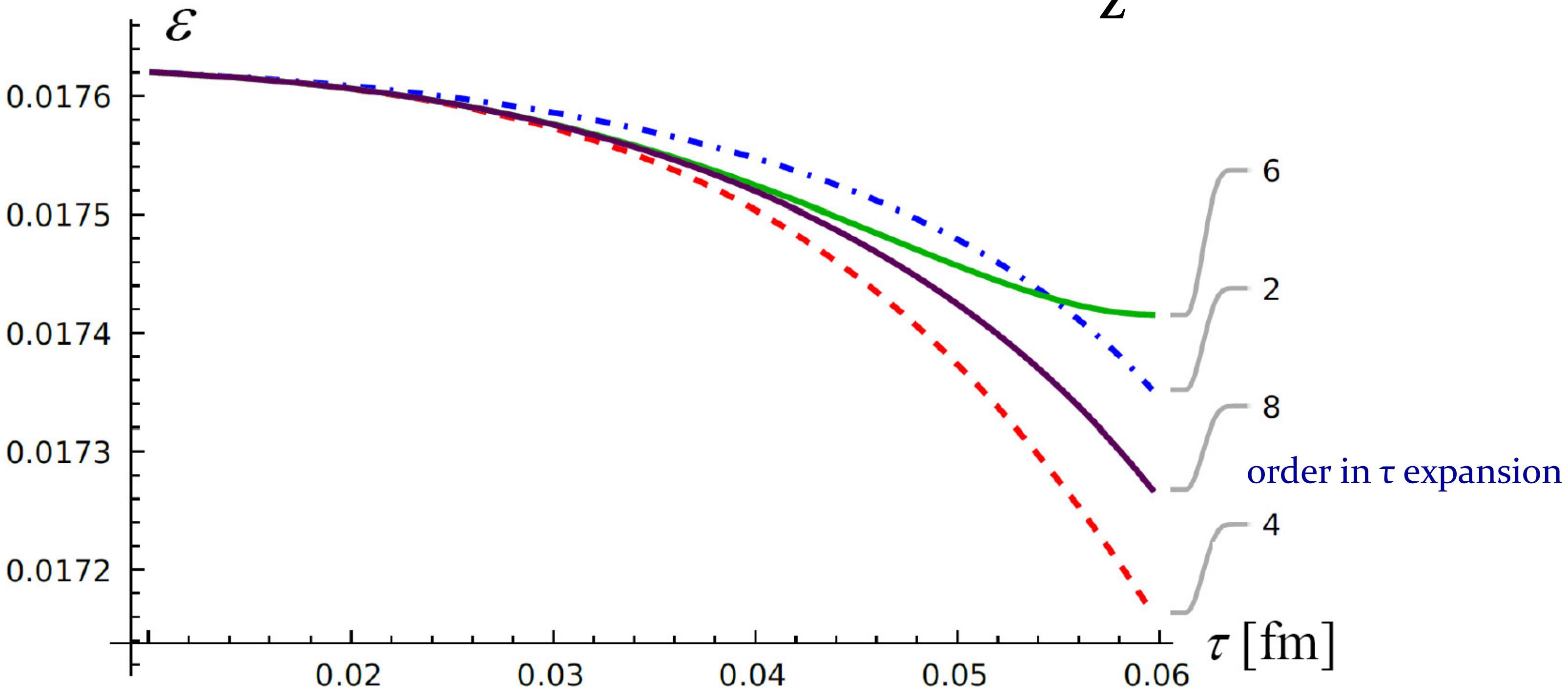
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Eccentricity



Pb-Pb collisions at b = 2 fm

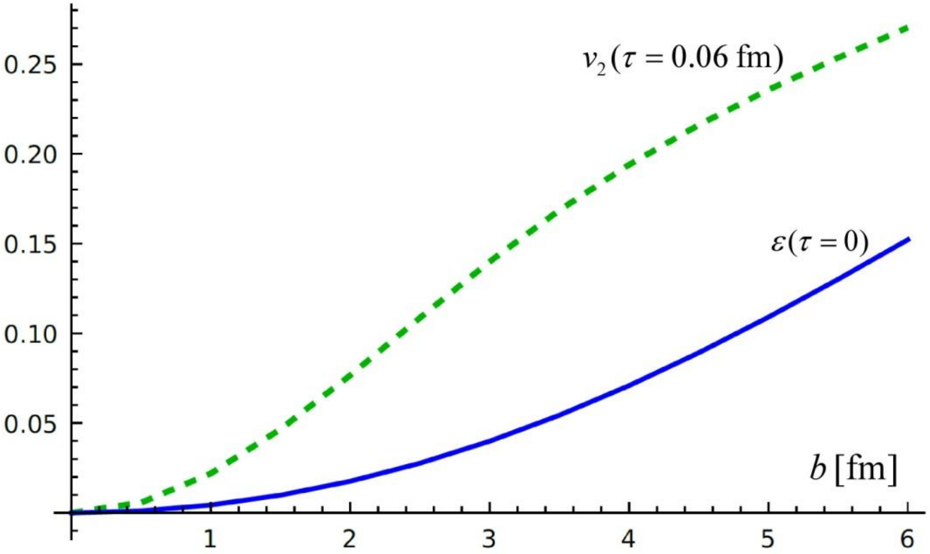
$$\varepsilon = \frac{\int d^2R \, \frac{R_x^2 - R_y^2}{\sqrt{R_x^2 + R_y^2}} \, T^{00}}{\int d^2R \sqrt{R_x^2 + R_y^2} \, T^{00}}$$

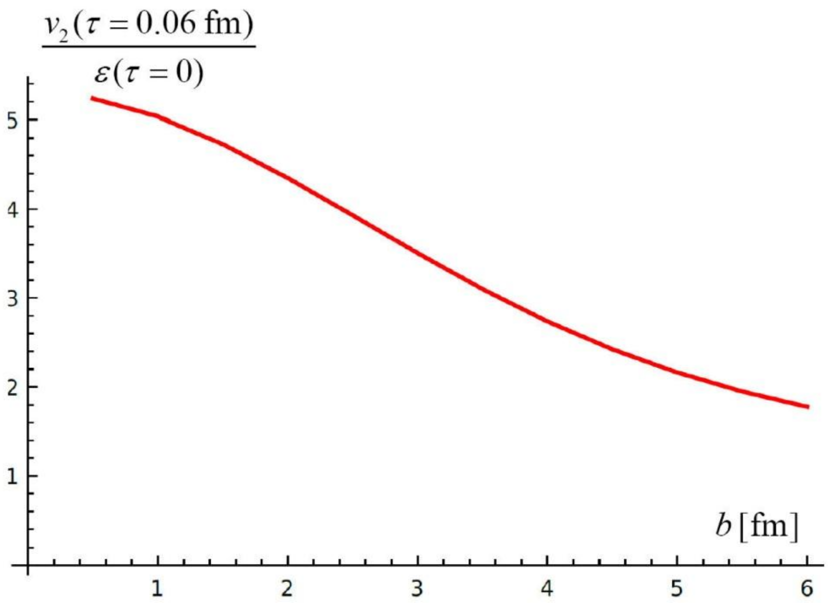


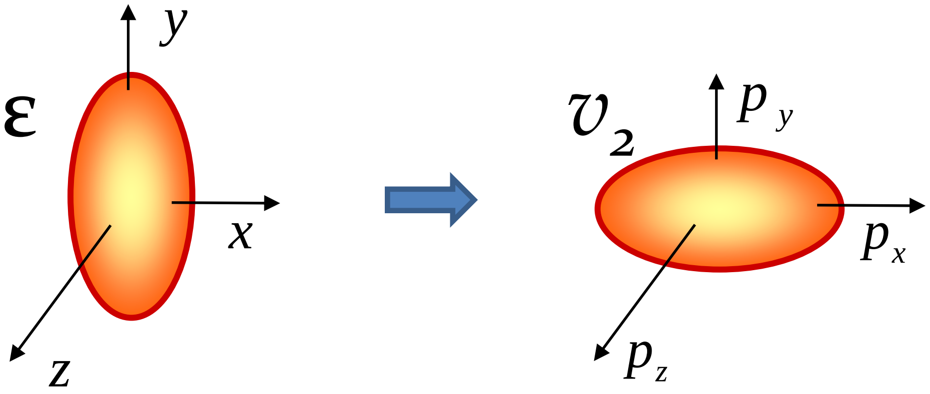
M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Hydrodynamic-like behavior

Pb-Pb collisions





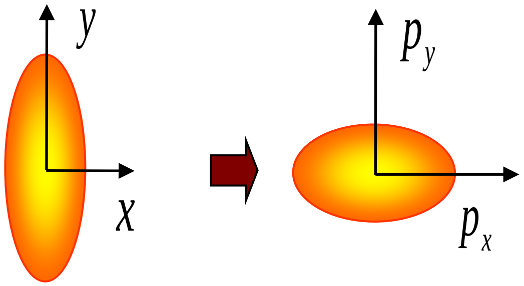


M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Equation of universal flow

$$\boxed{T^{0x} \approx -\frac{1}{2} t \frac{\partial T^{00}}{\partial x}}$$

Energy flow is generated by gradient of energy density.



Assumptions:

- $\partial_\mu T^{\mu\nu} = 0$
- Energy momentum tensor is mostly diagonal.
- The system is boost-invariant.

Glasma in ultrarelativistic collisions

$$T^{\mu\nu}(t=0) = \mathrm{diag}(\varepsilon, \varepsilon, \varepsilon, -\varepsilon)$$

short-time evolution

For glasma the equation of universal flow is satsified exactly order by order in proper time expansion.

J. Vredevoogd & S. Pratt, Phys. Rev. C **79**, 044915 (2009);

M. Carrington, St. Mrówczyński &J.-Y. Ollitrault, Phys. Rev. C **110**, 054903 (2024)

Mapping on hydrodynamic $T^{\mu\nu}$

$$T^{\mu\nu}_{\text{glasma}}(\tau, \mathbf{x}_T) \quad vs. \quad T^{\mu\nu}_{\text{hydro}}(\tau, \mathbf{x}_T)$$

Eigenvalue problem: $$\boxed{T^{\mu\nu}_{\text{glasma}} w_\nu = \lambda w^\mu}$$

Ideal hydrodynamics

$$\boxed{T^{\mu\nu}_{\text{hydro}} = (\varepsilon + p)u^\mu u^\nu - p g^{\mu\nu}} \qquad T^{\mu}_{\text{hydro}\ \mu} = 0 \quad \Rightarrow \quad p = \frac{1}{3}\varepsilon$$

$$T^{\mu\nu}_{\text{hydro}} u_\nu = \varepsilon u^\mu$$

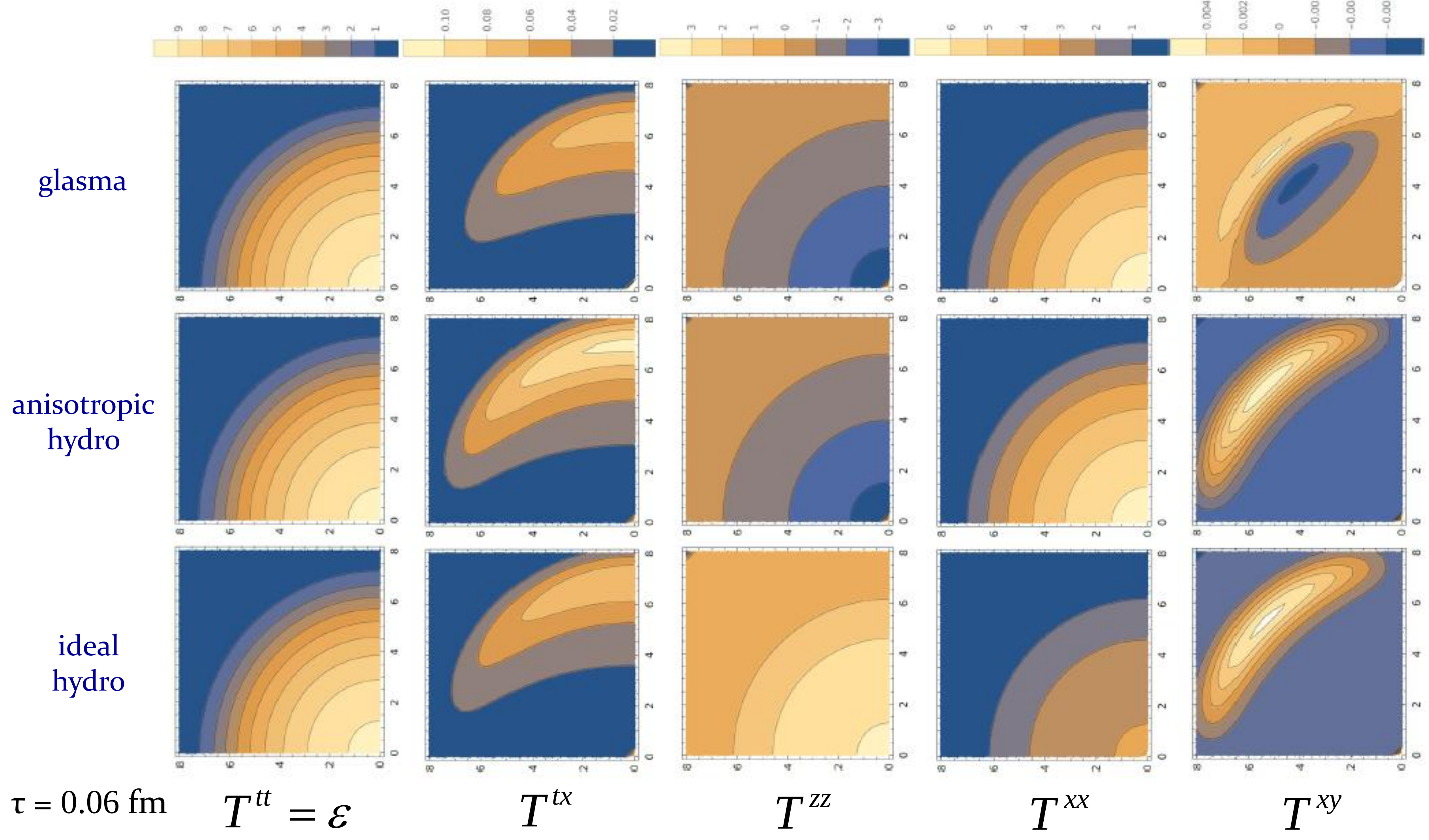
Anisotropic hydrodynamics

$$\boxed{T^{\mu\nu}_{\text{hydro}} = (\varepsilon + p_T)u^\mu u^\nu - p_T g^{\mu\nu} - (p_T - p_L) z^\mu z^\nu} \qquad T^{\mu}_{\text{hydro}\ \mu} = 0 \quad \Rightarrow \quad p_L = \varepsilon - 2 p_T$$

$$T^{\mu\nu}_{\text{hydro}} u_\nu = \varepsilon u^\mu, \qquad T^{\mu\nu}_{\text{hydro}} z_\nu = -p_L z^\mu$$

W. Florkowski & R. Ryblewski, Phys. Rev. C **83**, 034907 (2011)
M. Martinez & M. Strickland, Nucl. Phys. A **848**, 183 (2010)

Mapping on hydrodynamic $T^{\mu\nu}$

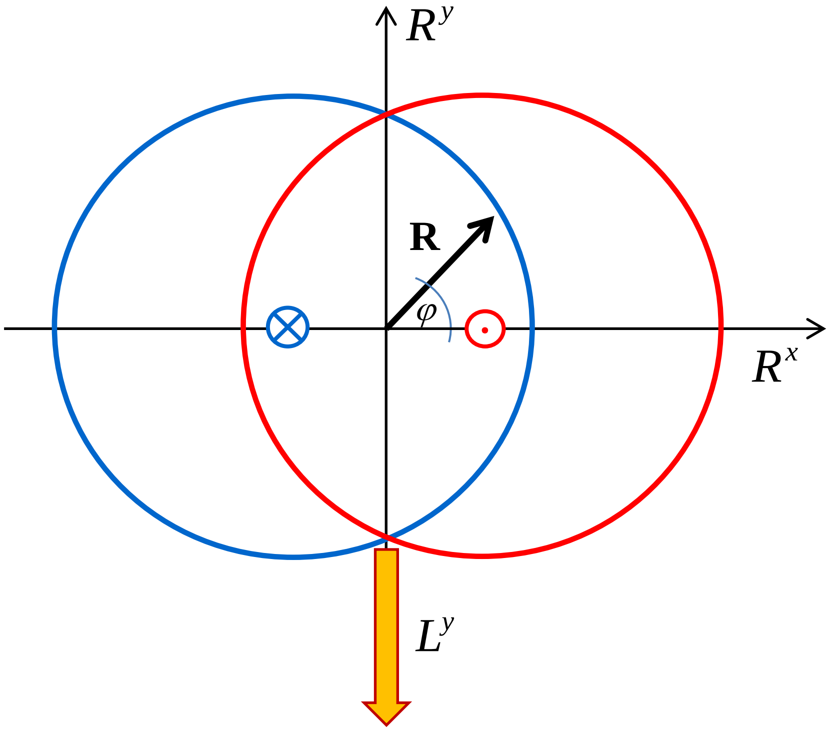


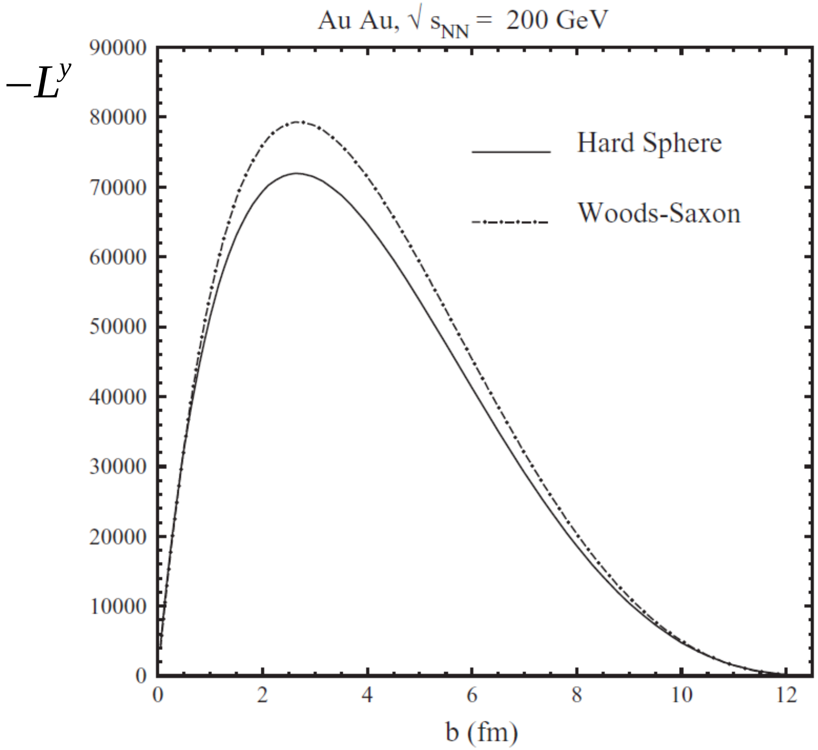
$\tau = 0.06$ fm

$b = 0, z = 0$

M. Carrington, St. Mrówczyński & J.-Y. Ollitrault, Phys. Rev. C **110**, 054903 (2024)

Angular momentum





$L^y \sim 10^7 \hbar$ @ LHC ?

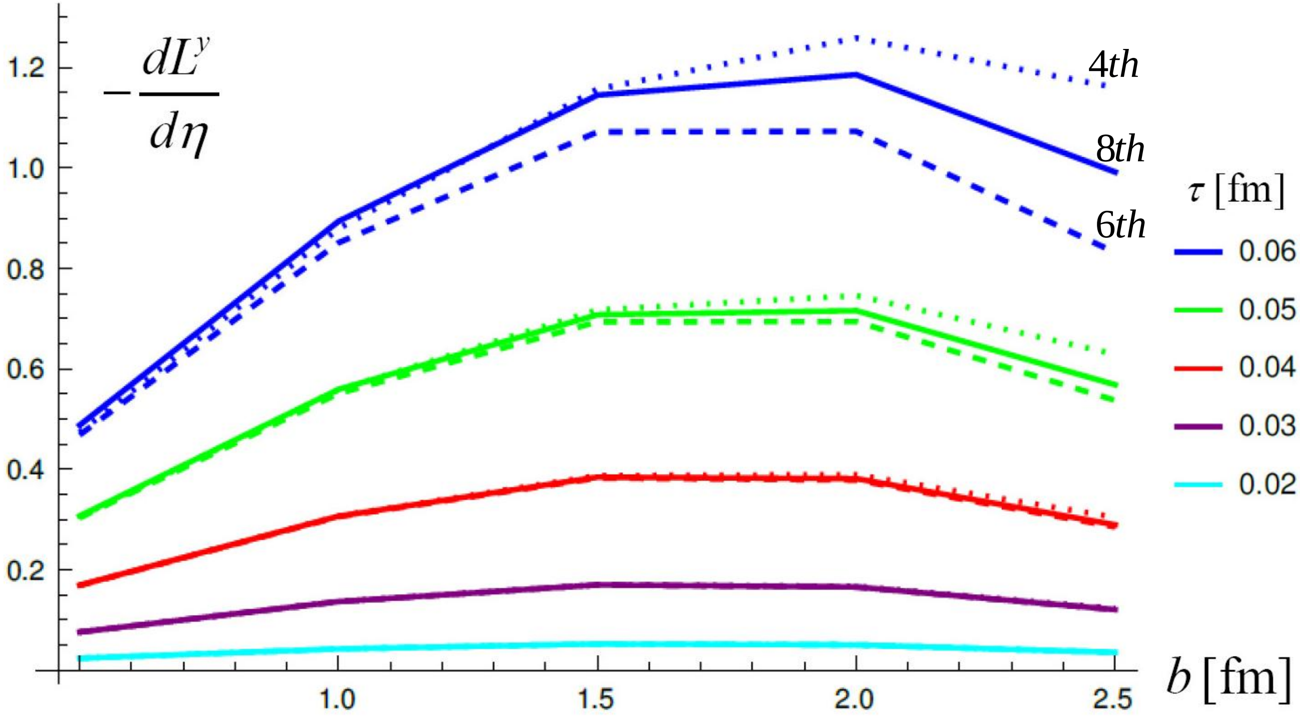
F. Becattini, F.Piccini, J. Rizzo, Physical Review C **77**, 024906 (2008)

Angular momentum cont.

Pb-Pb collisions

$$\frac{dL^\eta}{d\eta} = -\tau^2 \int d^2R\, R^x\, T^{\tau\eta}$$

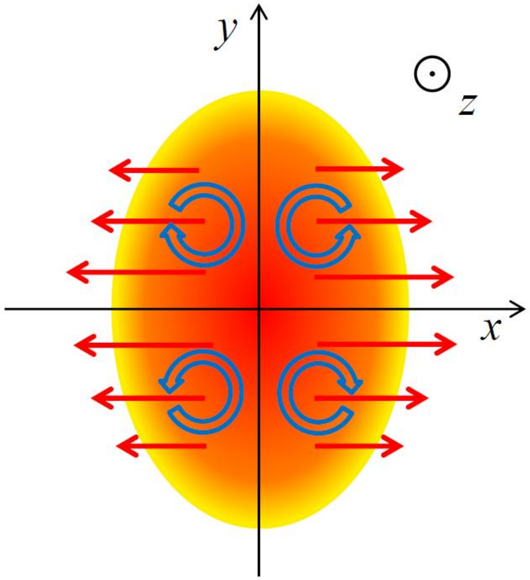
Milne coordinates

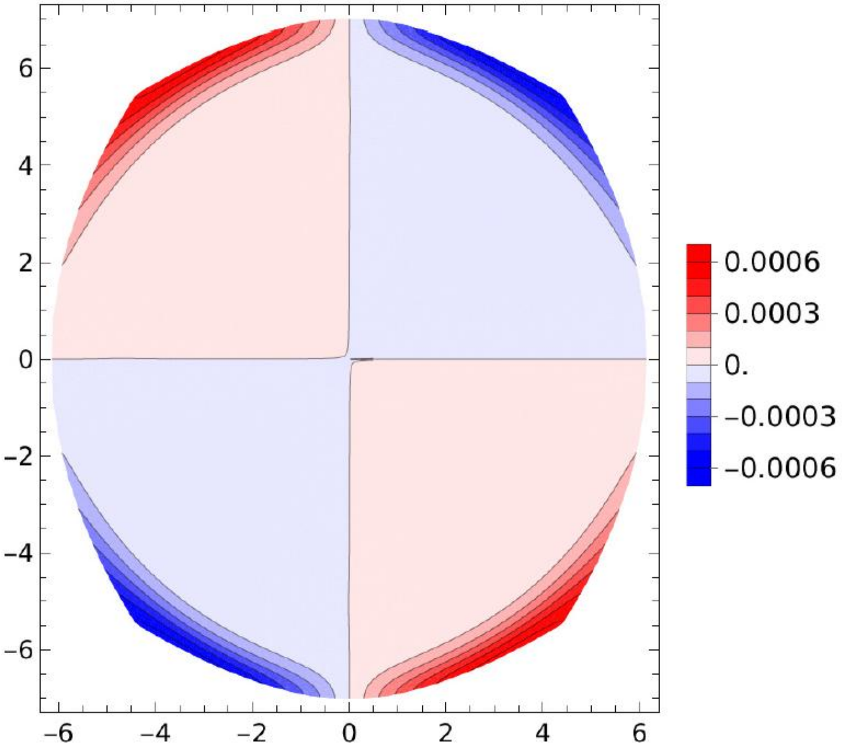


Glasma does not rotate!

M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Vorticity





$$V^{i}(\vec{r}) \equiv \frac{T^{0i}(\vec{r})}{T^{00}(\vec{r})}$$

ω^{z} at $b = 2$ fm & $\tau = 0.06$ fm in τ^{8} order

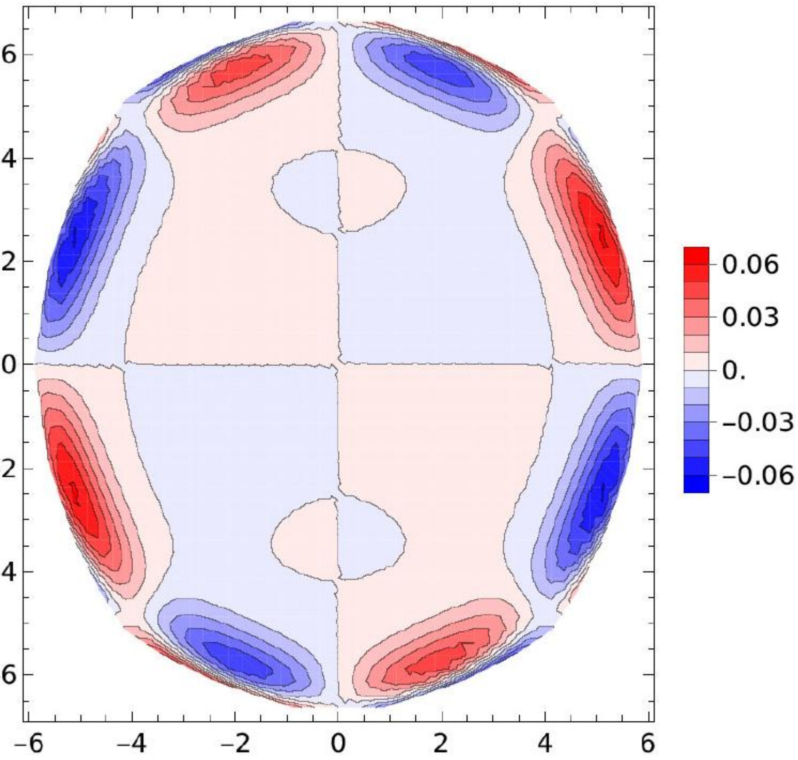
$$\omega(\vec{r}) = \nabla \times \vec{V}(\vec{r})$$

$$\omega_{\mathrm{th}}(\vec{r}) = \nabla \times \frac{\vec{u}(\vec{r})}{T(\vec{r})}$$

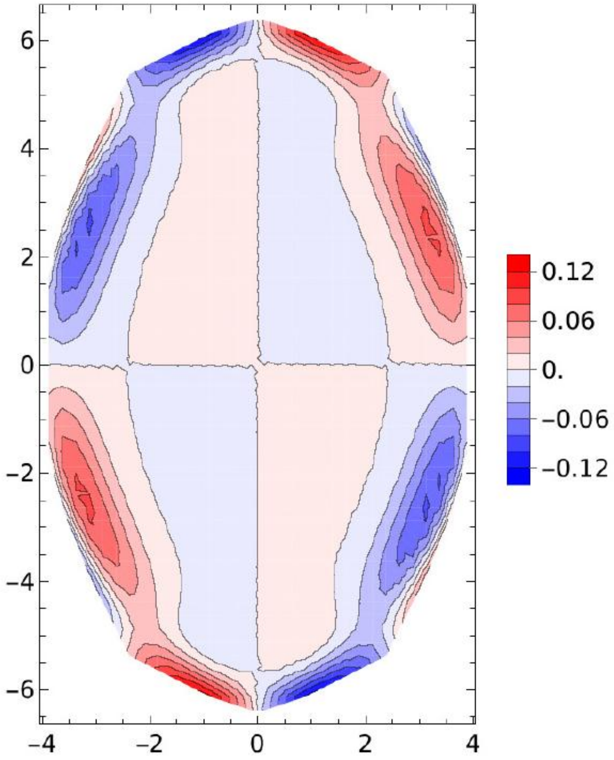
M. Carrington & St. Mrówczyński, arXiv:2505.07324 , Phys Rev. C in print

Local angular momentum

$$\frac{dL^z(\vec{r}_0)}{d\eta} = -\tau \int_{\Delta^2} d^2r\left((r^y - r_0^y)T^{0x} - (r^x - r_0^x)T^{0y}\right)$$



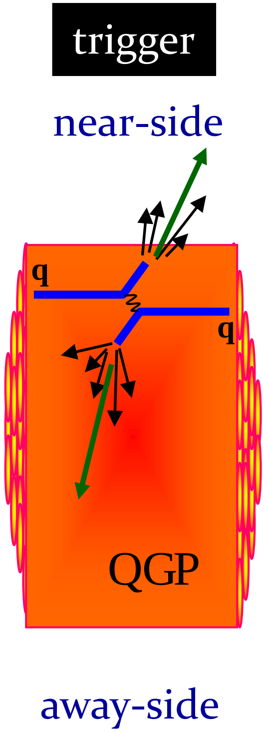
$b = 2$ fm



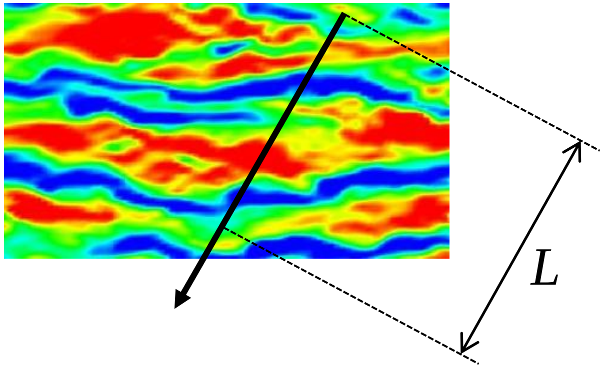
$b = 6$ fm

M. Carrington & St. Mrówczyński, arXiv:2505.07324, Phys Rev. C in print

Jet quenching in glasma



How hard probes propagate through the glasma?



$\frac{dE}{dx}$ - collisional energy loss

$\hat{q}$ - transverse momentum broadening

$\frac{dE^{\mathrm{rad}}}{dx} = -\frac{1}{8}\alpha_s N_c \hat{q} L$ - radiative energy loss

Fokker-Planck equation of hard probes

$$\overbrace{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)}^{\text{drift}} n(t,\mathbf{r},\mathbf{p}) = \overbrace{\left(\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j+\nabla_p^i Y^i(\mathbf{v})\right)}^{\text{collisions}} n(t,\mathbf{r},\mathbf{p})$$

$n(t,\mathbf{r},\mathbf{p})$ - distribution function of hard probes

$$\mathbf{v}\equiv\frac{\mathbf{p}}{E_{\mathbf{p}}},\quad \nabla_p^i\equiv\frac{\partial}{\partial p_i}$$

$$X^{ij}(\mathbf{v}),\ Y^i(\mathbf{v}) \quad \Rightarrow \quad \begin{cases} \dfrac{dE}{dx}=-\dfrac{\mathrm{v}^i}{\mathrm{v}}Y^i(\mathbf{v}) & \text{collisional energy loss} \\[2ex] \hat{q}=\dfrac{2}{\mathrm{v}}\left(\delta^{ij}-\dfrac{\mathrm{v}^i\mathrm{v}^j}{\mathrm{v}^2}\right)X^{ji}(\mathbf{v}) & \text{momentum broadening} \end{cases}$$

$$n(t,\mathbf{r},\mathbf{p})=n_{\mathrm{eq}}(\mathbf{p})\sim e^{-\frac{E_{\mathbf{p}}}{T}} \qquad \Leftrightarrow \qquad \boxed{Y^j(\mathbf{v})=\frac{\mathrm{v}^i}{T}X^{ij}(\mathbf{v})}$$

solves FK equation

Origin of Fokker-Planck Equation

Boltzmann equation

Diffusive approximation

$$p >> \Delta p$$

p – momentum of a hard probe

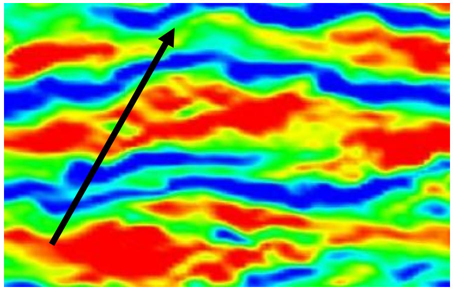
Δp – momentum transfer in a single collision

Fokker-Planck equation

- How to obtain a Fokker-Planck equation for glasma?

Apply the *quasilinear* method known in plasma physics.

Fokker-Planck equation of hard probes in glasma



$$\begin{cases} \hat{q} = \dfrac{2}{\mathrm{v}}\left(\delta^{ij} - \dfrac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right) X^{ji}(\mathbf{v}) \\ \dfrac{dE}{dx} = -\dfrac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}T} X^{ij}(\mathbf{v}) \end{cases}$$

$$X^{ij}(\mathbf{v}) = \frac{1}{2N_c}\int_0^t dt' \left\langle F_a^i(t,\mathbf{r}) F_a^j(t',\mathbf{r}')) \right\rangle$$

$$\mathbf{r}' \equiv \mathbf{r} - \mathbf{v}(t-t') \qquad \mathbf{F}_a(t,\mathbf{r}) = g\left(\mathbf{E}_a(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}_a(t,\mathbf{r})\right)$$

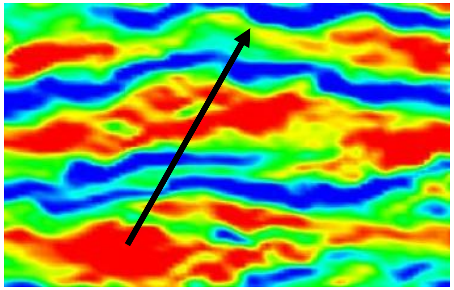
E^i, B^i - chromoelectric and chromomagnetic fields

two-point correlators

St. Mrówczyński, European Physical Journal A **54**, 43 (2018)

Momentum broadeing of hard probes

$$\hat{q} = \frac{1}{\mathrm{v}}\left(\delta^{ij} - \frac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right)\frac{\left\langle \Delta p^i \Delta p^j \right\rangle}{\Delta t}\Bigg|_{\Delta t \to 0}$$



Wong equations

$$\begin{cases} \dfrac{d\mathbf{r}(t)}{dt} = \mathbf{v}(t) \\[2ex] \dfrac{d\mathbf{p}(t)}{dt} = q_a \mathbf{F}_a(t,\mathbf{r}(t)) \end{cases}$$

Lorentz force

$$\mathbf{F}_a(t,\mathbf{r}) = g\left(\mathbf{E}_a(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}_a(t,\mathbf{r})\right)$$

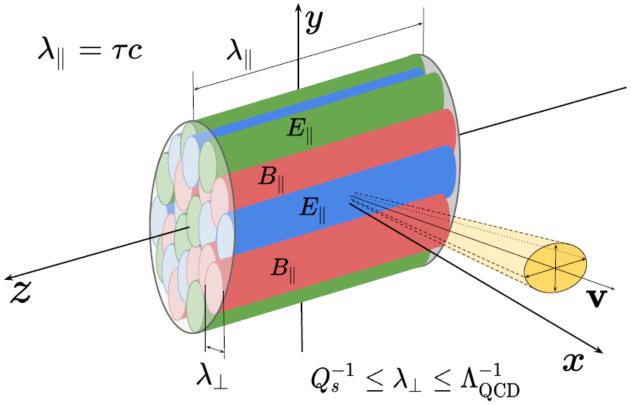
$$\Delta p^i = \int_0^t dt' q_a F_a^i\left(t',\mathbf{r}(t')\right)$$

$$\left\langle \Delta p^i \Delta p^j \right\rangle = \int_0^t dt' \int_0^t dt'' \underbrace{\int Dq\, q_a q_b}_{= \delta^{ab}\begin{cases}\frac{1}{2} & \text{quark} \\ N_c^2 - 1 & \text{gluon}\end{cases}} \left\langle F_a^i\left(t',\mathbf{r}(t')\right) F_b^j\left(t'',\mathbf{r}(t'')\right)\right\rangle$$

$$\boxed{\hat{q} = \frac{2}{\mathrm{v}}\left(\delta^{ij} - \frac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right) X^{ji}(\mathbf{v})}$$

M. Carrington, B. Friesen & St. Mrówczyński, arXiv:2604.02050, Phys. Rev. C in print

Hard probes in glasma - $\hat{q}$



$$\hat{q} = \frac{2}{\mathrm{v}}\left(\delta^{ij} - \frac{\mathrm{v}^i \mathrm{v}^j}{\mathrm{v}^2}\right) X^{ji}(\mathbf{v})$$

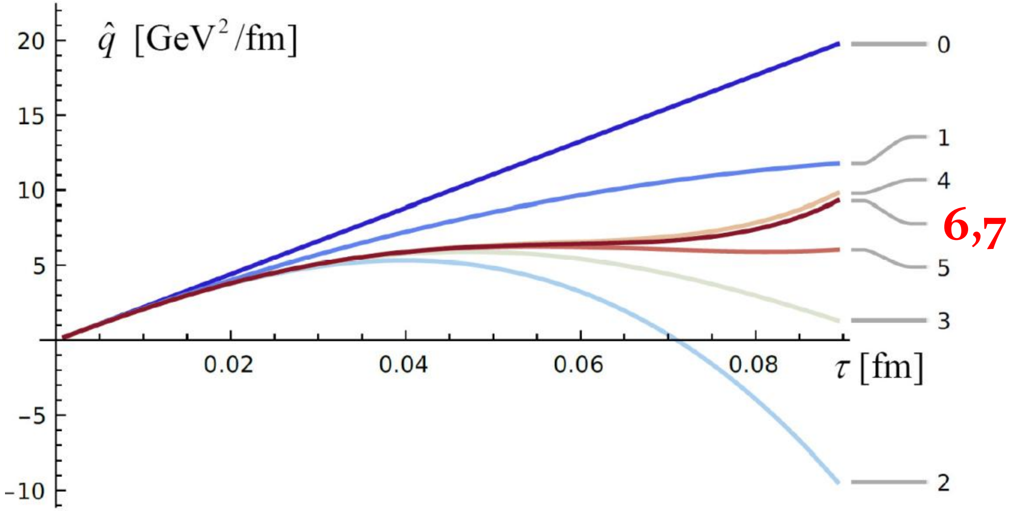
$N_c = 3, \quad g = 1$

$Q_s = 2\ \mathrm{GeV}$

$m = 0.2\ \mathrm{GeV}$

$\mathrm{v} = \mathrm{v}_\perp = 1$

order in
τ expansion



6,7

M. Carrington, W. Cowie, B. Friesen, St. Mrówczyński & D. Pickering, Phys. Rev. C **108**, 054903 (2023)

Guage dependence

$$X^{ij}(\mathbf{v}) = \frac{1}{2N_c}\int_0^t dt' \left\langle F_a^i(t,\mathbf{r}) F_a^j(t',\mathbf{r}')) \right\rangle$$

$$\mathbf{F}_a(t,\mathbf{r}) = g\left(\mathbf{E}_a(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}_a(t,\mathbf{r})\right)$$

Gauge transformation (in adjoint representation)

$$x \equiv (t,\mathbf{r})$$

$$F_a^i(x) \to U_{ab}(x) F_b^i(x) = F_b^i(x) U_{ba}^\dagger(x)$$

$$U_{ca}^\dagger(x) U_{ab}(x) = \delta^{bc}$$

$$\begin{cases} F_a^i(x) F_a^j(x) \to U_{ab}(x) F_b^i(x) F_c^j(x) U_{ca}^\dagger(x) = F_b^i(x) F_b^j(x) & \text{gauge invariant} \\ F_a^i(x) F_a^j(y) \to U_{ab}(x) F_b^i(x) F_c^j(y) U_{ca}^\dagger(y) = U_{ca}^\dagger(x) U_{ab}(y) F_b^i(x) F_c^j(y) & \text{gauge dependent} \end{cases}$$

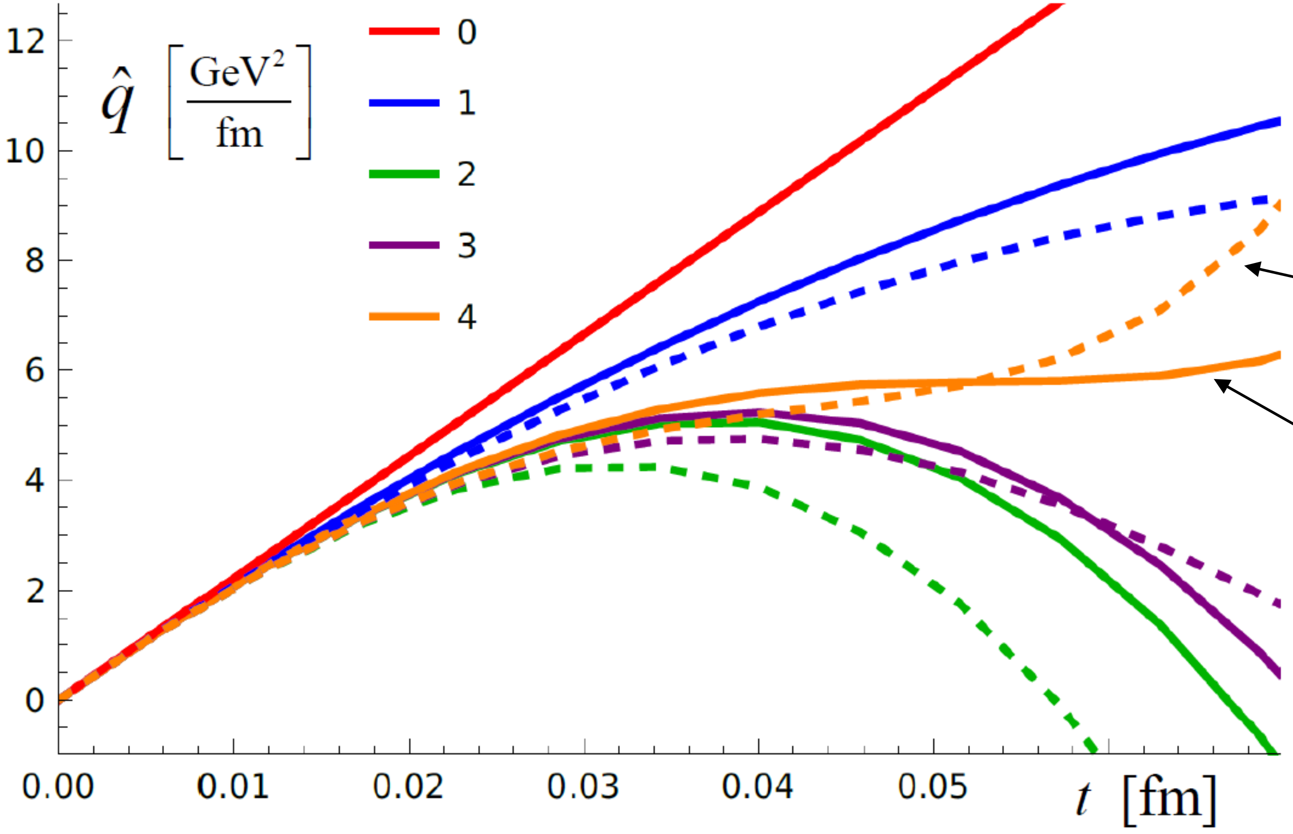
$$X^{ij}(\mathbf{v}) = \frac{1}{2N_c}\int_0^t dt' \left\langle F_a^i(t,\mathbf{r})\Omega_{ab}(t,\mathbf{r}\,|\,t',\mathbf{r}') F_b^j(t',\mathbf{r}')) \right\rangle$$

gauge invariant

$$\Omega(t,\mathbf{r}\,|\,t',\mathbf{r}') = P\exp\left(ig \int_{(t',\mathbf{r}')}^{(t,\mathbf{r})} ds_\mu A_c^\mu(s) T^c \right)$$

$$\Omega(x\,|\,y) \to U(x)\Omega(x\,|\,y)U^\dagger(y)$$

Gauge independent $\hat{q}$



gauge independent

gauge dependent

M. Carrington, B. Friesen & St. Mrówczyński, arXiv:2604.02050, Phys. Rev. C in print

Glasma impact on jet quenching

Glasma

$$\hat{q}_{\max} = 6 \ \ \mathrm{GeV}^2 / \mathrm{fm}$$

$$t_{\max} = 0.06 \ \ \mathrm{fm}$$

Equilibrium QGP

$$\hat{q} = 3T^3$$

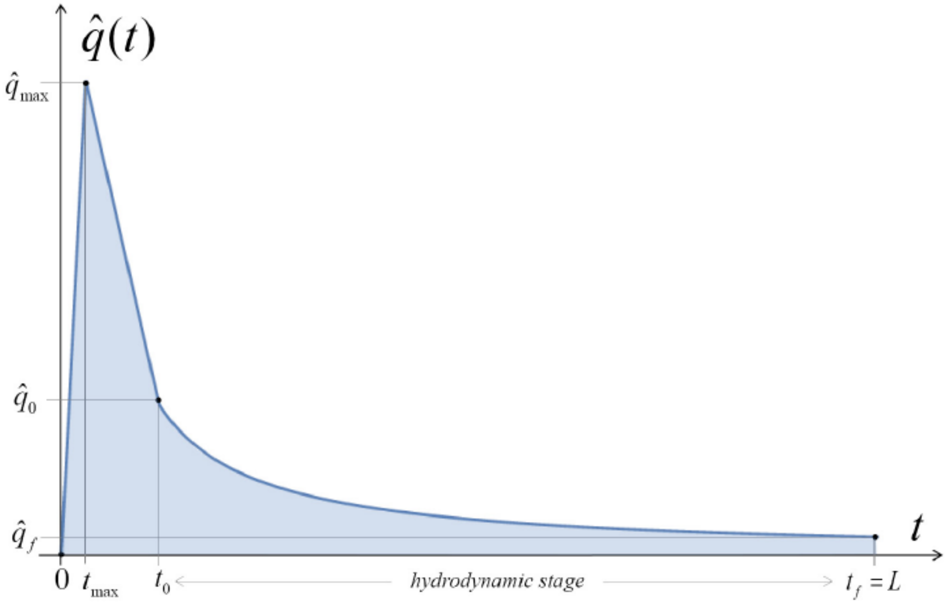
$$t_0 = 0.6 \ \mathrm{fm}$$

$$T_0 = 450 \ \mathrm{MeV}$$

$$\hat{q}_0 = 1.4 \ \ \mathrm{GeV}^2 / \mathrm{fm}$$

$$T = T_0 \left(\frac{t_0}{t} \right)^{1/3}$$

$$L = 10 \ \mathrm{fm}$$



$$\Delta p_T^2 \Big|^{\text{non-eq}} = \int_0^{t_0} dt \, \hat{q}(t)$$

$$\Delta p_T^2 \Big|^{\text{eq}} = \int_{t_0}^{L} dt \, \hat{q}(t)$$

$$\boxed{\frac{\Delta p_T^2 \Big|^{\text{non-eq}}}{\Delta p_T^2 \Big|^{\text{eq}}} = 0.93}$$

S. Cao et al. [JETSCAPE], Physical Review C **104**, 024905 (2021),
C. Shen, U. Heinz, P. Huovinen and H. Song, Physical Review C **84**, 044903 (2011).

Conclusions

- The glasma evolves in a hydrodynamic-like way.
- The glasma's global orbital momentum is small, the system does not rotate.
- There is a local angular momentum of glasma along the beam consistent with a sign of the Λ polarization.
- In spite of its short lifetime the glasma provides a significant contribution to the jet quenching.