Dynamics of Weakly Coupled Unstable Quark-Gluon Plasma

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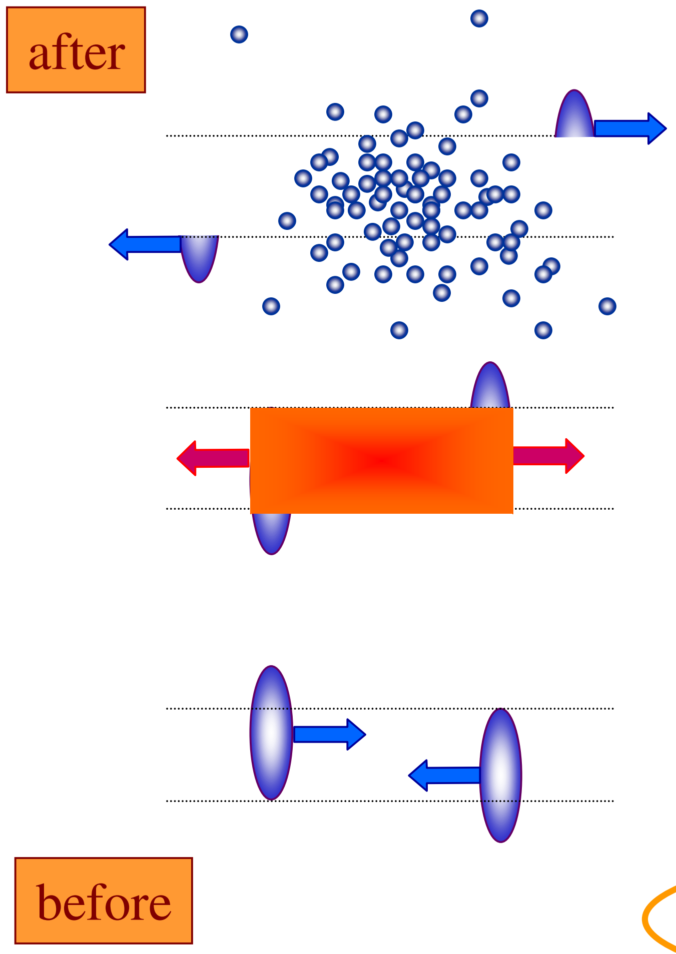
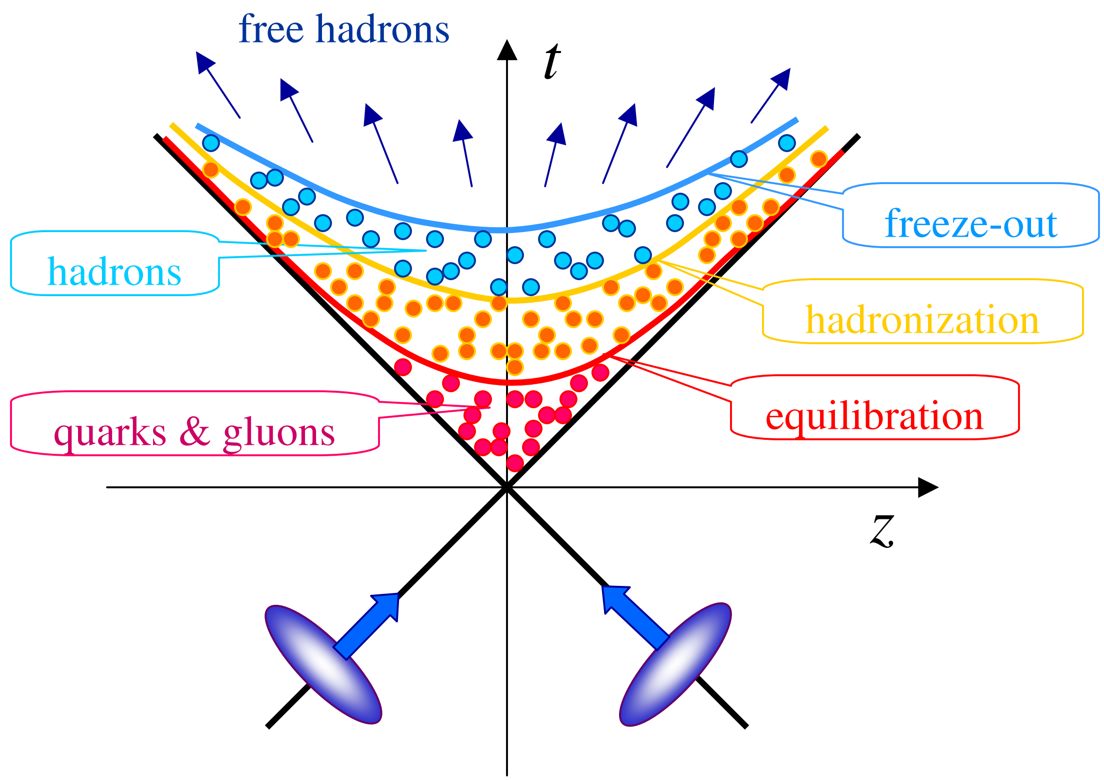
- Fluctuations of chromodynamic fields
- Momentum broadening of a fast parton - $\hat{q}$
- Process of equilibration

Happy birthday, Berndt!

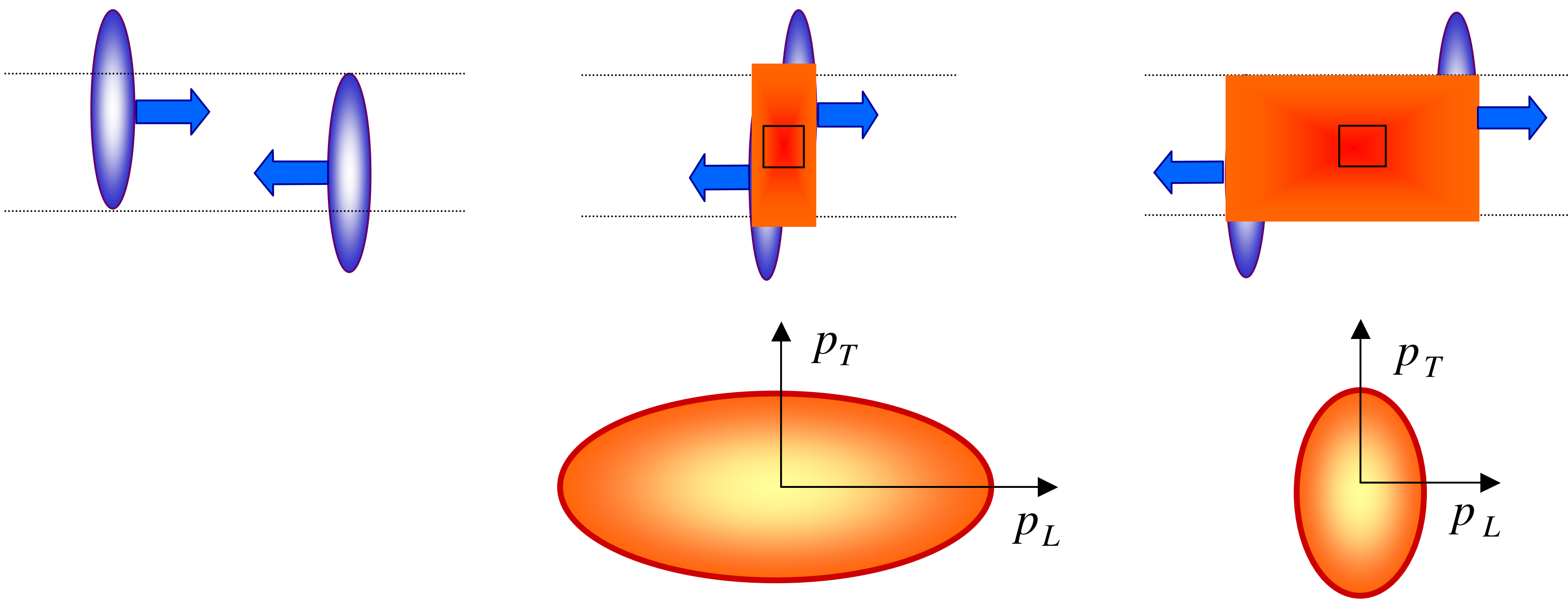
Scenario of relativistic heavy-ion collisions





At the early stage QGP is
out of equilibrium & weakly coupled

Anisotropic QGP @ RHIC



'prolate' 'oblate'

Anisotropic QGP is unstable due to magnetic plasma modes

Why fields in QGP are important?

Viscosity of magnetized plasma

$$\frac{1}{\eta} = \frac{1}{\eta_A} + \frac{1}{\eta_C}$$

collisional viscosity

$$\eta_C \sim \frac{T^3}{\alpha_s^2 \ln(1/\alpha_s)}$$

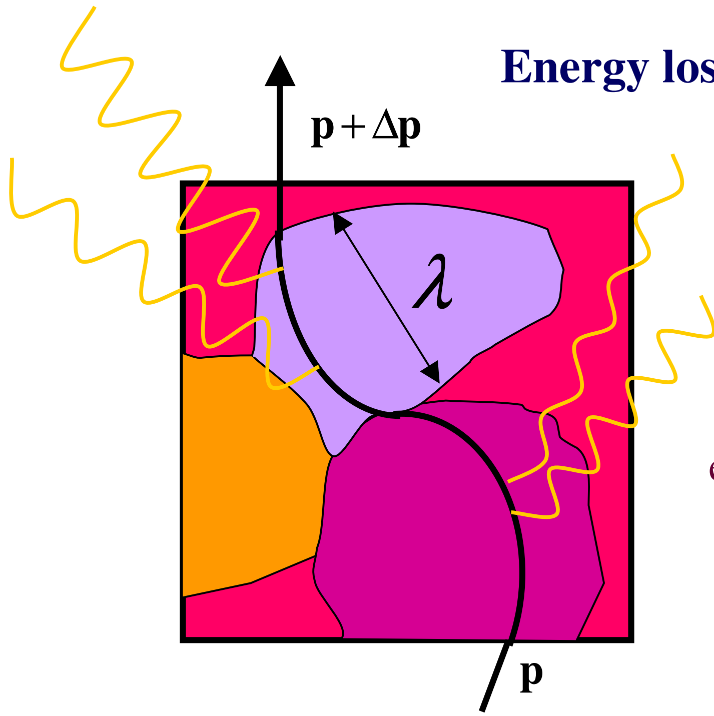
anomalous viscosity

$$\eta_A \sim \frac{1}{g^2 \langle \mathbf{B}^2 \rangle \lambda}$$

Viscosity of magnetized QGP can be small

M. Asakawa, S.A. Bass and B. Müller, Prog. Theor. Phys. **116**, 725 (2006)

Energy loss in magnetized plasma



enhanced energy loss due to synchrotron radiation

Magnetized QGP can be very opaque

B.G. Zakharov, JETP Lett. 88, 475 (2008)

How to compute field correlators in unstable plasma?

- Equilibrium methods are not applicable
- We deal with the initial value problem

The kinetic theory method by Klimontovich & Silin, Rostoker, Tsytovich, see E.M. Lifshitz and L.P. Pitaevskii, *Physical Kinetics*

St. Mrówczynski, Acta Phys. Pol. **B39** (2008) 941 - Electromagnetic Fluctuations
St. Mrówczynski, Phys. Rev. **D77** (2008) 105022 - Chromodynamic Fluctuations

Transport equations

fundamental	$p_\mu D^\mu Q - \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu Q\} = C[Q, \overline{Q}, G]$	quarks
	$p_\mu D^\mu \overline{Q} + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu \overline{Q}\} = \overline{C}[Q, \overline{Q}, G]$	antiquarks
adjoint	$p_\mu \mathcal{D}^\mu G - \frac{g}{2} p^\mu \{\mathcal{F}_{\mu\nu}, (x) \partial_p^\nu G\} = C_g[Q, \overline{Q}, G]$	gluons

free streaming

mean-field force

collisions

$$D^\mu \equiv \partial^\mu - ig[A^\mu, ...], \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

$$D_\mu F^{\mu\nu} = j^\nu[Q, \overline{Q}, G]$$

mean-field generation

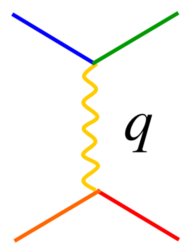
collisionless limit: $C = \overline{C} = C_g = 0$

Time scale of collisional processes

Time scale of processes driven by parton-parton scattering

$$t_{\text{hard}} \sim \frac{1}{g^4 \ln(1/g)\,T}$$

$$t_{\text{soft}} \sim \frac{1}{g^2 \ln(1/g)\,T}$$



hard scattering: $q \sim T$

soft scattering: $q \sim gT$

Time scale of collective phenomena

$$t_{\text{collec}} \sim \frac{1}{g\,T}$$

$$g^2 \ll 1 \quad \Rightarrow \quad t_{\text{hard}} \gg t_{\text{soft}} \gg t_{\text{collec}}$$

The instabilities are fast if QGP is weakly coupled

Small fluctuations

The distribution function of quarks

fluctuation

$$Q(t,\mathbf{r},\mathbf{p}) = Q_0(\mathbf{p}) + \delta Q\,(t,\mathbf{r},\mathbf{p})$$

stationary colorless state $\quad Q_0^{ij}(\mathbf{p}) = \delta^{ij} n(\mathbf{p})$

$$|Q_0(\mathbf{p})| >> |\delta Q(t,\mathbf{r},\mathbf{p})|, \quad |\nabla_p Q_0(\mathbf{p})| >> |\nabla_p \delta Q(t,\mathbf{r},\mathbf{p})|$$

$$\mathbf{E}(t,\mathbf{r}), \mathbf{B}(t,\mathbf{r}), A^0(t,\mathbf{r}), \mathbf{A}(t,\mathbf{r}) \sim \delta Q(t,\mathbf{r},\mathbf{p})$$

quarks only, inclusion of antiquarks and gluons: $n(\mathbf{p}) \rightarrow n(\mathbf{p}) + \bar{n}(\mathbf{p}) + 2N_c n_g(\mathbf{p})$

Linearized equations

Transport equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\delta Q(t,\mathbf{r},\mathbf{p}) - g\left(\mathbf{E}(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{r})\right)\nabla_p n(\mathbf{p}) = 0$$

Yang-Mills (Maxwell) equations

$$\nabla\cdot\mathbf{E}(t,\mathbf{r}) = \rho(t,\mathbf{r}), \qquad \nabla\cdot\mathbf{B}(t,\mathbf{r}) = 0,$$

$$\nabla\times\mathbf{E}(t,\mathbf{r}) = -\frac{\partial\mathbf{B}(t,\mathbf{r})}{\partial t}, \quad \nabla\times\mathbf{B}(t,\mathbf{r}) = \mathbf{j}(t,\mathbf{r}) + \frac{\partial\mathbf{E}(t,\mathbf{r})}{\partial t}$$

$$\begin{cases} \rho_a(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\,\mathrm{Tr}\left[\tau^a\delta Q(t,\mathbf{r},\mathbf{p})\right], \\ \mathbf{j}_a(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\,\mathbf{v}\,\mathrm{Tr}\left[\tau^a\delta Q(t,\mathbf{r},\mathbf{p})\right], \end{cases}$$

gauge dependence discussed *a posteriori*

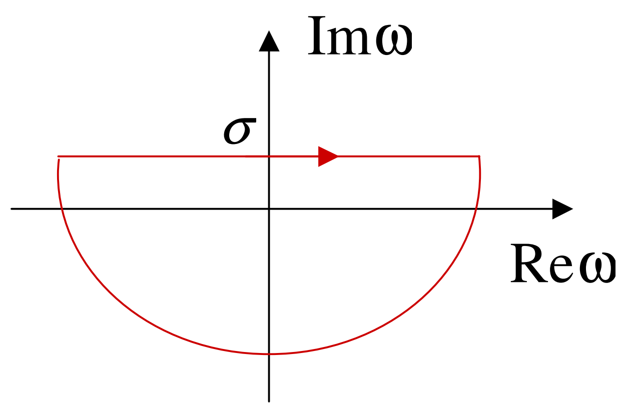
Initial value problem

$$\delta Q(t=0,\mathbf{r},\mathbf{p}) = \delta Q_0(\mathbf{r},\mathbf{p}),$$
$$\mathbf{E}(t=0,\mathbf{r},\mathbf{p}) = \mathbf{E}_0(\mathbf{r},\mathbf{p}), \quad \mathbf{B}(t=0,\mathbf{r},\mathbf{p}) = \mathbf{B}_0(\mathbf{r},\mathbf{p})$$

One-sided Fourier transformations

$$\begin{cases} f(\omega,\mathbf{k}) = \int_0^\infty dt \int d^3r\, e^{i(\omega t-\mathbf{kr})} f(t,\mathbf{r}) \\ f(t,\mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3}\, e^{-i(\omega t-\mathbf{kr})} f(\omega,\mathbf{k}) \end{cases}$$

$$0 < \sigma \in R$$



Transformed linear equations

Transport equation

$$-i(\omega - \mathbf{v}\cdot\mathbf{k})\delta Q(\omega,\mathbf{k},\mathbf{p}) - g\big(\mathbf{E}(\omega,\mathbf{k}) + \mathbf{v}\times\mathbf{B}(\omega,\mathbf{k})\big)\nabla_p n(\mathbf{p}) = \delta Q_0(\mathbf{k},\mathbf{p})$$

Yang-Mills (Maxwell) equations

$$i\mathbf{k}\cdot\mathbf{E}(\omega,\mathbf{k}) = \rho(\omega,\mathbf{k}), \qquad i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0,$$
$$i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = i\omega\mathbf{B}(\omega,\mathbf{k}) + \mathbf{B}_0(\mathbf{k}),$$
$$i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = \mathbf{j}(\omega,\mathbf{k}) - i\omega\mathbf{E}(\omega,\mathbf{k}) - \mathbf{E}_0(\mathbf{k})$$

$$\begin{cases} \rho_a(\omega,\mathbf{k}) = -g\int \frac{d^3p}{(2\pi)^3}\,\mathrm{Tr}\left[\tau^a\delta Q(\omega,\mathbf{k},\mathbf{p})\right], \\ \mathbf{j}_a(\omega,\mathbf{k}) = -g\int \frac{d^3p}{(2\pi)^3}\,\mathbf{v}\,\mathrm{Tr}\left[\tau^a\delta Q(\omega,\mathbf{k},\mathbf{p})\right], \end{cases}$$

Solution

$$\left[-\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})\right]E^j(\omega,\mathbf{k}) = -g\omega\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{v}\cdot\mathbf{k}}\delta Q_0(\mathbf{k},\mathbf{p})$$

$$-i\frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{v}\cdot\mathbf{k}}\frac{\mathbf{v}\times\mathbf{B}_0(\mathbf{k})}{\omega}\cdot\nabla_p n(\mathbf{p}) + i\omega E_0^i(\mathbf{k}) - i\left(\mathbf{k}\times\mathbf{B}_0(\mathbf{k})\right)^i$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

Isotropic system

$$\varepsilon^{ij}(\omega,\mathbf{k}) \equiv \varepsilon_L(\omega,\mathbf{k})\frac{k^i k^j}{\mathbf{k}^2} + \varepsilon_T(\omega,\mathbf{k})\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

$$\left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k}) = \frac{1}{\omega^2\varepsilon_L(\omega,\mathbf{k})}\frac{k^i k^j}{\mathbf{k}^2} + \frac{1}{\omega^2\varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

Fluctuations of E field

The solution

$$E^i(\omega,\mathbf{k}) = \left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k})\underbrace{\left[\ldots..\ \delta Q_0(\mathbf{k},\mathbf{p}) + \ldots..\ \mathbf{E}_0(\mathbf{k}) + \ldots..\ \mathbf{B}_0(\mathbf{k})\right]^j}_{\text{initial values}}$$

The correlation function

$$\begin{aligned}\left\langle E^i(\omega,\mathbf{k})E^j(\omega',\mathbf{k}')\right\rangle = \left(\Sigma^{-1}\right)^{ik}(\omega,\mathbf{k})\left(\Sigma^{-1}\right)^{jl}(\omega',\mathbf{k}')\Big[\ldots..\ \left\langle \delta Q_0(\mathbf{k},\mathbf{p})\delta Q_0(\mathbf{k}',\mathbf{p}')\right\rangle \\ + \ldots..\ \left\langle \delta Q_0(\mathbf{k},\mathbf{p})E_0^m(\mathbf{k}')\right\rangle + \ldots..\ \left\langle \delta Q_0(\mathbf{k},\mathbf{p})B_0^m(\mathbf{k}')\right\rangle \\ + \ldots..\ \left\langle E_0^m(\mathbf{k})E_0^n(\mathbf{k}')\right\rangle + \ldots..\ \left\langle E_0^m(\mathbf{k})B_0^n(\mathbf{k}')\right\rangle \\ + \ldots..\ \left\langle B_0^m(\mathbf{k})B_0^n(\mathbf{k}')\right\rangle\Big]^{kl}\end{aligned}$$

$\langle \cdots \rangle$ - statistical ensemble average

Initial values

Using Maxwell equations

$\mathbf{E}_0(\mathbf{k})$, $\mathbf{B}_0(\mathbf{k})$, $\rho_0(\mathbf{k})$, $\mathbf{j}_0(\mathbf{k})$ can be expressed through $\delta Q_0(\mathbf{k},\mathbf{p})$

Initial fluctuations

color indices $i, j, k, l = 1, 2, \ldots, N_c$

$$\left\langle \delta Q_0^{ij}(\mathbf{r},\mathbf{p})\, \delta Q_0^{kl}(\mathbf{r}',\mathbf{p}') \right\rangle = ?$$

Assumption

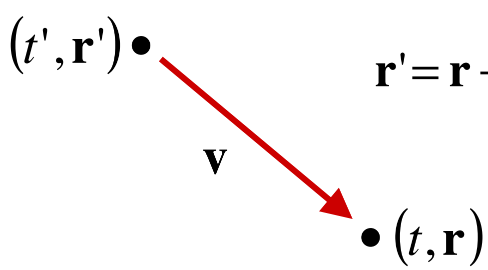
The initial fluctuations are given by $\left\langle \delta Q^{ij}(t=0,\mathbf{r},\mathbf{p})\, \delta Q^{kl}(t'=0,\mathbf{r}',\mathbf{p}') \right\rangle_{\text{free}}$

colorless state

$$\delta Q^{ij}(t,\mathbf{r},\mathbf{p}) \equiv Q^{ij}(t,\mathbf{r},\mathbf{p}) - \left\langle Q^{ij}(t,\mathbf{r},\mathbf{p}) \right\rangle = Q^{ij}(t,\mathbf{r},\mathbf{p}) - \delta^{ij} n(\mathbf{p})$$

Classical limit

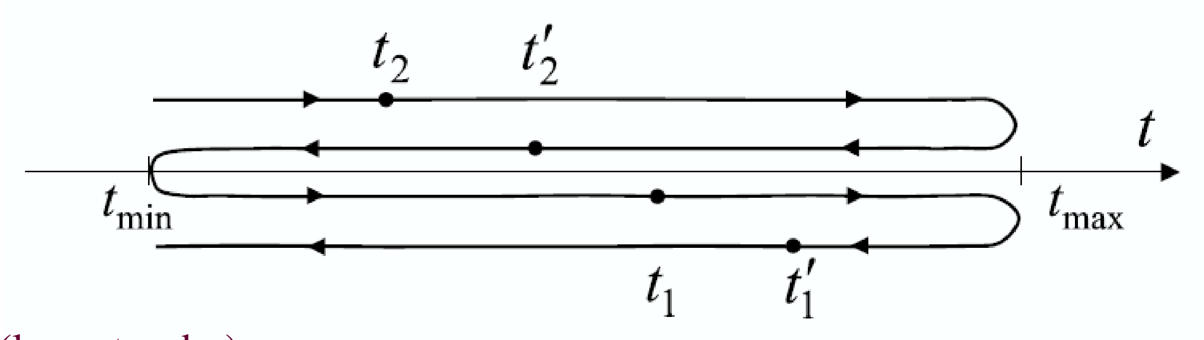
$$\left\langle \delta Q^{ij}(t,\mathbf{r},\mathbf{p})\, \delta Q^{kl}(t',\mathbf{r}',\mathbf{p}') \right\rangle_{\text{free}} = \delta^{il}\delta^{jk}(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')(2\pi)^3\delta^{(3)}\big(\mathbf{r}'-\mathbf{r}-\mathbf{v}(t'-t)\big)n(\mathbf{p})$$



$$\mathbf{r}' = \mathbf{r} + \mathbf{v}(t'-t)$$

Fluctuations of free distribution functions cont.

$$\left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle = \left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle$$



Wick theorem (lowest order)

$$\left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle = \left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\right)\right\rangle\left\langle T_c\left(\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle + \left\langle T_c\left(\varphi_j^*(x'_1)\varphi_k(x_2)\right)\right\rangle\left\langle T_c\left(\varphi_l^*(x'_2)\varphi_i(x_1)\right)\right\rangle$$

$$\left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle = \left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\right\rangle\left\langle \varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle + \left\langle \varphi_j^*(x'_1)\varphi_k(x_2)\right\rangle\left\langle \varphi_i(x_1)\varphi_l^*(x'_2)\right\rangle$$

Fluctuations in isotropic (stable) system

$$\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle = \frac{g^2}{2}\delta^{ab}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\int\frac{d^3p}{(2\pi)^3}n(\mathbf{p})F(\omega,\mathbf{k},\omega',\mathbf{k}',\mathbf{p})$$

colorless background

translational invariance

$F(\omega,\mathbf{k},\omega',\mathbf{k}',\mathbf{p})$ has poles at:

particle-wave resonance

$$\begin{cases}\omega-\mathbf{v}\cdot\mathbf{k}=0\\ \omega'-\mathbf{v}'\cdot\mathbf{k}'=0\end{cases}$$

collective longitudinal modes

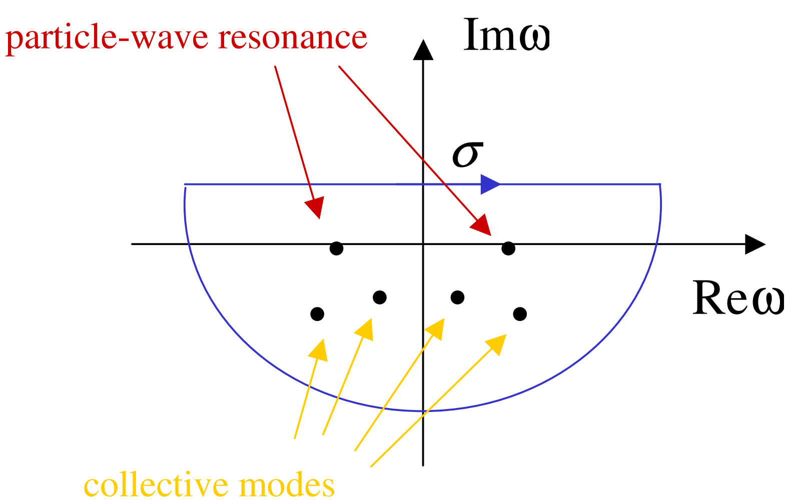
$$\begin{cases}\varepsilon_L(\omega,\mathbf{k})=0\\ \varepsilon_L(\omega',\mathbf{k}')=0\end{cases}$$

collective transverse modes

$$\begin{cases}\omega^2\varepsilon_T(\omega,\mathbf{k})-\mathbf{k}^2=0\\ \omega'^2\,\varepsilon_T(\omega',\mathbf{k}')-\mathbf{k}'^2=0\end{cases}$$

Fluctuations in isotropic (stable) system

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle = \int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi}\int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega'}{2\pi}\int\frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\,e^{-i(\omega t+\omega' t'-\mathbf{k}\mathbf{r}-\mathbf{k}'\mathbf{r}')}\times\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle$$



$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle \sim f(\mathbf{r}-\mathbf{r}')$$

$$\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle \sim \delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle = \begin{pmatrix}\text{collective}\\ \text{modes}\end{pmatrix}\left(e^{-\gamma t}\ \text{or}\ e^{-\gamma t'}\right) + \begin{pmatrix}\text{particle-wave}\\ \text{resonance}\end{pmatrix} f(t-t')$$

$$\gamma \equiv \text{Im}\,\omega > 0$$

Fluctuations in equilibrium system

Long time limit $t, t' \to \infty \quad \left\langle E_a^i(t,\mathbf{r}) E_b^j(t',\mathbf{r}') \right\rangle_\infty = f(t'-t, \mathbf{r}'-\mathbf{r})$

$$\left\langle E_a^i(t,\mathbf{r}) E_b^j(t',\mathbf{r}') \right\rangle_\infty = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i\left(\omega(t-t')-\mathbf{k}(\mathbf{r}-\mathbf{r}')\right)} \left\langle E_a^i E_b^j \right\rangle_{\omega,\mathbf{k}}$$

fluctuation spectrum

Fluctuation dissipation relation

$$\left\langle E_a^i E_b^j \right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab} T \omega^3 \left[\frac{k^i k^j}{\mathbf{k}^2} \frac{\operatorname{Im} \varepsilon_L(\omega,\mathbf{k})}{|\omega^2 \varepsilon_L(\omega,\mathbf{k})|^2} + \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) \frac{\operatorname{Im} \varepsilon_T(\omega,\mathbf{k})}{|\omega^2 \varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2|^2} \right]$$

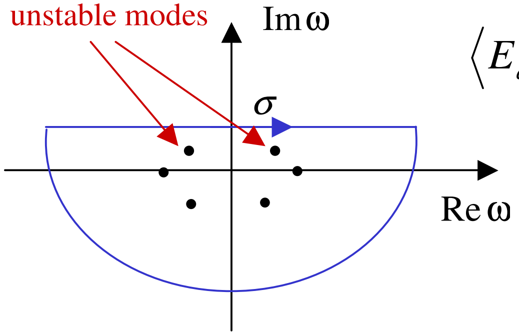
$$\left\langle B_a^i B_b^j \right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab} T \omega \left(\mathbf{k}^2 \delta^{ij} - k^i k^j \right) \frac{\operatorname{Im} \varepsilon_T(\omega,\mathbf{k})}{|\omega^2 \varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2|^2}$$

Fluctuations in unstable systems

Two-stream system

$$n(\mathbf{p}) = (2\pi)^3 n\left[\delta^{(3)}(\mathbf{p}-\mathbf{q}) + \delta^{(3)}(\mathbf{p}+\mathbf{q})\right]$$

Longitudinal electric field: $\omega_+(\mathbf{k})$ - stable mode, $\omega_-(\mathbf{k})$ - unstable mode



$$\left\langle E_a^i(\omega,\mathbf{k}) E_b^i(\omega',\mathbf{k}') \right\rangle = \frac{g^2}{2}\delta^{ab}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\frac{\mathbf{k}\cdot\mathbf{k}'}{\mathbf{k}^2\mathbf{k}'^2}$$
$$\times \frac{1}{\varepsilon_L(\omega,\mathbf{k})}\frac{1}{\varepsilon_L(\omega',\mathbf{k}')}\int\frac{d^3p}{(2\pi)^3}\frac{n(\mathbf{p})}{(\omega-\mathbf{v}\cdot\mathbf{k})(\omega'-\mathbf{v}'\cdot\mathbf{k}')}$$

broken time translational invariance

$$\left\langle E_a^i(t,\mathbf{r}) E_b^i(t',\mathbf{r}') \right\rangle_{\text{unstable}} = \frac{g^2}{2}\delta^{ab} n\int\frac{d^3k}{(2\pi)^3}\frac{e^{-i\mathbf{k}(\mathbf{r}-\mathbf{r}')}}{\mathbf{k}^2}\frac{1}{\left(\omega_+^2-\omega_-^2\right)^2}\frac{\left(\gamma_\mathbf{k}^2+(\mathbf{k}\mathbf{u})^2\right)^2}{\gamma_\mathbf{k}^2}$$
$$\times\left[\left(\gamma_\mathbf{k}^2+(\mathbf{k}\mathbf{u})^2\right)\cosh\left(\gamma_\mathbf{k}(t+t')\right)+\left(\gamma_\mathbf{k}^2-(\mathbf{k}\mathbf{u})^2\right)\cosh\left(\gamma_\mathbf{k}(t-t')\right)\right]$$

$\mathbf{u}\equiv\frac{\mathbf{q}}{E_\mathbf{q}}$, $\gamma_\mathbf{k}\equiv\text{Im}\,\omega_-(\mathbf{k})$

Gauge dependence

Generic correlation function: $L_{ab}(x,x') \equiv \langle H_a(x) K_b(x') \rangle$

Infinitesimal gauge transformation

$$H_a(x) \to H_a(x) + f_{abc}\lambda_b(x) H_c(x)$$

$$L_{ab}(x,x') \to L_{ab}(x,x') + f_{acd}\lambda_c(x) L_{db}(x,x') + f_{bcd}\lambda_c(x') L_{ad}(x,x')$$

colorless background

Actual correlation function: $L_{ab}(x,x') \equiv \delta^{ab} L(x,x')$

$$L_{ab}(x,x') \to \left(\delta^{ab} + f_{acb}\lambda_c(x) + f_{bca}\lambda_c(x')\right) L(x,x')$$

$L_{aa}(x,x') = \left(N_c^2 - 1\right) L(x,x')$ **- gauge invariant!**

Conclusions I

$$\left.\begin{array}{l}\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle_{\text{unstable}}\\ \left\langle B_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle_{\text{unstable}}\\ \left\langle B_a^i(t,\mathbf{r})B_b^j(t',\mathbf{r}')\right\rangle_{\text{unstable}}\end{array}\right\} \sim e^{2\gamma t}$$

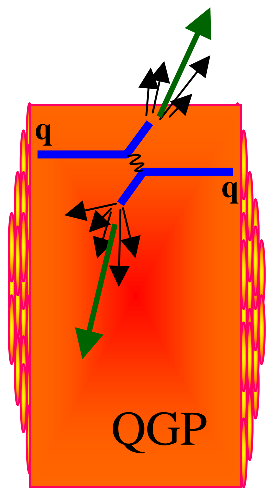
γ - growth rate of the fastest mode

- unstable QGP is generically time dependent
- there are strong fields in unstable QGP

Jet quenching @ RHIC

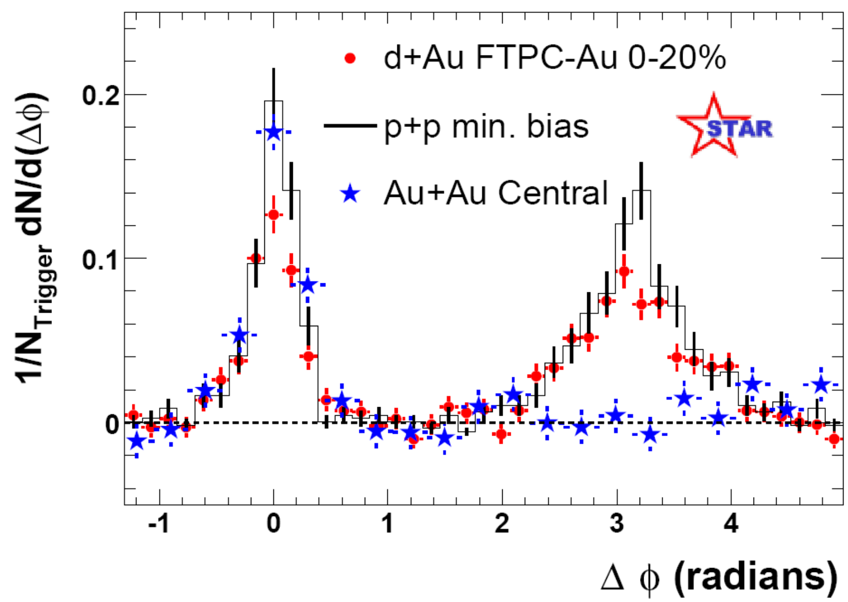
trigger

near-side



away-side

near-side away-side



Away-side jet is suppressed in central collisions

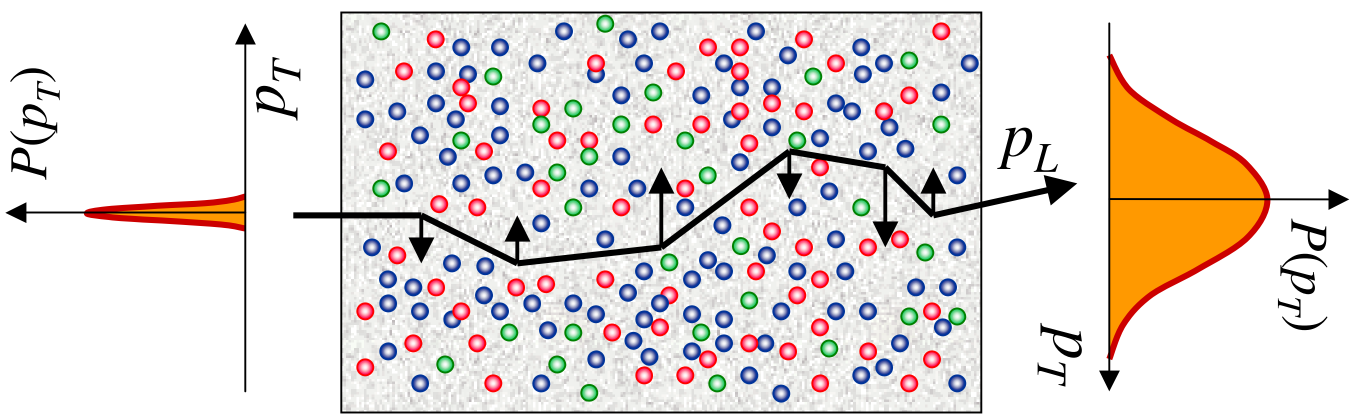
J. Adams et al. [STAR Collaboration], Nucl. Phys. **A757**, 102 (2005)

Momentum transverse broadening

Radiative energy loss of a fast parton is controlled by

$$\hat{q} \equiv \frac{d\left\langle \Delta p_T^2(t) \right\rangle}{dt}$$

Baier, Dokshitzer, Mueller, Peigne & Schiff 1996



From experiment: $\underbrace{0.5\ \mathrm{GeV}^2/\mathrm{fm}}_{\text{pQGP}} \leq \hat{q} \leq \underbrace{15\ \mathrm{GeV}^2/\mathrm{fm}}_{?}$

Is QGP strongly coupled ?

Momentum broadening in anisotropic QGP

- P. Romatschke, Phys. Rev. **C75**, 014901 (2007)
- R. Baier and Y. Mehtar-Tani, Phys. Rev. **C78**, 064906 (2008)

Anisotropic (unstable) QGP was treated as a static medium

$\hat{q} = \text{const}$

Unstable QGP is generically time dependent

Numerical simulations $\hat{q} \sim e^{2\gamma t}$

- A. Dumitru, Y. Nara, B. Schenke & M. Strickland, Phys. Rev. **C78**, 024909 (2008); B. Schenke, M. Strickland, A. Dumitru, Y. Nara & C. Greiner, Phys. Rev. **C79**, 034903 (2009)

Fast parton in QGP

Wong's equation of motion

$$\begin{cases} \dfrac{dx^\mu(\tau)}{d\tau} = u^\mu(\tau) \\ \dfrac{dp^\mu(\tau)}{d\tau} = gQ_a(\tau)\, F_a^{\mu\nu}(x(\tau))\, u_\nu(\tau) \\ \dfrac{dQ_a(\tau)}{d\tau} = -gf^{abc}\, p_\mu(\tau)\, A_b^\mu(x(\tau))\, Q_c(\tau) \end{cases}$$

Initial value problem

Gauge condition

$$p_\mu(\tau)\, A_b^\mu\big(x(\tau)\big) = 0 \quad \Rightarrow \quad Q_a(\tau) = \text{const}$$

Parton travels with constant velocity: $u^\mu = (\gamma, \gamma\mathbf{v}) = \text{const}$

$$p^\mu(\tau) = p^\mu(0) + gQ_a \int_0^\tau d\tau'\, F_a^{\mu\nu}(x(\tau'))\, u_\nu$$

A. Majumder, B. Müller & St. Mrówczyński, Physical Review **D80**, 125020 (2009)

Langevin problem

$$\left\langle p^{\mu}(\tau)\, p^{\nu}(\tau)\right\rangle = p^{\mu}(0)\, p^{\nu}(0) + g^2 C \int_0^{\tau} d\tau_1 \int_0^{\tau} d\tau_2 \left\langle F_a^{\mu\nu}(x(\tau_1)) F_a^{\sigma\rho}(x(\tau_2))\right\rangle u_{\sigma} u_{\rho}$$

- $p^{\mu}(0) = \left(E, 0, 0, p_z\right)$
- parton travels with speed of light along axis z: $\mathbf{v}(t) = \mathrm{const} = (0,0,1)$

$$C \equiv \begin{cases} \dfrac{1}{2N_c} & - \text{ quark} \\ \dfrac{N_c}{N_c^2 - 1} & - \text{ gluon} \end{cases}$$

$$\left\langle \Delta p_T^2(t)\right\rangle = \left\langle p_x^2(t)\right\rangle + \left\langle p_y^2(t)\right\rangle = g^2 C \int_0^t dt_1 \int_0^t dt_2 \left\langle \left(F_a^{\mu 0}(x(t_1)) - F_a^{\mu 3}(x(t_1))\right)\left(F_a^{\nu 0}(x(t_2)) - F_a^{\nu 3}(x(t_2))\right)\right\rangle$$

$$\begin{aligned}\left\langle \Delta p_T^2(t)\right\rangle = g^2 C \int_0^t dt_1 \int_0^t dt_2 \Big\{ & \left\langle E_a^x(x(t_1)) E_a^x(x(t_2))\right\rangle + \left\langle E_a^y(x(t_1)) E_a^y(x(t_2))\right\rangle - \left\langle E_a^x(x(t_1)) B_a^y(x(t_2))\right\rangle \\ & + \left\langle E_a^y(x(t_1)) B_a^x(x(t_2))\right\rangle - \left\langle B_a^y(x(t_1)) E_a^x(x(t_2))\right\rangle - \left\langle B_a^x(x(t_1)) E_a^y(x(t_2))\right\rangle \\ & + \left\langle B_a^x(x(t_1)) B_a^x(x(t_2))\right\rangle + \left\langle B_a^y(x(t_1)) B_a^y(x(t_2))\right\rangle \Big\}\end{aligned}$$

Momentum broadening in equilibrium QGP

space-time translational invariance

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle_\infty = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}\, e^{-i(\omega(t-t')-\mathbf{k}(\mathbf{r}-\mathbf{r}'))}\left\langle E_a^i E_b^j\right\rangle_{\omega,\mathbf{k}}$$

fluctuation spectrum

$\mathbf{r}(t)=(0,0,t)$ - parton's trajectory

$$\int_0^t dt_1\int_0^t dt_2 e^{-i(\omega-k_z)(t_1-t_2)} = \frac{4}{(\omega-k_z)^2}\sin\left(\frac{(\omega-k_z)t}{2}\right) \xrightarrow[t\to\infty]{} 2\pi t\,\delta(\omega-k_z)$$

$$\hat{q} = 2g^2 C_R T\int\frac{d^3k}{(2\pi)^3}\frac{k_T^2}{k_z\mathbf{k}^2}\left[\frac{\operatorname{Im}\varepsilon_L(k_z,\mathbf{k})}{|\varepsilon_L(k_z,\mathbf{k})|^2} + \frac{k_z^2 k_T^2\operatorname{Im}\varepsilon_T(k_z,\mathbf{k})}{|k_z^2\varepsilon_T(k_z,\mathbf{k})-\mathbf{k}^2|^2}\right]$$

$$\hat{q} \approx \frac{g^2}{2\pi} C_R\, m_D^2\, T\ln(1/g)$$

$$C_R \equiv \begin{cases} \frac{1}{2} & -\text{ quark} \\ N_c & -\text{ gluon}\end{cases}$$

Moore & Teaney 2005; Baier & Mehtar-Tani 2008

Momentum broadening in unstable QGP

Two-stream system

longitudinal electric fields only

$$\left\langle \Delta p_T^2(t) \right\rangle = g^2 C \int_0^t dt_1 \int_0^t dt_2 \left\{ \left\langle E_a^x(x(t_1)) E_a^x(x(t_2)) \right\rangle + \left\langle E_a^y(x(t_1)) E_a^y(x(t_2)) \right\rangle \right\}$$

$$\left\langle \Delta p_T^2(t) \right\rangle \approx \frac{g^4}{4} C_R\, n \int \frac{d^3k}{(2\pi)^3}\, e^{2\gamma_{\mathbf{k}} t}\, \frac{k_T^2 \left(\gamma_{\mathbf{k}}^2 + (\mathbf{ku})^2\right)^3}{\mathbf{k}^4 \left(\omega_+^2 - \omega_-^2\right)^2 \gamma_{\mathbf{k}}^2 \left(k_z^2 + \gamma_{\mathbf{k}}^2\right)}$$

A. Majumder, B. Müller & St. Mrówczyński, Physical Review **D80**, 125020 (2009)

Conclusions II

- $\hat{q} \sim e^{2\gamma t}$

- QGP is weakly coupled but $\hat{q}$ can be large

Equilibration in quasi-linear approach

The quasi-linear kinetic theory of weakly turbulent plasma

A.A. Vedenov, E. P. Velikhov and R.Z. Sagdeev,
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A.A. Vedenov, Atomnaya Energiya **13**, 5 (1962) [in Russian];
J. Nucl. Energy C **5**, 169 (1963).

E.M. Lifshitz and L.P. Pitaevskii, *Physical Kinetics*

Application to QGP: St. Mrówczyński & B. Müller, Physical Review **D80**, 065021 (2010)

Equilibration of quarks

The distribution function of quarks

fluctuating part

$$Q(t,\mathbf{r},\mathbf{p}) = \langle Q(t,\mathbf{r},\mathbf{p})\rangle + \delta Q\,(t,\mathbf{r},\mathbf{p})$$

regular colorless part

$$\langle Q(t,\mathbf{r},\mathbf{p})\rangle = n(t,\mathbf{r},\mathbf{p})I$$

$$|\,n\,| >> |\,\delta Q\,|, \quad |\,\nabla_p n\,| >> |\,\nabla_p \delta Q\,|$$

$$\left|\frac{\partial n}{\partial t}\right| << \left|\frac{\partial\, \delta Q}{\partial t}\right|, \quad |\,\nabla n\,| << |\,\nabla\, \delta Q\,|$$

$$\langle \mathbf{E}\rangle = 0, \quad \langle \mathbf{B}\rangle = 0, \quad \mathbf{E}, \mathbf{B}, A^0\mathbf{A} \sim \delta Q$$

Quarks in fluctuating background

$$Q(t,\mathbf{r},\mathbf{p}) = n(t,\mathbf{r},\mathbf{p})I + \delta Q\,(t,\mathbf{r},\mathbf{p})$$

longitudinal electric fields only

$$\left(D^0 + \mathbf{v}\cdot\mathbf{D}\right)Q - g\mathbf{E}\cdot\nabla_p Q = 0$$

$\mathrm{Tr}\langle\cdots\rangle$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)n(t,\mathbf{r},\mathbf{p}) - \frac{g}{N_c}\mathrm{Tr}\left\langle \mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta Q(t,\mathbf{r},\mathbf{p})\right\rangle = 0$$

‘fluctuations control the bulk’

Balescu-Lenard collision term for isotropic plasma

quite some work

$$-\frac{g}{N_c}\mathrm{Tr}\left\langle \mathbf{E}(t,\mathbf{r})\cdot\nabla_p \delta Q(t,\mathbf{r},\mathbf{p})\right\rangle = \ldots\ldots\ldots = \nabla_p \mathbf{S}[n,\bar{n},n_g]$$

$$S^i[n,\bar{n},n_g] = \int \frac{d^3p'}{(2\pi)^3} B^{ij}(\mathbf{v},\mathbf{v}')[f(\mathbf{p}')\nabla_p^j n(\mathbf{p}) - n(\mathbf{p})\nabla_{p'}^j f(\mathbf{p}')]$$

$$f(\mathbf{p}) \equiv n(\mathbf{p}) + \bar{n}(\mathbf{p}) + 2N_c n_g(\mathbf{p})$$

$$B^{ij}(\mathbf{v},\mathbf{v}') = \frac{g^4}{8}\frac{N_c^2-1}{N_c}\int \frac{d^3k}{(2\pi)^3}\frac{k^i k^j}{\mathbf{k}^4}\frac{2\pi\delta(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}'))}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})|^2}$$

Landau collision term $(\varepsilon_L = 1)$

$$B^{ij}(\mathbf{v},\mathbf{v}') \approx \frac{g^4 \ln(1/g)}{8}\frac{N_c^2-1}{N_c}\frac{1}{|\mathbf{v}-\mathbf{v}'|}\left(\delta^{ij} - \frac{(v^i - v'^i)(v^j - v'^j)}{(\mathbf{v}-\mathbf{v}')^2}\right)$$

Fokker-Planck collision term for isotropic plasma

$$\frac{g}{N_c}\mathrm{Tr}\left\langle \mathbf{E}(t,\mathbf{r})\cdot\nabla_p \delta Q(t,\mathbf{r},\mathbf{p})\right\rangle = [\nabla_p^i X^{ij}(\mathbf{v}) + \nabla_p^i Y^i(\mathbf{v})]n(\mathbf{p})$$

$$X^{ij}(\mathbf{v}) \equiv \frac{g^4}{8}(N_c^2-1)\int\frac{d^3p'}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{k^ik^j}{\mathbf{k}^4}\frac{2\pi\delta(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}'))}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})|^2}f(\mathbf{p}')$$

$$Y^i(\mathbf{v}) \equiv \frac{g^4}{8}(N_c^2-1)\int\frac{d^3p'}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{k^i}{\mathbf{k}^4}\frac{2\pi\delta(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}'))}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})|^2}\mathbf{k}\cdot\nabla_{p'}f(\mathbf{p}')$$

Fokker-Planck equation for two-stream system

Two-stream system $$f(\mathbf{p}) = (2\pi)^3 \rho \left[\delta^{(3)}(\mathbf{p}-\mathbf{q}) + \delta^{(3)}(\mathbf{p}+\mathbf{q})\right]$$

Longitudinal electric field: $\omega_+(\mathbf{k})$ - stable mode, $\omega_-(\mathbf{k}) = i\gamma_{\mathbf{k}}$ - unstable mode

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla - \nabla_p^i X^{ij}(\mathbf{v}) + \nabla_p^i Y^i(\mathbf{v})\right) n(t,\mathbf{r},\mathbf{p}) = 0$$

$$X^{ij}(\mathbf{v}) \equiv \frac{g^4}{4}\frac{N_c^2-1}{N_c}\rho \int \frac{d^3k}{(2\pi)^3}\frac{k^i k^j}{\mathbf{k}^4}\frac{(\gamma_{\mathbf{k}}^2 + (\mathbf{k}\cdot\mathbf{u})^2)^3}{(\omega_+^2+\gamma_{\mathbf{k}}^2)^2\gamma_{\mathbf{k}}(\gamma_{\mathbf{k}}^2+(\mathbf{k}\cdot\mathbf{v})^2)} \sinh(2\gamma_{\mathbf{k}} t)$$

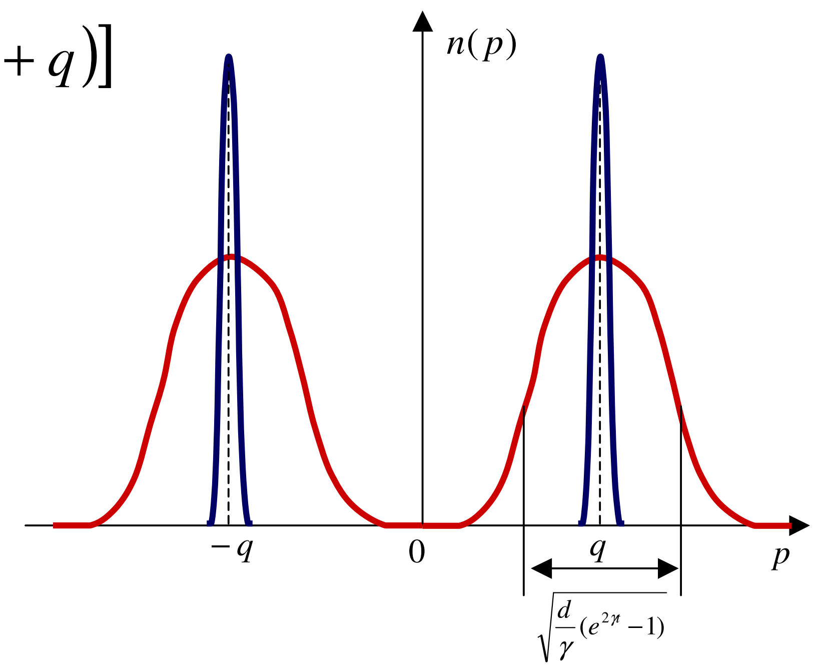
$$Y^i(\mathbf{v}) = 0$$

$$\mathbf{u} \equiv \frac{\mathbf{q}}{E_{\mathbf{q}}}$$

Evolution of the two-stream system

1D problem

$$n_0(p) = (2\pi)^3 \rho\left[\delta(p-q) + \delta(p+q)\right]$$



$$\frac{\partial n(t,p)}{\partial t} = D(t)\,\frac{\partial^2 n(t,p)}{\partial p^2}$$

$$D(t) = d\,e^{2\gamma t}$$

$$n(t,p) = \rho\sqrt{\frac{2\pi\gamma}{d\,(e^{2\gamma t}-1)}}\left\{\exp\left[-\frac{\gamma(p-q)^2}{2d\,(e^{2\gamma t}-1)}\right] + \exp\left[-\frac{\gamma(p+q)^2}{2d\,(e^{2\gamma t}-1)}\right]\right\}$$

Conclusions

Weakly coupled but unstable QGP is

- generically time dependent;
- there are strong fields;
- $\hat{q} \sim e^{2\gamma t}$ and can be large;
- evolves fast towards equilibrium.