Quasiquarks in two stream system

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(Received 26 March 2002; published 20 June 2002)

We study the collective quark excitations in an extremely anisotropic system of two interpenetrating streams of the quark-gluon plasma. In contrast with the gluon modes, all quark ones appear to be stable in such a system. Even more, the quark modes in the two-stream system are very similar to those in the isotropic plasma.

DOI: 10.1103/PhysRevD.65.117501 PACS number(s): 12.38.Mh, 11.10.Wx

The quark-gluon plasma exhibits a rich spectrum of collective excitations. During the past two decades a lot of effort has been paid to studying the equilibrium plasma and the excitations are rather well understood in this case; see [1] for an extensive review. Much less is known about the collective modes in the nonequilibrium quark-gluon plasma such as the parton system generated at the early stage of ultrarelativistic heavy ion collisions at the BNL Relativistic Heavy Ion Collider (RHIC) or CERN Large Hadron Collider (LHC). The parton momentum distribution is not isotropic but strongly elongated along the beam [2,3]. Therefore, specific color fluctuations, instead of being damped, can exponentially grow and noticeably influence the temporal evolution of the whole system. In a series of papers [4–6] it has been argued that there are indeed very fast unstable transverse gluon modes in such a parton system. The analysis [5,6] has been performed within the semiclassical transport theory [7,8]. However, it has been later shown that one gets equivalent results within the hard loop approach [9] extended to the anisotropic systems [10]. In the same paper [10], the quark self-energy for an arbitrary momentum distribution has been computed at one loop level and the dispersion relation of quarks in the anisotropic plasma has been also briefly discussed.

The aim of this Brief Report is to consider for the first time the quark modes in the anisotropic plasma. We start with an extremely anisotropic system of two interpenetrating parton streams. The gluon modes have been discussed in such a system in several papers [4,11–14] and the instabilities have been found. We show here that in contrast to the gluon excitations all quasiquark modes in the two-stream system are stable and rather similar to those in the isotropic plasma. At the end we argue that our results obtained for two streams approximately hold for any system of strongly elongated momentum distribution.

The quark dispersion relations are determined by the poles of the quark propagator or equivalently are found as solutions of the equation

$$\det[\not{k}-\Sigma(k^\mu)]=0, \tag{1}$$

where $k^\mu=(\omega,\mathbf{k})$ is the mode's four momentum; the mass of the bare quark is assumed to vanish. The quark self-energy $\Sigma(k^\mu)$ has been found [10] within the hard loop approximation at the one loop level in the following form:

$$\Sigma(k^\mu)=\frac{g^2}{16}C_F\int\frac{d^3p}{(2\pi)^3}\,\frac{f(\mathbf{p})}{|\mathbf{p}|}\,\frac{\gamma_0-\mathbf{v}\cdot\boldsymbol{\gamma}}{\omega-\mathbf{v}\cdot\mathbf{k}+i0^+}, \tag{2}$$

where g is the coupling constant of QCD with N_c colors and $C_F\equiv(N_c^2-1)/N_c$; $\mathbf{v}\equiv\mathbf{p}/|\mathbf{p}|$ is the parton velocity and $\gamma^\mu=(\gamma^0,\boldsymbol{\gamma})$ are Dirac matrices; $f(\mathbf{p})\equiv n(\mathbf{p})+\bar{n}(\mathbf{p})+2n_g(\mathbf{p})$ and $n(\mathbf{p})$, $\bar{n}(\mathbf{p})$, and $n_g(\mathbf{p})$ denote distribution functions of, respectively, quarks, antiquarks and gluons. The effective distribution function f is arbitrary though symmetric, i.e., $f(-\mathbf{p})=f(\mathbf{p})$.

Since the spinor structure of Σ given by Eq. (2) is very simple, i.e., $\Sigma(k^\mu)=\gamma^0\Sigma^0(\omega,\mathbf{k})-\boldsymbol{\gamma}\boldsymbol{\Sigma}(\omega,\mathbf{k})$, the dispersion equation (1) simplifies to

$$[\omega-\Sigma^0(\omega,\mathbf{k})]^2-[\mathbf{k}-\boldsymbol{\Sigma}(\omega,\mathbf{k})]^2=0. \tag{3}$$

Let us now consider an extremely anisotropic system of two interpenetrating streams with the distribution function of the form

$$f(\mathbf{p})=(2\pi)^3\rho[\delta^{(3)}(\mathbf{p}-\mathbf{q})+\delta^{(3)}(\mathbf{p}+\mathbf{q})], \tag{4}$$

where the parameter ρ is related to the parton density in the stream. The vector $\mathbf{q}$ is assumed to be parallel to the z axis. Substituting Eq. (4) into Eq. (2) one immediately finds the self-energies

$$\begin{aligned}\Sigma^0(\omega,\mathbf{k})&=m^2\frac{\omega}{\omega^2-k_z^2},\\ \Sigma^x(\omega,\mathbf{k})&=\Sigma^y(\omega,\mathbf{k})=0,\\ \Sigma^z(\omega,\mathbf{k})&=m^2\frac{k_z}{\omega^2-k_z^2},\end{aligned} \tag{5}$$

where $m^2\equiv g^2C_F\rho/(8|\mathbf{q}|)$, and gets the following dispersion equation:

$$\frac{\omega^2(\omega^2-k_z^2-m^2)^2}{(\omega^2-k_z^2)^2}-k_T^2-\frac{k_z^2(\omega^2-k_z^2-m^2)^2}{(\omega^2-k_z^2)^2}=0, \tag{6}$$

with $k_T\equiv\sqrt{k_x^2+k_y^2}$. Equation (6) is easily solved providing two quark $(\omega_\pm>0)$ and two antiquark $(\omega_\pm<0)$ modes:

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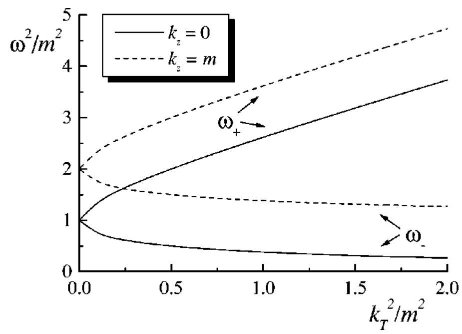

FIG. 1. $\omega_{\pm}^2$ as a function of k_T for $k_z=0$ and $k_z=m$.

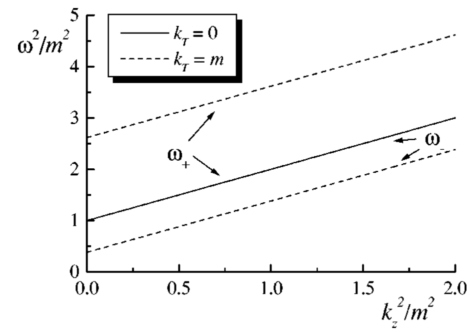

FIG. 2. $\omega_{\pm}^2$ as a function of k_z for $k_T=0$ and $k_T=m$.

$$\omega_{\pm}^2(k_T,k_z)=m^2+k_z^2+\frac{1}{2}k_T^2\pm\sqrt{m^2k_T^2+\frac{1}{4}k_T^4}$$
$$\cong m^2+k_z^2+\frac{1}{2}k_T^2\pm mk_T, \tag{7}$$

where the second approximate equality holds for $m \gg k_T$. Since the modes are pure real, there is no instability. The dispersion relations (7) are illustrated in Figs. 1 and 2.

We are going to compare now the quark modes in the two-stream system (7) to those in the isotropic plasma studied, in particular, in [15,16]. In contrast to the derivations in [15,16], we do not assume that the system is in the thermodynamic equilibrium. We only demand the isotropy of the parton momentum distribution, i.e., the distribution function which enters the self-energy (2) is of the form $f(\mathbf{p})=f(|\mathbf{p}|)$. Then, the angular and momentum integrals in Eq. (2) factorize and one finds

$$\Sigma^0(\omega,\mathbf{k})=\frac{M^2}{2k}\left[\ln\left|\frac{\omega+k}{\omega-k}\right|-i\pi\Theta(k-\omega)\right]$$
$$=\frac{M^2}{\omega}[1+\mathcal{O}(k^2/\omega^2)], \tag{8}$$

$$\mathbf{\Sigma}(\omega,\mathbf{k})=\frac{M^2}{k^2}\mathbf{k}-\frac{M^2\omega}{2k^3}\left[\ln\left|\frac{\omega+k}{\omega-k}\right|-i\pi\Theta(k-\omega)\right]\mathbf{k}$$
$$=\mathcal{O}(k^2/\omega^2)\frac{M^2}{k^2}\mathbf{k}, \tag{9}$$

where $k\equiv|\mathbf{k}|$ and

$$M^2\equiv\frac{g^2}{32\pi^2}C_F\int_0^\infty dp\,p\,f(p).$$

In the case of a massless baryonless plasma in thermal equilibrium M is proportional to the system's temperature.

Substituting the self-energies (8),(9) into the dispersion equation (3), one finds the quark modes which in the long wavelength limit ($\omega \gg k$) can be computed analytically and read

$$\omega_{\pm}^2(k)\cong M^2+\frac{1}{2}k^2\pm Mk. \tag{10}$$

As seen, the quark long wavelength modes in the two-stream system (7) and in the isotropic plasma (10) are very similar to each other. The ω_+ mode is nearly the same in the two systems for any wavelength. However, there is a qualitative difference in the case of the ω_- mode. In equilibrium, ω_- initially decreases with k according to Eq. (10); then there is a shallow minimum at $k\approx M$ and further ω_- monotonically grows. In the two-stream system, ω_- is a monotonically decreasing function of k_T. Since the phase velocity of the ω_- mode is greater than the velocity of light for sufficiently large k_T, the quasiquarks can emit gluons due to the Cherenkov mechanism. The relevance of the phenomenon for the nonequilibrium plasma from the early stage of heavy-ion collisions at RHIC and LHC is under study.

One observes that the form of the quark self-energy (5) and consequently the dispersion relations (7) hold not only for a highly simplified distribution function (4) but for any strongly elongated momentum distribution. Indeed, if $\langle p_z^2\rangle \gg \langle p_T^2\rangle$ the velocities under the integral (2) can be approximated as $v_z\cong 1$ and $v_{x,y}\cong 0$. Then, one immediately gets the formulas (5) with

$$m^2=\frac{g^2}{16}C_F\int\frac{d^3p}{(2\pi)^3}\frac{f(\mathbf{p})}{|\mathbf{p}|}.$$

We conclude our considerations as follows. In contrast to the gluon modes, the quark ones are all stable in the two-stream system. In the long wavelength limit, the quasiquark properties in the strongly anisotropic and isotropic systems are qualitatively the same. The ω_- mode can be responsible for the Cherenkov radiation.

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