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Plasma instability at the initial stage of ultrarelativistic heavy-ion collisions *

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Hard and semihard processes lead to a copious production of partons at the initial stage of ultrarelativistic heavy-ion collisions. Since the parton momentum distribution is strongly anisotropic the system can be unstable with respect to the specific plasma modes. The conditions of instability are found and the characteristic time of its development is estimated. Relevance of the phenomenon for heavy-ion collisions at RHIC and LHC is briefly discussed.

Hard and semihard processes are expected to lead to a copious production of partons, mainly gluons, in ultrarelativistic heavy-ion collisions [1–8]. The number of gluons generated at the initial stage of central Au–Au collision is estimated [6] as 570 at RHIC ($\sqrt{s}$ = 200 GeV per N–N collision) and 8100 at LHC ($\sqrt{s}$ = 6000 GeV per N–N collision). The temporal evolution of such a many-parton system has been discussed in the series of papers [7], where binary parton–parton interactions associated with the initial and final state radiation have been taken into account, while the mean field effects have been completely neglected. However, the electron-ion plasma studies show that the system dynamics can be dominated by the mean-field interaction when the particle momentum distribution is strongly anisotropic [9]. Such a system is usually unstable with respect to specific plasma oscillations and its behaviour is then essentially collective.

Since the longitudinal momenta of partons produced at the initial stage of ultrarelativistic nucleus–nucleus collision are, in average, much greater than the transverse momenta, one expects instability of such a many-parton system. Using the Penrose criterion [9] we discuss the stability conditions and then we find explicitly the unstable mode. The characteristic time of the instability development is estimated, and finally we discuss the relevance of our results for heavy-ion collisions at RHIC and LHC.

As it has been shown within the kinetic theory of the quark-gluon plasma [10,11], the general dispersion equation of the (small) plasma oscillations coincides with that one of the electrodynamic plasma and reads [11,12]

$$\det|\boldsymbol{k}^2\delta^{ij} - k^ik^j - \omega^2\epsilon^{ij}(\omega,\boldsymbol{k})| = 0\,, \qquad (1)$$

where $\boldsymbol{k}$ is the wave vector and ω is the frequency. ϵ^{ij} is the chromoelectric permeability tensor, which in the collisionless limit is

$$\epsilon^{ij}(\omega,\boldsymbol{k}) = \delta^{ij} + \frac{2\pi\alpha_s}{\omega}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega - \boldsymbol{k}\cdot\boldsymbol{v} + i0^+}\frac{\partial n(\boldsymbol{p})}{\partial p^l} \times\left[\left(1-\frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right)\delta^{lj} + \frac{k^l v^j}{\omega}\right], \qquad (2)$$

where α_s is the strong coupling constant and

$$n(\boldsymbol{p}) \equiv n_q(\boldsymbol{p}) + \bar{n}_q(\boldsymbol{p}) + 6n_g(\boldsymbol{p}) \qquad (3)$$

with $n_q, \bar{n}_q$ and n_g being the distribution function of quarks, antiquarks and gluons normalized in such a way that the quark and gluon densities are

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$$\rho_q = 3\int \frac{d^3p}{(2\pi)^3} n_q(\boldsymbol{p}), \quad \rho_g = 8\int \frac{d^3p}{(2\pi)^3} n_g(\boldsymbol{p}) .$$

The quarks and gluons are assumed massless and consequently the parton velocity $\boldsymbol{v}$ equals $\boldsymbol{p}/|\boldsymbol{p}|$. The plasma is assumed initially colorless, homogeneous but not isotropic. It should be also stressed that in spite of the similarity to the electrodynamic formulae, eqs. (1), (2) take into account the essential non-abelian effect i.e. the gluon–gluon coupling [11].

The solutions $\omega(\boldsymbol{k})$ of eq. (1) are stable when $\mathrm{Im}\,\omega < 0$ and unstable when $\mathrm{Im}\,\omega > 0$. In the first cases the amplitude exponentially decreases in time while in the second one there is an exponential growth. It is usually difficult to find the solutions of eq. (1) because of the complicated structure of the chromodielectric tensor (2). However, the stability analysis can be performed without solving eq. (1) explicitly.

In our further discussion, the distribution function is chosen in the form

$$n(y, p_\perp, \phi) = \frac{1}{2Y}\Theta(Y-y)\Theta(Y+y)h(p_\perp)\frac{1}{p_\perp \mathrm{ch}\, y} , \tag{4}$$

where y, $p_\perp$ and ϕ denote parton rapidity, transverse momentum and azimuthal angle, respectively. eq. (4) gives the parton number distribution $(\mathrm{d}N/\mathrm{d}^2p_\perp \mathrm{d}y)$ which is flat in the rapidity interval $(-Y, Y)$.

When the momentum distribution is monotonously decreasing function of $\boldsymbol{p}|$, as it is the case of (4), the longitudinal modes, those with the wave vector $\boldsymbol{k}$ parallel to the chromoelectric field $\boldsymbol{E}$, are stable [9]. Thus, we look for instabilities among transversal modes. When the instability occurs the kinetic energy of particles is converted into the field energy. Since the energy of the parton motion along the beam direction, which we identify with the z-axis, exceeds the perpendicular energy, the instability is expected to appear when the chromoelectric field is along the z-axis while the wave vector is transversal to it. Thus, we will consider the configuration

$$\boldsymbol{E} = (0,0,E), \quad \boldsymbol{k} = (k,0,0). \tag{5}$$

Let us mention that the unstable mode, the so-called filamentation instability, has been found for this configuration in the two-stream system of the quark gluon plasma [12,13]. It is also interesting to note that the electron–ion plasma from the pinch experiments is "hotter" in the transverse direction (due to magnetic squeezing) and the instability appears in the configuration where the electric field is perpendicular and the wave vector longitudinal to the pinch axis.

With the configuration (5) the dispersion equation (1) simplifies to

$$H(\omega) \equiv k^2 - \omega^2\epsilon^{zz}(\omega, k) = 0 . \tag{6}$$

The Penrose criterion states that eq. (6) has unstable solutions if $H(0) < 0$ [9].

Substituting the distribution function (4) into the dielectric tensor (2) one finds an analytical expression of $H(0)$. In the limit $\mathrm{e}^Y \gg 1$ it reads

$$H(0) = k^2 - \chi^2 ,$$

with

$$\chi^2 \equiv -\frac{\alpha_s}{4\pi}\frac{\mathrm{e}^Y}{Y}\int \mathrm{d}p_\perp \left(h(p_\perp) + p_\perp \frac{\mathrm{d}h(p_\perp)}{\mathrm{d}p_\perp}\right) .$$

After partial integration of the second term one gets

$$\chi^2 = \frac{\alpha_s}{4\pi}\frac{\mathrm{e}^Y}{Y} p_\perp^{\mathrm{min}} h(p_\perp^{\mathrm{min}}) , \tag{7}$$

where $p_\perp^{\mathrm{min}}$ is the minimal transverse momentum and the function $h(p_\perp)$ is assumed to decrease faster than $1/p_\perp$ when $p_\perp \to \infty$.

As seen, the sign of $H(0)$ is (for sufficiently small k^2) determined by the transverse momentum distribution at the minimal momentum. There are unstable modes (5) if $p_\perp^{\mathrm{min}} h(p_\perp^{\mathrm{min}}) > 0$. Unfortunately, it is hard to estimate this quantity. The point is that the perturbative calculations are not reliable when $p_\perp^{\mathrm{min}} \to 0$. We consider two possibilities.

(i) One treats perturbatively only partons with $p_\perp > p_\perp^{\mathrm{min}} = 1$–$2$ GeV. Those with lower momenta are assumed to form colorless clusters or strings due to nonperturbative interaction. Then, only the partons with $p_\perp > p_\perp^{\mathrm{min}}$ contribute to the dielectric tensor (2), and $p_\perp^{\mathrm{min}} h(p_\perp^{\mathrm{min}})$ is positive. Consequently, $H(0)$ is negative for sufficiently small k^2.

(ii) The perturbative approach is naively extended to $p_\perp^{\mathrm{min}} = 0$. Then, the quantity $p_\perp^{\mathrm{min}} h(p_\perp^{\mathrm{min}})$ blows up since $h(p_\perp)$ diverges at $p_\perp = 0$ as $p_\perp^{-6}$ [2] [#1]. In such a case $H(0)$ is negative for any k^2.

Although we cannot draw a firm conclusion there are good reasons to believe that $H(0) < 0$ for $k^2 < \chi^2$. Thus, we look for the unstable solutions of eq. (6) which is

$$k^2 - \omega^2 + \omega_0^2 - \frac{\alpha_s}{4\pi^2} \int_0^\infty dp_\perp\, p_\perp^2 \times \int_{-Y}^{Y} dy\, \mathrm{sh}^2 y \left(\frac{\partial n}{\partial p_\perp} - \frac{\partial n}{\partial y} \frac{\mathrm{sh} y}{p_\perp \mathrm{ch} y} \right) \times \int_0^{2\pi} \frac{d\phi \cos\phi}{\mathrm{ch} y\,(\omega/k) - \cos\phi + i0^+} = 0, \tag{8}$$

with the plasma frequency ω_0 expressed in the limit $e^Y \gg 1$ as

$$\omega_0^2 = \frac{\alpha_s}{8Y} \int dp_\perp h(p_\perp). \tag{9}$$

As will be shown below, the plasma frequency gives the frequency of the stable mode when $k \to 0$.

We solve eq. (8) in the two limiting cases $|\omega/k| \gg 1$ and $|k/\omega| \gg 1$. In the first case the azimuthal integral is approximated as

$$\int_0^{2\pi} \frac{d\phi \cos\phi}{a - \cos\phi + i0^+} = -\frac{\pi}{a^2} + \mathcal{O}(a^{-4}),$$

and the integration over y from eq. (8) can be performed analytically. When $e^Y \gg 1$ the resulting equation is

$$k^2 - \omega^2 + \omega_0^2 + \eta^2 \frac{k^2}{\omega^2} = 0, \tag{10}$$

where

$$\eta^2 \equiv \frac{\alpha_s}{16Y} \int dp_\perp \left(\tfrac{1}{4} h(p_\perp) - p_\perp \frac{dh(p_\perp)}{dp_\perp} \right). \tag{11}$$

[#1] The calculations from [6] for Au–Au collisions at LHC show $h(p_\perp) \sim (p_\perp + m_\perp)^{-6.4}$ with $m_\perp = 2.9$ GeV. Then, $h(0)$ is finite. However, the finite value of $h(0)$ is the result of cut-off parameters used in the computational procedure.

The solutions of eq. (10) are

$$\omega_\pm^2 = \tfrac{1}{2} [k^2 + \omega_0^2 \pm \sqrt{(k^2 + \omega_0^2)^2 + 4\eta_0^2 k^2}\,].$$

One sees that $\omega_+^2 \geqslant 0$ and $\omega_-^2 \leqslant 0$. Thus, there is a pure real mode ω_+, which is stable, and two pure imaginary modes ω_-, one of them being unstable.

Let us focus our attention on the unstable mode which can be approximated as

$$\omega_-^2 \cong \begin{cases} -(\eta^2/\omega_0^2) k^2 & \text{for } k^2 \ll \omega_0^2, \\ -\eta^2 & \text{for } k^2 \gg \omega_0^2. \end{cases} \tag{12}$$

One should keep in mind that eq. (12) holds only for $|\omega/k| \gg 1$. We see that ω_- can satisfy this condition for $k^2 \ll \omega_0^2$ if $\eta^2 \gg \omega_0^2$ and for $k^2 \gg \omega_0^2$ if $\eta^2 \ll \omega_0^2$.

Assuming that $h(p_\perp) \sim p_\perp^{-\beta}$, one finds from eq. (11)

$$\eta^2 = \frac{1 + 4\beta}{8} \omega_0^2. \tag{13}$$

Since $\beta \cong 6$ [2,6], $\eta^2 \cong 3\omega_0^2$.

Let us now discuss the second case when $|k/\omega| \gg 1$. Then, the azimuthal integral is approximated as

$$\int_0^{2\pi} \frac{d\phi \cos\phi}{a - \cos\phi + i0^+} = -2\pi + \mathcal{O}(a),$$

and after integration over y in eq. (8) we immediately get the dispersion relation

$$\omega^2 = k^2 - \chi^2, \tag{14}$$

with χ^2 given by eq. (7). As previously we have assumed that $e^Y \gg 1$, eq. (14) provides a real mode for $k^2 > \chi^2$ and two imaginary modes for $k^2 < \chi^2$. Since the solution (14) must satisfy the condition $|k/\omega| \gg 1$, it holds only for $k^2 \gg |k^2 - \chi^2|$.

The dispersion relation of the unstable mode in the whole domain of wave vectors is schematically shown in fig. 1, where the solutions (12) and (14) are combined.

The characteristic time of instability development is given by $1/\mathrm{Im}\,\omega$. As seen in fig. 1, $|\mathrm{Im}\,\omega| < \eta$. Thus, we define the minimal time as $\tau_{\min} = 1/\eta$. Let us estimate $\tau_{\min}$.

We start with the estimate of the plasma frequency. Approximating $\int dp_\perp h(p_\perp)$ as $\int dp_\perp p_\perp h(p_\perp)/\langle p_\perp \rangle$

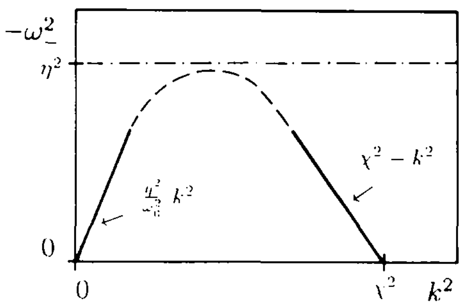

Fig. 1. Schematical view of the dispersion relation for the unstable mode.

with $\langle p_\perp \rangle$ being the mean parton transverse momentum, the plasma frequency (9) can be written as

$$\omega_0^2 \cong \frac{\alpha_s \pi}{6 Y r_0^2 A^{2/3}} (N_q + N_{\bar{q}} + \tfrac{9}{4} N_g) , \tag{15}$$

where N_q, $N_{\bar{q}}$, and N_g is the number of quarks, antiquarks and gluons, respectively, produced in the volume, which has been estimated in the following way. Since we are interested in the central collisions, the volume corresponds to a cylinder of the radius $r_0 A^{1/3}$ with $r_0 = 1.1$ fm and A being the mass number of the colliding nuclei. Using the uncertainty principle argument, the length of the cylinder has been taken as $1/\langle p_\perp \rangle$, which is the formation time of parton with the transverse momentum $\langle p_\perp \rangle$.

Neglecting quarks and antiquarks in eq. (15) and substituting there $N_g = 570$ for the central Au–Au collision at RHIC ($Y = 2.5$) and $N_g = 8100$ for the same colliding system at LHC ($Y = 5.0$) [6], we get

$$\omega_0 = 0.81\ c/\text{fm} \quad \text{for RHIC} ,$$

$$\omega_0 = 2.2\ c/\text{fm} \quad \text{for LHC}$$

for $\alpha_s = 0.1$. Using eq. (13) with $\beta = 6$ one finds

$$\tau_{\min} = 0.70\ \text{fm}/c \quad \text{for RHIC} ,$$

$$\tau_{\min} = 0.26\ \text{fm}/c \quad \text{for LHC} .$$

The estimated time of instability development is shorter than the characteristic time of nucleus–nucleus interaction, which is at least a few fm/c. This means that a color fluctuation, which appears at the initial stage of the collision, with the chromoelectric field along the beam and the wave vector perpendicular to it has a chance to grow converting the longitudinal energy to the transverse one. It is known from the studies of the electron–ion plasma that such an energy transport is very effective [9]. Thus, the appearance of the instability is expected to speed up equilibration of the system leading to a more isotropic momentum distribution [#2].

At the end let us briefly speculate on possible experimental consequences of the plasma instability. Since the longitudinal energy is converted into the transverse one when the instability grows, the broadening of the transverse momentum distribution is expected. The instability initiates as a random color fluctuation. Thus, there should be nuclear collisions where the unstable mode develops and the collisions without this mode. The azimuthal orientation of the wave vector should also change from collision to collision. Therefore, the instability is expected to magnify fluctuations of particle transverse momenta, which can be studied by means of the event-by-event analysis.

[#2] Since the collective modes discussed here are due to the mean field interaction, they do not produce entropy. Therefore, the instability contributes to the equilibration indirectly reducing relative parton momenta and consequently increasing the rate of collisions.

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