25 June 1998

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 430 (1998) 9–14

Hadronic matter compressibility from event-by-event analysis of heavy-ion collisions

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Received 19 December 1997; revised 6 March 1998
Editor: J.-P. Blaizot

Abstract

We propose a method to measure the hadronic matter compressibility by means of the event-by-event analysis of heavy-ion collisions at high energies. The method, which utilizes the thermodynamical relation between the compressibility and the particle number fluctuations, requires a simultaneous measurement of the particle source size, temperature and particle multiplicity.

PACS: 25.75.+r; 24.10.-k; 24.60.Ky
Keywords: Relativistic heavy-ion collisions; Thermal model; Fluctuations

Large acceptance detectors allows one for a detailed analysis of individual collisions of heavy-ions at high-energies. Due to hundreds or even thousands of particles produced in these collisions, variety of powerful statistical methods can be applied. Then, such event-by-event studies can provide valuable dynamical information which is otherwise hardly available. For example, we have shown in [1], see also [2], that the correlation between particle multiplicity and their average transverse momentum, which is observed in proton-proton interactions, causes sizeable fluctuations of the total transverse momentum of particles produced in a single nucleus-nucleus collision if such a collision is a superposition of independent nucleon-nucleon interactions. The fluctuations from Pb-Pb collisions at 158 GeV per nucleon studied in NA49 experiment have appeared to be noticeably smaller than the properly normalized fluctuations from the proton-proton interactions at the same collision energy [3]. Thus, a substantial role of the secondary interactions in heavy-ion collisions has been proven without a reference to any collision model.

Stodolsky [4] and Shuryak [5] have made another interesting proposal of the event-by-event analysis. They have adopted a standard assumption that the hadron matter from the nuclear collisions is in thermodynamical equilibrium. Then, according to the well known thermodynamical relation, the temperature fluctuations can be treated as a measure of the heat capacity of the hadronic matter. Shuryak [5] has also briefly considered the

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PII: S0370-2693(98)00492-4

relation, which couples the particle number fluctuations to the particle number derivative with respect to the chemical potential. In this paper we follow a similar line of reasoning and present a method to determine the hadronic matter compressibility via the multiplicity fluctuations.

The method is based on the thermodynamical relation, see e.g. [6,7], which expresses the dispersion of the particle number N in a volume V through the isothermal compressibility as

$$\frac{\langle N\rangle^2}{\langle N^2\rangle - \langle N\rangle^2} = -\frac{V^2}{T}\left(\frac{\partial p}{\partial V}\right)_{T,\langle N\rangle}, \tag{1}$$

where T is the temperature and p the pressure. In principle, the relation (1) allows one to find $\partial p/\partial V$ as a function of $\langle N\rangle$, V and T and then to reconstruct the equation of state. Below we discuss how to realize this program in heavy-ion reactions.

There are many sorts of hadrons (π^+, π^0, π^-, K^0, $\bar{K}^0$,.....) in the final state of nuclear collisions. Therefore, our first task is to generalize Eq. (1) to the system of k components. To solve this problem one has to assume that the hadron system at freeze-out is not only in the thermal but in the chemical equilibrium as well. The distinction between the thermal and the chemical freeze-out is discussed at the end of this paper. The extensive analysis [8,9] of the experimental data from AGS and SPS accelerators shows that the assumption of the chemical equilibrium is very well satisfied in heavy-ion collisions at least for nonstrange particles.

To derive the relation analogous to (1) we follow [6] and write the grand canonical sum for the k-component system as

$$\Xi(z_1,\dots z_k,V,T) = \sum_{N_1,\dots N_k} z_1^{N_1}\dots z_k^{N_k}\, Q_{N_1,\dots N_k}(V,T),$$

where z_i is the fugacity of the i-th component and $Q_{N_1,\dots N_k}(V,T)$ is the canonical partition function. Keeping in mind that

$$Q_{N_1,\dots N_k}(V,T) = e^{-\beta F(N_1,\dots N_k,V,T)}$$

with $\beta \equiv T^{-1}$ and $F(N_1,\dots N_k,V,T)$ being the system free energy, one finds that the probability $P_{N_1,\dots N_k}$ to find $\{N_1,\dots N_k\}$ particles is proportional to

$$P_{N_1,\dots N_k} \sim z_1^{N_1}\dots z_k^{N_k}\, e^{-\beta F(N_1,\dots N_k,V,T)}.$$

In the case of a hadronic system the particle numbers $\{N_1,\dots N_k\}$ are not conserved. Nevertheless the free energy $F(N_1,\dots N_k,V,T)$ with the numbers of particles $\{N_1,\dots N_k\}$ being fixed is still of physical meaning. However, this is $F(\langle N_1\rangle,\dots \langle N_k\rangle,V,T)$, which corresponds to the thermodynamical (thermal and chemical) equilibrium. The average particle numbers $\{\langle N_1\rangle,\dots \langle N_k\rangle\}$ are found as a minimum of $F(N_1,\dots N_k,V,T)$. If the particles carry charges, say $+$ and $-$, and the total charge Q is conserved, then the average values $\langle N^+\rangle$ and $\langle N^-\rangle$ are also found as a minimum of F but the additional constraint $\langle N^+\rangle - \langle N^-\rangle = Q$ must be imposed.

When we deal with the particles obeying quantum statistics, the approach described above, where the average particle numbers are found as a minimum of $F(N_1,\dots N_k,V,T)$ is rather inconvenient because there is no compact expression for $F(N_1,\dots N_k,V,T)$ even for the ideal gas. Then, we introduce the chemical potentials. However, this is a technical problem and not the matter of principles.

After this comment we expand the free energy around the average particle numbers and take into account only the first three terms i.e.

$$\begin{aligned} F(N_1,\dots N_k,V,T) \cong{}& F(\langle N_1\rangle,\dots \langle N_k\rangle,V,T) + \sum_i \left.\frac{\partial F(N_1,\dots N_k,V,T)}{\partial N_i}\right|_{N_1=\langle N_1\rangle,\dots N_k=\langle N_k\rangle} (N_i - \langle N_i\rangle) \\ &+ \frac{1}{2}\sum_{i,j} \left.\frac{\partial^2 F(N_1,\dots N_k,V,T)}{\partial N_i \partial N_j}\right|_{N_1=\langle N_1\rangle,\dots N_k=\langle N_k\rangle} (N_i - \langle N_i\rangle)(N_j - \langle N_j\rangle). \end{aligned} \tag{2}$$

Due to the relations

$$\mu_i \stackrel{\text{def}}{=} \frac{\partial F(N_1,\dots N_k,V,T)}{\partial N_i}\bigg|_{N_1=\langle N_1\rangle,\dots N_k=\langle N_k\rangle} \quad \text{and} \quad z_i \equiv e^{\beta\mu_i},$$

where μ_i is the chemical potential of the i-th component, one finds

$$P_{N_1,\dots N_k} \sim \exp\left[-\frac{1}{2}\sum_{i,j}\Lambda_{ij}\big(N_i-\langle N_i\rangle\big)\big(N_j-\langle N_j\rangle\big)\right], \tag{3}$$

with the matrix Λ given as

$$\Lambda_{ij} = \beta\,\frac{\partial^2 F(N_1,\dots N_k,V,T)}{\partial N_i \partial N_j}\bigg|_{N_1=\langle N_1\rangle,\dots N_k=\langle N_k\rangle}. \tag{4}$$

As well known, see e.g. [10], the moment matrix M_{ij} of the normal distribution (3) equals

$$M_{ij} \equiv \langle\big(N_i-\langle N_i\rangle\big)\big(N_j-\langle N_j\rangle\big)\rangle = (\Lambda^{-1})_{ij}. \tag{5}$$

We are now going to relate the matrix Λ to the compressibility. Since the free energy is an extensive quantity, which can be expressed as

$$F(N_1,\dots N_k,V,T) = Vf(\rho_1,\dots \rho_1,T),$$

with $\rho_i \equiv N_i/V$, one shows that

$$p(N_1,\dots N_k,V,T) \stackrel{\text{def}}{=} -\frac{\partial F(N_1,\dots N_k,V,T)}{\partial V} = -\frac{F(N_1,\dots N_k,V,T)}{V} + \sum_i \rho_i \frac{\partial F(N_1,\dots N_k,V,T)}{\partial N_i}$$

and then

$$-\frac{\partial p(N_1,\dots N_k,V,T)}{\partial V} = \sum_{i,j}\rho_i\,\rho_j \frac{\partial^2 F(N_1,\dots N_k,V,T)}{\partial N_i \partial N_j}. \tag{6}$$

Using Eqs. (5) and (6) we get the final result

$$\sum_{i,j}\langle N_i\rangle\langle N_j\rangle(M^{-1})_{ij} = -\frac{V^2}{T}\left(\frac{\partial p}{\partial V}\right)_{T,\langle N\rangle_1,\dots\langle N\rangle_k}. \tag{7}$$

The relation (7) essentially simplifies when the system components are independent from each other. Then, the matrices Λ (4) and M (5) are diagonal ($M_{ij}=\Lambda_{ij}=0$ for $i \neq j$) and

$$\sum_i \frac{\langle N_i\rangle^2}{\langle N_i^2\rangle - \langle N_i\rangle^2} = -\frac{V^2}{T}\left(\frac{\partial p}{\partial V}\right)_{T,\langle N\rangle_1,\dots\langle N\rangle_k}. \tag{8}$$

The question whether $\langle(N_i-\langle N_i\rangle)(N_j-\langle N_j\rangle)\rangle = 0$ for $i \neq j$ can be answered experimentally.

As already mentioned it is somewhat unusual in the statistical hadron physics to consider the thermodynamic quantities, like the compressibility from Eqs. (7), (8), at the fixed hadron average numbers. Computation of such quantities is straightforward if the hadrons are assumed to obey the Boltzmann statistics. In fact, this is a quite good approximation at the freeze-out stage when the hadron gas is expected to be rather dilute.

We consider as an example the classical multi-component van der Waals gas. The equation of state is taken in the form

$$p\left(V - \sum_i \langle N_i\rangle v_i\right) = \sum_j \langle N_j\rangle T,$$

where the parameter v_i is related the volume of the $i-$sort hadron. The r.h.s. of Eq. (7) or (8) then equals

$$-\frac{V^2}{T}\left(\frac{\partial p}{\partial V}\right)_{T,\langle N\rangle_1,\langle N\rangle_k,}=\frac{\sum\limits_j\langle N_j\rangle}{\left(1-\sum\limits_i\frac{\langle N_i\rangle v_i}{V}\right)^2}\cong\left(1+2\sum_j\frac{\langle N_i\rangle v_i}{V}\right)\sum_j\langle N_j\rangle, \tag{9}$$

where the last approximate equality assumes smallness of the van der Waals correction. When $v_i\to 0$ we get the ideal gas limit. The compressibility of the classical ideal gas is obtained from the relation (7) if the moment matrix M (5) equals $M_{ij}=\delta^{ij}\langle N_i\rangle$.

When the particles obey quantum statistics one usually introduces the chemical potentials to compute the thermodynamical quantities. Then, the pressure is a function of $\{\mu_1,.....\mu_k\}$ (not of $\{\langle N_1\rangle,...\langle N_k\rangle\}$) and we express the derivative at fixed $\{\langle N_1\rangle,...\langle N_k\rangle\}$, which is present in Eqs. (7), (8), through the derivative at fixed $\{\mu_1,.....\mu_k\}$. This is done by means of the respective jacobians, see e.g. [7]. We give here the formula for a single component system

$$\left(\frac{\partial p}{\partial V}\right)_{T,\langle N\rangle}=\left(\frac{\partial p}{\partial V}\right)_{T,\mu}-\frac{\left(\frac{\partial p}{\partial\mu}\right)_{T,V}\left(\frac{\partial\langle N\rangle}{\partial V}\right)_{T,\mu}}{\left(\frac{\partial\langle N\rangle}{\partial\mu}\right)_{T,V}},$$

and apply it to the ideal gas of pions which are bosons with three internal degrees of freedom. Then,

$$\langle N(\mu,V,T)\rangle=3V\int\frac{d^3p}{(2\pi)^3}\left(e^{\beta(E_p-\mu)}-1\right)^{-1},$$

$$p(\mu,V,T)=-3T\int\frac{d^3p}{(2\pi)^3}\ln\left(1-e^{-\beta(E_p-\mu)}\right),$$

where $E_p\equiv\sqrt{m^2+\boldsymbol{p}^2}$ with m being the pion mass and $\boldsymbol{p}$ its momentum. For massless pions with $\mu=0$, which corresponds to the chemical equilibrium, we get the r.h.s of eq. (1) in the form

$$-\frac{V^2}{T}\left(\frac{\partial p}{\partial V}\right)_{T,\langle N\rangle}=\frac{18\zeta^2(3)}{\pi^4}VT^3=\frac{6\zeta(3)}{\pi^2}\langle N\rangle\cong 0.71\langle N\rangle, \tag{10}$$

where $\zeta(z)$ is the Riemann function and $\zeta(3)\cong 1.202$. Comparing Eqs. (9) and (10) one sees that the particle number fluctuations are larger in the ideal gas of bosons than in the classical one.

Let us now discuss how to use Eq. (7) or (8) to determine the hadron matter compressibility. The l.h.s. of these equations is fully determined by the particle multiplicity but to extract $\partial p/\partial V$ one also needs to deduce T and V in individual collisions. It is not a trivial task to get these three quantities.

One should first realize that the final state characteristics correspond to the so-called thermal freeze-out when the whole system disintegrates due to the switching-off the inter-particle interactions. However, the system chemical composition is fixed earlier, at the so-called chemical freeze-out when the interactions, which change the particle numbers, are no longer effective. The particle multiplicity, the system volume and temperature, which enter Eqs. (7), (8) refer to the chemical freeze-out. Therefore, these quantities, which are obtained from the final state characteristics and consequently correspond to the thermal freeze-out, should be further recalculated to get their values at the chemical freeze-out. One can refer here to the procedure described in e.g. [9], where the system volume and temperature at the chemical freeze-out are inferred from the multiplicities of the different sort particles.

The second but related difficulty lies in the fact that a sizeable fraction of the final state particles originate from the decays of hadron resonances. Since the particle multiplicity from Eqs. (7), (8) is that at the chemical freeze-out, the final state multiplicity should be recalculated to reconstruct the resonance contribution. This can be achieved within the thermodynamical model which takes into account hadron resonances, see e.g. [8,9].

The system temperature at the thermal freeze-out is usually found as an inverse slope of the transverse mass spectrum ($m_T = \sqrt{m^2 + p_T^2}$ with p_T being the particle transverse momentum). This is however the 'effective' temperature which incorporates the effect of transverse collective motion of the hadronic matter [11] and that of the resonance decays [12]. The actual temperature can be disentangled by the simultaneous analysis of different sort particles [11,12]. The procedure is not very exact but the temperature is presumably measurable within 10% accuracy.

The system volume in nuclear reactions is controlled due to the collision centrality selection. In this way one can change the volume in a rather broad range. One usually deduces the particle source size at the thermal freeze-out by means of the particle interferometry measurements, see e.g. [13]. The high multiplicity of relativistic nuclear collisions allows one for the event-by-event interferometry analysis. However, one should keep in mind that the hadron system decay is a dynamic process and the concept of the system volume is then not very precise. The hadron resonances also complicate the analysis, see e.g. [14]. In spite of these difficulties, we still believe that the system volume can be measured with an accuracy which enables one to extract the hadron matter compressibility from Eq. (7).

How the data should be collected and processed? First of all one should eliminate or at least reduce the trivial fluctuations due to the impact parameter variation. This can be achieved by the trigger condition. The collision centrality is well known to be strongly correlated to the transverse or forward energy observed in the collision. Therefore, selecting the transverse or/and forward energy from a narrow interval we can collect a sample of collisions of a similar geometry. Then, one determines the system volume, temperature and hadron multiplicity for every event. The events are split into subsamples in such a way that a subsample contains the events with coinciding or close temperatures and volumes. For every subsample one constructs the multiplicity distribution and computes the l.h.s. of Eq. (7) or (8). Then, one gets the compressibility for a given V and T. Combining the results obtained for several subsamples we get the compressibility as a function of volume and temperature. The range of V and T will be presumably rather small. To get $\partial p / \partial V$ in a broader domain one should vary the collision geometry by means of the trigger, change the projectile-target system and the collision energy.

In summary, we have proposed a method to determine the hadronic matter compressibility due to the event-by-event analysis of the particle multiplicity fluctuations in heavy-ion collisions. The thermodynamical relation, which connects the two quantities in the multi-component system, has been derived. As an illustration we have considered the compressibility of the classical van der Waals gas and the quantum gas of massless pions. Since the method requires not only the measurement of the particle multiplicity but of the system volume and its temperature as well, the problems related to such measurements have been briefly discussed. Finally, the procedure of data analysis has been suggested.

I am very grateful to Marek Gaździcki, Mark I. Gorenstein and Edward V. Shuryak for critical reading of the manuscript and fruitful discussions.

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