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# On the Ideal Gas of Tachyons.

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In this paper we consider properties of the ideal gas of classical (nonquantum) tachyons [1-3].

One can say that it is senseless to study tachyons if their existence has not been experimentally confirmed. In our opinion, there are two possibilities to refute this objection.

1) In order to propose the experiment that could give an answer to the question on the existence of tachyons, theoretical investigations are needed, because we already know that properties of tachyons (if they exist) are quite different from those of particles slower than light—bradyons. It should be stressed that a theoretical support of the majority of experiments which have been done is very weak. So, their result cannot be counted as conclusive [1].

2) Even if we *a priori* assume that tachyons do not exist, a study of their properties can be valuable, since it makes our understanding of the theory of bradyons deeper.

We propose the reader to choose one of the above possibilities and read what follows.

Firstly we formulate some basic notions of the thermodynamics of tachyons following Huang's book [4]. We start from the microcanonical ensemble and the postulate of « *a priori* equal probabilities » that are the same as in bradyon physics. Also without any differences we define the entropy of a system and, through the enetropy, the temperature of a subsystem. In this way we are led to the canonical ensemble and the

(1) For review of tachyons see E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209 (1974); see also ref. (2,3).
(2) *Proceedings of Session « Tachyons, Monopoles and Related Topics », Erice, 1976*, edited by E. RECAMI (Amsterdam, 1978).
(3) P. CALDIROLA and E. RECAMI: « *Causality and Tachyons in Relativity* » in *Italian Studies in the Phylosophy and Science*, edited by M. L. DALLA CHIARA (Dordrecht, 1980), p. 249-298.
(4) K. HUANG: *Statistical Mechanics* (New York, N.Y., 1963).

partition function of $N$ particles

$$Q_N(V,T) \stackrel{\text{def}}{=} \int \frac{d^{3N}p\, d^{3N}q}{N!} \exp\left[-\frac{H(\bar{p}_1 \dots \bar{p}_N, \bar{q}_1 \dots \bar{q}_N)}{T}\right],$$

where $V$ is the volume of the system, $T$ the temperature and $H(\bar{p}_1 \dots \bar{p}_N, \bar{q}_1 \dots \bar{q}_N)$ the *Hamiltonian of the system* of $N$-particles which depends on their momenta, $\bar{p}$, and positions, $\bar{q}$. Integration is taken over momenta greater than the rest masses of tachyons, $m$. We use units where $k = c = \hbar = 1$.

The thermodynamical quantities: pressure, $P$, entropy, $S$ and energy, $U$, are defined through Helmholtz' free energy, $F$, which is related to $Q_N$ by the equation

$$F \stackrel{\text{def}}{=} -T \ln Q_N(V,T)\,, \tag{1}$$

$$P \stackrel{\text{def}}{=} -\left(\frac{\partial F}{\partial V}\right)_T, \qquad S \stackrel{\text{def}}{=} -\left(\frac{\partial F}{\partial T}\right)_V, \qquad U \stackrel{\text{def}}{=} F + TS\,. \tag{2}$$

In the case of free tachyons

$$H(\bar{p}_1, \dots, \bar{p}_N, \bar{q}_1, \dots, \bar{q}_N) = \sum_{i=1}^{N} \sqrt{\bar{p}_i^2 - m^2}$$

and

$$Q_N(V,T) = \frac{1}{N!}\left[\int d^3p \exp - \frac{\sqrt{\bar{p}^2 - m^2}}{T}\right]^N \cdot$$

$$\cdot \int\limits_{|\bar{p}|\geqslant m} d^3p \exp - \frac{\sqrt{\bar{p}^2 - m^2}}{T} = 4\pi m^3 \int_0^\infty dt \cosh^2 t \sinh t \exp\left[-z \sinh t\right] = 4\pi m^3 \frac{1}{z} S_{02}(z),$$

where $z = m/T$ and $S_{02}(z)$ is the so-called Lommel function [5]. Finally we get

$$Q_N(V,T) = \frac{4\pi m^2 T V S_{02}(m/T)]^N}{N}\,.$$

For the gas of relativistic bradyons the partition function is similar in form, although the Lommel function $S_{02}(z)$ has to be changed into the Macdonald function $K_2(z)$ [6,7].

Using (1) and (2), we find the equation of state which is the same as for bradyons, namely

$$pV = NT\,.$$

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(5) I. S. GRADSHTEYN and I. M. RYZHIK: *Tables of Integrals, Series and Products*, edited by A. JAFFREY (New York, N.Y., 1965).

(6) F. JUTTNER: *Ann. Phys. (N.Y.)*, **34**, 856 (1911).

(7) S. CHANDRASEKHAR: *An Introduction to the Study of Stellar Structure*, Chapt. X (New York, N.Y., 1939).

Let us notice that at zero temperature the pressure is zero. In the case of tachyons it is not a trivial property if we remember that zero-energy tachyons carry momenta equal to their masses. So, at first sight we could expect that, similar to the Fermi-Dirac gas, the value of pressure at zero temperature would be finite.

The energy of tachyon gas is expressed by the formula

$$U = NT\left(1 - z\,\frac{S'_{02}(z)}{S_{02}(z)}\right) = NT\left(3z\,\frac{S_{-13}(z)}{S_{02}(z)} - 1\right).$$

In two extreme cases, $z \gg 1$ and $1/z \gg 1$, $S_{02}(z)$ can be approximated

$$S_{02}(z) = \begin{cases} = \dfrac{z^2}{2} + O(\ln z)\,, & \text{for } \dfrac{1}{z} \gg 1\,, \\[2ex] = \dfrac{1}{z} + O\left(\dfrac{1}{z^2}\right), & \text{for } z \gg 1\,. \end{cases}$$

So, we find

$$U = \begin{cases} = NT\left(3 + O\left(\dfrac{m^2}{T^2}\ln\dfrac{m}{T}\right)\right), & \text{for } T \gg m\,, \\[2ex] = NT\left(2 + O\left(\dfrac{T}{m}\right)\right), & \text{for } T \ll m\,. \end{cases}$$

The high-temperature limit is the same as for bradyons [6,7]. It is connected with the fact that high-energy tachyons and bradyons behave as luxons. The low-temperature limit is obviously different, since for bradyons

$$U = N(\tfrac{3}{2}T + m)\,.$$

Specific heat is defined as follows:

$$C_v = \frac{1}{N}\left(\frac{\partial U}{\partial T}\right)_V$$

and is described by

$$C_v = 1 + z^2\,\frac{S''_{02}(z)}{S_{02}(z)} - \left(z\,\frac{S'_{02}(z)}{S_{02}(z)}\right)^2 = 3z^2\left(\frac{5S_{-24}(z) - (1/z^2)S_{-13}(z)}{S_{02}(z)} - \frac{3S^2_{-13}(z)}{S^2_{02}(z)}\right) - 1\,,$$

$$C_v = \begin{cases} = 3 + O\left(\dfrac{m^2}{T^2}\ln\dfrac{m}{T}\right), & \text{for } T \gg m\,, \\[2ex] = 2 + O\left(\dfrac{T}{m}\right), & \text{for } T \ll m\,. \end{cases}$$

In the table we show the numerically found values of $C_v$ and $U/pV$ *vs.* $m/T$. Let

TABLE I.

| $m/T$ | $C_v$ | $U/pV$ |
|---|---|---|
| 0 | 3.000 | 3.000 |
| 0.1 | 3.004 | 2.995 |
| 0.2 | 3.014 | 2.983 |
| 0.5 | 2.052 | 2.917 |
| 1 | 3.073 | 2.768 |
| 2 | 2.930 | 2.505 |
| 3 | 2.755 | 2.337 |
| 5 | 2.427 | 2.171 |
| 10 | 2.151 | 2.054 |
| 20 | 2.043 | 2.014 |
| $\infty$ | 2.000 | 2.000 |

us notice that specific heat is not a monotonical function of $m/T$ and at $m/T \simeq 1$ there is a weak maximum. An analogous table for bradyons is presented in ref. (7).

We conclude that all properties of the classical gas of tachyons and the gas of bradyons are similar and no new phenomena have been found in the case of tachyons.