The Phase Space of Tachyons (*).

ST. MRÓWCZYŃSKI
JINR - Dubna, USSR
Institute of Nuclear Problems - Warsaw, Poland

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Summary. — The phase space of quantized systems that contain tachyons has been investigated. Interpretation difficulties and unexpected divergences are found when we consider the volume of Lorentz-invariant phase space. These problems can be overcome, however, at the expense of Lorentz invariance.

PACS. 14.80. – Other and hypothetical particles.

1. – Introduction.

The aim of this paper is to discuss the phase space (PS) of quantized systems that contain bradyons and tachyons, slower and faster than light particles (**). At the beginning we present essential features of the concept of tachyons that we assume in this paper (1). We would like to introduce the nonpractitioners and to avoid possible misunderstandings that could arise since several methods of solving the well-known causal paradoxes were found and several concepts of tachyons were consequently proposed (2-4).

(*) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.
(**) Some results of this paper were presented in our previous article: ST. MRÓWCZYŃSKI: *Lett. Nuovo Cimento*, **36**, 340 (1983).
(1) E. RECAMI and R. MIGNANI: *Riv. Nuovo Cimento*, **4**, 209 (1974).
(2) O. M. P. BILANIUK, V. K. DESHPANDE and E. C. G. SUDARSHAN: *Am. J. Phys.*, **30**, 718 (1962).
(3) A. F. ANTIPPA: *Nuovo Cimento A*, **10**, 389 (1972).
(4) H. LEMKE: *Phys. Lett. A*, **72**, 409 (1979).

The tachyon is described by the spacelike four-momentum p lying on the single-sheeted hyperboloid

$$p^\mu p_\mu = -m^2 ,$$

where m is the real tachyon mass. From the above statement it follows that the tachyon energy

$$E = \sqrt{\bar{p}^2 - m^2}$$

is real because $|\bar{p}| \geqslant m$. The reinterpretation principle (RP) ([2]) is assumed according to which the negative-energy tachyon travelling backward in time has to be regarded as a positive-energy antitachyon travelling forward in time. In the version more suitable for this paper the RP can be formulated as follows: the negative-energy tachyon in the final (initial) state of the reaction has to be treated as a positive-energy antitachyon in the initial (final) state of the reaction.

Although progress towards understanding the tachyon dynamics has been made ([5]), no satisfactory field-theoretical model has been found. For a rewiev of this interesting problem see ref. ([6,7]). In such a situation it could be valuable to study the PS properties of tachyons, since some problems of field models arise in these considerations, but in a strongly simplified version. On the other hand, in the absence of a dynamical theory the knowledge of PS could be interesting because in the threshold regions PS effects are expected to be dominant.

2. – The Lorentz-invariant phase-space.

We define the volume of the Lorentz-invariant PS in the following explicitly invariant form:

$$L^N(S, \pm m_1^2, \dots, \pm m_N^2) = \int \prod_{i=1}^{N} \mathrm{d}^4 p_i \, \delta(p_i^2 \mp m^2) \, \delta^{(4)}\Big(P^\mu - \sum_{i=1}^{N} p_i^\mu\Big) \tag{1}$$

with $P^\mu P_\mu = S$,

$$p_i^2 = p_i^\mu p_{i\mu} .$$

Upper signs are for bradyons and lower ones for tachyons.

([5]) *Proceedings of Session « Tachyons, Monopols and Related Topics » Erice 1976*, edited by E. RECAMI (Amsterdam, 1978).

([6]) K. KAMOI and S. KAMEFUCHI: *Prog. Theor. Phys.*, **45**, 1646 (1971).

([7]) A. L. CAREY, C. M. EY and C. A. HURST: *Hadronic J.*, **2**, 1021 (1979).

The deltas under multiplication mark «keep» particles on the mass shell, while the four-dimensional delta secures the conservation of four-momentum P^μ. In (1) we have to integrate over positive and negative energies of particles. In the case of bradyons the distinction of particles with negative and positive energies is Lorentz invariant, since bradyons and antibradyons lie on two separated sheets of the hyperboloid. So, we can, without destroying invariance, neglect the negative-energy part. Using the equality

$$\delta(p^2 \mp m^2) = \frac{\delta(p_0 - \sqrt{\bar{p}^2 \pm m^2}) + \delta(p_0 + \sqrt{\bar{p}^2 \pm m^2})}{2\sqrt{\bar{p}^2 \pm m^2}}$$

after integration over p_0, we get the well-known definition of invariant PS

$$L^N(S, m_1^2, \ldots, m_N^2) = \int \prod_{i=1}^{N} \frac{d^3 p_i}{2E_i}\, \delta^{(4)}\left(P^\mu - \sum_{i=1}^{N} p_i^\mu\right),$$

where $E_i \equiv p_{0i} = \sqrt{\bar{p}_i^2 + m_i^2}$.

In the tachyon case it is not possible to remove the negative energies without violation of Lorentz invariance, since the hyperboloid of the tachyon is single-sheeted. By invoking the RP, we transfer the negative energies from the final to the initial state. However, PS defined in (1) by such a procedure loses its final states of the reaction where the four-momentum P^μ is conserved. Let us con-

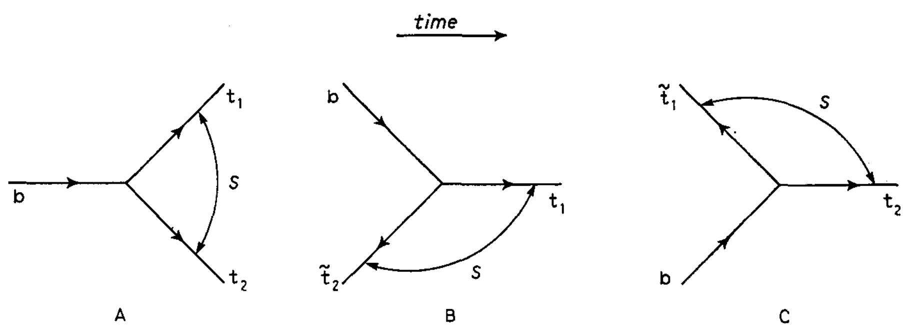

Fig. 1. – The decay of a bradyon b into tachyons t_1 and t_2 seen by three different observers A, B and C. Due to the RP, the tachyon can be transformed from the final to the initial state by Lorentz transformations.

sider the reaction which for different observers could be presented by the graphs shown in fig. 1. The two-particle PS defined in (1) suitable for this situation is a measure of the states of a system of two tachyons which can occur in the final and the initial states as well. The four-momenta of tachyons fulfil the condition

$$(\pm p_1^\mu \pm p_2^\mu)(\pm p_{1\mu} \pm p_{2\mu}) = S\,.$$

We see that Mandelstam variable S has different meanings for different observers; for A it is the square of energy of tachyons in their centre-of-mass frame (c.m.), while for B and C it is the four-momentum transfer between tachyons.

For symmetrically treating tachyons and bradyons we do not exclude the negative energies of bradyons from (1). Such an exclusion does not violate the Lorentz invariance of (1). However, covariance under superluminal transformations (boosts over the light velocity barrier that change spacelike to timelike vectors and *vice versa*, see, *e.g.*, ref. ([1])) would be lost, since such transformations do not conserve the sign of the zero component of the four-vector. In the next section we consider PS that has an ordinary meaning; however, it is neither Lorentz invariant nor covariant under superluminal transformations.

In the case of two particles L^2 is decomposed into four parts:

$$L^2 = L^2_{++} + L^2_{+-} + L^2_{-+} + L^2_{--} ,$$

where the marks $+$, $-$ indicate the sign of the energy of each particle. The final results are:

1) *Bradyon-bradyon*

$$L^2(S, m_1^2, m_2^2) = \begin{cases} A(S, m_1^2, m_2^2) , & (m_1 + m_2)^2 \leqslant S , \\ 0 , & (m_1 - m_2)^2 \leqslant S < (m_1 + m_2)^2 , \\ A(S, m_1^2, m_2^2) , & 0 \leqslant S < (m_1 - m_2)^2 , \\ +\infty , & S < 0 , \end{cases}$$

where

$$A(S, \pm m_1^2, \pm m_2^2) \equiv \frac{\pi}{2} \sqrt{\frac{(S \mp m_1^2 \mp m_2^2)^2 - 4(\pm m_1^2)(\pm m_2^2)}{S^2}} .$$

2) *Bradyon-tachyon or tachyon-tachyon*

$$L^2(S, \pm m_1^2, \pm m_2^2) = \begin{cases} A(S, \pm m_1^2, \pm m_2^2) , & S \geqslant 0 , \\ +\infty , & S < 0 . \end{cases}$$

The divergences that arise at $S < 0$ are linear and come from L^2_{+-} and L^2_{-+} terms. For finding L^N with $N > 2$, the recurrence formula (2) is introduced

$$(2) \qquad L^{N+1}(S, \pm m_1^2, \dots, \pm m_N^2, \pm m_{N+1}^2) = \int d^4 p_{N+1}\, \delta(p_{N+1}^2 \mp m_{N+1}^2)\, L^N(S, \pm m_1^2, \dots, \pm m_N^2) ,$$

where $P^\mu P_\mu = S$.

The divergences found for L^2 make that L^N for $N > 2$ is *divergent* for any S. This unexpected result shows that there arise not only the interpretation difficulties quoted previously when we try to build the formalism of tachyons which is a simple extension of methods for particles slower than light. The above divergence are of a completely different nature than those in QED, for example, since they come from pure kinematics, but do not depend on interaction phenomena. They arise for tachyons and bradyons as well, because such divergences are related to the way the negative energies are taken into account.

3. – The noninvariant phase space.

We consider PS that has an ordinary meaning and has been obtained from (1) by removing the negative-energy parts:

$$L^N_>(P^\mu, \pm m_1^2, \dots, \pm m_N^2) = \int \prod_{i=1}^{N} \frac{\mathrm{d}^3 p_i}{2E_i} \delta^{(4)}\Big(P^\mu - \sum_{i=1}^{N} p_i^\mu\Big) .$$

As noted, when N particles are bradyons and luxons (massless particles), L^N is Lorentz invariant. The results for L^N, where N equals merely two, are so complicated in form that we present here some particular cases only, and general results (any two-particle system in any reference frame) are confined to the appendix. In the centre-of-mass frame ($P^\mu = (\sqrt{S}, 0)$, S in c.m. is always positive for any kind of particles), we have the following:

$$L^2_>\big((\sqrt{S}, 0), \pm m_1^2, \pm m_2^2\big) = \begin{cases} A(S, \pm m_1^2, \pm m_2^2) , & E_0^2 \leqslant S , \\ 0 , & S < E_0^2 , \end{cases}$$

where E_0 is the threshold energy and

$$E_0^2 = \begin{cases} (m_1 + m_2)^2 & \text{for two bradyons ,} \\ m_1^2 + m_2^2 & \text{for bradyon and tachyon ,} \\ |m_1^2 - m_2^2| & \text{for two tachyons .} \end{cases}$$

The function A has been determined previously.

The existence of threshold in the systems containing tachyons is connected with the fact that the momentum of the tachyon cannot be smaller than its mass; this corresponds to the property of the bradyon the energy of which is always larger than its mass. The volume of PS for bradyons is a continuous and increasing function of energy, but when the system contains at least one tachyon, this function « jumps » at thresholds from zero to a finite value and then decreases with energy. In fig. 2 it is shown the volume of PS when the masses of particles are the same and equal to m. Let us notice that in this case there is no threshold in the two-tachyon system and L^2 goes to infinity when S reduces to zero.

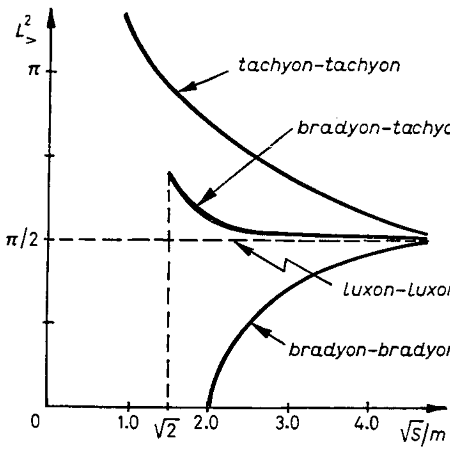

Fig. 2. – The volume of PS $L^2_>$ in c.m. for particles with equal masses as a function of energy.

Such a property can lead to peculiar vacuum instability, since the vacuum could decay into real (not virtual) tachyons, say tachyon-antitachyon pairs. These tachyons carry nearly zero energy but momenta higher than their masses. The possibility and some consequences of this vacuum instability have been discussed previously ([8]), but this PS aspect of the quoted phenomena has not been examined.

The volume of PS for bradyon-luxon and tachyon-luxon systems is presented in fig. 3. When we consider the reaction

$$\mathrm{B} \to \mathrm{T} + \mathrm{L}\,,$$

where B denotes bradyon, T tachyon and L luxon, we find that the most favourable configuration from the point of view of kinematics arises when the rest masses of B and T are the same and the velocity of the tachyon is infinite (the tachyon carries zero energy). If there exists the « tachyon electron », e_t, as an analog of the « bradyon electron », e_b, *i.e.* the ordinary electron, the above observation means that the following transitions or even

([8]) R. MIGNANI and E. RECAMI: *Phys. Lett. B*, **65**, 148 (1976).

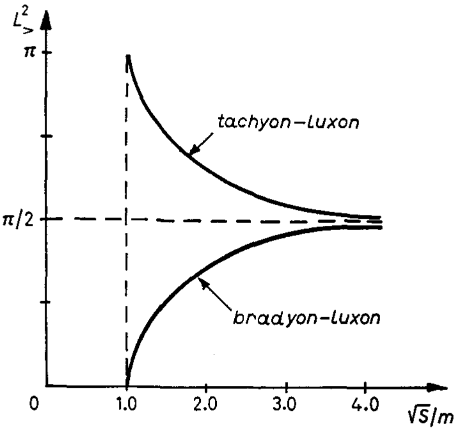

Fig. 3. – The volume of PS $L^2_>$ in c.m. for tachyon-luxon and bradyon-luxon systems as a function of energy.

oscillations, noticed in [9],

$$e_b \leftrightarrow e_t + \gamma$$

could occur. γ denotes the photon or, more generally, the luxon.

We have to stress that the above physical interpretation is valid in the c.m. only.

As an example of the volume of PS in any reference frame, we present a system of two tachyons with equal masses:

$$L^2_>(\mathcal{P}^\mu, -m^2, -m^2) =$$

$$= \begin{cases} S < 0\,, & \\ \dfrac{\pi}{2}\dfrac{\varepsilon}{\mathcal{P}}\,, & S^2 + 4m^2\varepsilon^2 \leqslant 4m^2\mathcal{P}^2\,, \\ \dfrac{\pi}{2}\dfrac{\varepsilon S + \sqrt{S^2 + 4m^2 S}\,\mathcal{P}}{\mathcal{P}S}\,, & S^2 \leqslant 4m^2\mathcal{P}^2 < S^2 + 4m^2\varepsilon^2\,, \\ 0 & 4m^2\mathcal{P}^2 \leqslant S^2\,, \\ \cdots\cdots\cdots\cdots\cdots\cdots & \\ S \geqslant 0\,, & \\ \dfrac{\pi}{2}\dfrac{\varepsilon}{\mathcal{P}}\,, & 4m^2\mathcal{P}^2 \geqslant S^2\,, \\ A(S, -m^2, -m^2) = \dfrac{\pi}{2}\sqrt{1 + \dfrac{4m^2}{S}}\,, & 4m^2\mathcal{P}^2 < S^2\,, \end{cases}$$

$$S = \mathcal{P}^\mu \mathcal{P}_\mu\,, \qquad \mathcal{P}^\mu = (\varepsilon, \overline{\mathcal{P}})\,, \qquad \mathcal{P} \equiv |\overline{\mathcal{P}}|\,.$$

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[9] H. LEMKE: *Phys. Rev. D*, **22**, 1342 (1980).

The differences of the volume of PS for the same reaction in different frames come from the fact that the energy of one or more particles in the configuration which conserves P^μ can be positive in one frame and negative in another one. So in one frame such a configuration is taken into consideration, while in the other one it is not. We see that the condition, for example

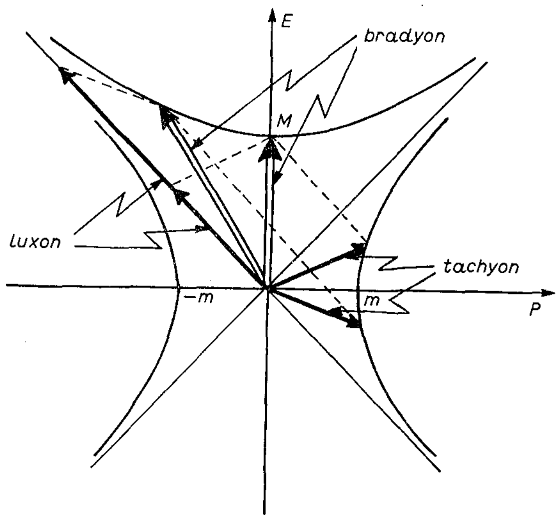

Fig. 4. – The decay of a bradyon with mass M into a luxon and a tachyon with mass m in the rest frame of the bradyon (c.m. of tachyons) and in the other moving frame. We see that the energy of the tachyon in the second frame is negative.

$4m^2\mathscr{P}^2 \geqslant S$, defines the reference frame, more precisely the class of reference frames. This situation is shown in fig. 4. However, $L^N_>$ is noninvariant, there are some reference frames where $L^N_>$ is the same.

For example,

$$L^2_>(\mathscr{P}^\mu, -m^2, 0) = \frac{\pi}{2}\left(1 + \frac{m^2}{S}\right),$$

if $(\varepsilon - \mathscr{P})^2 \geqslant m^2$ and $S \geqslant 0$. See the appendix.

$L^N_>$ for $N > 2$ can be calculated from the following recurrence formula:

$$(3) \qquad L^{N+1}_>(\mathscr{P}^\mu, \pm m_1^2, \ldots, \pm m_{N+1}^2) = \int \frac{d^3 p_{N+1}}{2E_{N+1}} L^N_>(\mathscr{P}^\mu - p^\mu_{N+1}, \pm m_1^2, \ldots, \pm m_N^2)\,.$$

We see that, for finding $L^{N+1}_>$ even in one frame, *e.g.* c.m., we have to know $L^N_>$ in any frame. Let us consider the two simplest cases of three-particle PS, namely two luxons and a bradyon and two luxons and a tachyon in the c.m.

$$L^3_>\left((\sqrt{S}, 0), 0, 0, \pm m^2\right) = \int \frac{d^3p}{2E} L^2_>(P^\mu - p^\mu, 0, 0)\,.$$

But $L^2_>(P^\mu, 0, 0) = \pi/2$, so, after integration, we get

$$L^3_>\left((\sqrt{S}, 0), 0, 0, \pm m^2\right) = \frac{\pi^2}{8} S\left(1 - \frac{m^4}{S^2} \pm 2\frac{m^2}{S}\ln\frac{m^2}{S}\right).$$

For luxons $L^N_>$ can be found for any N. The recurrence formula (3) is simplified to the form

$$L^{N+1}_>(S, 0, \ldots, 0) = 2\pi \int_0^{\sqrt{S}/2} dE\, E L^N_>(S - 2\sqrt{S}E, 0, \ldots, 0)$$

and by induction it can be proved that

$$L^N_>(S, 0, \ldots, 0) = \left(\frac{2}{\pi}\right)^{N-1} \frac{S^{N-2}}{(N-2)!(N-1)!}\,.$$

We conclude that it is easy to construct PS that is free of the difficulties discussed in the previous section, however at the expense of Lorentz invariance. We accept the view of the authors of (7) that «the first major problem to be overcome in developing a quantum theory of tachyons is in reconciling the apparent conflict between Lorentz invariance and the need to have only positive energies capable of being observed to the theory».

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The author would like to express his thanks to Prof. I. BIRULA-BIAŁYNICKI for helpful discussions.

Appendix

1) *tachyon-tachyon*, $m_1 \leqslant m_2$

$L^2_{>}(\mathscr{P}^\mu, -m_1^2, -m_2^2) =$

$\frac{\pi}{2}\frac{\varepsilon}{\mathscr{P}}$,		$a^2 + 4\varepsilon^2 m_2^2 \leqslant 4m_2^2\mathscr{P}^2$	$S < 0$
$\frac{\pi}{2}\frac{\varepsilon S + \sqrt{a^2 + 4Sm_2^2}\mathscr{P}}{\mathscr{P}S}$,	$b < 0$,	$a^2 + 4\varepsilon^2(s - a) \leqslant 4m_2^2\mathscr{P}^2 < a^2 + 4\varepsilon^2 m_2^2$	$S < 0$
$\frac{\pi}{2}\frac{\varepsilon a + \sqrt{a^2 + 4Sm_2^2}\mathscr{P}}{2\mathscr{P}S}$,	$0 \leqslant b$	$a^2 + 4\varepsilon^2(s - a) \leqslant 4m_2^2\mathscr{P}^2 < a^2 + 4\varepsilon^2 m_2^2$	$S < 0$
$\frac{\pi}{2}\frac{\varepsilon a + \sqrt{a^2 + 4Sm_2^2}\mathscr{P}}{2\mathscr{P}S}$,		$a^2 \leqslant 4m_2^2\mathscr{P}^2 < a^2 + 4\varepsilon^2(S - a)$	$S < 0$
0,		$4m_2^2\mathscr{P}^2 < a^2$	$S < 0$
$A(S, -m_1^2, -m_2^2)$,	$m_2^2 - m_1^2 < (\varepsilon - \mathscr{P})^2$	$4m_1^2\mathscr{P}^2 < b^2 + 4\varepsilon^2(S - b)$	$0 \leqslant S$
$\frac{\pi}{2}\frac{\varepsilon a + \sqrt{b^2 + 4Sm_1^2}\mathscr{P}}{2\mathscr{P}S}$,	$(\varepsilon - \mathscr{P})^2 \leqslant m_2^2 - m_1^2 < (\varepsilon + \mathscr{P})^2$	$4m_1^2\mathscr{P}^2 < b^2 + 4\varepsilon^2(S - b)$	$0 \leqslant S$
0,	$(\varepsilon + \mathscr{P})^2 \leqslant m_2^2 - m_1^2$	$4m_1^2\mathscr{P}^2 < b^2 + 4\varepsilon^2(S - b)$	$0 \leqslant S$
$\frac{\pi}{2}\frac{\varepsilon a + \sqrt{b^2 + 4Sm_1^2}\mathscr{P}}{2\mathscr{P}S}$,		$b^2 + 4\varepsilon^2(S - b) \leqslant 4m_1^2\mathscr{P}^2 < b^2$	$0 \leqslant S$
$\frac{\pi}{2}\frac{\varepsilon}{\mathscr{P}}$,		$b^2 \leqslant 4m_1^2\mathscr{P}^2$	$0 \leqslant S$

$a \equiv S + m_1^2 - m_2^2, \quad b \equiv S - m_1^2 + m_2^2, \quad S = \mathscr{P}^\mu\mathscr{P}_\mu, \quad \mathscr{P}^\mu = (\varepsilon, \overline{\mathscr{P}}), \quad \mathscr{P} \equiv |\overline{\mathscr{P}}|$

2) *tachyon-bradyon*

$$
L^2(\mathscr{P}^\mu, -m_1^2, m_2^2) = \begin{cases}
\dfrac{\pi}{2}\dfrac{\varepsilon}{\mathscr{P}}\,, & a^2 + 4m_1^2\varepsilon^2 \leqslant 4m_1^2\mathscr{P}^2 & S < 0 \\[2ex]
\dfrac{\pi}{2}\,\dfrac{\varepsilon a + \sqrt{a^2 + 4Sm_1^2\mathscr{P}}}{2\mathscr{P}S}\,, & a^2 \leqslant 4m_1^2\mathscr{P}^2 < a^2 + 4m_1^2\varepsilon^2 & \\[2ex]
0\,, & 4m_1^2\mathscr{P}^2 < a^2 & \\[2ex]
A(S, -m_1^2, m_2^2)\,, & m_1^2 + m_2^2 < (\varepsilon - \mathscr{P})^2 & 0 \leqslant S \\[2ex]
\dfrac{\pi}{2}\,\dfrac{\varepsilon a + \sqrt{b^2 - 4Sm_2^2\mathscr{P}}}{2\mathscr{P}S}\,, \quad 4m_2^2\mathscr{P}^2 < 4\varepsilon^2(b - S) - b^2 & (\varepsilon - \mathscr{P})^2 \leqslant m_1^2 + m_2^2 < S & \\[2ex]
A(S, -m_1^2, m_2^2)\,, \quad 4\varepsilon^2(b - S) - b^2 \leqslant 4m_2^2\mathscr{P}^2 & & \\[2ex]
\dfrac{\pi}{2}\,\dfrac{\varepsilon a + \sqrt{b^2 - 4Sm_2^2\mathscr{P}}}{2\mathscr{P}S}\,, \quad 4m_2^2\mathscr{P}^2 < 4\varepsilon^2(b - S) - b^2 & S \leqslant m_1^2 + m_2^2 < (\varepsilon + \mathscr{P})^2 & \\[2ex]
0\,, \quad 4\varepsilon^2(b - S) - b^2 \leqslant 4m_2^2\mathscr{P}_2^2 & & \\[2ex]
0\,, & (\varepsilon + \mathscr{P})^2 \leqslant m_1^2 + m^2 &
\end{cases}
$$

$a \equiv S - m_1^2 - m_2^2\,, \qquad b \equiv S + m_1^2 + m_2^2\,, \qquad S = \mathscr{P}^\mu\mathscr{P}_\mu\,, \qquad P^\mu = (\varepsilon, \overline{\mathscr{P}})\,, \qquad \mathscr{P} \equiv |\overline{\mathscr{P}}|$

● RIASSUNTO (*)

È stato studiato lo spazio delle fasi dei sistemi quantizzati che contengono tachioni. Si trovano difficoltà d'interpretazione e divergenze inaspettate quando si considera il volume dello spazio delle fasi invariante di Lorentz. Questi problemi si possono superare, comunque, a spese dell'invarianza di Lorentz.

(*) *Traduzione a cura della Redazione.*

Фазовое пространство тахионов.

Резюме (*). — Исследуется фазовое пространство квантованных систем, которые содержат тахионы. Возникают трудности интерпретации и неожиданные расходимости, когда мы рассматриваем объем Лоренц-инвариантного фазового пространства. Однако эти проблемы могут быть преодолены за счет Лоренц-инвариантности.

(*) *Переведено редакцией.*