Modeling nonperturbative structure of the pion

Research objective (scientific problem aimed to be solved by the proposed project, scientific questions and hypotheses)

Theoretical understanding of the structure of hadrons is one of the main goals of the physics of strong interactions. As the problem is inherently nonperturbative, concerning the low-momentum domain where the interactions are strong and no strict analytic tools are available, it needs dedicated methods which are applicable in this regime. Such methods involve first of all the lattice quantum chromodynamics (QCD), where recently a tremendous progress has been accomplished, providing new increasingly accurate and rich data of the intricate hadron structure.

On the other side, there are various phenomenological approaches based on specific features of QCD, such as effective chiral quark models (the Nambu–Jona-Lasinio model and its descendants), based on the phenomenon of the breaking of chiral symmetry, models based on confinement (various string or flux-tube models), on the parton-hadron duality principle, on the AdS/CFT correspondence, etc. These effective approaches have proven to be successful in reproducing, at least approximately, many experimental features (including the lattice data) of hadrons, thus contributing to our understanding of the challenging problems of nonperturbative QCD.

Presently, the lattice QCD is believed to provide us with reliable results, and in this respect it may be considered as “theoretical experiment” with fully controllable conditions, such as the lattice spacing, volume corrections, finite quark masses, and limited statistics. These are the ideal conditions to which effective models should be compared to. However, since the lattice simulations incorporate all aspects of QCD in a numerical fashion, they do not give us any clear hint what leading physical mechanisms are responsible for specific properties of the observables in question. In contrast, effective models, which incorporate only certain but crucial aspects of QCD, do provide us with a better intuitive insight and theoretical picture of the underlying physics. This is also because they are technically much simpler, and of course do not suffer from the cumbersome artifacts of the lattice discretization.

The choice of the pion (or more generally, a meson which is a Goldstone boson) as an object of our study, whose properties are usually very difficult to measure experimentally, is dictated by the fact that both the lattice QCD simulations and the effective models are much simpler for the pion than for baryons, where the physical data are more abundant. Nevertheless, as stated above, lattice data can be considered, to a large degree of confidence, as “experimental” results allowing us to test effective approaches in their full diversity.

Just recently, new and very interesting lattice QCD data for the correlation properties of the pion became available from the Regensburg group (RQCD). In particular, correlators of two currents of various quantum numbers [1] in the pion state have been computed, with a ramification to the double parton distribution functions (dPDFs) [2]. These data open a new scope of investigations, with a detailed and complete view on a class of hadronic four-point Green’s functions. This is perhaps the first time that a direct *ab initio* access has been accomplished to these intricate and poorly understood physical objects.

The main goal of the proposed project is to explore and understand these lattice data, and also, whenever possible, to go beyond, with the help of simple theoretical models. We thus plan to evaluate the correlation functions in the pion in two basic and complementary approaches: i) in chiral quark models (see [3] and references therein), and ii) in the meson dominance model [4] (which is a natural extension of Sakurai's vector-meson dominance principle). The results will be compared to the lattice simulations in a detailed manner, including all possible channels (quantum numbers of the currents) and with varied pion mass. The two models and our methods are described in more detail in the *Methods of research* section.

A topic directly related to the correlation functions are dPDFs of the pion. In general, the issue of dPDFs (of the nucleon) has recently received a lot of attention due to potential relevance of the double parton scattering (DPS) at the Large Hadron Collider (LHC). On the lattice, there is presently access to the lowest Mellin moment of dPDFs [2], with more data from RQCD announced to be released shortly. Importantly, the link to dPDFs gives a possibility to test to what degree works the frequently assumed approximation of factorization into the product of two single parton distribution functions (PDFs). This carries a practical significance for the phenomenological analyses of DPS. Whereas these objects encode crucial information and set constraints on the structure of the simpler single parton distributions via marginalization to the additional Bjorken variable, our present understanding of dPDFs is so far rather limited.

We plan to carry out model evaluation of the pion dPDF and its Mellin moments, and compare the results to the present and future lattice data. The comparison requires a suitable QCD evolution from the model scale up to the scale of the lattice, see the *Methods of research* section.

Apart for the two-current observables, we will also study the matrix elements of single bilocal currents in the pion state, for the reason that such objects have recently acquired a lot of theoretical attention. What we have in mind here is the evaluation of the Wigner distributions – the "mother" of all other quantities used in the field, such as the generalized parton distribution (GPDs), the transverse-momentum distributions (TMDs), and further the PDFs and various form factors (charge, gravitational, etc.). Moreover, appropriate moments of the Wigner distributions yield also the angular-momentum observables, providing the spin decomposition in the hadron. We thus plan to evaluate the Wigner distributions, and the related Husimi distributions, applying our models and methods. The model results will be most likely analytic or accessible through extremely simple numerical calculations, thus their simplicity will shed light on the involved theoretical issues underlying their physical features.

Finally, we wish to explore a theoretical concept related to the above-mentioned meson dominance, namely, the parton-hadron duality realized within the Regge approach. These models invoke infinitely many hadronic intermediate states (resonances) in the evaluation of matrix elements. In the proposed project we plan to determine whether in the high-energy limit, as seen in the deep inelastic scattering (DIS), the partonic substructure of hadrons could actually be described solely in terms of infinitely many colorless resonances. For definiteness, we plan to consider DIS of the pion, and will assume linear Regge spectra, supported experimentally [5]. If we answer this question affirmatively, a direct extension to all kinds of partonic calculations will become possible providing a completely new tool to study nonperturbative aspects of hadronic structure. Otherwise, there will be a flagrant deviation from the parton-hadron duality, which would deserve to take a serious notice. From the point of view of the completeness of the hadronic spectrum, it might hint to yet another reformulation of the phase transition in the high-energy physics environment. Therefore, this problem should be faced squarely, since any answer will shed definite light on the still mysterious intertwining between the hadronic and partonic degrees of freedom.

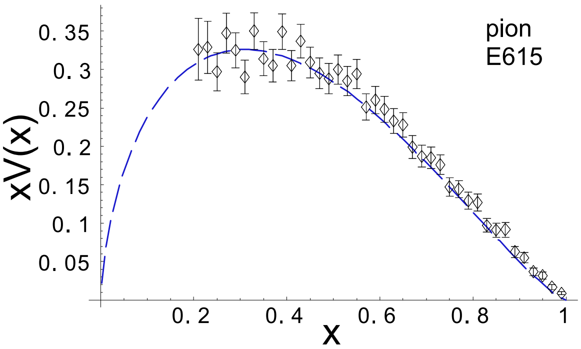

FIG. 1. The valence quark distribution function (PDF) of the pion (multiplied with x), plotted against the Bjorken x variable. One observes a very good agreement of the model predictions, evolved from the low quark-model energy scale to the scale of 2 GeV, to the data derived from Drell-Yan E615 experiment at Fermilab [6].

Significance of the project (state of the art, justification for tackling scientific problems by the proposed project, justification for the innovative nature of the research, the impact of the project's results on the development of the research field and scientific discipline)

The very latest lattice QCD results that we are hoping to explain with simple models represent, no doubt, the state of the art of the field. As already stated in the previous section, many features of the pion will be accessible predominantly on the lattice (and not in actual experiments), which supports dedicated theoretical studies of the lattice data.

The status of the chiral quark models in view of the planned tasks of the project is also very comfortable, after many years of fruitful and successful work done by the PI and the other proponents. Chiral quark models (amended with QCD evolution) indeed work surprisingly well for the soft matrix elements needed for the description of the high-energy processes, with a long list of results for the pion structure: parton distribution function (PDF) [7–9] (cf. Fig. 1 to assess the quality of comparison of the model to, this time real, experimental data), light-cone wave function (LCWF) and parton distribution amplitudes (PDA) [10–14], generalized parton distributions (GPD) [15–17], pion-photon transition distribution amplitude (TDA) [18, 19] and form factor (TFF) [20], gravitational [21] and transversity [22, 23] form factors, or the equal-time quark wave functions [24].

Another recently developed and very promising direction of the QCD lattice is based on the so-called parton quasi-distributions [25] or pseudo-distributions [26]. Our chiral quark model study [27, 28] yielded a very favorable comparison to the recent lattice data, as can be concluded from Fig. 2. The state of the art of the recent lattice results based on the quasi-distribution approach is presented in a recent review [29]. In all, it is rewarding that so many of the features analyzed both experimentally as well as on the lattice can be reproduced with so few ingredients, and almost parameter free, within a model rooted in a quantum field theoretic setup.

The proposed new studies of the two-current correlations and dPDFs in the chiral quark models will apply similar techniques that led to the above-mentioned successful results, but will involve nontrivial evaluations of quark box diagrams (four-point Green's functions), which carries some degree of complexity. Focusing on the Mellin moments of dPDFs (following [2]) will lead to much simpler results. The issue of factorizability into single PDFs, or departures thereof, should be possible to settle in simple terms in the model framework.

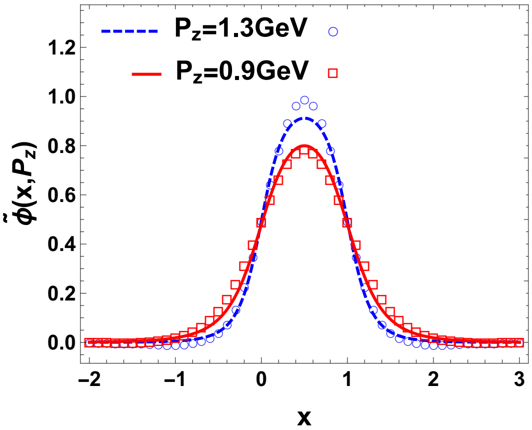

FIG. 2. Valence quark quasi-distribution in the pion, $\tilde{\phi}$, plotted as a function of the fraction of the pion's longitudinal momentum, P_z, carried but the valence quark (taken from [27]).

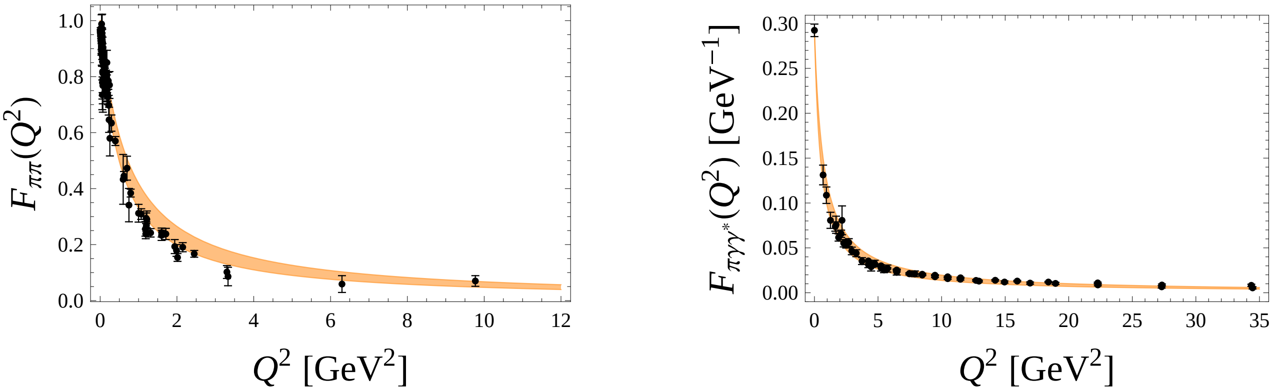

FIG. 3. The pion charge form factor (left) and the pion-photon transition form factor (right), obtained from the meson-dominance approach, plotted against the space-like momentum transfer and compared to experimental data (figure taken from [30]).

The meson-dominance approach to the soft-matrix elements follows from the concepts presented in [4], where the currents of given quantum numbers are saturated with a few lowest meson states, but still reproduce the high-energy QCD behavior by the so-called short-distance constraints. The method was used successfully in [30] to describe various pion form factors obtained from the experiment (cf. Fig. 3) and from lattice simulations (cf. Fig. 4). For instance, quite remarkably, the pion spin-2 gravitational form factor in the right panel of Fig. 4 is very nicely saturated with $f_2(1320)$ meson. This feature highlights the fact that most of these tediously computed form factors actually probe meson masses which agree, within estimated errors, with the values provided in the Particle Data Group review, a source of information of a completely different nature!

The application of the meson-dominance approach to the four-point functions, such as the two-current correlators in the pion state, requires also insertions of intermediate meson resonance states, which needs an extension of the model. One successful approach, which we wish to adopt here, is based on the large-N_c (N_c denotes the number of colors) Regge model of hadronic spectra. In the present context, the Regge approach was applied successfully to evaluate meson correlators [31], the pion DA, the pion-photon TFF [32, 33], the pion electromagnetic form factor [34], or the notorious σ meson [35]. In addition, a systematic study of the Regge light-quark non-strange mesons has been presented in [36].

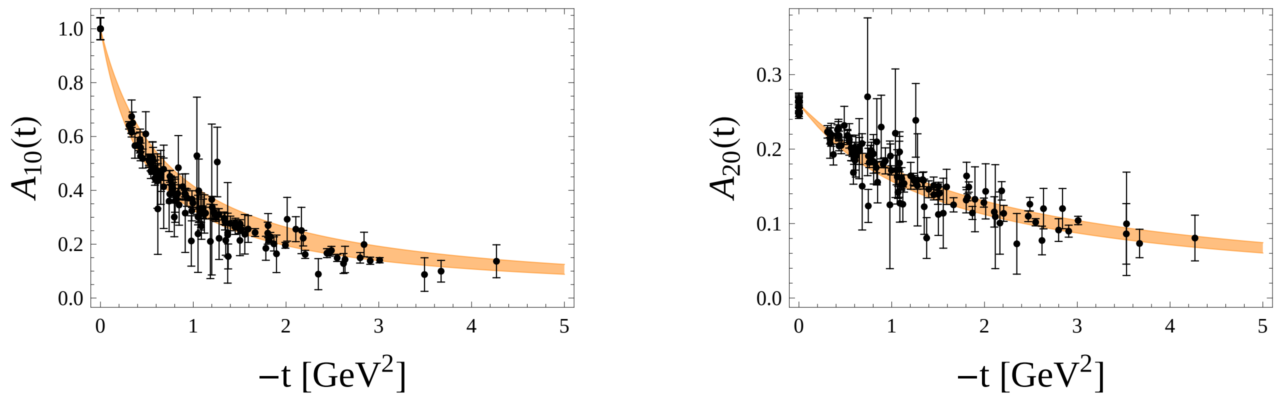

FIG. 4. Electromagnetic pion form factor $A_{10}(t)$ (left) and the quark part of the pion spin-2 gravitational form factor $A_{20}(t)$, plotted against the space-like momentum transfer, evaluated in the meson dominance approach and compared to the lattice data [37] (figure taken from [30]).

The issue of applying the model with Regge spectra is related to the task, where we plan to analyze deep inelastic scattering (DIS) in terms the parton-hadron duality. One of the most suggestive concepts in high-energy physics is the Bjorken scaling in DIS of electrons and neutrinos off protons. Such scaling is compatible with a partonic substructure of the nucleon in a first approximation, but is subject to radiative gluon corrections that involve a logarithmic violation of the Bjorken scaling. The corrections can be computed systematically in QCD in a perturbative manner assuming that the final states are quarks and, in general, they are experimentally met. However, at energies below the Bjorken limit what is observed experimentally are not quarks but baryonic resonances. It would be conceptually very important to show whether or not the parton-hadron duality can be implemented to the description of DIS in terms of purely hadronic degrees of freedom, and if yes, what predictive power it would carry for other observables.

The proposed task to compute the Wigner and Husimi partonic distributions inscribes into the long list of theoretical efforts to better intuitively understand the structure of hadrons. One invokes here the 3D tomographic imaging via the impact-parameter representation of GPDs [38], or the interpretation of generalized form factors in the hadron via pressure and surface tension [39]. The analytic nature of our results for the quark distributions in the pion will be of clear merit, allowing us to challenge the typical assumptions of the factorizability of the transverse and longitudinal distributions, or to verify the desired theoretical features (such as previously tested polynomiality of the generalized form factors, positivity of distributions, etc.)

We stress that our results, to be obtained both in chiral quark models and in the meson dominance/Regge model, will have analytic or semi-analytic character, which is always very useful in theoretical analysis, as physical features may be assessed in a simpler and intuitive way compared to numerical studies.

The potential significance and impact of the expected results of the correlation and dPDF tasks is related to a better understanding of the features of the rich lattice QCD data. Simple models provide useful benchmarks to which the lattice simulations may be compared to. In addition, in models one may easily investigate the effects of the change in pion mass (which on the lattice is usually above the physical value), or test the evolution of scale-dependent quantities from renormalization. Needless to say, lattice artifacts are absent on the model side. From the model perspective, we find it important to assess the validity and application range for the leading-N_c chiral quark models and the approaches

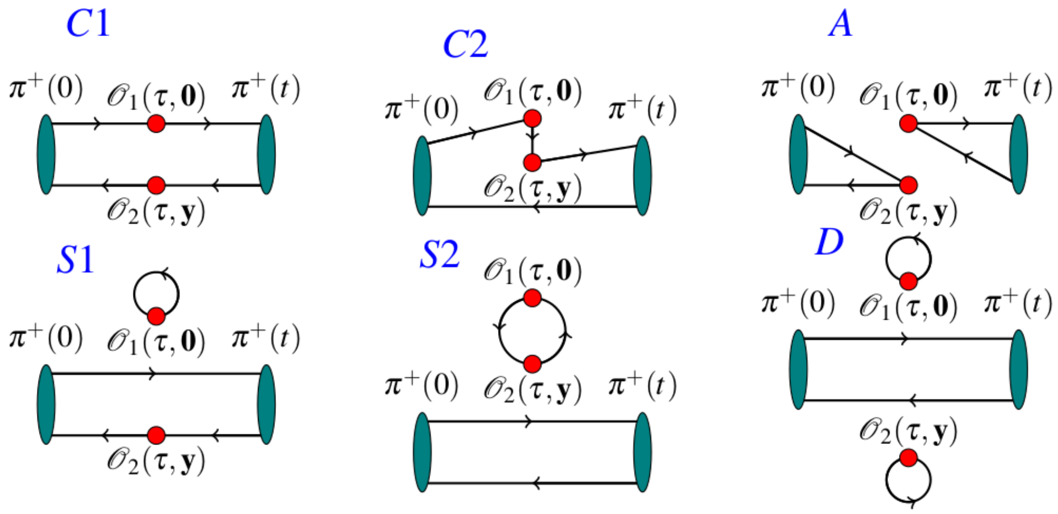

FIG. 5. Diagrams of various quark-line topologies for the two-current correlators considered in the lattice QCD study of [1] (figure taken from [2]).

based on meson dominance with Regge spectra. The new data pose a challenge to these conceptually attractive models, bringing them to the grounds where they have not been used up to now.

Our proposal is innovative, as the proposed tasks have not been carried out, nor even considered, in previous research. It will hopefully provide useful explanations for the current data and a further outreach to the physicists working on lattice QCD.

Concerning social aspects of our proposal and its impact on development of this branch of science in Poland, the project may be relevant for the formed lattice groups by young scientists in Poznań (Krzysztof Cichy) and at the Jagiellonian University (Piotr Korcyl), through the broadening of the landscape of hadronic topics relevant for lattice QCD. In addition, we foresee possibilities of suggesting studies of interesting aspects of the lattice calculations which will be triggered by the proposed model analyses.

Work plan (outline of the work plan, specific research goals, results of initial research, risk analysis)

The project consists of the following mutually related research tasks:

1. Analysis of current correlators in the pion within chiral quark models and in the meson dominance approach

The basic object of the analysis proposed in this task is the two-current correlator

$$\Pi_{ij}^{ab}(p,y) = \langle \pi^a(p)|J_i(y)J_j(0)|\pi^b(p)\rangle, \tag{1}$$

where $|\pi^a(p)\rangle$ is the pion state of isospin a and four-momentum p, and $J_i(y) = \bar{q}(y)\Gamma_i q(y)$ is a color singlet quark current with a Dirac and isospin structure i (for instance, $\Gamma_i = \gamma_\mu$). The diagrams contributing to the evaluation of $\Pi_{ij}^{ab}(p,y)$ in lattice QCD are shown in Fig. 5. It is useful that these diagrams, of different topology of the quark lines, may be analyzed independently, as similar distinction can be done in model studies.

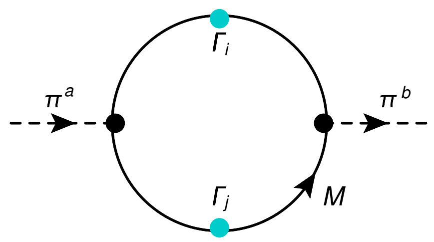

FIG. 6. Box diagram in the chiral quark model corresponding to diagram $C1$ in Fig. 5. The light blue dots correspond to the external current insertions.

In chiral quark models at the leading-N_c level, these QCD diagrams are replaced with corresponding quark-loop diagrams. Our methodology thus amounts to computing diagrams such as the box diagram of Fig. 6. The quark propagators, $i/(\not{p} - M)$, follow from the spontaneously broken chiral symmetry, and the external pion states are on the mass shell. We will carry out the calculation in the kinematics of Fig. 5, where the external current insertions are at equal Euclidean time, and integration over the space coordinate y is performed.

In effective chiral quark models, regularization is necessary to suppress the high-energy dynamics, not explicitly accounted for in the model. As this feature is generic for all our tasks, we describe it in the *Methods of research* section. We will then compute the properly regularized diagrams, such as that of Fig. 6, in all relevant channels: scalar-scalar, pseudoscalar-pseudoscalar, vector-vector, and axial vector-axial vector. Next, we will compare the obtained results to the lattice QCD data from [1]. Several issues are of particular relevance: first, it is of primary interest to see if the results match at all in various channels. Second, one may investigate contributions of particular graphs. Finally, we can check the quark mass (or the pion mass) dependence, which is of practical significance, as the values of the pion mass in lattice QCD implementations are usually above the physical point.

Another line of investigation in this task invokes the meson dominance and the large-N_c Regge model of the mesonic spectra. As mentioned in the previous section, the approach is remarkably successful in the description of hadronic form factors. In the present case, the application would involve (infinite) sums over intermediate meson states, with appropriately chosen couplings to the external currents. We remark here that the standard chiral perturbation theory (χPT) does not describe the lattice results, cf. Figs. 23 and 24 in [1]. However, the inclusion of resonances helps to some extent for the case of Fig. 23 and not for Fig. 24, which makes the case for amendment of the approach with higher-mass meson states. This is precisely the focal point of our task.

2. Evaluation of double parton distributions of the pion in chiral quark models

Our studies here will be complementary to the analysis of the previous task. The quark dPDFs of the pion are defined as [40]

$$F_{ij}^{ab}(x_1, x_2, \vec{y}) = 2p^+ \int \frac{dz_1^-}{2\pi}\,\frac{dz_2^-}{2\pi}\,dy^-\; e^{i(x_1 z_1^- + x_2 z_2^-)p^+}\,\langle \pi^a(p)|\,\mathcal{O}_i(0,z_1)\mathcal{O}_j(y,z_2)\,|\pi^b(p)\rangle, \qquad (2)$$

where x_1 and x_2 are momentum fractions of the partons and y is their relative transverse distance. The color-singlet quark bilinear operators are

$$\mathcal{O}_i(y,z) = \bar{q}(y - \tfrac{1}{2}z)\Gamma_i \mathcal{P}(y - \tfrac{1}{2}z, y + \tfrac{1}{2}z) q(y + \tfrac{1}{2}z)\Big|_{z^+=y^+=0,\ \vec{z}=\vec{0}}, \qquad (3)$$

where $\mathcal{P}$ is the Wilson gauge link operator, and Γ_i stands for a Dirac and isospin matrix (e.g, $\Gamma_i = \gamma^+, \sigma^{+\perp}$, etc.)

Let us mention here that at the energy scales reached at the LHC, multiparton interactions bring in a noticeable contribution in high-energy processes involving nucleons. DPS is frequently described by taking the product of two independent single parton scattering processes, which assumes that the partonic correlation effects can be neglected. The issue of this factorization was investigated in a simple valon model in [41], with the conclusion that it does not really hold in the region where the conservation of the longitudinal momentum is relevant.

The core of the evaluation of the present task is again the diagram of Fig. 6, but now with a constrained integration over the + light-cone momentum component in the quark loop. To satisfy the requirements of the Lorentz invariance, the convenient method of Appendix A of [17], based on the Schwinger representation of the quark propagators, will be used.

Similarly to the case of single PDFs, dPDFs are scale dependent and need proper QCD evolution. This can be carried out in the Mellin space for the simple case when no external momentum flows through the current insertions (we note that the general schemes to evolve dPDF with a momentum flow are still under development, as they involve the complication of the Wilson gauge link operators). In our study of the QCD evolution, we will proceed along the lines of [42] and references therein. As already mentioned, the present lattice QCD studies provide only the lowest Mellin moment of the pion dPDF, namely

$$M(p, y_\perp) = \frac{1}{2p^+} \int dy^- \, \Pi_{ij}^{ab}(p, y)\Big|_{y^+=0}, \tag{4}$$

where $\Pi_{ij}^{ab}(p, y)$ is defined in Eq. (1). Importantly, in the case of the Mellin moments, the quark-antiquark operators become local and hence the Wilson gauge-link operators do not appear, therefore the leading-order DGLAP evolution can be applied [42].

Factorization of the two-current correlations, if it holds, would mean that

$$\Pi_{ij}^{ab}(p, y) \overset{?}{\simeq} \int \frac{d^4p'}{(2\pi)^4} \delta(p'^2 - m_h^2) \langle \pi^a(p)| J_i(y) |h(p')\rangle \langle h(p')| J_j(0) |\pi^b(p)\rangle, \tag{5}$$

where $|h(p')\rangle$ is a hadronic state that couples in the given i, j channels. Its validity was checked on the lattice [2], with the outcome that it fails at small parton distances, as well as at large distances. We will probe the factorization in our approach by directly comparing the model evaluation of the right- and left-hand sides of Eq. (5).

We will also study the higher moments of dPDFs of the pion in view of the announced upcoming lattice QCD analyses.

We remark that up to now the only model evaluations of dPDFs of the pion have been carried out using the pion wave functions from the light-front approach [43] and from the AdS-CFT soft-wall model [44].

3. Evaluation of the partonic Wigner distribution of the pion in chiral quark models

Modern objects describing the hadron structure follow from the Wigner distributions (here given for the quarks):

$$W_i(x,\vec{b}_\perp,\vec{k}_\perp)=\frac{1}{2}\int \frac{dz^- d^2z_\perp}{(2\pi)^3}\frac{d^2\Delta_\perp}{(2\pi)^2}e^{i(xp^+z^- -\vec{k}_\perp\cdot\vec{z}_\perp)}\langle p+\tfrac{\Delta}{2}|\bar{q}(b-z/2)\Gamma_i\mathcal{P}q(b+z/2)|p-\tfrac{\Delta}{2}\rangle, \tag{6}$$

where, in the notation of Eq. (3), $\mathcal{P}\equiv\mathcal{P}(b-z/2,b+z/2)$. Here x has the interpretation of the pion's light-cone momentum fraction carried by the quark, $\vec{b}_\perp$ is the variable Fourier-conjugated to the external momentum transfer $\Delta_\perp$, and $\vec{k}_\perp$ is the relative transverse momentum between the quark and the anti-quark. The related Husimi distributions [45] are defined as Gaussian transforms of the Wigner distributions

$$H_i(x,\vec{b}_\perp,\vec{k}_\perp;\ell)\equiv\frac{1}{\pi^2}\int d^2b'_\perp d^2k'_\perp e^{-\frac{1}{\ell^2}(\vec{b}_\perp-\vec{b}'_\perp)^2-\ell^2(\vec{k}_\perp-\vec{k}'_\perp)^2}W_i(x,\vec{b}'_\perp,\vec{k}'_\perp), \tag{7}$$

where ℓ is a parameter of dimension of length (although the Husimi distributions carry the same information as the Wigner distributions, they are known to have better properties, such as positivity). As already stated, upon integration over $\vec{k}_\perp$ or $\vec{b}_\perp$, the Wigner distributions yield GPDs or TMDs, respectively, which have been objects of immense theoretical activity in the field.

The classification of the Wigner distributions of the pion has been presented in [46]. We will focus on the leading-twist contributions in various channels. The planed evaluation in chiral quark models amounts again to a one-quark loop diagram with two pion vertices and one external current, now evaluated with constrained + and $\perp$ component of the quark momentum in the loop. The anticipated results will be analytic and should be simple, in particular before the regularization is imposed. This will allow us for a simple check of the required formal features. The open issue here involves, as for all quantities with unintegrated transverse momentum, the presence of the Wilson gauge link operators and the corresponding complication in the QCD evolution. Exploration of these problems, which are currently being developed by several groups, are beyond the scope of this proposal.

We note that an evaluation of the Wigner distributions of the pion in a model with specific light-cone wave functions from [47] was recently presented in [48]. Our planned analysis, to be carried out covariantly at the one-quark-loop level, is of a different character.

4. Parton-hadron duality in large-N_c Regge models

The theoretical correspondence between final states formed in DIS with quarks and gluons or with hadron resonances is known as the quark-hadron or parton-hadron duality [49]. The most successful descriptions of the data collected initially in SLAC and more recently in TJLAB confirm this duality beyond its expected applicability [50, 51], a feature which is not understood. This phenomenon implies scaling rules for the Regge masses of excited baryons that are experimentally verified [30, 36]. Also, in a recent papers [52, 53] an interesting scaling relationship was derived, based on a dominance of quarks pairing into diquarks, which is seen in the spectrum of excited baryons.

In the case of the pion, what makes DIS interesting in the present context is that the quark content of hadrons can be deduced in the Bjorken limit, where scaling sets in, and thus the parton-hadron duality can be exploited. Under these conditions the universal underlying spin-$\frac{1}{2}$ nature of partons is encoded through the Callan-Gross relation, known to hold up to radiative corrections. Historically, this was the main reason to reject the old field algebra of Lee and Zumino, based on the rather fruitful idea of the vector meson dominance (VMD), where, however, partons had spin 1.

On the other hand, in the limit of large number of colors, QCD reduces to a theory of weakly interacting stable mesons and glueballs with no explicit reference to quark degrees of freedom [54, 55]. In fact, quark model relations, which work well, turn out to be the true large-N_c results, so this limit may be regarded as the model-independent formulation of the more restricted quark model, and hence an explicit realization of the parton-hadron duality. An intriguing and crucial question is how the spin-$\frac{1}{2}$ nature of hadrons, as encoded through the Callan-Gross relation, can be manifest in terms of mesonic resonances.

In the low and intermediate energy domain, the large N_c-limit has led to a QCD resurgence of VMD; model-independent results may be deduced if the short-distance constraints, inferred directly from the operator product expansion in QCD at high momenta, are imposed and only the lowest resonances need to be kept [4]. Deviations thereof are expected to be suppressed by inverse powers of the high-energy cut-off [56]. Then, unlike the case of the (local) chiral quark models, important chiral symmetry constraints, such as the second Weinberg sum rule, can be fulfilled. Moreover, an impressive parameter reduction of resonance masses, widths, and couplings in the chiral limit is achieved in terms of just f_π, the pion weak decay constant. As a by-product the low-energy constants of the effective Lagrangian in Chiral Perturbation Theory are produced as pure scale-free numbers. Lattice calculations going up to $N_c = 17$ fall in the bulk part of these purely analytic predictions, suggesting that this large-N_c-based inquiry into parton-hadron duality may still provide further simple and interesting results.

In the present project we plan to analyze the opposite, i.e., the high energy limit of the correlators, as seen in DIS. We will search for relations motivated by large-N_c, where the quark substructure of hadrons could be described solely in terms of infinitely many hadronic resonances. For definiteness, we will consider DIS on the pion, for which the Drell-Yan data exist [6] (see also [57] for a reanalysis and a comprehensive recent review). We will also assume a linear and universal Regge model behavior of the spectra, which is well fulfilled experimentally [5], particularly when the resonance widths are taken into account [30].

Using the same general scheme, we have successfully implemented the quark-hadron duality principle for the pion DA [32]. The pertinent question, to which the answer is by no mean obvious, is whether or not the parton-hadron duality can be implemented in DIS, while preserving the underlying spin-$\frac{1}{2}$ nature of quarks. Our analysis of this task will follow the approach used in [32, 53]

We remark that within the holographic approach to QCD based on the AdS/CFT correspondence, the Callan-Gross relation is not fulfilled [58], which poses a fundamental problem for these models.

Methods of research (manner of conducting research, methods, techniques and tools, data reduction and treatment schemes, equipment and devices used in the research)

Our methodology will be based on both analytic calculations in sufficiently simple cases, as well as on numerical simulations to be carried out in more involved elements of our approach, e.g., the

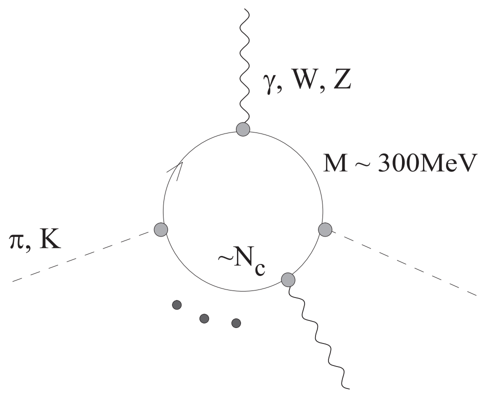

FIG. 7. One-quark-loop diagram with external Goldstone (π, K) or gauge (γ. W, Z) bosons, describing a leading-N_c quark process in the soft external momenta regime. The spontaneous chiral symmetry breaking attributes a large mass ($M \sim 300$MeV) to the quark.

QCD evolution. The specific methods to be used in the project can be divided into groups related to the chiral quark models, and to the meson dominance/parton-hadron duality analysis.

In general, the calculation in chiral quark models in the large-N_c limit are based on the evaluation of a generic one-loop diagram of Fig. 7, where due to the spontaneous chiral symmetry breaking, the quarks are massive. The external momenta must be sufficiently soft, such that the $q\bar{q}$ production channel is not open.

As already mentioned, chiral quark models require a suitable regularization, which is a way of introducing the necessary high-energy cut-off to the effective theory. In our study we will use the Nambu–Jona-Lasinio model with Pauli-Villars or the proper-time regularization, and the Spectral Quark Model [59], as these regularization schemes preserve the necessary Lorentz and gauge invariance.

Another key element is the QCD evolution, which is necessary to elevate the model results from the low-energy quark-model energy scale (around 320 MeV) to the much higher scales of the experiments or the lattice simulations. The standard method, based on the Mellin moments, is presented in detail in [17] for the single parton distributions, and in [42] for dPDFs.

The cartoon of the calculation in the meson dominance/parton-hadron duality model approach is shown in Fig. 8. The external wavy lines represent the meson-dominated currents, the extended bar represents the sum over the (infinitely many) intermediate mesonic states, and the blobs indicate the appropriate mesonic couplings. The associated crossed diagram is not displayed. As a general rule, we observe that the formal expression of the large-N_c parton-hadron duality is the feature that the one-loop integration of the chiral quark models turns into the infinitely many resonances of the tree level diagrams of the Regge model, cf. Fig. 6 vs Fig. 8. This has practical and non-trivial consequences on the high-energy side which we plan to investigate.

More involved analytic (symbolic) calculations will be carried out with the help of Mathematica, including the `FeynCalc` package to evaluate the Dirac traces for the one-quark loop processes. The license for Mathematica 11 (or higher) is available from Jan Kochanowski University. Some parts of our numerical analysis, such as the QCD evolution of the transverse-momentum unintegrated objects, will require running dedicated `FORTRAN` or `c++` codes. The numerics will be done on the computers purchased within the project, as well as on other available resources.

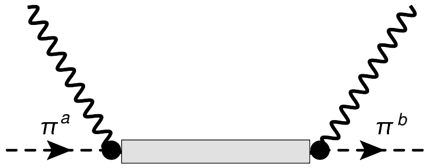

FIG. 8. The diagram for the meson-dominance evaluation of the two-current correlators in the pion state. The wavy lines indicate the external currents which may be saturated with meson states, whereas the solid extended bar represents the sum over the intermediate mesonic states, which exhibit Regge spectra. The blobs indicate appropriate mesonic couplings.

The planned research team will consist, apart for the PI, of two prominent researchers in the field: Prof. Enrique Ruiz Arriola from University of Granada, Spain, and Prof. Michał Paraszałowicz from the Jagiellonian University. Prof. Ruiz Arriola has a long track record of collaboration with the PI, with 63 jointly published papers and conference proceedings (according to the SPIRES database). Many of these papers are listed in the literature section of this proposal. He is an unquestionable expert in the field. So is Prof. Praszałowicz, who is an author of numerous seminal works on chiral quarks models, also in regard of the properties of the pion [11, 13, 60, 61]. The joint collaboration effort proposed in this project guarantees, in my view, a proper realization of the research tasks and a potential further important outreach. The organization of our collaborative work will be based on regular mutual visits between Spain and Poland, necessary to develop the key details of the proposed analyses, as well as via intense Internet communications.

Conclusion

The proposed project addresses important and timely topics of modern particle physics. The nonperturbative structure of the pion, manifest in two-current correlations, double parton distributions, or the partonic transverse-momentum distributions, has been actively pursued in very recent lattice QCD studies, providing more and more accurate and theoretically challenging results. We wish to address these issues from the point of view of simple QCD-based approaches: the chiral quark models, which in the past proved to be very successful in the description of other features of the pion, and in the meson dominance/parton-hadron duality approach at large number of colors, which intertwines both ends of the energy spectrum: the low-energy side, where chiral symmetry dominates and the high-energy side, where the asymptotic freedom sets in. By carrying out the tasks of the proposal we are hoping to get more valuable and intuitive physics insight into these difficult nonperturbative aspects of the dynamics of strong interactions, as well as to provide further outreach to the lattice-QCD community.

Literature (list of literature referencing all positions included by the author in the short and full description of the research project, with the full bibliographic data)

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