Partonic quasi-distributions of the pion in chiral quark models

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Outline

- **Parton distributions** – basic properties of hadrons
- **Soft matrix elements**, accessible from effective low-energy models of QCD
- **Chiral quark models** of the pion
- **Parton quasi-distributions**, designed for Euclidean QCD lattices

- Results and predictions for quasi-distributions of the pion from chiral models
Introduction
Parton distribution

\[ Q^2 = -q^2, \quad x = \frac{Q^2}{2p \cdot q}, \quad Q^2 \to \infty \]

Factorization of soft and hard processes, Wilson’s OPE

\[ \langle J(q)J(-q) \rangle = \sum_i C_i(Q^2; \mu) \langle O_i(\mu) \rangle \]

Twist expansion \( \rightarrow \)

\[ F(x, Q) = F_0(x, \alpha(Q)) + \frac{F_2(x, \alpha(Q))}{Q^2} + \ldots \]
Parton distribution

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Twist expansion \( \rightarrow \) \( F(x, Q) = F_0(x, \alpha(Q)) + \frac{F_2(x, \alpha(Q))}{Q^2} + \ldots \)

constrained light-cone momentum \( \delta(k^+, xP^+) \)

\[ k^+ = k^0 + k^3, \quad x \in [0, 1] \]
Distribution amplitude (DA) of the pion

Enters various measures of exclusive processes, e.g., pion-photon transition form factor
Field-theoretic definition
(here for quarks in the pion, leading twist)

Parton Distribution Function (DF):

\[
V(x) = \int \frac{dz^-}{4\pi} e^{ix P^+ z^-} \left\langle P | \bar{\psi}(0) \gamma^+ [0, z] \psi(z) | P \right\rangle \bigg|_{z^+ = 0, z^\perp = 0}
\]

Parton Distribution Amplitude (DA):

\[
\phi(x) = \frac{i}{F_\pi} \int \frac{dz^-}{2\pi} e^{i(x-1) P^+ z^-} \left\langle P | \bar{\psi}(0) \gamma^+ \gamma_5 [0, z] \psi(z) | \text{vac} \right\rangle \bigg|_{z^+ = 0, z^\perp = 0}
\]

(isospin suppressed)

\( P \) - pion momentum,
\( v^\pm \equiv v^0 \pm v^3 \) - light-cone basis
\([z_1, z_2] = \exp \left(-ig_s \int_{z_1}^{z_2} d\xi \lambda^a A^+_a(\xi) \right)\) - Wilson’s gauge link
\( x \) - fraction of the light-cone mom. \( P^+ \) carried by the quark, \( x \in [0, 1] \)
Remarks

- Only *indirect* experimental information for the *pion* distributions:
- DF from Drell-Yan in E615, DA from dijets in E791 and from exclusive processes involving pions
- Impossibility to implement PDF or PDA on the euclidean lattices, only lowest moments can be obtained
- However, there exist (largely forgotten) simulations on *transverse* lattices – discussed later
Quasi-distributions
Parton quasi-distributions (quarks in the pion)

Parton Quasi-Distribution Function (QDF):

\[ \tilde{V}(y; P_3) = \int \frac{dz_3^3}{4\pi} e^{iyP_3 z_3^3} \langle P | \bar{\psi}(0) \gamma^3 [0, z] \psi(z) | P \rangle \bigg|_{z^0 = 0, z^\perp = 0} \]

Parton Quasi-Distribution Amplitude (PDA):

\[ \tilde{\phi}(y; P_3) = \frac{i}{F_\pi} \int \frac{dz_3^3}{2\pi} e^{i(y-1)P_3 z_3^3} \langle P | \bar{\psi}(0) \gamma^+ \gamma_5 [0, z] \psi(z) | \text{vac} \rangle \bigg|_{z^0 = 0, z^\perp = 0} \]

\( y \) - fraction of \( P_z \) carried by the quark
Analogy, but: dependence on \( P_3 \), difference in support - \( y \) is not constrained
Miracle:

\[ \lim_{P_3 \to \infty} \tilde{V}(x; P_3) = V(x), \quad \lim_{P_3 \to \infty} \tilde{\phi}(x; P_3) = \phi(x) \]
QDF and QDA in the momentum representation

Constrained longitudinal momenta, but $y \in (-\infty, \infty)$
(partons can move “backwards”)

\[ \delta(k^2 - y^2 p^2) \]
Chiral quark models
Chiral quark models

- Point-like interactions
- Soft matrix elements with pions (and photons, $W$, $Z$)
- One-quark loop, regularization:
  1) Pauli-Villars (PV)
  2) Spectral Quark Model (SQM) - implements VMD

Evaluated at the quark model scale (where constituent quarks are the only degrees of freedom)
Chiral quark models

- Point-like interactions
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  1) Pauli-Villars (PV)
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Need for evolution
Gluon dressing, renorm-group-improved
Scale and evolution

QM provide non-perturbative result at a low scale $Q_0$

$$F_i(x, Q_0)\big|_{\text{model}} = F_i(x, Q_0)\big|_{\text{QCD}}, \quad Q_0 \text{ – the matching scale}$$

Determination of $Q_0$ via momentum fraction: quarks carry 100% of momentum at $Q_0$. One adjusts $Q_0$ in such a way that when evolved to $Q = 2$ GeV, the quarks carry the experimental value of 47%.

LO DGLAP evolution:

$$Q_0 = 313^{+20}_{-10} \text{ MeV}$$

[Davidson, Arriola 1995]:

$$q(x; Q_0) = 1$$
Older results from chiral quark models w/ evolution
Pion non-singlet DF, QM vs. E615

LO DGLAP evolution to the scale $Q^2 = (4 \text{ GeV})^2$:

- Points: Fermilab E615, Drell-Yan
- Curve: QM evolved to $Q = 4 \text{ GeV}$
Pion non-singlet DF, QM vs. transverse lattice

points: transverse lattice
[Dalley, van de Sande 2003]
yellow: QM evolved to 0.35 GeV
pink: QM evolved to 0.5 GeV
dashed: GRS param. at 0.5 GeV
points: E791 data from dijet production in $\pi + A$

solid line: QM at $Q = 2$ GeV

dashed line: asymptotic form $6x(1-x)$ at $Q \to \infty$
Pion DA, QM vs. transverse lattice

points: transverse lattice data [Dalley, van de Sande 2003]

line: QM at $Q = 0.5 \text{ GeV}$
NEW: Quasi-distributions from QM
Analytic formulas (in the chiral limit)

SQM:

\[ \tilde{\phi}(y, P_z) = V(y, P_z) = \frac{1}{\pi} \left[ \frac{2m_\rho P_z y}{m_\rho^2 + 4P_z^2 y^2} + \text{arctg} \left( \frac{2P_z y}{m_\rho} \right) \right] + (y \to 1 - y) \]

(similar simplicity for PV NJL)

Satisfy the proper normalization

\[ \int_{-\infty}^{\infty} dy \tilde{\phi}(y, P_z) = \int_{-\infty}^{\infty} dy V(y, P_z) = 1, \quad \int_{-\infty}^{\infty} dy 2yV(y, P_z) = 1 \]

and the limit

\[ \lim_{P_z \to \infty} \tilde{\phi}(y, P_z) = \lim_{P_z \to \infty} V(y, P_z) = \theta[y(1 - y)] \]
QDA and QDF from chiral quark models

(a) Quark QDA of the pion in the NJL model (for $m_\pi = 0$) at various values of $P_z$, plotted vs. the longitudinal momentum fraction $y$

(b) The same, but for the valence quark QDF multiplied (conventionally) with $2y$
Comparison to lattice

Quark QDA of the pion in NJL (a) and SQM (b) (for $m_\pi = 310$ MeV), plotted vs. the longitudinal momentum fraction $y$, evaluated with $P_z = 0.9$ and 1.3 GeV and compared to the lattice data at $\mu = 2$ GeV (LaMET)
Evolution of QDF
Relation $k_T$-unintegrated quantities (TMA, TMD)

Radyushkin’s formula [2016]

$$
\tilde{\phi}(y, P_z) = \int_{-\infty}^{\infty} dk_1 \int_{0}^{1} dx \, P_z \text{TMA}(x, k_1^2 + (x - y)^2 P_z^2).
$$

QDA can be obtained from TMA via a double integration!

Analogously

$$
\tilde{V}(y, P_z) = \int_{-\infty}^{\infty} dk_1 \int_{0}^{1} dx \, P_z \text{TMD}(x, k_1^2 + (x - y)^2 P_z^2).
$$
Evolution of unintegrated DF

UDF or TMD

Kwieciński’s method [2003], one-loop CCFM
DGLAP-like evolution, diagonal in $b$-space conjugate to $k_T$
For the non-singlet case:

\[ Q^2 \frac{\partial f(x, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 dz \, P_{qq}(z) \times \left[ \Theta(z - x) \, J_0[(1 - z)Qb] \left( f\left(\frac{x}{z}, b, Q\right) - f(x, b, Q) \right) \right] \]
Results of evolution of pion QDF in $Q$ at fixed $P_z$

\begin{align*}
P_z &= 1\text{GeV} \\
Q^2 &= Q_0^2 \\
Q^2 &= 4\text{GeV}^2 \\
Q^2 &= 100\text{GeV}^2
\end{align*}

\begin{align*}
P_z &= 2\text{GeV} \\
Q^2 &= Q_0^2 \\
Q^2 &= 4\text{GeV}^2 \\
Q^2 &= 100\text{GeV}^2
\end{align*}

Strength moved to lower $y$ as $Q$ increases
Changing $P_z$ at fixed $Q$

$P_z \to \infty$ limit achieved fastest at large $y \sim 0.6 - 0.9$
Conclusions
Conclusions

- Model evaluation of quasi-distributions of the pion (at the quark-model scale)
- Very simple analytic results, all consistency conditions met
- Exemplification of definitions and methods
- Results at finite $P_z$ acquire meaning, can be (favorably) compared to QDA from lattice QCD
- For QDF of the pion predictions made for various $Q$ (Kwieciński’s evolution) and $P_z$
- $P_z \sim 1$ GeV, accessible on the lattice, may not be sufficient for assessment of the $P_z \to \infty$ limit
- Convergence fastest for intermediate $y$, suggesting the domain where lattice may work best

- Recent activity also on related objects: quasi-distributions, Ioffe-time distributions ...