... between $\frac{d}{dx}$ and $\int$

Derivative of a product is simple: $\frac{d}{dx}(f(x)\cdot g(x)) = f'g + fg'$

composite f $\qquad \frac{d}{dx}(f(g(x))) = \frac{dg}{dx}\frac{df}{dg}$

Derivative is an operation which does not generate new function... if we start from $\{\sin x, \cos x, e^x, x^n, \dots\}$ → by deriving we still get something which is a combination of elementary functions.

But for the integration it is not so.

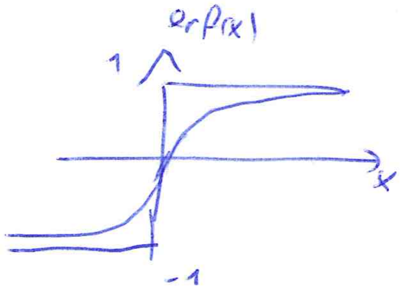
$\int f(x)g(x)dx$ cannot be expressed as a function of $\int f\,dx$ and $\int g\,dx$, f, g

$\int f(g(x))$ " " " " " " " " " " " "

Integration can be used to define or 'create' new functions.

No way to solve $\int e^{-x^2}dx$! (Remind it... before solving an $\int$ ask yourself: is this doable???)

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-z^2}dz$$



Important in statistic (evaluation of errors).
Another example:

$$li(x) = \int_0^x \frac{dz}{\ln z}$$

We can thus generate an "∞" of new functions.

Derivation of parameters

$$\int_a^b x e^{-x} dx$$

(I take a definite integral in this case ... I explain later why).

One could solve this integral by parts, but we show here a different wa[illegible]

$$\int_a^b x e^{-x} dx = \left(\int_a^b x e^{-\alpha x} dx \right)_{\alpha = 1} =$$

(Notice that $g(\alpha) = \int_a^b x e^{-\alpha x} d[illegible]$ is a well defined function of α)

$$= \left[-\frac{d}{d\alpha} \int_a^b e^{-\alpha x} dx \right]_{\alpha=1} = \left[-\frac{d}{d\alpha} \left(-\frac{1}{\alpha} e^{-\alpha x} \right)_a^b \right]_{\alpha = 1}$$

$$= \left[+\frac{d}{d\alpha} \left(\frac{1}{\alpha} e^{-\alpha b} - \frac{1}{\alpha} e^{-\alpha a} \right) \right]_{\alpha=1} = \left[-\frac{1}{\alpha^2} e^{-\alpha b} - \frac{b}{\alpha} e^{-\alpha b} + \frac{1}{\alpha^2} e^{-\alpha a} + \frac{a}{\alpha} e^{-\alpha a} [illegible]\right.$$

$$= e^{-a} - e^{-b} + a e^{-a} - b e^{-b}.$$