

1) CONSIDER THE FUNCTION

$$f(x, y) = x^3 + 2y^2 + 3xy$$

a) DETERMINE THE EXTREMA OF $f(x, y)$ AND THEIR NATURE (MAXIMA, MINIMA, OR SADDLE POINT).

b) DETERMINE THE EXTREMA OF f UNDER THE CONSTRAINT

$$x = 1$$

(USE BOTH THE PARAMETRIC DESCRIPTION AND THE LAGRANGE MULTIPLIER TECHNIQUE)

c) JUST AS IN (b) BUT WITH THE CONSTRAINT

$$x = y$$

2) a) DETERMINE THE EXTREMA OF THE FUNCTION

$$S(x, y) = -x \ln x - y \ln y$$

UNDER THE CONSTRAINT

$$x + y = 1$$

WHICH OTHER EXTREMA DO WE HAVE IF, IN ADDITION, WE IMPOSE ALSO THAT

$$0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1$$

DO IT BY USING THE PARAMETRIC FORM AS FOR INSTANCE $\begin{cases} x = t \\ y = 1 - t \end{cases}$

AND BY USING THE LAGRANGE MULTIPLIER.

b) FIND THE MAXIMUM OF $S = - \sum_{m=1}^N p_m \ln p_m$ WITH THE CONSTRAINT

$$p_1 + p_2 + \dots + p_N = 1 \quad (\text{HINT: USE ONLY THE LAGRANGE MULTIPLIER})$$

3) WRITE DOWN THE MATRIX FORM OF \hat{Q}
(I.E. THE ELEMENTS $\langle m | \hat{Q} | m \rangle$) FOR THE EXER. 1 AND 2
OF SHEET 1.

(1) WAS THE HARMONIC OSCILLATOR AND (2) THE 3-STATE BASIS $\{ |E_1\rangle, |E_2\rangle \}$