

Ein effektives chirales Modell der QCD mit
Vektormesonen, Dilaton und Tetraquarks: Physik
im Vakuum und bei nichtverschwindender Dichte
und Temperatur

Habilitation

von

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Diese Habilitationsschrift wird per kumulativem Verfahren verfasst. Aus diesem Grund besteht sie aus zwei Teilen.

Teil I ist eine detaillierte Zusammenfassung der theoretischen Arbeiten, die in einem thematischen Zusammenhang mit dem Habilitationsthema

„Ein effektives chirales Modell der QCD mit Vektormesonen, Dilaton und Tetraquarks: Physik im Vakuum und bei nicht verschwindender Dichte und Temperatur“

stehen. Sie ist in deutscher Sprache geschrieben.

Teil II enthält die Veröffentlichungen, die für das Thema der Habilitation relevant sind. Die Artikel wurden im Zeitraum 2006-2011 in internationalen Zeitschriften publiziert.

TEIL I

Zusammenfassende Diskussion

Ein effektives chirales Modell der QCD mit Vektormesonen, Dilaton und Tetraquarks: Physik im Vakuum und bei nichtverschwindender Dichte und Temperatur

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Zusammenfassung

Diese Arbeit befasst sich mit den Eigenschaften von Hadronen, die als gebundene Zustände von Quarks und Gluonen auftreten. Zunächst wird eine allgemeine Studie der hadronischen Modelle der QCD präsentiert: hierbei spielen Eigenschaften im Limes einer großen Zahl N_c von Quark Farbtransformationen und die Diskussion über die Zusammensetzung von Resonanzen eine wichtige Rolle. Mit Hilfe eines einfachen Modells werden auch einige komplexe Aspekte der niederenergetischen Hadronphysik diskutiert.

Nach dieser allgemeinen Beschreibung wird die explizite Konstruktion eines Modells mit den relevanten niederenergetischen Freiheitsgraden beschrieben: neben den üblichen pseudoskalaren und skalaren Mesonen werden auch die vektoriellen und axialvektoriellen Mesonen berücksichtigt. Überlegungen, die auf der Dilatationsinvarianz der QCD beruhen, erlauben, die Form des effektiven Modells zu vereinfachen: nur Terme mit (exakt) Dimension 4 dürfen auftreten. Als Nebenprodukt dieses Verfahrens ist der skalare Glueball automatisch in das chirale Modell eingeschlossen. Die Konstruktion des Modells ist ganz allgemein für eine beliebige Anzahl N_f von Quark-Flavors gültig. Resultate für den Fall $N_f = 2$ werden hier ausführlich präsentiert.

Die Frage, ob auch zusätzliche mesonartige Zustände wie Tetraquarks berücksichtigt werden müssen, ist im Zentrum einer langjährigen wissenschaftlichen Debatte. Es ist durchaus möglich, dass die leichten skalaren Resonanzen gar keinen Quark-Antiquark-Konfigurationen entsprechen, sondern Tetraquark-Feldern. In dieser Arbeit wird die entsprechende mathematische Sprache für Tetraquark-Felder präsentiert, die ihre Einführung in das chirale Modell erlaubt. Die daraus entstehenden Mischungssphänomene werden dargelegt.

Die Baryonen werden ebenfalls im Modell berücksichtigt. Die Entstehung der Nukleonmasse und die Frage über die Natur des chiralen Partners des Nukleons können im Rahmen des chiralen Modells beantwortet werden. Es wird gezeigt, dass nicht nur das chirale Kondensat, sondern auch das Gluon-Kondensat und womöglich das Tetraquark-Kondensat auf eine nicht vernachlässigbare Weise zur Nukleonmasse beitragen.

Als letzter Schritt werden erste Studien bei nicht-verschwindenden Dichte und Temperatur durchgeführt. Dabei wird der Einfluss eines skalaren Tetraquarks auf den chiralen Phasenübergang untersucht. Die dichteabhängigen und temperaturabhängigen Mischungsprozesse, der Verlauf von Tetraquark- und Quarkonium-Kondensat und der Massen bei wachsender Dichte/Temperatur werden studiert.

Zuletzt werden die vielfältigen Ausblicke dieser Arbeit präsentiert.

1 Einleitung

Ein großer Teil des ‘Particle Data Group’ Buches [1] widmet sich einer Zusammenfassung der Eigenschaften von hadronischen Resonanzen. Viele Experimente sind notwendig gewesen, um Informationen über diese kurzlebigen ($\tau \sim 10^{-20}$ s) Zustände zu gewinnen.

Die grundlegende Theorie, die Quantenchromodynamik (QCD), beschreibt Quarks und Gluonen, die die Bestandteile von Hadronen sind. Sie basiert auf einer exakten, lokalen $SU_c(3)$ Farbgruppe, deren Eichfelder die acht Gluonen sind. Außerdem sind sechs Quarkflavors bekannt: die Flavors u, d, s sind die leichten Quarks mit nackten Quarkmassen $m_u = 1.5\text{--}3.3$ MeV, $m_d = 5\text{--}10$ MeV, $m_s = 104^{+26}_{-34}$ MeV; die Flavors c, b, t sind die schweren Quarks mit nackten Quarkmassen $m_c = 1.27^{+0.07}_{-0.11}$ GeV, $m_b = 4.20^{+0.17}_{-0.07}$ GeV, $m_t = 171.2 \pm 2.1$ GeV.

Trotz bedeutender theoretischer Fortschritte wurde die QCD analytisch nicht gelöst. Obwohl die Bewegungsgleichungen der Felder bekannt sind, erlaubt ihre Nicht-Linearität keine exakte Lösung. Überdies versagt im Niederenergiebereich auch die Störungstheorie, da die ‘laufende’ Kopplungskonstante wächst. Eine damit verbundene und zentrale, obwohl mathematisch nicht bewiesene, Eigenschaft der QCD ist Confinement¹: nur ‘farblose’ Objekte (invariant unter einer $SU_c(3)$ -Transformation) werden in der Natur realisiert. Farbige Zustände, wie Quarks und Gluonen, aber auch wie Diquarks, sind keine asymptotischen Zustände, die in einem Detektor gemessen werden können.

Die asymptotischen physikalischen Zustände sind die Hadronen, die in Mesonen und Baryonen eingeteilt werden. Um sie genau zu definieren, muss man beachten, dass jedes Quark eine Baryonenzahl $B_q = 1/3$ und jedes Gluon hingegen eine Baryonenzahl $B_g = 0$ trägt. Dann gilt die folgende Klassifizierung:

- Die Mesonen sind farblose Zustände, die eine Gesamtbaryonenzahl $B = 0$ tragen. Quarkonia $|\bar{q}q\rangle$, wie z.B. die Pionen, sind Mesonen: $B_{\bar{q}q} = B_q + B_{\bar{q}} = \frac{1}{3} - \frac{1}{3} = 0$. Es gibt aber andere Möglichkeiten: Gluebälle $|gg\rangle$, d.h. gebundene Zustände aus zwei (oder mehr) Gluonen, und Tetraquarks $|(\bar{q}q)(qq)\rangle$, die aus einem Diquark und einem Antidiquark bestehen, tragen auch $B = 0$ und sind somit Mesonen. Nicht zuletzt gibt es mesonische Moleküle $|(\bar{q}q)(\bar{q}q)\rangle$, die gebundene Zustände aus zwei $|\bar{q}q\rangle$ Objekten sind.
- Die Baryonen sind farblose Zustände, die eine Gesamtbaryonenzahl $B = 1$ tragen. Zustände der Form $|qqq\rangle$, die aus drei Quarks bestehen wie z.B. das Proton, sind Baryonen. Auch in diesem Fall gibt es andere Möglichkeiten: Pentaquarks $|(qq)(qq)\bar{q}\rangle$ (Diquark-Diquark-Antiquark), und Moleküle der Form $|(qqq)(\bar{q}q)\rangle$ (bestehend aus drei Quarks und einem Quark-Antiquark).

Die Existenz von $|\bar{q}q\rangle$ und $|qqq\rangle$ gilt als erwiesen. Im Niederenergiebereich, in dem nur die leichten Quarks $q = u, d, s$ berücksichtigt werden, gibt es Multipletts von Zuständen, die aus der approximierten Flavorsymmetrie folgen. Wenn die schweren Quarks $q = c, b, t$ betrachtet werden, findet man auch die erwarteten Quark-Antiquark-Zustände. Dennoch gibt es sowohl für die leichten Quarks als auch für die schweren Quarks Objekte, die nicht in das ‘naive’ Quark-Antiquark-Bild passen. Die Frage, wie solche Objekte interpretiert werden und welche Rolle sie spielen, ist besonders spannend und aktuell. In dieser Arbeit wird die Aufmerksamkeit auf den mesonischen und baryonischen Niederenergiebereich für die Fälle $N_f = 2$ und $N_f = 3$ beschränkt, wobei N_f die Anzahl der berücksichtigten Flavors ist. $N_f = 2$ bedeutet, dass nur die Quarks u und d berücksichtigt werden, so als ob das s -Quark sehr schwer wäre. Obwohl das nicht der realistische Fall ist, sind Studien für $N_f = 2$ wichtig und wegweisend. $N_f = 3$ bedeutet, dass all die leichten Quarks u, d und s mitgenommen werden. Die schweren Quarks

¹Die mathematische Lösung dieses Problems wird mit einer Million Dollar belohnt, siehe die Webseite des Clay-Institutes: http://www.claymath.org/millennium/Yang-Mills_Theory/

c, b , und t sind schwer genug, um für die Zwecke der hadronischen Niederenergiephysik ($\lesssim 2$ GeV) vernachlässigt zu werden.

Da die QCD analytisch (noch) nicht lösbar ist, ist die Benutzung von effektiven Methoden sowohl im Vakuum als auch bei endlicher Temperatur und Dichte notwendig, um eine theoretisch konsistente Beschreibung der hadronischen Resonanzen zu bekommen. Besonders wichtig ist die Frage, welche Kriterien ein effektives Modell erfüllen soll, das nicht nur $|\bar{q}q\rangle$ und $|qqq\rangle$ Zustände enthält, sondern auch einige von den oben aufgelisteten zusätzlichen Mesonen wie Tetraquarks und Gluebälle. Außerdem kann die Frage untersucht werden, wie diese zusätzlichen Felder die Physik bei endlicher Temperatur und Dichte beeinflussen. Ein wichtiges Ziel dieser Studie ist, die Resultate von Experimenten mit Schwerionenkollisionen besser zu verstehen und interpretieren.

Diese Arbeit beruht auf den Artikeln in Refs. [2, 3, 4, 5, 6, 7, 8, 9, 10, 11], die einen wichtigen Teil meiner Forschungsarbeiten in den letzten vier Jahren darstellen. Sie bietet die Möglichkeit, die verschiedenen Themen in einem einheitlichen Kontext zu präsentieren. Das wiederum erlaubt auch, neue Aspekte zu diskutieren und Ausblicke für künftige Projekte zu schildern.

Die Arbeit ist wie folgt strukturiert:

- Abschnitt 2 beruht auf [2, 3]: die Eigenschaften der QCD für $N_c \gg 3$, wobei N_c die Farbanzahl ist (Large- N_c Limes), werden zusammengefasst und eine allgemeine Diskussion über die Form einer hadronischen Theorie der QCD und über die dynamische Generierung von Resonanzen wird präsentiert.
- Abschnitt 3 beruht auf [4, 5]: die Zusammenfassung der QCD-Eigenschaften und ihre Anwendung zur Erstellung einer mesonischen Theorie, die sowohl (pseudo)skalare als auch (axial)vektorielle Mesonen enthält, wird dargelegt. Überdies wird das skalare Dilaton/Glueball-Feld vom Anfang an eingekoppelt. Unterschiedliche Zuordnungen der skalaren Resonanzen werden untersucht.
- Abschnitt 4 beruht auf [6, 7, 8]: Die Eigenschaften von Diquarks und von Tetraquarks werden zusammengefasst. Tetraquark-Felder werden in das chirale Modell eingebaut und beschreiben zusätzliche Skalarfelder.
- Abschnitt 5 beruht auf [9, 10]: das Nukleon und sein chiraler Partner werden eingeführt. Der Ursprung der Nukleonenmasse wird beschrieben. Die mögliche Rolle eines leichten Tetraquark-Feldes für Kernmaterie wird diskutiert. Erste Resultate bei nicht verschwindender Dichte werden präsentiert.
- Abschnitt 6 beruht auf [11]: der chirale Phasenübergang bei wachsender Temperatur wird für eine vereinfachte Form des Modells studiert, wobei das Tetraquark die leichteste skalare Resonanz beschreibt. Die Konsequenzen dieses Szenarios für die thermodynamischen Eigenschaften des Systems werden dargelegt.
- Abschnitt 7: die Zusammenfassung der Arbeit und Ausblicke werden präsentiert.

Zuletzt ist eine Anmerkung über die Referenzen wichtig: es werden einige relevante Artikel im Laufe dieser Arbeit zitiert, jedoch wird auf die Referenzen in Refs. [2, 3, 4, 5, 6, 7, 8, 9, 10, 11] verwiesen, um eine vollständige Liste der Arbeiten über die diskutierten Themen zu bekommen.

2 Allgemeine Diskussion über hadronische Theorien: der Large- N_c Limes und die dynamische Generierung/Rekonstruktion der Resonanzen

2.1 Die Suche nach einer hadronischen Theorie und der Large- N_c Limes

In diesem Abschnitt wird das Problem der Konstruktion hadronischer Theorien ganz allgemein beschrieben.

Die Lagrange-Dichte der QCD als Funktion der acht Gluon-Felder A_μ^a und der N_f Quark-Felder q_i mit entsprechenden Quarkmassen m_i ($i = 1, \dots, N_f$) lautet

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (i\gamma^\mu D_\mu - m_i) q - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}, \quad D_\mu = \partial_\mu - ig_0 A_\mu^a t^a, \quad (1)$$

wobei die Größe $G_{\mu\nu}^a$ der gluonische Feld-Tensor ist:

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_0 f^{abc} A_\mu^b A_\nu^c, \quad a, b, c = 1, \dots, 8. \quad (2)$$

Der Parameter g_0 beschreibt die Quark-Gluon- und Gluon-Gluon-Wechselwirkungen. Die Matrizen t^a lauten $t^a = \lambda^a/2$, wobei λ^a die üblichen Gell-Mann-Matrizen sind. Die Konstanten f^{abc} sind die Strukturkonstanten der Gruppe $SU(3)$. Es sei angemerkt, dass, im sogenannten ‘chiralen Limes’ $m_i = 0$, der dimensionslose Parameter g_0 der einzige Parameter der QCD-Lagrange-Dichte ist.

Die Lagrange-Dichte in Gl. (1) ist invariant unter den lokalen Farbtransformationen der Eichgruppe $SU(3)_c$ und, im chiralen Limes $m_i = 0$, unter den globalen chiralen Transformationen der Gruppen $SU(N_f)_R \times SU(N_f)_L$. Die genaue mathematische Diskussion dieser Symmetrien, mit besonderer Aufmerksamkeit auf die chirale Symmetrie, wird in Abschnitt 3.2 präsentiert.

Wie in der Einführung schon erwähnt, wurden die Felder der QCD Lagrange-Dichte (Quarks und Gluonen) nicht direkt gemessen und können wegen des Confinement auch prinzipiell nicht gemessen werden. Man soll also eine effektive Theorie erstellen, die die messbaren hadronischen Teilchen als Freiheitsgrade enthält. Das wiederum wirft die folgende Frage auf: Welche Felder sollten ein effektives Modell der QCD bilden? Welche Zustände sollte man hingegen als dynamisch generierte Zustände betrachten? Das ist in den letzten Jahren ein besonders wichtiges und häufig diskutiertes Thema der hadronischen Forschergemeinde.

Die Frage kann ganz allgemein wie folgt formuliert werden: wie soll man aus der QCD-Lagrange-Dichte $\mathcal{L}_{QCD}$ eine konsistente Theorie $\mathcal{L}_{had}$ mit hadronischen Freiheitsgraden konstruieren?

$$\mathcal{L}_{QCD} \xrightarrow{?} \mathcal{L}_{had}. \quad (3)$$

Das ist generell nicht bekannt. Es gibt aber einen wichtigen Limes, in dem das, obwohl nur ansatzweise, möglich ist. Wenn man anstatt mit der Farbgruppe $SU_c(3)$ mit der erweiterten Gruppe $SU_c(N_c)$ arbeitet, finden besondere Vereinfachungen im Limes $N_c \rightarrow \infty$ statt². Das ist der sogenannte ‘Large- N_c ’ Limes der QCD, der u.a. von ’t Hooft und Witten entwickelt wurde [12, 13]. Die Tatsache, dass die QCD im Niederenergiebereich eine stark gekoppelte Theorie ist, wo keinerlei störungstheoretische Methoden anwendbar sind, war der Grund und der Anreiz für die Suche nach neuen Entwicklungsschemen. Der daraus entstandene Large- N_c Limes wurde in vielen Bereichen der QCD in den letzten 30 Jahren benutzt und spielt weiter eine fundamentale Rolle in der Modellbildung und im Verständnis der fundamentalen Eigenschaften der QCD.

²Dazu muss auch angenommen werden, dass g_0 wie $N_c^{-1/2}$ skaliert. Das ist aber eine natürliche Annahme, wie im folgenden Abschnitt erklärt wird.

Das Verhalten der relevanten Größen im Large- N_c Limes wird zusammengefasst:

- Quark-Antiquark- und Glueball-Mesonmassen sind konstant für $N_c \rightarrow \infty$:

$$M_{|\bar{q}q\rangle} \propto N_c^0, \quad M_{|gg\rangle} \propto N_c^0.$$

Es ist eine wichtige Eigenschaft, dass die Massen von Quarkonia und Gluebällen N_c -unabhängig sind.

- Die Wechselwirkungsamplitude zwischen n Quarkonia $|\bar{q}q\rangle$ skaliert wie

$$A_{n-|\bar{q}q\rangle} \propto N_c^{-\frac{n-2}{2}}.$$

Das bedeutet, dass die Amplitude für eine n -Meson-Streuung kleiner und kleiner für wachsende Farbzahl N_c wird. Ein konkretes und wichtiges Beispiel ist die Zerfallsamplitude für ein Quark-Antiquark-Meson, das in zwei Quark-Antiquark-Mesonen zerfällt. Diese Situation entspricht dem Fall $n = 3$, d.h. $A_{\text{Zerfall}} \propto N_c^{-1/2}$. Das wiederum impliziert, dass die Breite $\Gamma \propto 1/N_c$ skaliert. Quark-Antiquark-Zustände werden schmaler für hohes N_c .

- Die Wechselwirkungsamplitude zwischen n Gluebällen $|gg\rangle$ skaliert wie

$$A_{n-|gg\rangle} \propto N_c^{-(n-2)}.$$

Diese Amplitude ist im Large- N_c Limes noch mehr unterdrückt als die Wechselwirkung zwischen Quark-Antiquark-Resonanzen.

- Eine gemischte Wechselwirkungsamplitude zwischen n Quark-Antiquark-Zuständen und m Gluebällen skaliert wie

$$A_{(n-|\bar{q}q\rangle)(m-|gg\rangle)} \propto N_c^{-(\frac{n}{2}+m-1)} \text{ für } n \geq 1 \text{ und } m \geq 1.$$

Das ist die allgemeinste Skalierung für Gluebälle und Quarkonia. Die Mischung zwischen einem Glueball und einem Quarkonium entspricht dem Fall $n = m = 1$, d.h. $A_{\text{Mischung}} \propto N_c^{-1/2}$. Das bedeutet, dass auch die Mischung zwischen Gluebällen und Quarkonia für $N_c \gg 1$ unterdrückt ist.

- Die Baryonen bestehen aus N_c Quarks für ein beliebiges N_c . Nur auf diese Weise kann man weiße baryonische Feldkonfigurationen konstruieren. Als Konsequenz wächst die Masse eines Baryons mit N_c :

$$M_B \propto N_c.$$

- Die Tetraquarks, im Sinne von vier-Quark Zuständen, ‘verschwinden’ im Large- N_c Limes. Dennoch ist ein anderer Limes möglich, in dem die Tetraquarks als Multiquark-Objekte im Large- N_c Limes interpretiert werden. Die genaue Beschreibung wird in Abschnitt 4 präsentiert.

Ganz allgemein suchen wir die effektive Theorie $\mathcal{L}_{had}(N_c, E_{\max})$ der QCD, wobei N_c die Anzahl der Farben ist und $E_{\max}$ die maximale Energie ist, bei der die effektive Theorie gültig ist. Im Prinzip könnte man die effektive hadronische Theorie für $E_{\max} \simeq M_{Planck}$ suchen (d.h. bis zur Gültigkeitsgrenze der QCD selbst³), aber man ist in der Regel nur an niederenergetischen Resonanzen, die eine Masse kleiner als $E_{\max} \simeq 2 \text{ GeV}$ besitzen, interessiert. (Folgende Nebenbemerkung ist interessant: wenn $\mathcal{L}_{had}(E_{\max} \simeq M_{Planck}, N_c)$ bekannt wäre, dann wäre es möglich, jedes $\mathcal{L}_{had}(E_{\max}, N_c)$ bei einer beliebigen Energie $E_{\max}$ durch Ausintegrieren aller Felder, die schwerer als $E_{\max}$ sind, zu bestimmen.)

Im Large- N_c Limes, $N_c \gg 1$, besteht die Theorie $\mathcal{L}_{had}(E_{\max}, N_c \gg 1)$ aus freien Quarkonium-Feldern ϕ_k und freien Glueball-Feldern G_h bis zu einer vorgegeben maximalen Energie $E_{\max}$:

$$\mathcal{L}_{had}(E_{\max}, N_c \gg 1) = \sum_{k=1}^{N_{\bar{q}q}} \left[\frac{1}{2} (\partial_\mu \phi_k)^2 - \frac{1}{2} M_{\bar{q}q,k}^2 \phi_k^2 \right] + \sum_{h=1}^{N_{gg}} \left[\frac{1}{2} (\partial_\mu G_h)^2 - \frac{1}{2} M_{G,h}^2 G_h^2 \right], \quad (4)$$

³Es wird hier angenommen, dass das Standardmodell bis M_{Planck} gültig ist. Es könnte auch sein, dass die Grenze kleiner ist und mit $E_{GUT} \simeq 10^{16} \text{ GeV}$ übereinstimmt. (Laut modernen Theorien könnte die Energiegrenze sogar 1 TeV sein). Das ändert nicht die Diskussion dieser Arbeit, wo nur niederenergetische Prozesse berücksichtigt werden.

wobei nur die Zustände mit Masse $\lesssim E_{\max}$ berücksichtigt wurden. Es ist auffällig, dass Baryonen *nicht* auftreten, da ihre Masse für $N_c \gg 1$ sicherlich größer als $E_{\max}$ ist. Unterhalb der maximalen Energie $E_{\max} \simeq 2$ GeV befinden sich mit Sicherheit die Quark-Antiquark-Felder mit Quantenzahlen $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}$, d.h. pseudoskalare, skalare, vektorielle und axialvektorielle Quarkonium-Mesonen. Außerdem wird auch ein skalarer 0^{++} Glueball mit einer Masse von etwa 1.5 GeV erwartet: das ist eine klare und wohldefinierte Vorhersage von QCD-Computersimulationen auf dem Gitter [14]. Für $N_c = 3$ sind die Wechselwirkungen zwischen Mesonen nicht vernachlässigbar. Die Lagrange-Dichte für $N_c = 3$ lautet

$$\begin{aligned} \mathcal{L}_{had}(E_{\max}, N_c = 3) = & \sum_{k=1}^{N_{\bar{q}q}} \left[\frac{1}{2} (\partial_\mu \phi_k)^2 - \frac{1}{2} M_{\bar{q}q,k}^2 \phi_k^2 \right] + \sum_{h=1}^{N_{gg}} \left[\frac{1}{2} (\partial_\mu G_h)^2 - \frac{1}{2} M_{G,h}^2 G_h^2 \right] \\ & + \mathcal{L}_{int}^{\bar{q}q-gg}(E_{\max}, N_c = 3) + \mathcal{L}_{new-mes}(E_{\max}, N_c = 3) + \mathcal{L}_{bar}(E_{\max}, N_c = 3), \end{aligned} \quad (5)$$

wobei folgendes gilt:

- (i) $\mathcal{L}_{int}^{\bar{q}q-gg}(E_{\max}, N_c = 3)$ beschreibt die Wechselwirkung zwischen den Quarkonia und Gluebällen. Wegen der oben aufgeführten Large- N_c Eigenschaften gilt $\mathcal{L}_{int}^{\bar{q}q-gg}(E_{\max}, N_c) \propto O(1/N_c)$.
- (ii) $\mathcal{L}_{new-mes}(E_{\max}, N_c = 3)$ enthält die kinetischen und Wechselwirkungsterme von zusätzlichen Mesonen, die bei $N_c = 3$ eine Masse unterhalb von $E_{\max}$ haben. Ein explizites Beispiel für solche Felder wird in Abschnitt 4 diskutiert. Dieser Term der Lagrange-Dichte verschwindet schneller als $O(1/N_c)$ im Large- N_c Limes.
- (iii) $\mathcal{L}_{bar}(E_{\max}, N_c = 3)$ beschreibt die kinetischen Terme und die Meson-Baryon Wechselwirkungen, die bei $N_c = 3$ eine Masse kleiner als $E_{\max}$ besitzen.

Wir kommen jetzt zu der Frage, ob auch mesonische Moleküle und/oder Tetraquarks in das effektive Modell durch den Term $\mathcal{L}_{new-mes}(E_{\max}, N_c = 3)$ eingeführt werden sollten. Die Antwort ist unterschiedlich und wird in den folgenden Unterabschnitten besprochen.

2.2 Mesonische Moleküle sollten *nicht* als Freiheitsgrade in $\mathcal{L}_{had}(E_{\max}, N_c = 3)$ auftreten

Um zu verstehen, warum mesonische Moleküle nicht in $\mathcal{L}_{had}(E_{\max}, N_c)$ auftreten sollten, ist es zunächst instruktiv, den nichthadronischen Fall der Quanten-Elektrodynamik (QED) zu diskutieren.

Die Lagrange-Dichte der QED besteht aus zwei Feldern: das Elektron und das Photon. Das Energie-Intervall, in dem die Theorie gültig ist, ist sehr groß: zwischen null und der Planckmasse. Bei einer Energie von etwa $2m_e$ gibt es zusätzliche Zustände: die Positronia. Diese Resonanzen sind nicht elementar, sondern gebundene, molekülartige Zustände, die aus einem Elektron und einem Positron bestehen. Theoretisch können Positronia mit Hilfe der Bethe-Salpeter-Gleichung beschrieben werden. Positronia sind nicht direkt in der QED-Lagrange-Dichte enthalten und sind keine Elementarteilchen. Sie sind dynamisch generierte Resonanzen, die aus der Wechselwirkung zwischen den in der Lagrange-Dichte vorhandenen Elektron- und Photonfeldern entstehen.

Das Deuteron stellt eine ähnliche physikalische Situation dar. Das Deuteron besteht aus einem Proton und einem Neutron. Wenn man also eine Theorie erstellt, sollte man nur das Proton und das Neutron, und eventuell Austauscheteilchen wie Pionen, berücksichtigen. Das Deuteron tritt als ein gebundener Zustand auf, der keinem Feld des Lagrangians entspricht⁴.

⁴Es ist eigentlich schon möglich, eine Lagrange-Dichte hinzuschreiben, in der das Deuteron und seine Konstituenten, das Proton und das Neutron, gleichzeitig auftreten. Jedoch muss man die sogenannte Zusammensetzungsbedingung einführen, die sicherstellt, dass das Deuteron kein elementares Teilchen der Theorie ist. Für eine genauere Diskussion siehe Ref. [2].

Diese Beispiele zeigen, was man im Grunde unter einem ‘dynamisch generierten Feld’ versteht. Es ist dann klar, dass Meson-Meson-Moleküle nicht direkt in $\mathcal{L}_{had}$ auftreten sollten, da sie ganz analog dem Positronium oder dem Deuteron sind. Molekülartige mesonische Zustände entstehen, falls sie existieren, durch die Large- N_c unterdrückten Wechslewirkungsterme der Lagrange-Dichte $\mathcal{L}_{int}^{\bar{q}q-gg}(E_{\max}, N_c = 3)$. Genau wie im QED-Fall muss man die Bethe-Salpeter-Gleichung für Meson-Meson Streuung lösen und prüfen, ob gebundene (oder quasi-gebundene) Zustände entstehen. Solche mesonischen Moleküle verschwinden im Large- N_c Limes, da $\mathcal{L}_{int}^{\bar{q}q-gg}(E_{\max}, N_c = 3)$ kleiner und kleiner wird. Die Anziehungskraft ist für $N_c \gg 1$ nicht ausreichend, um solche Zustände zu generieren.

Es gibt eine wichtige Bedingung, die mit der möglichen Entstehung eines mesonischen Moleküls betrachtet werden muss: die Masse des neu entstandenen Zustands muss kleiner als $E_{\max}$ sein, damit man über eine ‘dynamisch generierte’ Resonanz sprechen kann. Die Energie $E_{\max}$ ist nämlich die maximale Energie, bei der die hadronische effektive Theorie gültig ist. Wenn die Molekülmasse $E_{\max}$ überschreitet, liegt dieser generierte Zustand nicht innerhalb der Gültigkeit der Theorie. Es kann keine Aussage über die Natur dieses Objektes gemacht werden, siehe Abschnitt 2.5 für eine explizite Rechnung.

2.3 Tetraquarks sollten als Freiheitsgrade in $\mathcal{L}_{had}(E_{\max}, N_c = 3,)$ auftreten

Ein Tetraquark kann *nicht* durch Meson-Meson-Wechselwirkungen erzeugt werden, weil die Farbstruktur eines Tetraquarks irreduzibel ist⁵. Ein Tetraquark besteht aus zwei farbigen Diquarks, die ein farbloses Objekt bilden, und nicht –wie die vor kurzem besprochenen mesonischen Moleküle– aus zwei farblosen Bestandteilen.

Im Gegensatz dazu beschreibt die Lagrange-Dichte $\mathcal{L}_{int}^{\bar{q}q-gg}(E_{\max}, N_c = 3)$ die Wechselwirkungen zwischen farblosen Quarkonia und Gluebällen. Ein daraus entstehender Zustand besteht aus zwei (oder mehr) weißen Objekten (d.h., ist farb-reduzibel) und ist daher kein Tetraquark.

Man kann folgern, dass die Tetraquarks direkt in $\mathcal{L}_{new-mes}(E_{\max}, N_c = 3)$ eingeschlossen werden müssen, vorausgesetzt, dass ihre Masse kleiner als $E_{\max}$ ist. Die Frage ist dann eher phänomenologischer Natur: sind die Tetraquarks leicht und schmal genug, um gemessen zu werden? Oder sind die (meisten) Tetraquarks so breit, dass sie im Kontinuum verschwinden? Wie später diskutiert, trifft die letzte Bemerkung zu: die meisten Tetraquarks sind zu breit, um gemessen zu werden. Es gibt eine mögliche, aber potentiell sehr wichtige, Ausnahme: die leichten skalaren Tetraquarks mit einer Masse kleiner als 1 GeV.

Im Large- N_c Limes verschwinden alle Tetraquarks. Es gibt jedoch eine Möglichkeit, eine konsistente und wohldefinierte Verallgemeinerung eines Tetraquarks zu bestimmen, die im Large- N_c Limes überlebt. Die Details werden in Abschnitt 4 beschrieben.

Diese Konzepte gelten nicht nur für Tetraquarks, sondern allgemein für jedes nicht farb-reduzible Objekt, wie Pentaquarks, Hexaquarks, usw. Wenn solche Zustände existieren, sollten sie direkt in der Lagrange-Dichte auftreten.

2.4 Niederenergetische effektive Theorien und Unitarisierungen

Oft ist man nur an physikalischen Prozessen interessiert, die im Niederenergiebereich stattfinden. Man muss dafür die Lagrange-Dichte $\mathcal{L}_{had}(E_{\max}, N_c = 3)$ für eine kleine Energie $E_{\max}$ bestimmen. Wegen der sogenannten spontanen Brechung der chiralen Symmetrie gibt es leichte Quark-Antiquark-Mesonen, die drei Pionen, die eine Masse $M_\pi = 139$ MeV besitzen. Die anderen Hadronen sind wesentlich

⁵Es sei bemerkt, dass die Farbstruktur eines Tetraquarks nicht lokal ist. Das Diquark und das Anti-Diquark sind selbst nicht punktförmig, liegen nämlich auseinander und ziehen sich auf eine ähnliche Art wie Quark und Antiquark an. Im lokalen Limes wäre es durchaus möglich, die Farbstrukturen von Quarkonia und Tetraquarks ineinander zu transformieren. Das entspricht aber nicht der physikalischen Situation.

schwerer als 200 MeV. Wenn man also $E_{\max} = E_{\chi PT} \simeq 300$ MeV setzt, enthält die Lagrange-Dichte $\mathcal{L}_{had}(N_c = 3, E_{\max} \simeq 300 \text{ MeV})$ nur die drei Pionfelder

$$\mathcal{L}_{\chi PT} = \mathcal{L}_{had}(E_{\max} = E_{\chi PT}, N_c = 3) = \sum_{k=1}^3 \left[\frac{1}{2} (\partial_\mu \pi_k)^2 - \frac{1}{2} M_\pi^2 \pi_k^2 \right] + \mathcal{L}_{int}^\pi, \quad (6)$$

wobei $\mathcal{L}_{int}^\pi$ die Wechselwirkung zwischen den Pionen beschreibt. Dies ist die Lagrange-Dichte der chiralen Störungstheorie (χPT) [15]. Der Term $\mathcal{L}_{int}^\pi$ enthält Operatoren, die aus Symmetrie-Überlegungen bestimmt werden können. Jedoch ist die Bestimmung der entsprechenden Kopplungskonstanten (die sogenannten ‘Low Energy Constants’, LECs) theoretisch nicht möglich. Diese Konstanten werden durch Vergleich mit den experimentellen Resultaten festgelegt. Es ist möglich, $\mathcal{L}_{int}^\pi$ als Summe von Termen mit anwachsender Anzahl von Ableitungen des Pion-Feldes auszudrücken. Je höhere Ableitungen mitgenommen werden, desto größer ist die Anzahl von Termen und von unbekannten LECs. Obwohl es prinzipiell möglich ist, bis zu einer beliebigen Ordnung zu gehen, ist die Benutzung der χPT von einem praktischen Standpunkt für die niedrigsten Ordnungen möglich.

Die Lagrange-Dichte $\mathcal{L}_{had}(N_c = 3, E_{\max})$ für $E_{\max} > 400$ MeV enthält nicht nur Pionen, sondern auch die Mesonen und Baryonen, die leichter als $E_{\max}$ sind. Wenn man zum Beispiel $E_{\max} \simeq 1$ GeV setzt, erhält man eine Lagrange-Dichte, die im Meson-Sektor außer den pseudoskalaren Mesonen auch die Vektormesonen enthält:

$$\mathcal{L}_{had}(E_{\max} \simeq 1 \text{ GeV}, N_c = 3) = \mathcal{L}_{\chi PT+VM}. \quad (7)$$

$\mathcal{L}_{\chi PT+VM}$ beschreibt die freien Teilchen und die gegenseitige Wechselwirkung [16]. Auch in diesem Fall ist es möglich, die Operatoren zu bestimmen, jedoch nicht die LECs. (Die skalaren Mesonen werden hier vernachlässigt, siehe die Diskussion in den nächsten Kapiteln).

Wenn $\mathcal{L}_{had}(E_{\max} \simeq 2 \text{ GeV}, N_c = 3)$ bekannt ist, kann $\mathcal{L}_{\chi PT}$ (oder $\mathcal{L}_{\chi PT+VM}$) aus $\mathcal{L}_{had}(E_{\max} \simeq 2 \text{ GeV}, N_c = 3)$ bestimmt werden, indem man alle Felder, die schwerer als $E_{\chi PT} \simeq 300$ MeV (oder $E_{\chi PT+VM} \simeq 1$ GeV) sind, ausintegriert. Ganz allgemein: die Kenntnis einer effektiven Theorie mit höherer Gültigkeitsgrenze erlaubt, eine effektive Theorie mit niedrigerer Gültigkeitsgrenze zu bestimmen.

Die Kenntnis von $\mathcal{L}_{\chi PT}$ erlaubt hingegen nicht, $\mathcal{L}_{had}(E_{\max} > E_{\chi PT}, N_c = 3)$ eindeutig zu bestimmen: die Extrapolation einer effektiven Theorie mit einer kleinen Gültigkeitsenergie zu einer anderen Theorie mit einer größeren Gültigkeitsenergie ist nicht eindeutig. Dennoch ist es möglich, aus einer Theorie mit einer maximalen Energie $E_{\max}$ Information außerhalb dieser Energie zu gewinnen. Es gibt sogenannte Unitarisierungsmethoden, d.h. ‘Extrapolationen’, die erlauben, Einsichten und Eigenschaften jenseits des Gültigkeitsbereichs $E_{\max}$ zu bekommen. Obwohl im Prinzip unendlich viele solche Methoden existieren, wurden bessere mathematische Verfahren in diesem Zusammenhang entwickelt. Wir beschreiben kurz drei solcher Methoden. (Eine vollständige und rigorose Beschreibung dieses Problems ist in den Referenzen von Ref. [2] präsentiert.)

Sei $S = 1 + iT$ die S-Matrix für den Streuprozess zweier Teilchen $a + b \rightarrow a + b$. Wegen der Unitarität der S-Matrix gilt: $S^\dagger S = 1$.

(i) *K*-Matrix Unitarisierung.

Die *T* Matrix kann als Reihe in der Kopplungskonstante α entwickelt werden:

$$T(n) = a_1 \alpha + a_2 \alpha^2 + \dots + a_n \alpha^n + O(\alpha^{n+1}).$$

Die S-Matrix $S = 1 + iT(n)$ ist dann nur bis zur Ordnung n unitär. Man kann aber S wie folgt unitarisieren:

$$S = 1 + iT(n) \Rightarrow S_K = \frac{1 + \frac{i}{2}T(n)}{1 - \frac{i}{2}T(n)}.$$

Dadurch wird die Unitarität von S_K erzwungen. Der Ausdruck S_K enthält auch Terme $\alpha^{m>n}$: das bedeutet, dass man über die ursprüngliche Gültigkeit hinausgeht. Verschiedene Variationen dieser Unitarisierung werden in Ref. [17] beschrieben.

(ii) Bethe-Salpeter-Unitarisierung.

Sei $T = T(1) = a_1\alpha$. Die Bethe-Salpeter-Unitarisierung erfolgt durch eine Resummation der Schleifen-Diagramme:

$$T_{BS} = T(1) + T(1)KT(1) + \dots = \frac{T(1)}{1 - K \cdot T(1)} ,$$

wobei K die sogenannte Schleifen-Funktion ist und explizit vom Impuls $p^2 = s$ abhängt. Der Prozess gilt natürlich auch, wenn $T = T(n)$ bis zu einer höheren Ordnung n bekannt ist. Für eine genauere Diskussion dieser Methode siehe z.B. Ref. [18] und Referenzen darin. Ein explizites Beispiel wird in Abschnitt 2.5 diskutiert.

(iii) IAM-Unitarisierung.

Man braucht die störungstheoretische Form bis (mindestens) Ordnung zwei,

$$T(n=2) = a_1\alpha + a_2\alpha^2 \rightarrow T_1 = a_1\alpha, T_2 = a_2\alpha^2 .$$

Die unitarisierte Form lautet (die Kanäle t und u werden vernachlässigt):

$$T_{\text{IAM}} \simeq T_1 (T_1 - T_2 - iT_1\sigma T_1)^{-1} T_1 ,$$

wobei $\sigma = \sqrt{\frac{p^2}{4} - m^2}$ und m die Masse aller Teilchen ist. Es ist wichtig zu beachten, dass diese Methode nur für $n \geq 2$ anwendbar ist. Zahlreiche Anwendungen dieser Unitarisierung sind in Refs. [19] zu finden.

Ein wichtiges Beispiel für diese Konzepte stammt aus dem schwachen Sektor des Standardmodells.

- Im schwachen Sektor des Standardmodells gibt es das nahezu masselose Neutrino, das leichte Elektron und, aufgrund der spontanen Symmetriebrechung im Higgs-Sektor, das schwere W -Boson ($M_W \sim 80$ GeV). Das W -Boson verursacht die Neutrino-Elektron-Wechselwirkung. Da das W -Boson so schwer ist, kann es ausintegriert werden. Man kann eine effektive Theorie erstellen, in der nur das Elektron und das Neutrino auftreten. Das ist die berühmte Fermi-Theorie der schwachen Wechselwirkung:

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} [\bar{\psi}_{\nu_e} \gamma^\mu (1 + \gamma^5) \psi_e]^2 ,$$

wobei ψ_e das Elektron und ψ_{ν_e} das Neutrino beschreibt. Die Kopplungskonstante $G_F \sim 1/M_W^2$ stammt von der Propagation eines virtuellen W -Bosons. Der Gültigkeitsbereich dieser Theorie ist natürlich dadurch beschränkt, dass die Ausintegration des W -Bosons stattgefunden hat. Die maximale Energie E_{max} ist somit wesentlich kleiner als M_W .

- Man stelle sich vor, nur die niederenergetische Fermi-Theorie $\mathcal{L}_F$ wäre bekannt. Kein W -Teilchen ist im Fermi-Lagrangian enthalten. Ist es möglich, auf die Existenz des W -Bosons nur aus dieser begrenzten Information zu schließen? Die Antwort ist positiv: durch (eine) Unitarisierung.
- Eine mögliche Unitarisierungsmethode besteht darin, eine Bethe-Salpeter-Gleichung für die Neutrino-Elektron-Streuung zu studieren. Obwohl das Resultat von einem physikalischen Cutoff abhängt (das ist ja zu erwarten, da die Fermi-Theorie nicht renormierbar ist) kann man tatsächlich aus der Elektron-Neutrino-Wechselwirkung ein W -Teilchen erzeugen. Man kann somit aus der Fermi-Theorie die Existenz des W -Bosons deduzieren. Wegen der Natur der BS-Gleichung könnte man auf die Idee kommen, das erzeugte W -Teilchen als einen gebundenen Zustand aus einem Elektron und einem Neutrino zu

interpretieren. Das ist aber offensichtlich falsch, da das W -Boson ein ‘fundamentales’ Teilchen ist. Dieses Beispiel zeigt, dass Vorsicht angebracht ist, wenn Unitarisierungsmethoden benutzt werden, um die Natur eines Teilchens zu interpretieren.

2.5 Ein konkretes Beispiel mit Hilfe eines einfachen Modells

In diesem Abschnitt wird ein einfaches Modell diskutiert, in dem all die vorherigen Themen anhand von Gleichungen besprochen werden können.

Man berücksichtigt eine einfache Lagrange-Dichte $\mathcal{L}_{\text{toy}}$ (wobei ‘toy’ aus dem englischen Ausdruck ‘toy model’ stammt) mit zwei Feldern φ (mit Masse m) und S (mit Masse $M_0 > 2m$). Eine Large- N_c Abhängigkeit wird eingeführt, damit beide Felder sich wie ‘Quark-Antiquark Felder’ verhalten. Die Idee ist einfach: das Feld S wird zuerst ausintegriert, um eine ‘effektive’ Lagrange-Dichte $\mathcal{L}_{\text{le}}$ zu bekommen, die nur vom Feld φ abhängt. Dann können wir uns fragen, ob das Feld S , das ausintegriert wurde, durch Unitarisierungsmethoden wiedererzeugt werden kann. Man kann sich auch fragen, was man über diese Resonanz S erfahren kann, wenn nur die niederenergetische Lagrange-Dichte bekannt wäre.

Die Lagrange-Dichte lautet [2, 3]:

$$\begin{aligned} \mathcal{L}_{\text{toy}}(E_{\text{max}}, N_c) = & \frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} (\partial_\mu S)^2 - \frac{1}{2} M_0^2 S^2 \\ & + g(N_c) S \varphi^2 - \frac{g(N_c)^2}{2M_0^2} \varphi^4 , \end{aligned} \quad (8)$$

wobei die Large- N_c Abhängigkeit durch $g(N_c) = g_0 \sqrt{3/N_c}$ ausgedrückt wird. Durch diese Wahl sind beide Massen N_c -unabhängig.

Die Wechselwirkung besteht aus zwei Termen: eine $S\varphi^2$ Wechselwirkung zwischen den zwei Feldern S und φ und eine φ^4 -Selbstwechselwirkung des φ -Feldes, deren Bedeutung später klar wird.

Der $S\varphi^2$ -Term generiert den Zerfall des Feldes S . Die Zerfallsbreite für den Prozess $S \rightarrow 2\varphi$ lautet

$$\Gamma = \frac{\sqrt{\frac{M_0^2}{4} - m^2}}{8\pi M_0^2} \left[\sqrt{2} g(N_c) \right]^2 . \quad (9)$$

Γ skaliert wie $g^2(N_c) \sim 1/N_c$, genau als ob φ und S Quarkonia wären. Es wird angenommen, dass die Lagrange-Dichte (8) bis zu einer hohen Energie gültig ist ($E_{\text{max}} \gg M_0$).

Der Propagator der Resonanz S wird durch Schleifen von φ -Teilchen modifiziert und nimmt die folgende Form an (auf dem resummierten 1-loop Niveau, siehe Abb. 1):

$$\Delta = i \left[p^2 - M_0^2 + (\sqrt{2}g)^2 \Sigma_\Lambda(p^2) \right]^{-1} , \quad (10)$$

wobei $\Sigma_\Lambda(p^2)$ der 1-Schleifen Beitrag ist, der durch einen Cutoff regularisiert wird (Details in Refs. [2, 3]). Die renormierte Masse M des Zustands S kann als die Nullstelle des Realteils von Δ^{-1} definiert werden:

$$M^2 - M_0^2 + (\sqrt{2}g(N_c))^2 \text{Re} \Sigma_\Lambda(M^2) = 0 . \quad (11)$$

Es ist klar, dass im Limes $N_c \rightarrow \infty$ gilt, dass $M \rightarrow M_0$, da $g(N_c) = g_0 \sqrt{3/N_c}$.

Die T -Matrix für die $\varphi\varphi$ -Streuung im s -Kanal (auf dem 1-Schleifen Niveau, Abb. 1) lautet:

$$T(p^2) = i(\sqrt{2}g)^2 \Delta = \frac{1}{-K^{-1} + \Sigma_\Lambda(p^2)} , \quad K = \frac{(\sqrt{2}g)^2}{M_0^2 - p^2} - \frac{(\sqrt{2}g)^2}{M_0^2} . \quad (12)$$

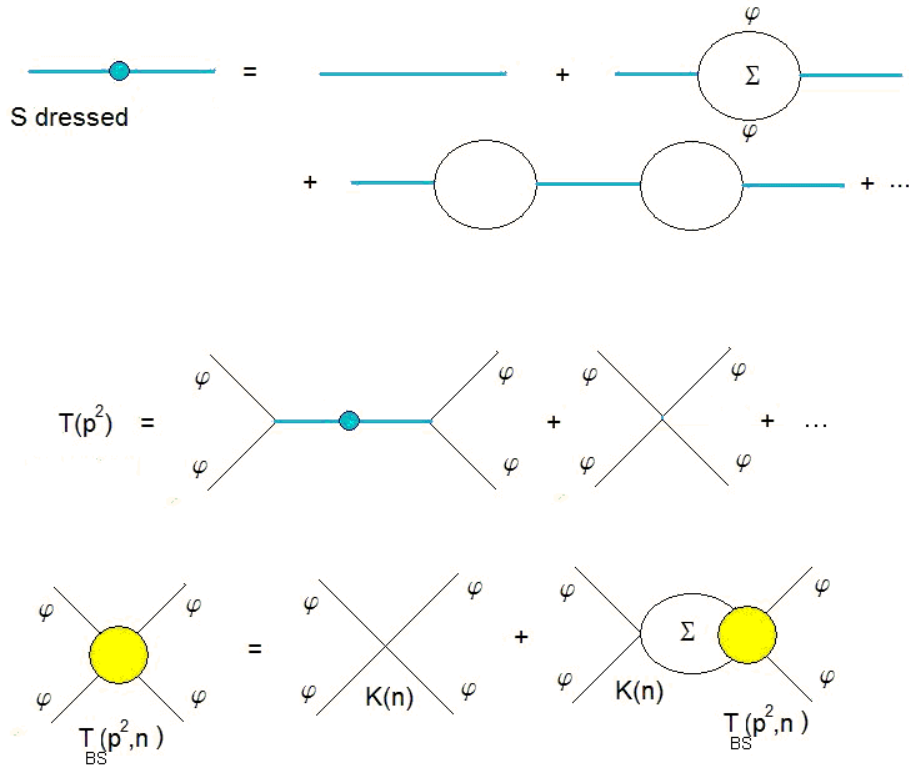


Abbildung 1: Modifikation der Resonanz S durch Schleifen von φ -Feldern. Graphische Darstellung der ‘exakten’ T -Matrix $T = T(p^2)$ von Gl. (12) und graphische Darstellung der BS-approximierten Form der ‘exakten’ T -Matrix $T_{BS} = T_{BS}(p^2, n)$ von Gl. (16).

Das ist für unsere Zwecke die ‘exakte’ T -Matrix des Problems⁶.

Jetzt wenden wir uns der Konstruktion eines effektiven niederenergetischen Modells von $\mathcal{L}_{\text{toy}}$ zu. Da das Feld S schwerer als das Feld φ ist, kann S ausintegriert werden, um eine Lagrange-Dichte herzuleiten, die nur von φ abhängt. Wir bekommen eine effektive Lagrange-Dichte $\mathcal{L}_{\text{le}}$, die bis zur Energie $E_{\text{le}} \lesssim 2m < M_0$ gültig ist:

$$\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c) = \frac{1}{2} (\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 + V, \quad V = \sum_{k=1}^{\infty} V^{(k)}, \quad (13)$$

wobei:

$$V^{(k)} = L^{(k)} \varphi^2 (-\square)^k \varphi^2, \quad L^{(k)} = \frac{g(N_c)^2}{2M_0^{2+2k}}. \quad (14)$$

Das effektive Potential V besteht aus einer Summe von Termen mit höheren Ableitungen. Der erste Term in der Entwicklung entspricht $k = 1$ und kein Term ohne Ableitungen ist anwesend. Dadurch ist die $\varphi\varphi$ -Streuung dem realistischen Fall der $\pi\pi$ -Streuung ähnlich. Der Grund dafür ist, dass ein φ^4 -Term in unser Modell von Gl. (8) eingefügt wurde: dadurch bleibt in der effektiven niederenergetischen Lagrange-Dichte $\mathcal{L}_{\text{le}}$ kein φ^4 -Term ohne Ableitungen übrig.

BS-Unitarisierung

Sei das Potential V bis zur Ordnung n approximiert: $V(n) = \sum_{k=1}^n V^{(k)}$. Durch eine Bethe-Salpeter-Studie, siehe Abb. 1, bekommt man die folgende approximierte T -Matrix T_{BS} :

$$T_{BS}(p^2, n) = -K(n) + K(n) \Sigma_\Lambda(p^2) T_{BS}(p^2, n), \quad (15)$$

$$T_{BS}(p^2, n) = \frac{1}{-K(n)^{-1} + \Sigma_\Lambda(p^2)}, \quad K(n) = \frac{(\sqrt{2}g)^2}{M_0^2} \sum_{k=1}^n \left(\frac{p^2}{M_0^2} \right)^k, \quad (16)$$

wobei $K(n)$ die ‘nackte’ Baumniveau-Amplitude ist, die aus der niederenergetischen effektiven Lagrange-Dichte $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ folgt. Sie ist die Summe aller Vierbeinterme bis zur Ordnung n . Man muss beachten, dass $T_{BS}(p^2, n)$ eine approximierte Form von $T(p^2)$ in Gl. (12) darstellt. Je größer n , desto besser die Näherung. Formell gilt:

$$\lim_{n \rightarrow \infty} T_{BS}(p^2, n) = T(p^2). \quad (17)$$

Wir können jetzt einen Pol der Bethe-Salpeter-Amplitude bestimmen; dieser Pol entspricht einer Resonanz, die *nicht* in der effektiven Lagrange-Dichte $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ enthalten ist. Wie der Limes in Gl. (17) zeigt, entspricht diese Resonanz dem Zustand S der ursprünglichen Lagrange-Dichte $\mathcal{L}_{\text{toy}}$. Es wurde insofern eine dynamische *Rekonstruktion* des Zustands S durch eine BS-Analyse der niederenergetischen Lagrange-Dichte $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ durchgeführt. Der Zustand S wurde zuerst ausintegriert, und dann aus der effektiven Lagrange-Dichte $\mathcal{L}_{\text{le}}$ wieder generiert. Eine numerische Studie wird in Abb. (2) gezeigt: die approximierte Form $T_{BS}(p^2, n = 1)$ (wobei $n = 1$ bedeutet, dass nur der erste Term in der Entwicklung berücksichtigt wurde) und die ‘exakte’ Funktion $T(p^2)$ werden verglichen. Man kann leicht beobachten, dass $T_{BS}(p^2, n = 1)$ tatsächlich $T(p^2)$ reproduzieren kann. Subtilitäten, die der Wahl des Cutoffs entsprechen, werden ausführlich in Ref. [2] besprochen.

Jetzt wird ein anderer Standpunkt betrachtet: wenn nur $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ bekannt wäre (d.h. die Form der ursprünglichen, exakten Lagrange-Dichte $\mathcal{L}_{\text{toy}}(E_{\text{max}}, N_c)$ wäre unbekannt), könnte man aus der BS-Rechnung -zu Unrecht- behaupten, dass der Pol der BS-Gleichung ein Molekül beschreibt. Es wurde

⁶Natürlich ist diese Form der T -Matrix nur in der 1-Schleifen-Näherung gültig. Selbst diese einfache Feldtheorie ist nicht exakt lösbar. Dennoch sind dank der Resummation alle erwünschten mathematischen Eigenschaften erfüllt. Sie ist aber ‘exakt’ nur im Vergleich zu der später diskutierten Bethe-Salpeter-approximierten Form.

nämlich aus einer BS-Studie bestimmt und hat den ‘Anschein’, ein molekülartiger Zustand aus zwei φ -Mesonen zu sein.

Diese Schlussfolgerung ist aber *nicht* korrekt: es ist eben von Anfang an klar, dass der Zustand S -per Konstruktion- ein fundamentaler (quarkoniumartiger) Zustand ist. Was ist dann falsch gelaufen? Man muss aufpassen, wenn man Informationen über die Resonanz S aus der effektiven niederenergetischen Lagrange-Dichte $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ gewinnen will: die Masse der Resonanz S ist größer als die Energiegrenze E_{le} des effektiven Modells $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$. Das bedeutet, dass die Natur dieser Resonanz ohne weitere Informationen nicht in diesem Rahmen bestimmt werden kann. Die Bethe-Salpeter-Methode soll hier zuerst als Unitarisierungsmethode verstanden werden. Sie ist *nicht* auf derselben theoretischen Ebene wie eine BS-Studie im Rahmen der QED-Lagrange-Dichte, die erlaubt, Positronia zu bestimmen. Im QED-Fall ist die Gültigkeitsgrenze der Energie sehr groß (viel größer als $2m_e$). Also ist die Interpretation der Positronia als Moleküle durchaus erlaubt und korrekt. In unserem Beispiel ist das nicht der Fall: die BS-Gleichung geht über die Gültigkeitsgrenze der effektiven Theorie hinaus. Obwohl die Existenz der Resonanz S korrekterweise vorhergesagt wird, kann keine Aussage über ihre Natur gemacht werden.

Es ist sehr instruktiv, eine Large- N_c Studie dieses einfachen Systems durchzuführen. Wenn die Zahl n festgehalten ist und der Limes $N_c \rightarrow \infty$ ausgeführt wird, stellt sich heraus, dass der korrekte Large- N_c Limes im Rahmen der BS-Unitarisierung nicht reproduziert werden kann. Es gilt nämlich:

$$T_{BS}(p^2, n) \stackrel{N_c \rightarrow \infty}{\simeq} -K(n) . \quad (18)$$

Dieses Resultat gilt für jedes n , da $K(n)$ wie $1/N_c$ skaliert und $\Sigma_\Lambda(p^2)$ N_c -unabhängig ist. Aber $K(n)$ ist ein Polynom der Ordnung n in p^2 und hat keinen Pol. Das wiederum bedeutet, dass die Masse M des Zustandes S unendlich ist. Dieses Resultat ist offensichtlich falsch, da $M = M_0$ für $N_c \rightarrow \infty$ sein muss. Dieses Beispiel zeigt, dass die BS-Studie, so wie sie hier formuliert wurde, nicht fähig ist, das korrekte Large- N_c Resultat wiederzugeben. Dieses Versagen ist besonders gefährlich, weil vom Standpunkt der effektiven Theorie $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ eine weitere Bestätigung zu existieren scheint, dass der Zustand S ein Molekül ist.

IAM-Unitarisierung: Wenn hingegen die sogenannte IAM-Unitarisierung gewählt wird, kann das korrekte Large- N_c Resultat reproduziert werden. Bis zur Ordnung $n = 2$ gilt:

$$T_{\text{IAM}} \simeq T_2 (T_2 - T_4 - iT_2 \sigma T_2)^{-1} T_2 , \quad (19)$$

wobei $\sigma = \sqrt{\frac{p^2}{4} - m^2}$. Da in unserem Fall $T_2 = -\frac{(\sqrt{2}g)^2}{M_0^4} p^2$ und $T_4 = \frac{(\sqrt{2}g)^2}{M_0^6} p^4$, folgt:

$$T_{\text{IAM}} \simeq (\sqrt{2}g)^2 \left[M_0^2 - p^2 - i(\sqrt{2}g)^2 \sigma \right]^{-1} . \quad (20)$$

Die letzte Gleichung stellt eine approximierte Form der T -Matrix dar, wenn g nicht zu groß ist ($M \simeq M_0$). Es ist einfach zu zeigen, dass man für hohes N_c das korrekte Resultat $M \rightarrow M_0$ und eine skalierende Zerfallsbreite $1/N_c$ bekommt. Das zeigt, dass die IAM-Unitarisierung den richtigen Large- N_c Limes reproduziert. Dennoch benötigt diese Unitarisierung zumindest zwei Terme in der Entwicklung der entsprechenden niederenergetischen Theorie.

K-Matrix-Unitarisierung

Die unitarisierte Form der S -Matrix lautet für $n = 1$

$$S_K = \frac{1 + i\sigma T(1)}{1 - i\sigma T(1)} . \quad (21)$$

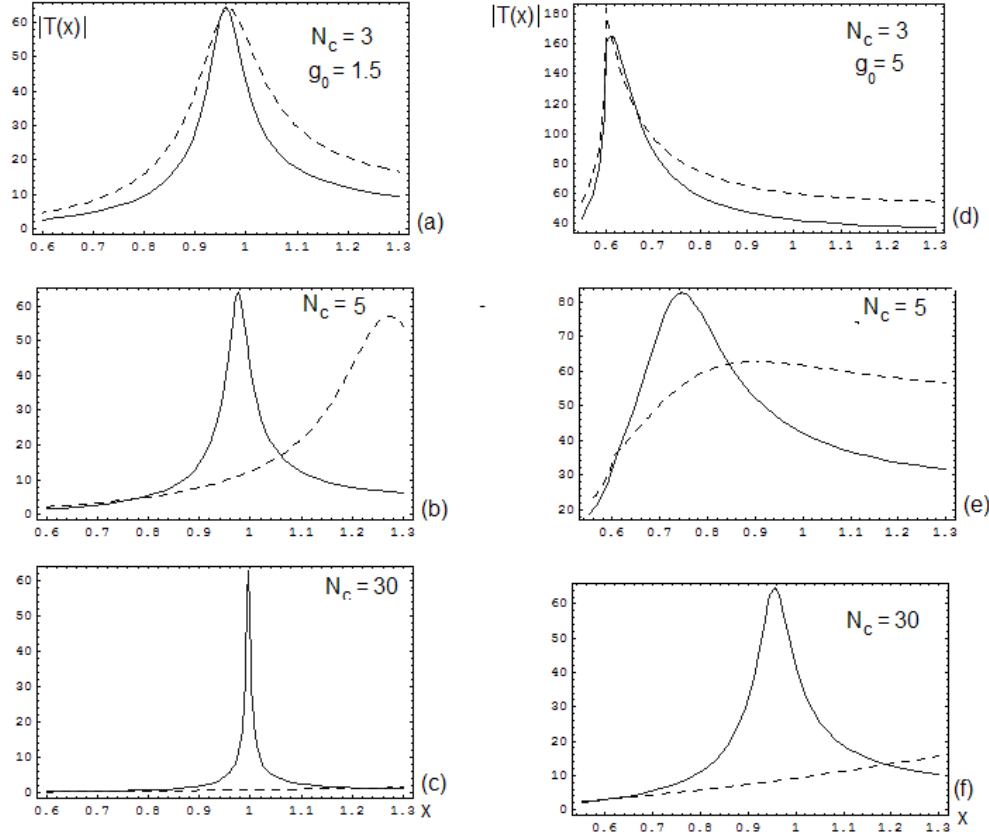


Abbildung 2: Betrag der Streumatrix T als Funktion der Variable $x = p^2$ für $N_c = 3, 5$ und 30 für die zwei Fälle $g_0 = 1.5$ GeV (links) und $g_0 = 5$ GeV (rechts). Die durchgezogene Kurve entspricht der ‘exakten’ T -Matrix von Gl. (12), die gestrichelte Linie der approxiierten Bethe-Salpeter T -Matrix T_{BS} von Gl. (16). Obwohl die zwei Kurven qualitativ für $N_c = 3$ übereinstimmen, ist das Verhalten für großes N_c ganz anders: während die richtige Kurve T im Einklang mit dem Large- N_c Limes schmaler wird, wird die BS-approximierte Kurve T_{BS} breiter und der Peak wandert nach rechts.

Der Pol der S -Matrix skaliert wie $p^2 \propto \sqrt{N_c}$, was nicht im Einklang mit dem richtigen Large- N_c Resultat ist. Diese Tatsache zeigt, dass die Eigenschaften der rekonstruierten Resonanz S entscheidend von der Wahl der Unitarisierungsmethode abhängt. Das ist im Sinne dieser Diskussion ein Grund mehr, vorsichtig zu sein, wenn Resonanzen als Pole von Unitarisierungsverfahren einer niederenergetischen Lagrange-Dichte diskutiert werde.

Analogie mit der realen, hadronischen Welt

Das besprochene Modell ist nützlich, um einige Eigenschaften der realen Welt zu verstehen. In Tabelle 1 wird eine Analogie dargestellt. $\mathcal{L}_{\text{toy}}$ entspricht der vollständigen hadronischen Lagrange-Dichte $\mathcal{L}_{\text{had}}(E_{\text{max}}, N_c)$ von Gl. (5) mit $E_{\text{max}} \simeq 2$ GeV, und die niederenergetische Lagrange-Dichte $\mathcal{L}_{\text{le}}$ entspricht der chiralen Störungstheorie χPT oder der chiralen Störungstheorie plus Vektormesonen $\chi PT + VM$.

Tabelle 1: Hadronen-Analogie

Modell	Hadronische Welt
$\mathcal{L}_{\text{toy}}(E_{\text{max}}, N_c)$	$\mathcal{L}_{\text{had}}(E_{\text{max}}, N_c)$
$\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$	$\mathcal{L}_{\chi PT}$, oder $\mathcal{L}_{\chi PT+VM}$
$L^{(k)}$	LECs
Feld S	Felder $a_1(1260)$, $f_2(1270)$, ...

Natürlich ist diese Analogie nur schematischer Natur. Sie trifft aber einen wichtigen Punkt. Das wahre Problem der heutigen niederenergetischen hadronischen Physik ist, dass $\mathcal{L}_{\text{had}}(E_{\text{max}}, N_c)$ nicht bekannt ist. Nur niederenergetische Lagrange-Dichten -wie die Chirale Störungstheorie $\mathcal{L}_{\chi PT}$ gültig bis $E_{\chi PT}$ oder die chirale Störungstheorie mit Vektormesonen $\mathcal{L}_{\chi PT+VM}$ gültig bis $E_{\chi PT+VM} \simeq 1$ GeV- sind aus Symmetriegründen bekannt. Sie bestehen aus einer unendlichen Reihe von Termen, genau wie unsere effektive Lagrange-Dichte $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$. Aber, im Gegensatz zu unserem Modell, wo die genaue Kenntnis der ursprünglichen Lagrange-Dichte $\mathcal{L}_{\text{toy}}(E_{\text{max}}, N_c)$ die Herleitung von $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ bis zu einer beliebigen Ordnung n erlaubt, sind in der realen hadronischen Welt $\mathcal{L}_{\chi PT}$ oder $\mathcal{L}_{\chi PT+VM}$ nur “teilweise” bekannt: wie zuvor schon angesprochen, sind die verschiedenen Terme der effektiven Lagrange-Dichte zwar bekannt, die entsprechenden Kopplungskonstanten $L^{(k)}$ sind aber unbekannt (da $\mathcal{L}_{\text{had}}(E_{\text{max}}, N_c)$ nicht bekannt ist). Diese Konstanten (LECs) müssen durch experimentelle Werte bestimmt werden. Diese Tatsache setzt auch eine klare Grenze der effektiven niederenergetischen Theorien: obwohl es prinzipiell möglich ist, bis zu einer beliebigen Ordnung n zu arbeiten, sind Rechnungen nur für die niedrigsten Ordnungen durchführbar.

In vielen Studien zur ‘dynamischen Generierung’ der Resonanzen mit BS-Gleichung wird nur der erste Term in der Entwicklung der effektiven Lagrange-Dichte in Betracht gezogen. Einige Resonanzen oberhalb 1 GeV wurden auf diese Weise bestimmt, wie z.B. $a_1(1260)$, $f_2(1270)$, $f_0(1370)$, ...: $a_1(1260)$ wird als $\pi\rho$ -Molekül, $f_2(1270)$ und $f_0(1370)$ werden als $\rho\rho$ -Moleküle interpretiert (siehe Ref. [18] und Referenzen darin). Diese Resonanzen entstehen als Pole in den BS-Gleichungen, die aus der Unitarisierung von $\mathcal{L}_{\chi PT+VM}$ entstehen. Wie mit Hilfe des obigen Modells erklärt, kann das ein Fall von ‘dynamischer Rekonstruktion’ sein, genau wie bei dem W -Meson in der elektroschwachen Theorie oder bei der Resonanz S in unserem einfachen Modell.

Die Resonanzen $a_1(1260)$, $f_2(1270)$ und $f_0(1370)$ können problemlos als Quark-Antiquark Zustände interpretiert werden [20]. Viele Studien mit dem ‘naiven’ Quark-Modell bestätigen eigentlich diese Einsicht. Die Resonanzen $f_2(1275)$, $f_2(1525)$, $a_2(1320)$ und $K_2(1430)$ stellen ein tensorielles $J^{PC} = 2^{++}$ Quark-Antiquark-Nonett dar: $f_2(1275) = \bar{n}n$, $f_2(1525) = \bar{s}s$, $a_2(1320) = u\bar{d}$, ... $K_2(1430) = s\bar{d}$, u.s.w. Die (beinahe) ideale Mischung und die genau gemessenen starken und elektromagnetischen

Zerfallsbreiten passen sehr gut in dieses Bild (siehe auch Ref. [21] und Referenzen darin). Ähnlich können $f_1(1285)$, $f_1(1510)$, $a_1(1260)$ und $K_1(1270)$ erfolgreich als das leichteste 1^{++} Nonett verstanden werden. Selbst der skalare Sektor zwischen 1-2 GeV (siehe später für eine genauere Beschreibung) ist mit dem Bild eines Quarkonium-Nonetts plus einem skalaren Glueball konsistent, obwohl die Situation noch nicht eindeutig ist [25].

Die geführte Diskussion zeigt, dass es keinen Konflikt zwischen auf Bethe-Salpeter basierten Rechnungen und den Standard-Quark-Modell-Rechnungen gibt. Sie beschreiben dieselben Objekte aus unterschiedlichen Perspektiven. Was hier kritisiert wird, ist nicht die Gültigkeit der Rechnungen selbst, sondern die daraus folgende molekülartige Interpretation vieler Resonanzen oberhalb 1 GeV, die viele Autoren vertreten haben. Sicherlich wird es in der Zukunft interessant (obwohl zugleich kompliziert), solche moderne Studien mit Hilfe der IAM-Methode im Large- N_c Limes zu wiederholen; wenn diese Resonanzen oberhalb 1 GeV tatsächlich Quarkonia (oder Gluebälle) sind, müssen sie schmaler werden. Die hier geschilderte Situation ist generell und gilt mit geringen Änderungen auch für Baryonen (z.B. was die Natur von $N(1535)$ betrifft, mehr darüber im Abschnitt 5) und auch im schweren Quark-Bereich.

Wir wenden uns den Prinzipien, die die Erstellung eines hadronischen Modells ermöglichen, zu. Das wird uns erlauben, eine explizite Form einer hadronischen Lagrange-Dichte für $E_{\max} \simeq 2$ GeV herzu-leiten.

3 Die Erstellung des Modells: Meson-Sektor

3.1 Ziel des Abschnittes

Die hadronische Lagrange-Dichte $\mathcal{L}_{had}(E_{\max}, N_c = 3)$ mit $E_{\max} \simeq 2$ GeV ist nicht analytisch aus dem QCD-Lagrange-Dichte $\mathcal{L}_{QCD}$ herleitbar, siehe Gl. (3). Um $\mathcal{L}_{had}(E_{\max}, N_c = 3)$ explizit zu konstruieren, ist eine genaue Betrachtung der Symmetrien der QCD notwendig. $\mathcal{L}_{had}$ soll nämlich dieselben Symmetrien (und Symmetriebrechungen) der ursprünglichen QCD-Lagrange-Dichte haben. Natürlich bleiben einige Konstanten der hadronischen Lagrange-Dichte nicht bestimmbar, da wir keine exakte Rechnung durchführen können.

Das Ziel dieses Abschnittes ist die Konstruktion eines mesonischen Modells, das nicht nur skalare und pseudoskalare Quarkonia enthält, sondern auch von Anfang an vektorielle und axialvektorielle Quarkonia. Außerdem wird ein Kriterium diskutiert, das nur eine endliche (und relativ kleine) Anzahl von Termen im hadronischen Potential erlaubt. Dieses Kriterium basiert auf den Eigenschaften der Dilatationssymmetrie und ihrer expliziten Brechung. Die Fluktuation des Dilaton-Feldes, der Glueball, wird auch von Anfang an eingekoppelt.

Da die genaue Beschreibung der Konstruktion des Modells noch nicht Teil einer Veröffentlichung ist, werden zuerst grundlegende Konzepte wiederholt und ihre Bedeutung für das chirale Modell dargelegt. Das Modell wird für eine beliebige Flavor-Anzahl N_f und Farbanzahl N_c konstruiert, was künftige Studien erleichtern wird. Die Resultate des expliziten Falles $N_f = 2$ werden zusammengefasst.

3.2 Symmetrien der QCD

Um die Diskussion dieses Abschnitts zu vereinfachen, wird die Lagrange-Dichte der QCD für eine beliebige Anzahl von Farben N_c und eine beliebige Anzahl von Quark-Flavors N_f in der Matrix-Form hingeschrieben:

$$\mathcal{L}_{QCD} = \text{Tr} \left[\bar{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{2} G_{\mu\nu} G^{\mu\nu} \right], \quad D_\mu = \partial_\mu - ig_0 A_\mu, \quad (22)$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_0 [A_\mu, A_\nu], \quad A_\mu = A_\mu^a t^a, \quad a, = 1, \dots, N_c^2 - 1, \quad (23)$$

wobei $i = 1, \dots, N_f$, die Matrizen t^a sind die Generatoren der Gruppe $SU(N_c)$ in der fundamentalen Darstellung, und f_{abc} sind die Strukturkonstanten. (Unten wird eine Zusammenfassung der mathematischen Eigenschaften dargelegt.)

Wir listen zuerst die Symmetrien von $\mathcal{L}_{QCD}$ und ihrer spontanen und expliziten Brechungen auf:

- (a) Lokale Farbsymmetrie $SU(N_c)$.
- (b) Spursymmetrie und ihre Anomalie, d.h. ihre Brechung durch Quantenfluktuationen.
- (c) Chirale Symmetrie $U(N_f)_R \times U(N_f)_L \equiv U(1)_V \times SU(N_f)_V \times U(1)_A \times SU(N_f)_A$.
- (d) $U(1)_A$ Anomalie. Die klassische $U(1)_A$ Symmetrie wird auch durch Quantenfluktuationen gebrochen.
- (e) Spontane Symmetriebrechung $SU(N_f)_V \times SU(N_f)_A \rightarrow SU(N_f)_V$.
- (f) Explizite Brechung von $U(1)_A$ und $SU(N_f)_A$ durch nicht verschwindende Quarkmassen.

Bevor wir ausführlich diese Symmetrien diskutieren, werden die Eigenschaften der Gruppen $U(N)$ und $SU(N)$ kurz zusammengefasst.

Gruppen $U(N)$ und $SU(N)$: Diese Gruppen sind im Rahmen der QCD allgegenwärtig (sowohl im Farb- als auch im Flavor-Raum). Ein Element der $U(N)$ -Gruppe ist eine komplexe $N \times N$ -Matrix, die die folgende Bedingung erfüllt:

$$U^\dagger U = U U^\dagger = 1_N. \quad (24)$$

U kann als

$$U = e^{i\theta_a t^a}, \quad a = 0, 1, \dots, N^2 - 1 \quad (25)$$

ausgedrückt werden, wobei die Matrizen t^a eine Basis von linearunabhängigen $N \times N$ -Matrizen darstellen. Es ist üblich, $t^0 = \frac{1}{\sqrt{2N}} 1_N$ zu setzen. Für die restlichen Matrizen t^a mit $a = 1, \dots, N^2 - 1$, wird in den Fällen $N = 2$ und $N = 3$ folgende, nützliche Wahl getroffen: $t^a = \frac{\tau^a}{2}$ für $a = 1, 2, 3$, wobei τ^a die Pauli-Matrizen sind, und $t^a = \frac{\lambda^a}{2}$ für $a = 1, \dots, 8$, wobei λ^a die Gell-Mann-Matrizen sind. Ganz allgemein werden die Matrizen so gewählt, dass folgende Normierung gilt:

$$\text{Tr} [t^a t^b] = \frac{1}{2} \delta^{ab} \quad \text{mit } a, b = 0, 1, \dots, N^2 - 1. \quad (26)$$

Eine $N \times N$ komplexe Matrix gehört zu der Untergruppe $SU(N)$, wenn sie folgende Eigenschaften erfüllt:

$$U^\dagger U = U U^\dagger = 1_N, \quad \det U = 1; \quad (27)$$

die zusätzliche Bedingung $\det U = 1$ wird hinzugefügt. Es ist deutlich, dass Matrizen in der Form $U = e^{i\theta_a t^a}$ mit $a = 1, \dots, N^2 - 1$ (wobei die Identitätsmatrix ausgelassen wird) zu der Gruppe $SU(N)$ gehören. Die Matrizen t^a mit $a = 1, \dots, N^2 - 1$ sind die Generatoren der Gruppe $SU(N)$ und erfüllen folgende Gleichung:

$$[t^a, t^b] = i f^{abc} t^c \quad \text{mit } a, b, c = 1, \dots, N^2 - 1, \quad (28)$$

wobei f^{abc} die (antisymmetrischen) Strukturkonstanten sind.

Die allgemeine Form einer Matrix U der $SU(N)$ -Gruppe enthält auch die sogenannten Elemente des Zentrums:

$$U = Z e^{i\theta_a t^a}, \quad a = 1, \dots, N^2 - 1, \quad (29)$$

wobei

$$Z = Z_n = e^{i\frac{2\pi n}{N}} 1_N, \quad n = 0, 1, 2, \dots, N - 1. \quad (30)$$

Die Zentrum-Elemente erfüllen auch die Bedingung $\det U = 1$. Die Zentrum-Gruppe wird auch als $Z(N)$ bezeichnet. Grob gesagt, es ist als ob einige Elemente der Gruppe $U(1)$ (von der Form $U = e^{-i\theta^0 t^0}$) bei $SU(N)$ in diskreter Form ‘weiterleben’.

Nach dieser kurzen Abschweifung über Gruppentheorie ist eine genauere Diskussion der schon aufgeführten Eigenschaften der QCD an der Reihe.

(a) *Lokale Farbsymmetrie* $SU(N_c = 3)_c$

Die $N_c \times N_c$ Matrix der Yang-Mills-Felder $A_\mu(x) = A_\mu^a(x) t^a$ und die N_f -Vektorspalten q_i ($q_i^t = (q_{i,1}, \dots, q_{i,N_c})$, $i = 1, \dots, N_f$) transformieren unter einer lokalen $SU(N_c)_c$ -Transformation wie folgt:

$$A_\mu(x) \rightarrow A'_\mu(x) = U(x) A_\mu(x) U^\dagger(x) - \frac{i}{g_0} U(x) \partial_\mu U^\dagger(x), \quad q_i \rightarrow U(x) q_i, \quad (31)$$

wobei

$$U(x) = e^{i\theta_a(x) t^a}, \quad a = 1, \dots, N_c^2 - 1 \quad (= 8 \text{ für } N_c = 3). \quad (32)$$

Der QCD -Lagrangian $\mathcal{L}_{QCD}$ ist invariant unter der lokalen Transformation (31). Diese lokale Symmetrie ist in einem hadronischen Modell automatisch erfüllt, da man von Anfang an mit farblosen Feldern (Farbsingulets) arbeitet. Dennoch ist sie nützlich: die Betrachtung des ‘Large- N_c ’ Limes, in dem die Gruppe $SU(N_c \gg 3)_c$ in Betracht gezogen wird, spielt eine zentrale Rolle in hadronischen Modellen. Obwohl die Felder automatisch $SU(N_c)_c$ -invariant sind, *skalieren* die Parameter des Modells mit Potenzen von N_c . *Es sind nicht die Felder, sondern die Parameter, die die Abhängigkeit von N_c in einem effektiven Modell der QCD tragen.*

Die $SU(N_c)_c$ -Gruppe involviert auch die Transformationen unter den Elementen des Zentrums, die in Gl. (31) nicht berücksichtigt wurden. Die entsprechenden Transformationen lauten

$$A_\mu \rightarrow A'_\mu = Z A_\mu Z^\dagger = A_\mu, \quad q_i \rightarrow Z q_i \quad (33)$$

mit

$$Z = Z_n = e^{i \frac{2\pi n}{N}} 1_N, \quad n = 0, 1, 2, \dots, N_c - 1. \quad (34)$$

Selbstverständlich lassen diese Transformationen der Lagrange-Dichte invariant. Wenn aber die Temperatur ins Spiel gebracht wird, finden zwei wichtige Phänomene statt: (i) bei großem T wird im Eichsektor (QCD ohne Quarks) die Zentrum-Symmetrie spontan gebrochen. Die spontane Brechung dieser Symmetrie signalisiert Deconfinement der Gluonen. (ii) Wenn Quarks dabei sind, ist die Zentrum-Gruppe keine exakte Symmetrie, da die notwendigen antisymmetrischen Randbedingungen für die Fermionfelder nicht erfüllt werden.

Obwohl die Zentrum-Symmetrie in den folgenden Überlegungen keine weitere Rolle spielen wird, ist es aus zwei Gründen wichtig, sie zu erwähnen: (i) sie ist ein wichtiger Bestandteil vieler moderner Forschungsarbeiten in der hadronischen Physik bei nicht verschwindender Temperatur und Dichte. Das erfolgt durch den Polyakov-Loop, der den Ordnungsparameter der spontanen Symmetriebrechung der Zentrum-Gruppe $Z(N_c)$ bei hohem T darstellt. (ii) Das hadronische Modell, das wir später diskutieren werden, kann leicht verallgemeinert werden, um den Polyakov-Loop mit zu berücksichtigen.

(b) *Spursymmetrie und Spuranomalie*

Die klassische Lagrange-Dichte der QCD ist im Limes $m_i \rightarrow 0$ invariant unter Raum-Zeit-Dilatationen. Das ist einfach zu verstehen, da kein dimensionsbehafteter Parameter in der Lagrange-Dichte auftaucht. Zuerst beschränken wir unsere Aufmerksamkeit auf den Eichsektor (QCD ohne Quarks):

$$\mathcal{L}_{YM} = -\frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu} \quad (35)$$

Die Dilatationstransformation lautet

$$x^\mu \rightarrow x'^\mu = \lambda^{-1} x^\mu, \quad A_\mu^{a'}(x') = \lambda A_\mu^a(x). \quad (36)$$

Es ist bekannt, dass schon im Eichsektor diese Symmetrie durch Quantenfluktuationen gebrochen wird. Es ist eben auf diesem Niveau (und nicht wegen der nackten Quarkmassen), dass der Skalierungsfaktor $\Lambda_{QCD} \simeq 250$ MeV entsteht und die Größenordnung aller hadronischen Prozesse bestimmt. Die Brechung der klassischen Skaleninvarianz stellt ein sehr wichtiges und tiefes Phänomen dar, das zentral für die gesamte QCD ist.

Man kann diesen sehr wichtigen Prozess auf eine einfache Weise erklären: der klassische Noetherstrom, der aus der Dilatationssymmetrie stammt, lautet

$$J^\mu = x_\nu T^{\mu\nu} \rightarrow \partial_\mu J^\mu = T^\mu_\mu = 0, \quad (37)$$

wobei $T^{\mu\nu}$

$$T^{\mu\nu} = \frac{\partial \mathcal{L}_{YM}}{\partial (\partial_\mu A_\rho)} \partial^\nu A_\rho - g^{\mu\nu} \mathcal{L}_{YM} + \text{'sym'} \quad (38)$$

der Energie-Impuls-Tensor der Yang-Mills Lagrange-Dichte ist. (Der Term 'sym' ist ein Symmetrisierungsterm, der in Eichtheorien gebraucht wird, um den Tensor symmetrisch zu machen).

Die explizite Berücksichtigung von gluonischen Schleifen (Quantenfluktuationen) bricht diese Symmetrie. Anstatt einer konstanten Kopplungskonstante g_0 entsteht durch Renormierung (d.h. durch Quanten-Schleifen-Berechnungen) eine energieabhängige 'laufende' Kopplung $g(\mu)$:

$$g_0 \xrightarrow{\text{Renormierung}} g(\mu), \quad (39)$$

wobei μ die Energieskala ist, bei der die Kopplung festgelegt wird (wie z.B. die Energie im Schwerpunktsystem in einem Streuprozess) .

Folgende Gleichung wird im Rahmen störungstheoretischer QCD hergeleitet:

$$\partial_\mu J^\mu = T_\mu^\mu = \frac{\beta(g)}{4g} G_{\mu\nu}^a G^{a,\mu\nu} \neq 0 \text{ wobei } \beta(g) = \mu \frac{\partial g}{\partial \mu} . \quad (40)$$

Auf dem 1-Schleifen-Niveau gilt:

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} = -bg^3 \text{ mit } b = \frac{11N_c}{48\pi^2} . \quad (41)$$

Wenn g konstant wäre ($g = g_0$), hätte man tatsächlich $\partial_\mu J^\mu = T_\mu^\mu = 0$. Das ist aber wegen der Renormierung nicht der Fall. Die Lösung der Differentialgleichung (41) lautet

$$g^2(\mu) = \frac{g_*^2}{1 + 2bg_*^2 \log \frac{\mu}{\mu_*}} . \quad (42)$$

Da in der Yang-Mills-Theorie (wie auch in der QCD mit Quarks, siehe unten) $b > 0$, wird die laufende Kopplungskonstante $g(\mu)$ kleiner, wenn μ wächst. Das ist die berühmte asymptotische Freiheit der QCD und Grund für den Nobelpreis im Jahr 2004. Andererseits wächst für kleine μ , also im Niederenergiebereich, die Kopplung $g(\mu)$ und somit die Intensität der Wechselwirkung zwischen Gluonen (und Quarks). Das ist einer der Gründe für die Entstehung von Confinement: die Gluonen (und die Quarks) können sich nicht frei bewegen, sondern sind in farbneutralen eingeschlossenen Zuständen gefangen. Es ist auch interessant zu bemerken, dass aus Gl. (42) folgt: $g \propto 1/\sqrt{N_c}$. Das ist die Grundlage für viele Large- N_c Resultate, die im vorherigen Abschnitt zusammengefasst wurden.

Die störungstheoretische Gleichung (42) hat einen Pol für

$$\mu_{Pol} = \Lambda_{Landau} = \Lambda_{YM} = \mu_* e^{\frac{-1}{2bg_*^2}} . \quad (43)$$

Natürlich ist eine störungstheoretische Rechnung im Niederenergiebereich nicht gültig. Diese Rechnung bedeutet nicht, dass $g(\mu = \Lambda_L)$ unendlich wird. Der wichtige Punkt ist, dass diese Rechnung eine Energieskala setzt. Sie impliziert, dass QCD für $\mu \sim \Lambda_{YM}$ stark gekoppelt wird und jede störungstheoretische Studie bei solchen Energien nutzlos macht. Leider kann Λ_{YM} theoretisch nicht bestimmt werden, da der Wert g_* bei einem vorgegebenen μ_* (wie z.B. der GUT-Energie-Skala $\mu_* = 10^{16}$ GeV) unbekannt ist. Dennoch bedeutet sie etwas sehr Wichtiges, indem sie zeigt, wie aus einer dimensionslosen Theorie eine endliche Dimension entsteht. Dieses Phänomen ist auch unter dem englischen Namen ‘Dimensional Transmutation’ bekannt.

Es ist üblich, die Funktion $g(\mu)$ mit Hilfe von Λ_{YM} auszudrücken:

$$g^2(\mu) = \frac{1}{2b \log \frac{\mu}{\Lambda_{YM}}} . \quad (44)$$

Die Brechung der Skalensymmetrie generiert auch ein Gluon-Kondensat: der Vakuumerwartungswert (vev) von Gluonfeldern verschwindet nicht und bildet das sogenannte Gluon-Kondensat:

$$\langle T_\mu^\mu \rangle = - \left\langle \frac{11N_c}{48} \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a,\mu\nu} \right\rangle \sim - \frac{11N_c}{48} (350-600 \text{ MeV})^4 . \quad (45)$$

Die numerischen Resultate wurden für $N_c = 3$ mit Hilfe von Gitter-Simulationen und QCD-Summenregeln berechnet [22]. Es ist insofern wichtig, dass hadronische Modelle dieses Kondensat berücksichtigen, siehe später für ein explizites Verfahren.

Wenn die Quarks berücksichtigt werden, transformieren sie unter einer Dilatationstransformation wie

$$q'(x') = \lambda^{3/2} q(x) , \quad (46)$$

da sie Dimension $3/2$ haben. Die Diskussion ist dem vorherigen Fall ähnlich, bis auf die Konstante b , die in Anwesenheit von Quarks

$$b = \frac{11N_c - 2N_f}{48\pi^2}$$

lautet. Es gilt $b > 0$ für $N_f < \frac{11}{2}N_c$. Das ist eine Bedingung, die in der Natur ($N_f = 6$ und $N_c = 3$) erfüllt ist.

(c) *Chirale Symmetrie* $U(N_f)_R \times U(N_f)_L$

Im Limes $m_i \rightarrow 0$ ist $\mathcal{L}_{QCD}$ unter folgender $U(N_f)_R \times U(N_f)_L$ -Transformation der Quark-Felder invariant:

$$q_i = q_{i,R} + q_{i,L} \rightarrow U_{R,ij}q_{j,R} + U_{L,ij}q_{j,L} , \quad (47)$$

mit

$$U_R \in U(N_f)_R , \quad U_L \in U(N_f)_L . \quad (48)$$

Die rechtshändigen und linkshändigen Spinoren $q_{i,R}$ und $q_{i,L}$ sind wie folgt definiert:

$$q_{i,R} = P_R q_i , \quad q_{i,R}^\dagger = q_i^\dagger P_R , \quad \bar{q}_{i,R} = \bar{q}_i P_L , \quad (49)$$

$$q_{i,L} = P_L q_i , \quad q_{i,L}^\dagger = q_i^\dagger P_L , \quad \bar{q}_{i,L} = \bar{q}_i P_R , \quad (50)$$

und

$$P_R = \frac{1}{2}(1 + \gamma_5) , \quad P_L = \frac{1}{2}(1 - \gamma_5) . \quad (51)$$

Das ist die berühmte chirale Symmetrie zwischen links- und rechtshändigen Quarks.

(d) $U(1)_A$ *Anomalie*

Die $U(1)_A$ -Transformation ist eine Untergruppe von $U(N_f)_L \times U(N_f)_R$. Sie entspricht der Wahl

$$U_A^{(1)} = U_R = U_L^\dagger = e^{i\nu t^0} , \quad (52)$$

d.h.

$$q_{i,R} \rightarrow e^{i\nu \frac{1}{\sqrt{2N_f}}} q_{i,R} , \quad q_{i,L} \rightarrow e^{-i\nu \frac{1}{\sqrt{2N_f}}} q_{i,L} \Rightarrow q \rightarrow e^{i\nu t^0 \gamma_5} q . \quad (53)$$

Diese Symmetrie wird auch von Quantenfluktuationen explizit verletzt. Deswegen wird sie auch Anomalie genannt. Die Divergenz des axialen Stroms

$$A_\mu^0 = \bar{q} \gamma^\mu \gamma^5 q = \sum_{i=1}^{N_f} \bar{q}_i \gamma^\mu \gamma^5 q_i \quad (54)$$

verschwindet nämlich wegen der Quantenfluktuationen *nicht*. Der Ausdruck lautet

$$\partial^\mu A_\mu^0 = -\frac{g^2 N_f}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} \neq 0 , \quad (55)$$

wobei der duale Gluon-Tensor

$$\tilde{G}^{a,\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}^a \quad (56)$$

eingeführt wurde.

(e) *Spontane Symmetriebrechung der chiralen Symmetrie*: $SU(N_f)_R \times SU(N_f)_L \rightarrow SU(N_f)_V$

Die spontane Symmetriebrechung der chiralen Symmetrie ist ein zentrales Phänomen in der niederenergetischen hadronischen Welt: sie erklärt, warum die Pionen (beinahe) masselos sind, und warum ihre Wechselwirkung schwach ist. Sie verursacht Massenunterschiede zwischen Multiplets und beeinflusst

viele Zerfallsmoden stark. Um sie genau zu verstehen, schreiben wir zuerst die chirale Symmetriegruppe der QCD wie folgt:

$$U(N_f)_R \times U(N_f)_L \equiv U(1)_V \times SU(N_f)_V \times U(1)_A \times SU(N_f)_A . \quad (57)$$

Die Transformation $U_V(1)$ entspricht der Wahl

$$U_1 = U_L = U_R = e^{i\theta t^0} , \quad (58)$$

$SU(N_f)_V$ entspricht der Wahl

$$U_V = U_L = U_R = e^{i\theta_a^V t^a} \quad (a = 1, \dots, N_f^2 - 1) , \quad (59)$$

und $SU(N_f)_A$ entspricht der Wahl

$$U_A = U_L = U_R^\dagger = e^{i\theta_a^A \lambda^a} \quad (a = 1, \dots, N_f^2 - 1). \quad (60)$$

Es ist wichtig zu beachten, dass $SU(N_f)_A$ keine Gruppe bildet, da das Produkt zweier Elemente kein neues Element der Menge ist. Es ist aber genau die Symmetrie unter den $SU(N_f)_A$ -Transformationen, die spontan gebrochen wird. Das Vakuum der QCD $|0_{QCD}\rangle$ ist nicht invariant unter dieser Transformation, was wegen des Goldstone-Theorems zur Folge hat, dass pseudoskalare Anregungen mit Masse Null entstehen (die oben erwähnten Pionen).

(f) *Explizite Symmetriebrechung durch die Quarkmassen*

Der Massenterm

$$\mathcal{L}_{mass} = \sum_{i=1}^{N_f} m_i \bar{q}_i q_i \quad (61)$$

in der QCD-Lagrange-Dichte bricht explizit viele der oben genannten Symmetrien.

Die Spuranomalie wird dadurch verursacht, dass die Massen dimensionsbehaftet sind. Die entstehende Verletzung ist aber deutlich kleiner als die Quantenverletzung im Eichsektor. In Anwesenheit der Quarkmassen lautet die Spur des Energie-Impuls-Tensors

$$T_\mu^\mu = \frac{\beta(g)}{4g} G_{\mu\nu}^a G^{a,\mu\nu} + \sum_{i=1}^{N_f} m_i \bar{q}_i q_i , \quad (62)$$

wobei der letzte Term die explizite Symmetriebrechung der Skalensymmetrie durch die nicht verschwindenden Quark-Massen beschreibt.

Die Symmetrie unter $U(N_f)_V \equiv U(1)_V \times SU(N_f)_V$ -Transformationen ist nur erhalten, wenn die Massen gleich sind: $m_1 = m_2 = \dots = m_{N_f}$. Sobald es einen Massenunterschied gibt, ist diese Symmetrie auch verletzt. Die Divergenzen der entsprechenden Ströme,

$$V_\mu^a = \bar{q} \gamma^\mu t^a q , \quad (63)$$

lauten

$$\partial^\mu V_\mu^a = i \bar{q} [\hat{m}, t^a] q \neq 0 , \quad (64)$$

wobei $\hat{m} = \text{diag}\{m_1, m_2, \dots, m_{N_f}\}$. Die $U_V(1)$ -Symmetrie, die der Wahl $a = 0$ entspricht, bleibt von dem Massenterm ‘verschont’. Sobald zwei Massen gleich sind, wie z.B. $m_1 = m_2$, gibt es andere Ströme, die erhalten bleiben.

Die Symmetrie unter $U_A(N_f) \equiv U_A(1) \times SU_A(N_f)$ -Transformationen wird verletzt, sobald $m_i \neq 0$. Sei

$$A_\mu^a = \bar{q} \gamma^\mu \gamma^5 t^a q , \quad (65)$$

dann gilt:

$$\partial^\mu A_\mu^a = i\bar{q}\gamma^5\{\hat{m}, t^a\}q \neq 0 \text{ für } a \neq 0. \quad (66)$$

Die explizite Brechung der $SU_A(N_f)$ -Symmetrie verursacht eine kleine, aber nicht verschwindende Masse der Pionen, die dann Quasi-Goldstone-Bosonen sind. Die Divergenz des Stroms für den Fall $a = 0$ besteht letztendlich aus zwei Beiträgen,

$$\partial^\mu A_\mu^0 = 2i\hat{m}\gamma^5 q - \frac{g^2 N_f}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu}, \quad (67)$$

wobei der erste klassisch ist und der zweite die $U_A(1)$ -Anomalie beschreibt. Die axiale Anomalie hat eine wichtige Folge im pseudoskalaren mesonischen Spektrum: das entsprechende isoskalare Meson (η' in der realen Welt) entspricht keinem Goldstone-Boson, da die Symmetrie ohnehin durch die Quantenfluktuationen verletzt ist.

Da diese Arbeit auf die leichten Quarks fokussiert ist, ist man überwiegend an den Fällen $N_f = 2$ und $N_f = 3$ interessiert. Für $N_f = 2$ ist die explizite Brechung durch die Quarkmassen klein: $m_{u,d} \ll \Lambda_{YM}$. Außerdem werden wir in dieser Arbeit in der $SU(2)_V$ -Näherung $m = m_u = m_d$ arbeiten: die kleinen Brechungseffekte sind proportional zu dem kleinen Massenunterschied $m_d - m_u$ und, obwohl interessant, wenn man bestimmte Prozesse studiert, sind sie für den Zweck dieser Arbeit vernachlässigbar. Für $N_f = 3$ ist der Wert der s -Masse (130 MeV) zwar noch klein im Vergleich zu vielen Resonanzen, dennoch ist er von derselben Größenordnung wie Λ_{YM} , $m_s \sim \Lambda_{YM} \gg m$. Das bedeutet, dass die explizite Brechung durch die s -Masse in der Regel nicht vernachlässigbar ist.

Als letzter Punkt beschreiben wir kurz ein physikalisches Bild, das im Rahmen des berühmten Nambu-Jona-Lasinio (NJL)-Modells entsteht [23] und das die Punkte (e) und (f) vereint. Die spontane Symmetriebrechung von $SU(N_f)_A$ kann als Folge der dynamischen Erzeugung einer effektiven Quarkmasse beschrieben werden. Die Wechselwirkung eines Quarks mit dem nicht-trivialen QCD-Vakuum bedeutet schematisch ($N_f = 2$ als Beispiel):

$$m \simeq 5 \text{ MeV} \rightarrow m^* \simeq 300 \text{ MeV} \gg m. \quad (68)$$

Mit solch großen Quarkmassen ist deutlich, dass die Symmetrie unter $SU(N_f)_A$ -Transformationen nicht einmal annähernd erfüllt ist: $\partial^\mu A_\mu^a \propto \{m^*, t^a\} \neq 0$. Diese massiven Quarks werden auch Quark-Konstituenten genannt. Sie sind effektive Quasi-Teilchen, die die Bestandteile von Mesonen und Baryonen darstellen. Zahlreiche Modelle benutzen erfolgreich diese Ideen, um die Phänomenologie der QCD im Niederenergiebereich von einem mikroskopischen Standpunkt zu erklären.

3.3 Konstruktion des Modells im mesonischen Sektor

Ein effektives, niederenergetisches hadronisches Modell muss die Eigenschaften (a)-(f) besitzen. Wir besprechen die Einzelheiten der Punkte (a)-(f) unter dem Standpunkt eines mesonischen Modells, das (pseudo)skalare und (axial)vektorielle Felder enthält. Der Zusammenhang zwischen diesen Symmetrien wird uns die Möglichkeit bieten, ein Kriterium zu formulieren, das das effektive Modell wesentlich vereinfacht.

3.3.1 Confinement

Wegen des Confinements arbeiten wir mit Meson-Feldern (Matrizen $\Phi, R, \dots$, siehe unten), die farbneutral sind: $\Phi \rightarrow \Phi, R \rightarrow R, \dots$. Aus diesem Grund ist die Symmetrie unter $SU(N_c)_c$ aus Abschnitt 3.2 vom Anfang an trivial erfüllt. Dennoch ist die Large- N_c Abhängigkeit in den Parametern versteckt: die Parameter skalieren mit inversen Potenzen von N_c , wie im Verlauf dieses Abschnittes erklärt wird.

3.3.2 Eichsektor

Wir brauchen auch auf dem confinierten Niveau die korrekte Beschreibung der Spuranomalie und der Entstehung einer Dimension im Modell. Wir führen zu diesem Zweck ein Skalarfeld G ein, das die Physik der Spuranomalie enthält.

Intuitiv gibt es die Entsprechung

$$G^4 \sim G_{\mu\nu}^a G^{a,\mu\nu}, \quad (69)$$

wobei G ein kollektives Feld ist, das die Gluonen beschreibt. Wie in Ref. [24] gezeigt wurde, erfüllt die folgende Lagrange-Dichte

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - V_{dil}(G), \quad (70)$$

mit

$$V_{dil}(G) = \frac{1}{4} \frac{m_G^2}{\Lambda_G^2} \left[G^4 \ln \left(\frac{G}{\Lambda_G} \right) - \frac{G^4}{4} \right] \quad (71)$$

die gewünschten Eigenschaften. Wegen des logarithmischen Terms ist die Dilatationssymmetrie

$$G(x) \rightarrow G'(x') = \lambda G(x) \text{ und } x^\mu \rightarrow \lambda^{-1} x^\mu$$

explizit gebrochen. Die Divergenz des Noether-Stroms lautet nun:

$$\partial_\mu J^\mu = T_\mu^\mu = -\frac{1}{4} \frac{m_G^2}{\Lambda_G^2} G^4. \quad (72)$$

Es gibt eine klare Entsprechung zu Gl. (40). Wenn man den Vakuumerwartungswert berechnet, gilt

$$\langle T_\mu^\mu \rangle = \left\langle -\frac{1}{4} \frac{m_G^2}{\Lambda_G^2} G^4 \right\rangle = -\frac{1}{4} \frac{m_G^2}{\Lambda_G^2} G_0^4 \equiv -\left\langle \frac{11N_c}{48} \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a,\mu\nu} \right\rangle. \quad (73)$$

Der Wert $G_0 = \Lambda_G$ entspricht dem Minimum des Potentials $V_{dil}(G)$. Das impliziert, dass

$$\langle T_\mu^\mu \rangle = -\frac{1}{4} m_G^2 \Lambda_G^2. \quad (74)$$

Wenn man das Feld G um G_0 verschiebt, $G \rightarrow G_0 + G$, und wie üblich eine Taylor-Entwicklung durchführt, kann die Masse des Feldes G bestimmt werden: sie lautet einfach m_G . Dieses Teilchen ist der berühmte skalare Glueball, d.h. der leichteste gebundene Zustand, der aus Gluonen besteht. Die Existenz dieses Zustandes wurde in zahlreichen Gitter-Simulationen bestätigt [14], und der numerische Wert lautet $m_G \sim 1.5\text{-}1.7$ GeV. Ein guter experimenteller Kandidat ist die Resonanz $f_0(1500)$: obwohl Beimischungen von Quarkonia erwartet werden (sie entstehen auf eine natürliche Weise, sobald Quarksfreiheitsgrade berücksichtigt werden), zeigen viele Modelle, dass diese Resonanz dominant aus Gluonen besteht [25] (siehe auch die Review-Artikel in Ref. ([26])).

Die Verbindung mit dem Yang-Mills-Sektor der QCD kann auch numerisch benutzt werden. Gittersimulationen und QCD-Summenregeln zeigen, dass

$$\left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a,\mu\nu} \right\rangle = (200\text{-}600 \text{ MeV})^4. \quad (75)$$

Das impliziert den folgenden Wert für Λ_G^2 :

$$\Lambda_G^2 = \frac{11}{4m_G^2} \left\langle \frac{\alpha_s}{2} G_{\mu\nu}^a G^{a,\mu\nu} \right\rangle \simeq \frac{(0.25\text{-}0.8 \text{ GeV})^4}{m_G^2} = (0.23\text{-}0.7 \text{ GeV})^2, \quad (76)$$

wobei im letzten Schritt der numerische Wert $m_G \simeq 1.5$ GeV benutzt wurde.

Die Diskussion über Gluebälle kann verallgemeinert werden: es wird nämlich nicht nur ein einziger Glueball erwartet, sondern eine unendliche Reihe mit wachsenden Massen. Der skalare Glueball ist

besonders wichtig, weil er der leichteste Glueball ist und weil er mit der Spuranomalie verbunden ist. Es ist aber nicht schwierig, weitere skalare und pseudoskalare Gluebälle mitzunehmen:

$$\mathcal{L} = \mathcal{L}_{dil} + \sum_k \alpha_k G_k^2 G^2 + \sum_{k,l} \beta_{k,l} G_k^2 G_l^2 , \quad (77)$$

wobei die Konstanten α_k die Wechselwirkungen mit dem leichtesten Glueball G und $\beta_{k,l}$ Selbstwechselwirkungen und gegenseitige Wechselwirkungen darstellen. (Gluebälle mit anderen Quantenzahlen können auch auf eine ähnliche Art eingeführt werden). Es wäre interessant, auf diese Weise das Glueballspektrum zu studieren; es ist aber nicht das Ziel dieser Arbeit. Da Gittersimulationen zeigen, dass weitere Gluebälle oberhalb 2 GeV liegen, können diese weiteren Zustände in den folgenden Erwägungen ausgelassen werden.

Als letzten Punkt beschreiben wir die Large- N_c Abhängigkeit der Parameter. Der Parameter m_G ist N_c unabhängig, damit die Glueballmasse korrekterweise wie N_c^0 skaliert. Der Skalenfaktor Λ_G hingegen skaliert wie N_c , damit die G^4 -Wechselwirkung wie $1/N_c^2$ skaliert. Zusammengefasst gilt:

$$m_G \propto N_c^0 , \Lambda_G \propto N_c . \quad (78)$$

Die Koeffizienten α_k und $\beta_{k,l}$ in Gl. (77) skalieren wie $1/N_c^2$.

3.3.3 Die Einführung von skalaren und pseudoskalaren Quark-Antiquark-Mesonen

Wir führen skalare und pseudoskalare Felder ein, die in der Meson-Matrix Φ eingebettet werden. Die Matrix, die die Quarkfelder beschreibt, lautet:

$$\Phi_{ij} \equiv \sqrt{2} \bar{q}_{j,R} q_{i,L} . \quad (79)$$

Die Äquivalenz $\equiv$ bedeutet, dass Φ und $\sqrt{2} \bar{q}_{j,R} q_{i,L}$ gleich unter chiralen Transformationen transformieren. Es bedeutet aber nicht, dass die Meson-Matrix Φ aus einem perturbativen Quark-Antiquark-Paar besteht. Die Matrix Φ ist ein nicht-perturbatives Objekt. Man kann sich die physikalische Situation wie folgt ausmalen: ein findet eine nicht-perturbative Modifikation der perturbativen Quarks durch Gluonen und Quark-Antiquark-Vakuumpaare statt, die den Quarks eine effektive Masse von ungefähr 300 MeV verleiht. Die Matrix Φ kann in guter Näherung als ein zusammengesetztes Objekt interpretiert werden, das aus einem effektiven Quark und einem effektiven Antiquark besteht. Es ist wichtig zu bemerken, dass die nichtperturbative Modifikation die Transformationseigenschaften unverändert lässt. Aus diesem Grund ist es ausreichend, die Identifizierung $\Phi_{ij} \equiv \sqrt{2} \bar{q}_{j,R} q_{i,L}$ zu machen, wenn man nur an den Transformationseigenschaften interessiert ist.

Da die Quarkfelder unter einer chiralen Transformation wie $q_{i,L} \rightarrow U_L q_{i,L}$, $q_{i,R} \rightarrow U_R q_{i,R}$ transformieren, transformiert die Matrix Φ wie folgt:

$$\Phi \rightarrow U_L \Phi U_R^\dagger . \quad (80)$$

[Die $U_V(1)$ -Transformation wird trivial: für $U_L = U_R = e^{i\theta t^0}$ gilt $\Phi \rightarrow \Phi$].

Die Matrix Φ kann auch als

$$\begin{aligned} \Phi_{ij} &\equiv \sqrt{2} \bar{q}_{j,R} q_{i,L} = \sqrt{2} \bar{q}_j P_L P_L q_i = \sqrt{2} \bar{q}_j P_L q_i \\ &= \frac{1}{\sqrt{2}} (\bar{q}_j q_i - \bar{q}_j \gamma^5 q_i) = \frac{1}{\sqrt{2}} (\bar{q}_j q_i + i \bar{q}_j i \gamma^5 q_i) \end{aligned} \quad (81)$$

geschrieben werden, wobei $P_L = \frac{1}{2}(1 - \gamma^5)$ und $P_R = \frac{1}{2}(1 + \gamma^5)$ die Chiralitätsprojektoren sind. Man erkennt in der letzten Gleichung den skalaren und den pseudoskalaren Strom:

$$S_{ij} \equiv \bar{q}_j q_i , \mathcal{P}_{ij} \equiv \bar{q}_j i \gamma^5 q_i , \quad (82)$$

und letztendlich

$$\Phi = \mathcal{S} + i\mathcal{P} . \quad (83)$$

Die Matrizen $\mathcal{S}$ und $\mathcal{P}$ sind hermitesch. Daher können sie auch als

$$\mathcal{S} = \mathcal{S}^{at^a}, \quad \mathcal{P} = \mathcal{P}^{at^a} \quad (84)$$

$$S^a \equiv \sqrt{2}\bar{q}t^aq, \quad P^a \equiv \sqrt{2}\bar{q}i\gamma^5t^aq. \quad (85)$$

ausgedrückt werden, wobei t^a die Generatoren der Gruppe $U(N_f)$ sind ($a = 0, 1, \dots, N_f^2 - 1$).

Eine wichtige Nebenbemerkung betrifft noch die Identifizierung $\Phi_{ij} \equiv \sqrt{2}\bar{q}_{j,R}q_{i,L}$ (und die daraus stammenden Identifizierungen in Gl. (85)). Eine Möglichkeit, die Verbindung zwischen dem nicht-perturbativen Objekt Φ_{ij} und den perturbativen Strömen $\sqrt{2}\bar{q}_{j,R}q_{i,L}$ besser zu verstehen, besteht darin, Φ_{ij} als nicht-lokale Zusammensetzung von $\sqrt{2}\bar{q}_{j,R}q_{i,L}$ darzustellen:

$$\Phi_{ij} = \int d^4y \sqrt{2}\bar{q}_{j,R}(x+y/2)q_{i,L}(x-y/2)f(y) , \quad (86)$$

wobei $f(y)$ eine nicht-perturbative Vertex-Funktion ist. (Der perturbative Limes entspricht der trivialen Wahl $f(y) = \delta(y)$.) Eine Klasse von quark-basierten Modellen beruht auf solchen nichtlokalen Feldkonfigurationen [27]. Es ist aber deutlich, dass die Flavor-Transformationen unverändert bleiben. (Das ist nicht der Fall für lokale Eichtransformationen; es ist aber möglich, den nicht-lokalen Strom lokal farbinvariant zu machen, siehe z.B. Ref. [28]. Dieser Punkt ist aber für unsere Studie nicht relevant.)

Tabelle 2: Transformationseigenschaften von $\mathcal{P}$, $\mathcal{S}$ und Φ .

	$\mathcal{P} = \frac{1}{\sqrt{2}} \sum_{i=0}^8 P^i \lambda_i$	$\mathcal{S} = \frac{1}{\sqrt{2}} \sum_{i=0}^8 S^i \lambda_i$	$\Phi = \mathcal{S} + i\mathcal{P}$
Elemente	$\mathcal{P}_{ij} \equiv \bar{q}_j i\gamma^5 q_i$	$\mathcal{S}_{ij} \equiv \bar{q}_j q_i$	$\Phi_{ij} \equiv \sqrt{2}\bar{q}_{j,R}q_{i,L}$
Ströme	$P^i \equiv \bar{q}i\gamma^5 \frac{\lambda_i}{\sqrt{2}} q$	$S^i \equiv \bar{q} \frac{\lambda_i}{\sqrt{2}} q$	$\Phi^i \equiv \sqrt{2}\bar{q}_R \frac{\lambda_i}{\sqrt{2}} q_L$
P	$-\mathcal{P}(x^0, -\mathbf{x})$	$\mathcal{S}(x^0, -\mathbf{x})$	$\Phi^\dagger(x^0, -\mathbf{x})$
C	$\mathcal{P}^t$	$\mathcal{S}^t$	Φ^t
$U(N_f)_V$	$U_V \mathcal{P} U_V^\dagger$	$U_V \mathcal{S} U_V^\dagger$	$U_V \Phi U_V^\dagger$
$U(N_f)_A$	$\frac{1}{2i} \left(U_A \Phi U_A - U_A^\dagger \Phi^\dagger U_A^\dagger \right)$	$\frac{1}{2} \left(U_A \Phi U_A + U_A^\dagger \Phi^\dagger U_A^\dagger \right)$	$U_A \Phi U_A$
$U(N_f)_R \times U(N_f)_L$	$\frac{1}{2i} \left(U_L \Phi U_R^\dagger - U_R \Phi^\dagger U_L^\dagger \right)$	$\frac{1}{2} \left(U_L \Phi U_R^\dagger + U_R \Phi^\dagger U_L^\dagger \right)$	$U_L \Phi U_R^\dagger$

Es wird zuerst der chirale Limes $m_i \rightarrow 0$ betrachtet. In diesem Fall ist die chirale Symmetrie der QCD exakt. Insofern muss auch eine Lagrange-Dichte $\mathcal{L}_\Phi$ für das Feld Φ hingeschrieben werden, die diese Symmetrie exakt erfüllt (die $U_A(1)$ -Anomalie wird zuerst vernachlässigt):

$$\mathcal{L}_\Phi = \text{Tr} \left[(\partial^\mu \Phi)^\dagger (\partial_\mu \Phi) - m_0^2 \Phi^\dagger \Phi - \lambda_2 (\Phi^\dagger \Phi)^2 \right] - \lambda_1 (\text{Tr}[\Phi^\dagger \Phi])^2 . \quad (87)$$

Das ist das übliche σ -Modell der QCD mit (pseudo)skalaren Quarkonia, das als wichtiges Mittel zur Erforschung der QCD gedient hat.

Normalerweise werden weitere Terme, wie z.B. $(\text{Tr}[\Phi^\dagger \Phi])^6$, nicht mitgenommen, da man die Renormierbarkeit der effektiven Theorie fordert. Diese Motivation kann aber nicht gelten: eine effektive Theorie ist nicht bis zu einer hohen Energieskala (wie M_{Planck}) gültig. Es spricht in dieser Hinsicht nichts dagegen, weitere Terme hinzuzufügen, die, obwohl nicht renormierbar, die Symmetrie erfüllen. In der chiralen Störungstheorie, wo die Energiegrenze so niedrig ist, dass nur die Pionen mitgenommen werden (siehe Abschnitt 2), werden tatsächlich -prinzipiell- unendlich viele Terme mit beliebiger

Dimension hingeschrieben, obwohl aus praktischen Gründen eine Rechnung nur bis zu der dritten Ordnung durchgeführt werden kann. Auch nicht-renormierbare Terme werden berücksichtigt, und ein Power-Counting-Schema wird eingeführt.

Die Situation ändert sich, wenn man auch das schon diskutierte Dilaton-Feld G mitnimmt. Das erlaubt, ein Kriterium zur Erstellung eines hadronischen Modells zu formulieren.

3.3.4 Ein Kriterium für die Konstruktion eines hadronischen Modells

Die vollständige Lagrange-Dichte mit dem Dilaton-Feld G , mit der Quarkonium-Matrix Φ und mit anderen Quarkonium-Feldern hat die schematische Form

$$\mathcal{L}_{had} = \mathcal{L}_{had}(G, \Phi, \dots) = \frac{1}{2}(\partial_\mu G)^2 - V_{dil}(G) + \text{Tr}[(\partial^\mu \Phi)^\dagger (\partial_\mu \Phi)] - V_{had}(G, \Phi, \dots), \quad (88)$$

wobei ... für weitere Felder steht. Sie können schwerere skalare und pseudoskalare Multipletts Φ_k mit $k = 1, 2, \dots$, weitere Glueball-Felder wie in Gl. (77), Felder mit anderen Quantenzahlen (wie die in Kürze zu betrachtenden (axial)vektoriellen Felder), usw. sein.

Wir fordern die Gültigkeit der folgenden zwei Bedingungen:

- (1) Im chiralen Limes ($m_i = 0$) soll es im $\mathcal{L}_{had}$ einen einzigen dimensionsbehafteten Parameter geben: Λ_G , der im Potential $V_{dil}(G)$ auftaucht. Dadurch wird die Spuranomalie, genau wie in der QCD, im Eichsektor generiert.
- (2) Das Potential $V_{had}(G, \Phi, \dots)$ soll keine Singularitäten haben, wenn die Felder einen endlichen Wert annehmen.

Diese zwei Bedingungen sind allgemein, dennoch schränken sie $\mathcal{L}_{had}$ auf eine entscheidende Art und Weise ein. Zum Beispiel verbietet Punkt (1) den Term

$$\alpha(\text{Tr}[\Phi^\dagger \Phi])^6, \quad (89)$$

weil der Parameter α Dimension Energie^{-2} hat.

Der analoge Term

$$\frac{\beta}{G^2}(\text{Tr}[\Phi^\dagger \Phi])^6 \quad (90)$$

enthält die dimensionslose Konstante β und ist daher im Einklang mit Punkt (1). Dadurch wird aber Punkt (2) verletzt: für $G = 0$ ist dieser Term singulär. Die Gültigkeit von Punkt (2) ist auch insofern wichtig, weil mit wachsender Temperatur das Gluonkondensat G_0 verschwindet, was die Anwesenheit von Termen wie $\frac{\beta}{G^2}(\text{Tr}[\Phi^\dagger \Phi])^6$ problematisch macht.

Man ist gezwungen, nur Terme mit Dimension exakt 4 mitzunehmen. Außerdem ist die Kopplung mit dem Dilaton-Feld festgelegt. Mit den Feldern G und Φ lautet die allgemeinste Lagrange-Dichte, in der die Bedingungen (1) und (2) (und natürlich die chirale Symmetrie) erfüllt sind :

$$\mathcal{L}_{G\Phi} = \frac{1}{2}(\partial_\mu G)^2 - V_{dil}(G) + \text{Tr}[(\partial^\mu \Phi)^\dagger (\partial_\mu \Phi) - aG^2 \Phi^\dagger \Phi - \lambda_2 (\Phi^\dagger \Phi)^2] - \lambda_1 (\text{Tr}[\Phi^\dagger \Phi])^2. \quad (91)$$

Der zweite Term $aG^2 \Phi^\dagger \Phi$ beschreibt auch die Wechselwirkung zwischen dem Glueball und den (pseudo)skalaren Mesonen. Die Verbindung mit $\mathcal{L}_\Phi$ von Gl. (87) ist klar:

$$m_0^2 = aG_0^2, \quad (92)$$

wobei -wie erfordert- a dimensionslos ist. (Wenn Quarkonia anwesend sind, ist $G_0 \simeq \Lambda_G$, aber nicht exakt gleich, da Mischungen auftreten.) Es ist daher nicht die Renormierbarkeit, sondern die Dilatationsinvarianz, die uns zwingt, nur Terme nur mit Ordnung (exakt) 4 mitzunehmen.

Die Large- N_c Abhängigkeit der Parameter steht fest:

$$a \propto N_c^{-2} \rightarrow m_0^2 \propto N_c^0 , \quad (93)$$

$$\lambda_2 \propto N_c^{-1} , \quad (94)$$

$$\lambda_1 \propto N_c^{-2} , \quad (95)$$

Der λ_2 -Term geht wie N_c^{-1} , da er eine Vierwechselwirkung von Quark-Antiquark-Zuständen beschreibt. Der λ_1 -Term wird um einen weiteren Faktor N_c unterdrückt und skaliert wie N_c^{-2} : das ist so, weil der Term aus dem Produkt von zwei getrennten Spuren gebildet ist. Auf dem mikroskopischen Quark-Gluon-Niveau sind weitere Large- N_c unterdrückte transversale Gluonen notwendig, um diesen Term zu generieren.

3.3.5 Die Einführung von vektoriellen und axialvektoriellen Quark-Antiquark-Mesonen

Die frühere Argumentation bezüglich $\mathcal{L}_{had}(G, \Phi, \dots)$ ist prinzipiell gültig, wenn alle mesonischen Felder mitberücksichtigt werden. Es handelt sich nämlich um eine unendliche Anzahl von Gluebällen und Quarkonia. Es ist aber natürlich zu erwarten, dass massive Felder (mit Masse $\gtrsim 2$ GeV) keinen großen Einfluss auf die leichten Resonanzen haben.

Die vektoriellen und axialvektoriellen Felder sind leicht ($\lesssim 1.4$ GeV) und müssen mitgenommen werden, damit eine konsistente und vollständige Beschreibung der mesonischen Niederenergiephysik erfolgen kann. In vielen Studien der hadronischen Welt wurden nur (pseudo)skalare Mesonen berücksichtigt und die (axial)vektoriellen Mesonen vernachlässigt. Das ist unserer Ansicht nach weder qualitativ noch quantitativ korrekt: die Einführung von (axial)vektoriellen Freiheitsgraden beeinflusst stark die physikalischen Eigenschaften von (pseudo)skalaren Mesonen. Es ist aus diesem Grund nicht möglich, diese Felder auszulassen. Dabei soll nicht vergessen werden, dass die Vektormesonen sogar leichter als die Skalarmesonen sind: sobald die Skalare eingeführt werden, was in einem linearen Schema natürlich ist, ist es nicht erlaubt, die Vektorfelder zu vernachlässigen. Die Axialvektormesonen treten dann als chirale Partner der Vektormesonen auf.

Eine erste allgemeine Studie des Problems hat es in Ref. [29] gegeben. In Ref. [30] wurde das chirale Modell für den Fall $N_f = 2$ in dem sogenannten chiral-lokalen Limes untersucht und in Ref. [31] die (axial)vektorielle Propagatoren berechnet wurden. Jedoch ist es bemerkenswert, dass ein hadronisches Modell, das auf der einfachen Idee der globalen chiralen Symmetrie und Skaleninvarianz basiert, erst neulich erforscht wurde, siehe Ref. [4, 5], deren Resultate wir noch später diskutieren werden.

Wir widmen uns der Konstruktion einer Lagrange-Dichte, die die vorher genannten Prinzipien erfüllt und die (Axial)Vektormesonen enthält. Auf dem mathematischen Niveau führt man die folgenden $N_f \times N_f$ Matrizen R_μ und L_μ ein:

$$R_{ij}^\mu \equiv \sqrt{2} \bar{q}_{j,R} \gamma^\mu \bar{q}_{i,R} = \frac{1}{\sqrt{2}} (\bar{q}_j \gamma^\mu \bar{q}_i - \bar{q}_j \gamma^5 \gamma^\mu \bar{q}_i) , \quad (96)$$

$$L_{ij}^\mu \equiv \sqrt{2} \bar{q}_{j,R} \gamma^\mu \bar{q}_{i,R} = \frac{1}{\sqrt{2}} (\bar{q}_j \gamma^\mu \bar{q}_i + \bar{q}_j \gamma^5 \gamma^\mu \bar{q}_i) . \quad (97)$$

Sie transformieren unter chiralen Transformation wie

$$R^\mu \rightarrow U_R R^\mu U_R^\dagger , L^\mu \rightarrow U_L L^\mu U_L^\dagger . \quad (98)$$

Die Matrizen R_μ und L_μ können wiederum als Funktion der vektoriellen und axialvektoriellen Matrizen V_μ und A_μ ausgedrückt werden

$$R^\mu = V^\mu - A^\mu , \quad (99)$$

$$L^\mu = V^\mu + A^\mu , \quad (100)$$

wobei:

$$V_{ij}^\mu \equiv \frac{1}{\sqrt{2}} \bar{q}_j \gamma^\mu \bar{q}_i = V^{\mu,a} t^a ; V^{\mu,a} \equiv \sqrt{2} \bar{q} \gamma^\mu t^a q ; \quad (101)$$

$$A_{ij}^\mu \equiv \frac{1}{\sqrt{2}} \bar{q}_j \gamma^5 \gamma^\mu \bar{q}_i = A^{\mu,a} t^a ; A^{\mu,a} \equiv \sqrt{2} \bar{q} \gamma^5 \gamma^\mu t^a q . \quad (102)$$

Die Tabellen 3 und 4 zeigen die Transformationen der Felder R^μ , L^μ und V_μ und A_μ .

Tabelle 3: Transformationen von R_μ und L_μ

	$R_\mu = \frac{1}{\sqrt{2}} \sum_{i=0}^8 R_\mu^i \lambda_i$	$L_\mu = \frac{1}{\sqrt{2}} \sum_{i=0}^8 L_\mu^i \lambda_i$
Elemente	$R_{ij}^\mu \equiv \sqrt{2} \bar{q}_{j,R} \gamma^\mu q_{i,R}$	$L_{ij}^\mu \equiv \sqrt{2} \bar{q}_{j,R} \gamma^\mu q_{i,R}$
Ströme	$R_\mu^i \equiv \bar{q}_R \gamma^\mu \frac{\lambda_i}{\sqrt{2}} q_R$	$L_\mu^i \equiv \bar{q}_L \gamma^\mu \frac{\lambda_i}{\sqrt{2}} q_L$
P	$g^{\mu\nu} L_\mu(x^0, -\mathbf{x})$	$g^{\mu\nu} R_\mu(x^0, -\mathbf{x})$
C	$-L_\mu^t$	R_μ^t
$U(N_f)_V$	$U_V R_\mu U_V^\dagger$	$U_V L_\mu U_V^\dagger$
$U(N_f)_A$	$U_A R_\mu U_A^\dagger$	$U_A^\dagger L_\mu U_A$
$U(N_f)_R \times U(N_f)_L$	$U_R R_\mu U_R^\dagger$	$U_L R_\mu U_L^\dagger$

Tabelle 4: Transformationseigenschaften von V_μ und A_μ

	$V_\mu = \frac{1}{\sqrt{2}} \sum_{i=0}^8 V_\mu^i \lambda_i$	$A_\mu = \frac{1}{\sqrt{2}} \sum_{i=0}^8 A_\mu^i \lambda_i$
Elemente	$V_{ij}^\mu \equiv \sqrt{2} \bar{q}_j \gamma^\mu q_i$	$A_{ij}^\mu \equiv \sqrt{2} \bar{q}_j \gamma^5 \gamma^\mu q_i$
Ströme	$V^i \equiv \bar{q} \gamma^\mu \frac{\lambda_i}{\sqrt{2}} q$	$A^i \equiv \bar{q} \gamma^5 \gamma^\mu \frac{\lambda_i}{\sqrt{2}} q$
P	$g^{\mu\nu} V_\mu(x^0, -\mathbf{x})$	$-g^{\mu\nu} A_\mu(x^0, -\mathbf{x})$
C	$-V_\mu^t$	A_μ^t

Es wird oft behauptet, dass die Parität eines Vektormesons negativ ist. Das stimmt, wenn man sich auf die räumlichen Komponenten bezieht: $\rho^i(t, \mathbf{x}) \rightarrow -\rho^i(t, -\mathbf{x})$. Die nullte Komponente transformiert hingegen mit positiver Parität: $\rho^0(t, \mathbf{x}) \rightarrow -\rho^0(t, -\mathbf{x})$. In einem dichten Medium kondensieren eben die Komponenten ω^0 und ρ^0 .

Die Lagrange-Dichte $\mathcal{L}_{AV}$, die die (Axial)Vektormesonen einschließt, folgt aus denselben Prinzipien wie der (pseudo)skalaren Sektor: nur Terme mit Dimension (exakt) 4 werden berücksichtigt. Sie kann als Summe von vier Termen aufgestellt werden:

$$\mathcal{L}_{AV} = \mathcal{L}_{2,AV} + \mathcal{L}_{3,AV} + \mathcal{L}_{4,AV} + \mathcal{L}_{PS,AV} , \quad (103)$$

wobei 2,3,4 die Anzahl der Felder signalisiert und $\mathcal{L}_{PS,AV}$ die Wechselwirkung mit den (pseudo)skalaren Mesonen darstellt.

Der Term $\mathcal{L}_{2,AV}$ lautet

$$\mathcal{L}_{2,AV} = -\frac{1}{4} \text{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{b}{2} G^2 \text{Tr}[(L^\mu)^2 + (R^\mu)^2] , \quad (104)$$

wobei die Feldtensoren $L^{\mu\nu}$ und $R^{\mu\nu}$ durch

$$L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu, \quad R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu \quad (105)$$

gegeben sind. Dieser Term beschreibt die Propagation der (Axial)Vektormesonen. Man erkennt aus $\mathcal{L}_{2,AV}$ einen Massenterm für diese Felder: wenn das Gluonfeld kondensiert, gilt

$$m_1^2 = bG_0^2, \quad (106)$$

wobei m_1 der Beitrag zur (axial)vektoriellen Masse ist, der aus dem Gluon-Kondensat G_0 stammt. Weitere Beiträge zur Masse der (Axial)Vektormesonen stammen vom Quark-Antiquark-Kondensat ϕ , siehe Ref. [4] für Details.

Die Lagrange-Dichten $\mathcal{L}_{3,AV}$ und $\mathcal{L}_{4,AV}$ enthalten 3- und 4-Punkt-Vertices von (Axial)Vektormesonen:

$$\mathcal{L}_{3,AV} = -2ig_2 (\text{Tr}[L_{\mu\nu}[L^\mu, L^\nu]] + \text{Tr}[R_{\mu\nu}[R^\mu, R^\nu]]) , \quad (107)$$

$$\begin{aligned} \mathcal{L}_{4,AV} = & g_3 \{ \text{Tr}[L^\mu L^\nu L_\mu L_\nu] + \text{Tr}[R^\mu R^\nu R_\mu R_\nu] \} + \\ & g_4 \{ \text{Tr}[L^\mu L_\mu L^\nu L_\nu] + \text{Tr}[R^\mu R_\mu R^\nu R_\nu] \} \\ & g_5 \text{Tr}[R^\mu R_\mu] \text{Tr}[L^\mu L_\mu] + \\ & g_6 \{ \text{Tr}[L_\mu L^\mu] \text{Tr}[L_\mu L^\mu] + \text{Tr}[R_\mu R^\mu] \text{Tr}[R_\mu R^\mu] \} . \end{aligned} \quad (108)$$

Es ist wichtig zu betonen, dass $\mathcal{L}_{4,AV}$ keinen Einfluss auf die Zerfälle hat, die wir später untersuchen werden.

Der letzte Term von Gl. (103) beschreibt die Wechselwirkung der (Axial)Vektormesonen mit den (Pseudo)Skalarmesonen:

$$\mathcal{L}_{PS,AV} = \text{Tr} [(ig_1(\Phi R^\mu - L^\mu \Phi))^\dagger (\partial^\mu \Phi)] + \text{Tr} [(\partial^\mu \Phi)^\dagger (ig_1(\Phi R^\mu - L^\mu \Phi))] \quad (109)$$

$$+ \text{Tr} [(ig_1(\Phi R^\mu - L^\mu \Phi))^\dagger (ig_1(\Phi R^\mu - L^\mu \Phi))] + \frac{h_1}{2} \text{Tr} [\Phi \Phi^\dagger] \text{Tr} [L_\mu L^\mu + R_\mu R^\mu] \quad (110)$$

$$+ h_2 \text{Tr} [\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi] + 2h_3 \text{Tr} [\Phi R_\mu \Phi^\dagger L^\mu] . \quad (111)$$

(Der g_1^2 -Term ist nicht unabhängig und könnte in die Konstanten h_2 und h_3 absorbiert werden, aber diese Form wird bevorzugt, weil der (chiral-)lokale Limes, der oft in der Literatur studiert wurde, einfacher ist, siehe später).

Die Large- N_c Abhängigkeit der Parameter lautet:

$$\begin{aligned} g_1, g_2, g & \propto N_c^{-1/2}, \\ h_2, h_3, g_3, g_4 & \propto N_c^{-1}, \\ h_1, g_5, g_6 & \propto N_c^{-2}, \\ b & \propto N_c^{-2} \rightarrow m_1^2 \propto N_c^0. \end{aligned} \quad (112)$$

3.3.6 $U(1)_A$ -Anomalie

Die $U(1)_A$ -Anomalie wird in Betracht gezogen, indem man den folgenden Term zur Lagrange-Dichte addiert:

$$\mathcal{L}_{U(1)_A} = c(\det \Phi^+ + \det \Phi) . \quad (113)$$

Dieser Term ist invariant unter $SU(N_f)_R \times SU(N_f)_L$, das die Transformation $\Phi \rightarrow U_L \Phi U_R^\dagger$ generiert. Das ist einfach zu beweisen, indem man die Eigenschaft $\det[ABC] = \det[A] \det[B] \det[C]$ zusammen mit der Einheit der Determinante einer $SU(N)$ -Matrix ausnutzt.

Der Term in Gl. (113) ist aber nicht invariant unter $U(1)_A$. Unter einer $U(1)_A$ -Transformation $\Phi \rightarrow e^{2i\nu t^0} \Phi$ bekommt man nämlich:

$$c(\det \Phi^+ + \det \Phi) \rightarrow c(e^{-i\nu\sqrt{2N_f}} \det \Phi^+ + e^{i\nu\sqrt{2N_f}} \det \Phi) . \quad (114)$$

Dieser Term trägt zur Masse der isoskalaren-pseudoskalaren Bosonen bei: die Masse verschwindet auch im chiralen Limes nicht. Diese Felder sind also keine Goldstone-Bosonen. Das ist verständlich, da die Symmetrie ohnehin auf dem hadronischen Niveau gebrochen ist. Im pseudoskalaren Spektrum ist tatsächlich die Resonanz η' (~ 1 GeV) wesentlich schwerer als die anderen pseudoskalaren Resonanzen: der Grund dafür ist die axiale Anomalie.

Der Parameter c skaliert im Large- N_c -Limes auf eine N_f -abhängige Weise: $c \propto N_c^{-N_f/2}$. Für $N_f \geq 2$ verschwindet er schneller als die übliche Skalierung der anderen Parameter. Das bedeutet, dass für Large- N_c die Anomalie vernachlässigbar ist. Das entsprechende Meson ist dann ein Goldstone-Boson für $N_c \gg 1$.

Es ist wichtig anzumerken, dass für $N_f \neq 4$ der Parameter c dimensionsbehaftet ist. Das ist eine Ausnahme der diskutierten Regel. Das ist aber möglich, da die Anomalie auch von dem Eichsektor stammt. Es ist in dieser Hinsicht nicht notwendig (obwohl nicht verboten), diesen Term mit einer passenden Potenz vom Dilatonfeld G zu multiplizieren. Der hier präsentierte Term in Gl. (114) ist nicht die einzige Möglichkeit, die axiale Anomalie zu beschreiben. Eine andere Möglichkeit erfolgt durch einen logarithmischen Term [32]. Weitere Studien dieses Sektors im Rahmen eines chiralen Modells sind möglich und interessant.

3.3.7 Spontane Symmetriebrechung

Um die spontane Symmetriebrechung zu diskutieren, reicht es für eine qualitative Studie, das Potential der Lagrange-Dichte $\mathcal{L}_{G\Phi}$ von Gl. (91) entlang der Wahl $\Phi = \sigma t^0$ zu studieren:

$$V(G, \sigma) = V_{dil}(G) + aG^2\sigma^2 + (\lambda_2 + \lambda_1)\sigma^4. \quad (115)$$

(In diesem Unterabschnitt wird die $U_A(1)$ -Anomalie vernachlässigt). Eine vollständige Studie erfordert die Suche der Minima. Für unsere Zwecke muss Folgendes klargestellt werden:

- $a > 0 \rightarrow$ Minimum für $G_0 \neq 0, \sigma_0 = 0$.
- $a < 0 \rightarrow$ Minimum für $G_0 \neq 0, \sigma_0 \neq 0$.

Wenn σ_0 nicht verschwindet, ist die spontane Symmetriebrechung der chiralen Symmetrie realisiert. Die Transformation $SU(N_f)_A$ lässt das Vakuum nicht invariant. In diesem Fall gibt es eigentlich unendlich viele Minima, wenn man die anderen Isospinrichtungen betrachtet. Die Erhaltung der Parität und der $SU(N_f)_V$ -Symmetrie erfordert aber, dass nur der Isospin-Singulett-Zustand σ kondensieren kann.

Man kann heuristisch wie folgt argumentieren: $G_0 \sim \Lambda_G$. (Das ist exakt, wenn $a = 0$, d.h. wenn der Glueball von den Quark-Antiquark-Zuständen entkoppelt). Dann gilt $\sigma_0 \sim \sqrt{\frac{-a}{\lambda_2 + \lambda_1}}$. Es wird deutlich, dass die spontane Symmetriebrechung durch die Brechung der Dilatationssymmetrie erzeugt wird. Die Dilatationssymmetrie ist in dieser Hinsicht die ‘treibende Kraft’ für alle Phänomene der niederenergetischen QCD.

3.3.8 Explizite Symmetriebrechung durch Quarkmassen

Die chirale Symmetriegruppe wird auch explizit durch die Quarkmassen gebrochen. Das Modell berücksichtigt diesen Punkt mit der Aufnahme des folgenden Terms:

$$\mathcal{L}_H = \text{Tr}[H(\Phi^+ + \Phi)] , \quad (116)$$

wobei

$$H = \text{diag}\{h_0^1, h_0^2, \dots, h_0^{N_f}\} \quad (117)$$

eine diagonale Matrix ist. Die Konstante h_0^k ist proportional zum Quadrat der k -ten Quarkmasse (z.B., $h_0^1 \propto m_u^2$). Die Large- N_c Abhängigkeit der Parameter lautet $h_0^i \propto N_c^{1/2}$.

Es ist leicht zu prüfen, dass im allgemeinsten Fall alle Symmetrien verletzt werden, bis auf die triviale $U(1)_V$ -Symmetrie. Ein wichtiger Limes ist $H = h_0 1_{N_f}$: die $SU(N_f)_V$ bleibt in diesem Fall erhalten. Im Fall $N_f = 2$ wird tatsächlich die Wahl $H = h_0 1_2$ getroffen.

Die Existenz einer (auch infinitesimalen) Quarkmasse impliziert, dass die Minima nicht mehr entartet sind. In dem vereinfachten Fall $H = h_0 1_2$ gibt es tatsächlich nur ein absolutes Minimum des Potentials: für $a < 0$ entspricht dieses Minimum $\sigma_0 > 0$.

Es ist auch notwendig, die explizite Symmetriebrechung im (axial)vektoriellen Kanal in Betracht zu ziehen. Das erfolgt durch einen ähnlichen Term

$$\frac{\alpha}{2} \text{Tr}[H(L^\mu)^2 + (R^\mu)^2], \quad (118)$$

wobei die Matrix H in Gl. (117) definiert wurde und α ein neuer Parameter ist. Wenn $H \propto 1_{N_f}$ hat dieser zusätzlicher Term keinen Einfluss, weil er mit dem zweiten Term von Gl. (104) übereinstimmt, wenn $G = G_0$ gesetzt wird. Wenn aber die explizite Flavor-Symmetriebrechung berücksichtigt wird, kann man durch Gl. (118) in Betracht ziehen, dass (axial)vektorielle Zustände, die aus schwere Quarks bestehen, auch eine größere Masse bekommen.

3.3.9 Die gesamte Lagrange-Dichte

Die bis zu diesem Punkt durchgeführte Analyse bringt uns zum Hauptresultat dieses Abschnittes. Man kann nun all die Teile zusammenfügen und die mesonische Lagrange-Dichte für eine beliebige Flavoranzahl N_f erstellen:

$$\mathcal{L}_{mes} = \mathcal{L}_{G\Phi} + \mathcal{L}_{AV} + \mathcal{L}_{U_A(1)} + \mathcal{L}_H. \quad (119)$$

Der Teil der Lagrange-Dichte, der für die Vakuumphysik relevant ist, wird explizit hingeschrieben:

$$\begin{aligned} \mathcal{L}_{mes} = & \frac{1}{2}(\partial_\mu G)^2 - V_{dil}(G) \\ & + \text{Tr} \left[(D^\mu \Phi)^\dagger (D_\mu \Phi) - a G^2 \Phi^\dagger \Phi - \lambda_2 (\Phi^\dagger \Phi)^2 \right] - \lambda_1 (\text{Tr}[\Phi^\dagger \Phi])^2 \\ & + c(\det \Phi^\dagger + \det \Phi) + \text{Tr}[H(\Phi^\dagger + \Phi)] \\ & - \frac{1}{4} \text{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{b}{2} G^2 \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + \frac{\alpha}{2} \text{Tr}[H(L^\mu)^2 + (R^\mu)^2] \\ & - 2ig_2 (\text{Tr}[L_{\mu\nu}[L^\mu, L^\nu]] + \text{Tr}[R_{\mu\nu}[R^\mu, R^\nu]]) \\ & + \frac{h_1}{2} \text{Tr}[\Phi \Phi^\dagger] \text{Tr}[L_\mu L^\mu + R_\mu R^\mu] + \\ & + h_2 \text{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi] + 2h_3 \text{Tr}[\Phi R_\mu \Phi^\dagger L^\mu] + \dots, \end{aligned} \quad (120)$$

wobei

$$D^\mu \Phi = \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu). \quad (121)$$

Der g_1 -Term wurde in einer kovarianten Ableitung absorbiert. Die 4-Felder-Wechselwirkungsterme zwischen Vektorfeldern haben keinen Einfluss auf die Vakuumphysik und werden in Gl. (120) nicht explizit hingeschrieben.

Im chiralen Limes gibt es in Gl. (120) folgende 12 für die Vakuumphysik relevante freie Parameter:

$$\Lambda_G, \xi = \frac{m_G}{\Lambda_G}, a, \lambda_1, \lambda_2, b, g_1, g_2, c, h_1, h_2, h_3, \quad (122)$$

wobei die einzigen dimensionsbehafteten Parameter Λ_G und c sind. Außerdem gibt es die diagonale Matrix $H = \text{diag}\{h_0^1, h_0^2, \dots, h_0^{N_f}\}$, die N_f dimensionsbehaftete Parameter enthält. Wenn man z.B. vom Fall $N_f = 2$ zum Fall $N_f = 3$ übergeht, gibt es nur einen zusätzlichen Parameter, h_0^3 . Der chirale-lokale Limes entspricht der folgenden Wahl der Parameter:

$$g_1 = g_2, \quad g_3 = \frac{g_1^2}{2}; \quad g_4 = -\frac{g_1^2}{2}; \quad g_5 = g_6 = 0. \quad (123)$$

Es ist aber bekannt, dass dieses chirale lokale Modell die physikalischen Werte von wichtigen Größen (wie z.B. die $\rho \rightarrow \pi\pi$ Zerfallsbreite) *nicht* korrekt beschreiben kann. Es ist ein üblicher Ausweg, Terme von Dimension > 4 mitzunehmen, um dieses Problem zu lösen. Dieser Weg verstößt aber gegen die vorher erwähnten Bedingungen aus Abschnitt 3.3.4, die aus der Dilatationssymmetrie stammen. Aus diesem Grund wird in dieser Arbeit die Hypothese der chiralen Lokalität nicht verwendet, weil sie zu restriktiv ist. Das ist ohnehin im Einklang mit der QCD-Lagrange-Dichte, in der die chirale Symmetrie global ist.

3.4 $N_f = 2$: Resultate und Diskussion

3.4.1 Resultate ohne Glueball

In Ref. [4] wurde der Fall $N_f = 2$ im Limes $m_G \rightarrow \infty$ (eingefrorener Glueball) ausführlich untersucht. Es gibt 16 mesonische Felder (4 skalare, 4 pseudoskalare, 4 vektorielle, 4 axialvektorielle Mesonen):

$$\begin{aligned} \Phi &= (\sigma + i\eta_N) t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}, \\ L^\mu &= (\omega^\mu + f_1^\mu) t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu) \cdot \vec{t}, \\ R^\mu &= (\omega^\mu - f_1^\mu) t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu) \cdot \vec{t}, \end{aligned}$$

wobei $t^0 = \frac{1}{2}1_2$ und $\vec{t} = \frac{1}{2}\vec{\tau}$ (Pauli-Matrizen). Die Identifikation der Felder ist unproblematisch in den 0^{++} , 1^{--} und 1^{++} Sektoren: die Felder $\vec{\pi}$ und η_N entsprechen dem Pion und dem $SU(2)$ -Gegenstück des η -mesons, $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$. Die Masse des Feldes η_N wird berechnet, indem man den physikalischen Fall $N_f = 3$ entmischt, siehe z.B. Ref. [33]. Man bekommt den Wert $m_{\eta_N} \simeq 700$ MeV. Die Felder ω^μ und $\vec{\rho}^\mu$ entsprechen den Resonanzen $\omega(782)$ und $\rho(770)$, und die Felder f_1^μ und $\vec{a}_1^\mu$ den Resonanzen $f_1(1285)$ und $a_1(1260)$.

Unglücklicherweise ist die Identifikation der skalaren Resonanzen $\{\sigma, \vec{a}_0\}$ nicht trivial und umstritten: dieses Thema hat sich zum eigenen Untergebiet der leichten hadronischen Physik entwickelt. Die zwei (meist diskutierten, aber nicht die einzigen) Möglichkeiten sind die Paare $\{f_0(600), a_0(980)\}$ und $\{f_0(1370), a_0(1450)\}$. Wir nennen diese zwei Zuordnungen jeweils Szenario I und Szenario II. Das richtige Szenario zu ermitteln ist von primärer Bedeutung nicht nur für die Vakuumphysik; auch die Phänomenologie bei nicht verschwindender Temperatur und Dichte hängt auf eine entscheidende Art und Weise davon ab.

Die Studie in Ref. [4] beruht auf bekannten experimentellen Größen, wie den Zerfallsbreiten $\rho \rightarrow \pi\pi$, $a_0 \rightarrow \pi\eta$ und den $\pi\pi$ -Streulängen, usw. Außerdem wurde auch das Photon in das Modell (durch minimale Kopplung) eingebaut: das ist wichtig, um die Berechnung des Zerfalls $a_1(1260) \rightarrow \pi\gamma$ durchführen zu können.

Folgende Erkenntnisse wurden gewonnen:

- Im Szenario I (leichtes σ , $M_\sigma \lesssim 500$ MeV) ist die Zerfallsbreite $\Gamma_{\sigma \rightarrow \pi\pi}$ bei Weitem zu klein: $\Gamma_{\sigma \rightarrow \pi\pi} \lesssim 200$ MeV. Das Experiment zeigt hingegen eine sehr breite Resonanz: $f_0(600)$, mit $\Gamma_{f_0(600) \rightarrow \pi\pi} \gtrsim 500$ MeV. (Aus diesem Grund wurde sogar lange von den Experimentalphysikern bezweifelt, ob eine solche Resonanz überhaupt existiert. Mittlerweile gilt die Existenz dieser leichten skalaren Resonanz als

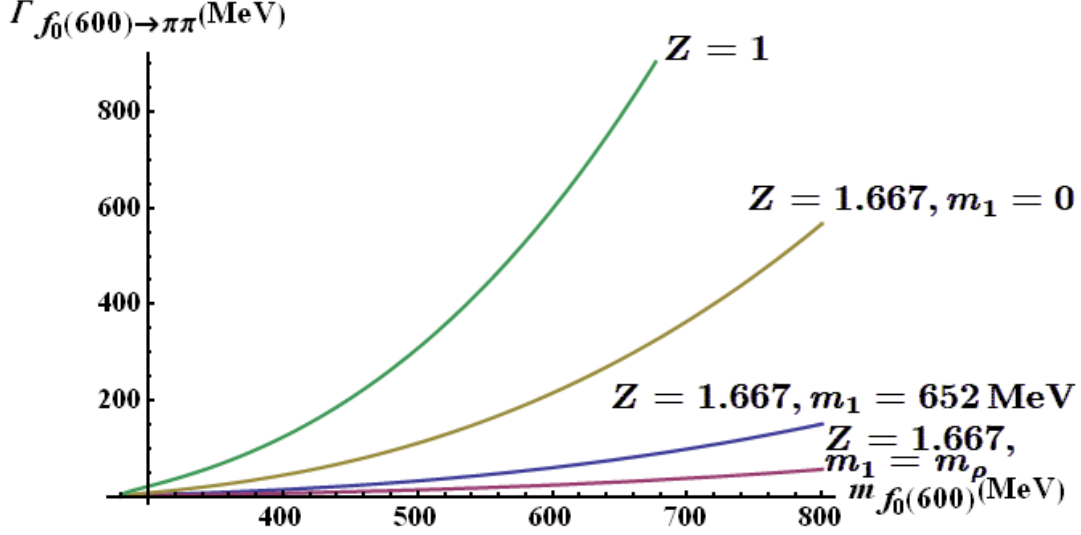


Abbildung 3: Zerfall $\Gamma_{\sigma \rightarrow \pi\pi}$ als Funktion der σ -Masse im Szenario I mit ($Z = 1.667$) und ohne Vektormesonen ($Z = 1$). Es ist deutlich, dass der realistische Fall $Z = 1.667$ eine wesentlich kleinere Breite als der künstliche Fall $Z = 1$ liefert. Die Abhängigkeit von dem Parameter m_1 (Beitrag zur ρ -Masse, der nicht vom Quark-Kondensat stammt, siehe Gl. (106)) wird auch geplottet.

erwiesen.) Im Rahmen unseres Modells ist es nicht möglich, die Streulängen und den Zerfall $\sigma \rightarrow \pi\pi$ gleichzeitig zu beschreiben. Dieses Resultat zeigt deutlich, dass Szenario I nicht bevorzugt ist.

- Der Grund für eine solche schmale (und unphysikalische) leichte σ -Resonanz in unserem Modell ist die besondere Rolle der (axial)vektoriellen Freiheitsgrade. Ohne Vektormesonen wäre das leichte σ -Meson hingegen sehr breit, $\Gamma_{\sigma \rightarrow \pi\pi} \simeq 500$ MeV, und wäre sogar im Einklang mit der $\pi\pi$ -Streuung.

- Die vorherige Behauptung mag auf den ersten Blick unverständlich erscheinen: warum braucht man überhaupt (Axial)Vektormesonen, wenn ohne sie die Identifikation $\sigma = f_0(600)$ so gut funktioniert? Die Antwort ist aber deutlich: die (Axial)Vektormesonen müssen wegen der Vollständigkeit des Modells berücksichtigt werden. Ohne sie würde ein wichtiger Bestandteil der Hadronenphysik fehlen, wie sie experimentell bestätigt ist. Man hätte die absurde Situation, dass $\Gamma_\rho = 0$ und $\Gamma_{a_1} = 0$.

- Eine zweite Frage tritt in diesem Zusammenhang natürlich auf: wie kann es sein, dass die Einführung von Resonanzen mit verschiedenen Quantenzahlen einen solchen riesigen Einfluss auf die (pseudo)skalaren Resonanzen hat? Die Antwort ist hier eher technischer Natur: durch spontane Brechung der chiralen Symmetrie entsteht ein Mischungsterm zwischen a_1 und π , der die Form $\sim g_1 \phi a_{1,\mu} \partial^\mu \pi$ hat. Diese Mischung erzeugt weitere Terme im (pseudo)skalaren Sektor und vor allem den $\sigma \rightarrow \pi\pi$ Zerfall. Eine destruktive Interferenz wird dadurch generiert, die die σ Zerfallsbreite wesentlich verkleinert. Das wiederum führt dazu, dass Szenario I keine gute Übereinstimmung mit dem Experiment liefert. Die Tatsache, dass in vielen Studien die (Axial)Vektormesonen nicht einbezogen wurden, ist der Grund dafür, dass die Identifikation $\sigma = f_0(600)$ oft vertreten wurde.

Diese Diskussion wird in den Abbildungen (3) und (4) zusammengefasst. Der Parameter Z tritt wegen der oben erwähnten a_1 - π Mischung auf. Z kann aus der Zerfallsbreite $a_1(1260) \rightarrow \pi\gamma$ bestimmt werden: $Z = 1.67 \pm 0.2$. Der Limes $Z \rightarrow 1$ ist theoretisch interessant, weil er die komplette Entkopplung von (pseudo)skalaren und (axial)vektoriellen Mesonen bedeutet.

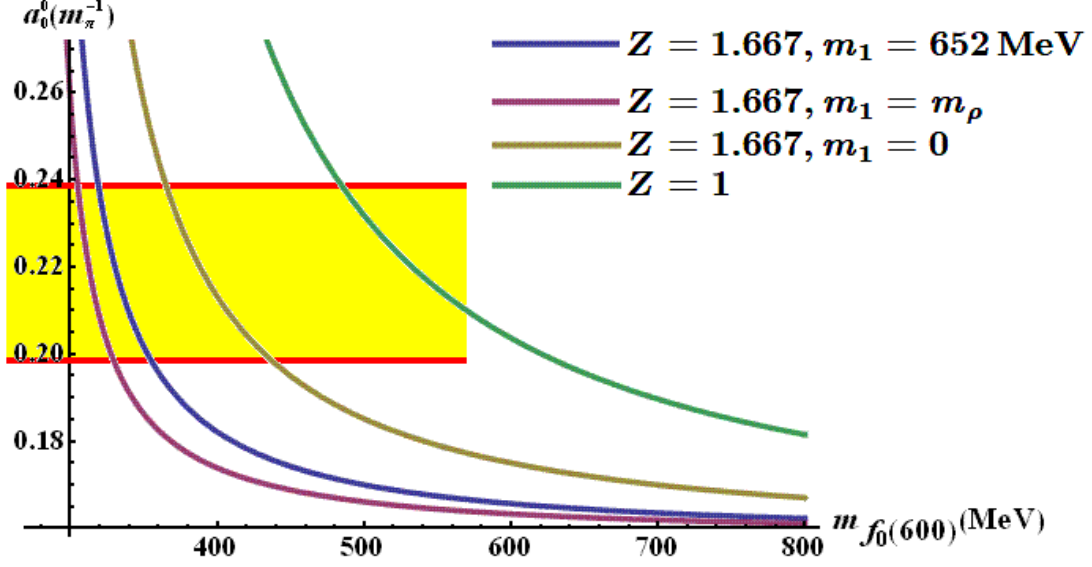


Abbildung 4: Streulänge a_0^0 als Funktion der σ -Masse im Szenario I mit ($Z = 1.667$) und ohne Vektormesonen ($Z = 1$). Der gelbe Balken ist das experimentelle Intervall. Es ist deutlich, dass der realistische Fall $Z = 1.667$ eine kleine σ -Masse bevorzugt. Das bedeutet aber eine (zu) kleine Zerfallsbreite für das σ -Teilchen. Die Abhängigkeit von dem Parameter m_1 (Beitrag zur ρ -Masse, der nicht vom Quark-Kondensat stammt) wird auch geplottet.

- Im Szenario II gilt die Zuordnung $\sigma = f_0(1370)$. In diesem Fall ist eine konsistente Beschreibung der Daten realisiert. Die Zerfallsbreite $\Gamma_{\sigma \rightarrow \pi\pi} = 300\text{-}500 \text{ MeV}$ ist im Einklang mit den jetzigen Beobachtungen. Das heißt, dass der chirale Partner des Pions mit $\sigma = f_0(1370)$ zu identifizieren ist. Auch dieses Resultat folgt aus der Einführung der Vektormesonen: ohne sie wäre ein solcher schwerer skalarer Zustand extrem breit ($\Gamma_{\sigma \rightarrow \pi\pi} > 1 \text{ GeV}$) und daher nicht im Einklang mit dem Experiment. Dieses Resultat des unphysikalischen Modells ohne (Axial)Vektormesonen hat auch dazu beigetragen, dass diese Zuordnung nur selten im Rahmen von chiralen Modellen untersucht wurde. Wie nun von unserem Standpunkt klar erscheint, hatten wichtige Teile des Puzzles gefehlt.

3.4.2 Resultate mit Glueball

Der erste Schritt, um das Modell zu verbessern, besteht darin, die Glueballmasse nicht als unendlich zu betrachten, sondern den Gitter-Wert von 1.5 GeV zu verwenden. In diesem Fall ist der Glueball nicht mehr ‘eingefroren’, sondern als zusätzlicher Freiheitsgrad des Modells zu betrachten. Oft wird der Glueball mit der Resonanz $f_0(1500)$ in Verbindung gesetzt.

Dieses Projekt wurde unternommen und die Resultate sind vorhanden [5]: das reine Quarkonium $\bar{n}n$ und der reine Glueball gg mischen. Szenario I funktioniert auch hier nicht, und man ist wie vorher an den skalaren Resonanzen oberhalb 1 GeV interessiert. Die Mischung zwischen dem skalaren Quarkonium und dem skalaren Glueball erzeugt die Resonanzen $f_0(1370)$ und $f_0(1500)$. Der Mischungswinkel kann durch einen Fit an den bekannten Zerfällen des Mesons $f_0(1500)$ bestimmt werden (siehe Details in Ref. [5]): der Zustand $f_0(1370)$ besteht überwiegend aus $\bar{n}n$ (etwa $\sim 75\%$) und aus gg (restliche 25%), und $f_0(1500)$ besteht überwiegend ($\sim 75\%$) aus gg und aus $\bar{n}n$ (restliche 25%). Das zeigt im Nachhinein, dass die zuvor beschriebene Studie ohne Glueball schon qualitativ korrekt war.

Des Weiteren zeigt die Studie in Ref. [5], dass $m_1^2 = bG_0^2 \sim m_\rho^2$ (siehe Gl. (106)): der Beitrag des Gluon-Kondensats zur Masse des ρ -Mesons ist dominant. Das bedeutet, dass es nicht erwartet wird, dass die ρ -Masse sich bei nicht verschwindender Dichte und Temperatur mit dem chiralen Kondensat skaliert. Ein weiteres interessantes Resultat dieser Arbeit ist die Bestimmung des Gluon-Kondensats: $\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a,\mu\nu} \rangle \simeq (600 \text{ MeV})^4$. Dieser Wert ist im Einklang mit der oberen Grenze von Gitter-Simulationen.

3.4.3 Ausblicke im mesonischen Sektor

Die hier präsentierte Studie über den Fall $N_f = 2$ ist nur der Anfang von weiteren Studien. Wir fassen diese Ausblicke kurz zusammen.

Schwache Eichbosonen

Die schwachen Eichbosonen können ins Modell eingebaut werden. Das erlaubt, wichtige Prozesse zu beschreiben: die hadronischen τ -Zerfälle ($\tau \rightarrow \nu\rho \rightarrow \nu\pi\pi$ und $\tau \rightarrow \nu a_1 \rightarrow \nu\rho\pi \rightarrow \nu\pi\pi\pi$) können somit berechnet werden. Dadurch können die Spektralfunktionen von ρ und a_1 theoretisch beschrieben werden, was weitere und genauere Tests des Modells erlauben wird [34].

Der Fall $N_f = 3$

Eine natürliche Verallgemeinerung ist die Studie des Falles $N_f = 3$ [35]. Die niederenergetische Hadronenphysik beruht eigentlich auf diesem Szenario: die Quark-Antiquark-Multipletts der PDG sind Nonetts. Es werden nur zwei zusätzliche Parameter im Vergleich zum Fall $N_f = 2$ benötigt, die den Einfluss der nackten s -Quarkmasse im (pseudo)skalaren und (axial)vektoriellen Sektoren beschreiben. Diese zwei Parameter sind h_0^3 von Gl. (117) und α von Gl. (118).

Dafür gibt es aber viel mehr experimentelle Werte, die zur Anpassung und Falsifizierung des Modells dienen können: $\phi \rightarrow \bar{K}K$, $K^* \rightarrow K\pi$, Die Mischung im skalaren-isoskalaren Sektor betrifft nun drei nackte Konfigurationen: $\bar{n}n$, gg und $\bar{s}s$, aus deren Mischung die Resonanzen $f_0(1370)$, $f_0(1510)$ und $f_0(1710)$ stammen.

Die Fälle $N_f \geq 4$

Es ist auch möglich, die schweren Quarks ins Modell einzubauen. Die Massenterme der schweren Quarks verursachen zwar eine starke explizite Brechung der chiralen Symmetrie, die Wechselwirkungsterme bleiben jedoch unberührt. Das ermöglicht, die Physik der schweren Quarks mit den leichten Quarks in einem chiralen Modell mit Vektorfeldern zu vereinen.

Der pseudoskalare Glueball

Ein weiteres Feld, das von theoretischen und experimentellen Interesse ist, ist der pseudoskalare Glueball. Laut Gitter-Simulationen besitzt dieser Zustand eine Masse von ungefähr 2.6 GeV [14]. Dieser Zustand kann durch die $U(1)_A$ -Anomalie an die (pseudo)skalare Felder gekoppelt werden. Die entsprechende Lagrange-Dichte lautet

$$\mathcal{L}_{\tilde{G}} = ic_{\tilde{G}\Phi} \tilde{G}(\det \Phi^+ - \det \Phi) , \quad (124)$$

wobei $\tilde{G}$ der pseudoskalare Glueball darstellt. Dadurch können Zerfälle und Mischungen berechnet werden. Wegen des geplanten PANDA Experiments an der GSI/Darmstadt [36], ist eine detaillierte Studie der Zerfälle des pseudoskalaren Glueballs relevant.

Nicht nur weitere Quarkonia werden gebraucht. Um die leichten skalaren Resonanzen zu beschreiben, müssen neue Freiheitsgrade eingeführt werden. Wie wir im nächsten Abschnitt diskutieren werden, erfüllen die Tetraquark-Zustände die nötigen Eigenschaften.

4 Tetraquark Felder

4.1 Werden Tetraquarks benötigt?

Die Resultate des vorherigen Abschnittes haben gezeigt, dass die Interpretation der leichten skalaren Resonanzen (Masse < 1 GeV) als Quark-Antiquark-Mesonen problematisch ist. Die jetzige experimentelle Lage zeigt in diesem Energiebereich ein gesamtes Nonett von Skalarzuständen: $f_0(600)$, $f_0(980)$, $a_0(980)$, und $k = K_0^*(800)$. Die natürliche Frage lautet, wie man solche Zustände interpretiert, wenn die Quarkonium-Option ausfällt.

Eine elegante und natürliche Erklärung dieser Resonanzen wurde in Ref. [37] präsentiert: sie werden als Tetraquark-Zustände interpretiert: es handelt sich um gebundene Zustände aus einem ‘guten’ Diquark und einem ‘guten’ Antidiquark, wobei ein gutes (Anti-)Diquark eine antisymmetrische Konfiguration aus zwei (Anti)Quarks in Isospin-Raum und Farbraum bedeutet. (Einzelheiten sind im nächsten Unterabschnitt zu finden.)

Für $N_f = 3$ gibt es drei gute Diquarks: $[d, s]$, $[u, s]$ und $[u, d]$, wobei der Kommutator die Antisymmetrie im Isospin-Raum darstellt. In diesem Szenario wird die Resonanz $f_0(600)$ (überwiegend) als $[u, d][\bar{u}, \bar{d}]$ und die Resonanzen $f_0(980)$ und $a_0^0(980)$ (überwiegend) als $\frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] + [d, s][\bar{d}, \bar{s}])$ und $\frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] - [d, s][\bar{d}, \bar{s}])$ interpretiert. Zu $a_0(980)$ gehören auch die zwei geladenen Teilchen $[u, s][\bar{d}, \bar{s}]$, $[d, s][\bar{u}, \bar{s}]$. Die Massenentartung von $f_0(980)$ und $a_0(980)$ ist wegen der Flavorstruktur ganz einfach und natürlich erklärt. Die kaonischen Konfigurationen $[u, d][\bar{d}, \bar{s}]$, $[u, d][\bar{u}, \bar{s}]$, entsprechen der Resonanz $k = K_0^*(800)$.

Wegen der (dynamischen) Quarkmassen $m_u^* \simeq m_d^* \simeq 300$ MeV $< m_s^* \simeq 500$ MeV kann man die folgende nicht typische, invertierte Massenordnung

$$M_{f_0(600)} < M_k < M_{f_0(980)} = M_{a_0(980)} \quad (125)$$

problemlos verstehen. Das ist für ein Quark-Antiquark-Nonett nicht möglich.

Auch die Phänomenologie der Zerfälle kann im Rahmen der Tetraquark-Interpretation korrekt beschrieben werden [6]. Das ist wiederum nicht der Fall, wenn die Quark-Antiquark-Interpretation untersucht wird, siehe die comparative Analyse in Ref. [8].

Natürlich können Mischungseffekte -vor allem im isoskalaren Sektor- stattfinden, die das Verständnis der Resonanzen schwerer machen. In dieser Hinsicht ist diese Diskussion nur ansatzweise korrekt und eine genauere Studie muss unternommen werden. Dafür ist es wichtig, die Tetraquark-Felder in einem chiralsymmetrischen Kontext zusammenzubringen. Das erfordert zunächst eine mathematische Studie dieses Problems.

4.2 Mathematische Beschreibung der skalaren Diquarks und Tetraquarks

Um Tetraquarks zu beschreiben, muss man zuerst Diquarks definieren. Wir sind nicht an all den möglichen Diquark-Objekten interessiert, sondern nur an den sogenannten “guten” Diquarks [37, 38]. Das sind skalare Objekte, die antisymmetrisch im Flavorraum und Farbraum sind. Ihre Wellenfunktion lautet schematisch:

$$|qq\rangle_{L=S=0} = |\text{Raum: } L=0\rangle |\text{Spin: } S=0\rangle |\text{Farbe: } \bar{3}_c\rangle |\text{Flavor: } \bar{N}_f\rangle; \quad J^P = 0^+ . \quad (126)$$

Der Ket $|\text{Flavor: } \bar{N}_f\rangle$ bedeutet eine antisymmetrische Wellenfunktion im Flavorraum. Im Fall $N_f = 2$ gibt es nur ein solches (gutes) Diquark: $\sqrt{\frac{1}{2}}[u, d]$. Im Fall $N_f = 3$ gibt es drei gute Diquarks:

$$\sqrt{\frac{1}{2}}[d, s], \quad \sqrt{\frac{1}{2}}[u, s], \quad \sqrt{\frac{1}{2}}[u, d]. \quad (127)$$

Für beliebiges N_f gibt es $N_f(N_f - 1)/2$ antisymmetrische Diquarks.

Im Farbraum ist das ähnlich: die Anzahl von Farben ist aber auf $N_c = 3$ fixiert⁷. Die entsprechenden antisymmetrischen Farbkonfigurationen für $|\text{Farbe: } \bar{3}_c\rangle$ lauten

$$\sqrt{\frac{1}{2}}[rot, blau], -\sqrt{\frac{1}{2}}[blau, grün], \sqrt{\frac{1}{2}}[grün, rot] . \quad (128)$$

Verschiedene Studien (Gitter, 1-Gluon-Austausch-Modelle, Dyson-Schwinger-Rechnungen [40]) haben tatsächlich gezeigt, dass das gute Diquark ein stabiles Objekt bildet, das eine wichtige Rolle in der mesonischen Phänomenologie spielen kann.

Um die Diquarks, und entsprechend die Tetraquarks, in einem chiralen Modell beschreiben zu können, ist es notwendig, eine korrekte mathematische Sprache zu entwickeln. Zu diesem Zweck berücksichtigen wir die folgende Diquark-Matrix D , die die guten Diquarks auf eine kompakte Art darstellt:

$$D_{ij} \equiv \sqrt{\frac{1}{2}} (q_j^t C \gamma^5 q_i - q_i^t C \gamma^5 q_j) = \sum_{i=1}^{N_f(N_f-1)/2} \varphi_i A^i \quad (129)$$

mit

$$(A^i)_{jk} = \varepsilon_{ijk}, \quad \varphi_i \equiv \sqrt{\frac{1}{2}} \varepsilon_{ijk} q_j^t C \gamma^5 q_k . \quad (130)$$

Die Buchstabe t steht für die Transposition im Dirac-Raum und C ist die Ladungskonjugationsmatrix ($C^{-1} \gamma^\mu C = (-\gamma^\mu)^t$, $C = i\gamma^2 \gamma^0$ in der Dirac-Darstellung). Die Matrizen A^i sind die antisymmetrischen $N_f \times N_f$ -Matrizen. Die Farbindizes, die formell mit den Flavorindizes identisch sind, werden hier weggelassen. Die Größen φ_i , die in der Dekomposition von D in der Basis der antisymmetrischen Matrizen A^i auftauchen, stellen die Diquark-Ströme dar. Die hermitesch-konjugierten Größen $\varphi_i^\dagger$ beschreiben die Antidiquark-Ströme.

In der Tabelle 5 werden die Eigenschaften der Diquarks zusammengefasst.

Tabelle 5: Eigenschaften der skalaren Diquarks.

	$D = \sum_{i=1}^{N_f(N_f-1)/2} \varphi_i A^i$	φ_i
Elemente/Ströme	$D_{ij} \equiv \sqrt{\frac{1}{2}} (q_j^t C \gamma^5 q_i - q_i^t C \gamma^5 q_j)$	$\varphi_i \equiv \sqrt{\frac{1}{2}} \varepsilon_{ijk} q_j^t C \gamma^5 q_k$
P	$D(t, -\mathbf{x})$	$\varphi_i(t, -\mathbf{x})$
C	$D^\dagger$	$\varphi_i^\dagger$
$SU(N_f)_V$	$U_V D U_V^t$	$\sqrt{\frac{1}{2}} \varepsilon_{ijk} U_{V,jj'} U_{V,kk'} q_j^t C \gamma^5 q_{k'} .$

Die daraus entstehenden Tetraquarks sind gebundene Zustände aus einem Diquark und einem Antidiquark. Es gibt $N_f^2(N_f - 1)^2/4$ solcher Felder, die mit Hilfe der hermiteschen $N_f \times N_f$ -Matrix

$$\mathcal{S}_{ij}^{[4q]} = \varphi_i^\dagger \varphi_j \quad (131)$$

hingeschrieben werden können. Das Zeichen $[4q]$ in $\mathcal{S}^{[4q]}$ weist auf die ‘Vierquark-Struktur’ hin und vermeidet Konfusion mit der zuvor eingeführten Matrix $\mathcal{S}$ der Quark-Antiquark-Zustände.

Wir beschreiben kurz die Unterschiede der Fälle für verschiedene N_f :

- $N_f = 1$: es existiert kein (gutes) Diquark, und daher kann kein Tetraquark konstruiert werden.
- $N_f = 2$: es gibt nur ein (gutes) Diquark: $\varphi = \sqrt{\frac{1}{2}}[u, d]$. Deswegen gibt es nur ein skalares Tetraquark-Feld:

$$\mathcal{S}^{[4q]} = \chi = \frac{1}{2}[u, d][\bar{u}, \bar{d}] = \varphi^\dagger \varphi .$$

⁷Die Verallgemeinerung zu einer beliebigen Anzahl von Farben N_c findet im Abschnitt 4.7 statt.

Das Feld χ wird (überwiegend) mit der Resonanz $f_0(600)$ identifiziert.

• $N_f = 3$: Es ist interessant zu beobachten, dass der Fall $N_f = 3$ besonders ist: es gibt die gleiche Anzahl von Quarks und von (guten) Diquarks. Die Diquarks φ_i lauten:

$$\varphi_1 = \sqrt{\frac{1}{2}}[d, s] \leftrightarrow \bar{u}, \varphi_2 = -\sqrt{\frac{1}{2}}[u, s] \leftrightarrow \bar{d}, \varphi_3 = \sqrt{\frac{1}{2}}[u, d] \leftrightarrow \bar{s}, \quad (132)$$

wobei die Korrespondenz $\leftrightarrow$ bedeutet, dass ein Diquark sich wie ein Antiquark in Bezug auf Flavortransformationen verhält. (Ganz analog geht es mit den Farbtransformationen.) In diesem besonderen Fall transformieren die Diquarks unter einer $SU_V(3)$ -Transformation wie

$$\varphi_i \rightarrow \varphi_k U_{ki}^\dagger. \quad (133)$$

Die Transformationseigenschaft ist genau die $SU(3)_V$ -Transformation von Antiquarks: $\bar{q}_i \rightarrow \bar{q}_k U_{ki}^\dagger$. Das ist die formell korrekte Weise, die Korrespondenz in Gl. (132) auszudrücken.

Im Flavorraum lautet $\mathcal{S}^{[4q]}$ explizit (siehe Gl. (129) und Gl. (130)):

$$\mathcal{S}^{[4q]} = \frac{1}{2} \begin{pmatrix} [\bar{d}, \bar{s}][d, s] & -[\bar{d}, \bar{s}][u, s] & [\bar{d}, \bar{s}][u, d] \\ -[\bar{u}, \bar{s}][d, s] & [\bar{u}, \bar{s}][u, s] & -[\bar{u}, \bar{s}][u, d] \\ [\bar{u}, \bar{d}][d, s] & -[\bar{u}, \bar{d}][u, s] & [\bar{u}, \bar{d}][u, d] \end{pmatrix} \quad (134)$$

$$= \begin{pmatrix} \sqrt{\frac{1}{2}}(f_B[4q] - a_0^0[4q]) & -a_0^+[4q] & k^+[4q] \\ -a_0^-[4q] & \sqrt{\frac{1}{2}}(f_B[4q] + a_0^0[4q]) & -k^0[4q] \\ k^-[4q] & -\bar{k}^0[4q] & \sigma_B[4q] \end{pmatrix} \quad (135)$$

$$\simeq \begin{pmatrix} \sqrt{\frac{1}{2}}(f_B - a_0^0(980)) & -a_0^+(980) & k^+ \\ -a_0^-(980) & \sqrt{\frac{1}{2}}(f_B + a_0^0(980)) & -k^0 \\ k^- & -\bar{k}^0 & \sigma_B \end{pmatrix}, \quad (136)$$

wobei in Gl. (136) die Identifikation mit den physikalischen Feldern gemacht wurde. Hierbei wurden mögliche Mischungen mit anderen skalaren Feldern, wie z.B. Quark-Antiquark-Felder, vernachlässigt.

Die Zustände

$$\sigma_B[4q] = \frac{1}{2}[u, d][\bar{u}, \bar{d}]$$

und

$$f_B[4q] = \frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] + [d, s][\bar{d}, \bar{s}])$$

stellen die *nackten* (ungemischten) skalar-isoskalaren Tetraquarks dar. Die Felder $\sigma_B[4q]$ und $f_B[4q]$ werden überwiegend mit den Resonanzen $f_0(600)$ und $f_0(980)$ identifiziert. Es soll aber nicht vergessen werden, dass Mischungen zwischen diesen Konfigurationen mit anderen skalaren Feldern wie dem skalaren Glueball und skalaren Quark-Antiquark-Zuständen sicherlich auftreten. Die genaue Studie solcher Mischungen ist ein wichtiges, aber zugleich schwieriges Projekt für die Zukunft, siehe auch die folgende Diskussion für erste Resultate.

Wenn man, wie in Ref. [6], die chirale Symmetrie nicht in Betracht zieht und sich nur auf Flavorsymmetrie $SU_V(N_f = 3)$, sowie C und P Symmetrien beschränkt, kann man die folgende invariante Lagrange-Dichte hinschreiben, die die Zerfälle eines Tetraquarks in zwei pseudoskalare Quarkonium-Mesonen (wie z.B. die Pionen) beschreibt:

$$\begin{aligned} \mathcal{L}_{\mathcal{S}^{[4q]}PP} &= -c_1 \text{Tr} [D\mathcal{P}^t D^\dagger \mathcal{P}] + c_2 \text{Tr} [DD^\dagger \mathcal{P}^2] \\ &= c_1 \mathcal{S}_{ij}^{[4q]} \text{Tr} [A^j \mathcal{P}^t A^i \mathcal{P}] - c_2 \mathcal{S}_{ij}^{[4q]} \text{Tr} [A^j A^i \mathcal{P}^2], \end{aligned} \quad (137)$$

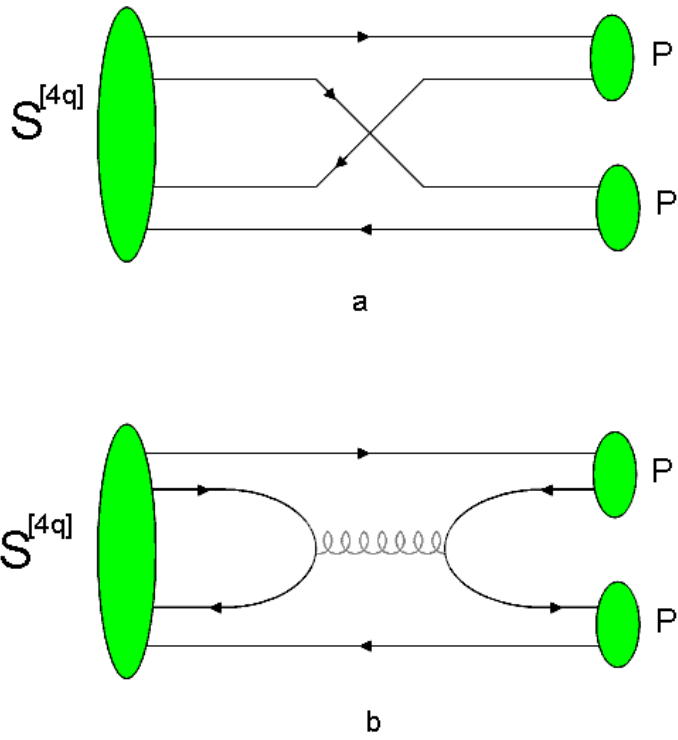


Abbildung 5: Tetraquark-Zerfälle: (a) Durch Quark-Antiquark-Austausch. (b) Durch Quark-Antiquark-Annihilation und einem zusätzlichen Gluon als Zwischenzustand.

wobei der dominante und der subdominante Term in der Large- N_c Entwicklung mitgenommen wurden. Die Terme sind proportional zu c_1 (Large- N_c dominant) und c_2 (Large- N_c unterdrückt). Die entsprechenden Zerfälle werden in den Abbildungen 5.a und 5.b gezeigt.

Die Zuordnung von Gl. (136), d.h. die Interpretation der leichten Skalare als Tetraquarks, kann die besonderen Eigenschaften dieser Resonanzen erklären: die (beinahe exakte) Massen-Entartung der Zustände $a_0(980)$ und $f_0(980)$ und die starke Kopplung von beiden Resonanzen an $\bar{K}K$ sind unmittelbare Konsequenzen. Außerdem ist die besonders starke Kopplung von $f_0(980)$ an $\bar{K}K$ auch eine Folge dieser Zuordnung: der Large- N_c unterdrückte Term proportional zu c_2 ist in diesem Kanal besonders stark. Das passiert wegen der Clebsch-Gordon-Koeffizienten für den Prozess $f_0(980) \rightarrow \bar{K}K$, siehe Details in Ref. [6] und die experimentellen Analysen von Ref. [39].

Die Lagrange-Dichte (137) ist zwar nützlich für eine phänomenologische Studie der Zerfälle, ist aber nicht chiralsymmetrisch. Man muss das Modell erweitern, um die Prinzipien vom Abschnitt 3.3.4 zu erfüllen. Als nächster Schritt muss das pseudoskalare Diquark beschrieben werden.

4.3 Pseudoskalare Diquarks

Das pseudoskalare Diquark hat die gleiche Flavor- und Farbstruktur $(\bar{N}_f, \bar{3}_c)$ wie das skalare Diquark. Es hat aber negative Parität, die aus der folgenden Wellenfunktion stammt:

$$|qq\rangle_{L=S=1} = |\text{space: } L=1\rangle |\text{spin: } S=1\rangle |\text{color: } \bar{3}_c\rangle |\text{flavor: } \bar{N}_f\rangle; \quad J^P = 0^- . \quad (138)$$

Das pseudoskalare Diquark wird durch die Matrix $\tilde{D}$ und durch die Ströme $\tilde{\varphi}_i$ beschrieben:

$$\tilde{D}_{ij} \equiv \sqrt{\frac{1}{2}} (q_j^t C q_i - q_i^t C q_j) = \sum_{i=1}^{N_f(N_f-1)/2} \tilde{\varphi}_i A^i; \quad \tilde{\varphi}_i \equiv \sqrt{\frac{1}{2}} \varepsilon_{ijk} q_j^t C q_k . \quad (139)$$

Die mathematischen Eigenschaften von $\tilde{D}$ und $\tilde{\varphi}_i$ sind, bis auf die Parität, genau wie in der Tabelle 5 angegeben.

4.4 Tetraquarks im chiralen Modell

Aus den Matrizen D und $\tilde{D}$ von Gl. (129) und (139) werden die Matrizen D_R und D_L definiert:

$$D_R = \sqrt{\frac{1}{2}} (\tilde{D} + D) = \sum_{i=1}^{N_f(N_f-1)/2} \varphi_i^R A^i, \quad \varphi_i^R = \sqrt{\frac{1}{2}} (\tilde{\varphi}_i + \varphi_i) , \quad (140)$$

$$D_L = \sqrt{\frac{1}{2}} (\tilde{D} - D) = \sum_{i=1}^{N_f(N_f-1)/2} \varphi_i^L A^i, \quad \varphi_i^L = \sqrt{\frac{1}{2}} (\tilde{\varphi}_i - \varphi_i) . \quad (141)$$

Die Transformationen der Matrizen D_R und D_L werden in der Tabelle 6 zusammengefasst. Die Transformation von D_R und D_L ist den vorher diskutierten Matrizen R_μ und L_μ ähnlich.

Tabelle 6: Eigenschaften der Matrizen D_R und D_L .

	$D_R = \sum_{i=1}^{N_f(N_f-1)/2} \varphi_i^R A^i$	$D_L = \sum_{i=1}^{N_f(N_f-1)/2} \varphi_i^L A^i$
Ströme	$\varphi_i^R \equiv \varepsilon_{ijk} q_j^t C P_R q$	$\varphi_i^L \equiv \varepsilon_{ijk} q_j^t C P_L q_k$
P	$-D_L(t, -\mathbf{x})$	$-D_R(t, -\mathbf{x})$
C	$D_R^\dagger$	$D_L^\dagger$
$SU(3)_V$	$U D_R U^t$	$U D_L U^t$
$U(3)_R \times U(3)_L$	$R D_R R^t$	$L D_L L^t$

In den relevanten Fällen $N_f = 2$ und $N_f = 3$ gibt es folgende Eigenschaften:

- $N_f = 2$. Es gilt: $\varphi^R \rightarrow \varphi^R$, $\varphi^L \rightarrow \varphi^L$. Es handelt sich also um invariante Objekte.
- $N_f = 3$. Unter chiralen $SU(3)_R \times SU(3)_L$ Transformationen gilt

$$\varphi_i^R \rightarrow \varphi_k^R R_{ki}^\dagger, \quad \varphi_i^L \rightarrow \varphi_k^L L_{ki}^\dagger, \quad (142)$$

wobei L und R unitäre Matrizen sind. Es wird deutlich, dass sich die Größen φ_i^R wie rechtshändige Antiquarks und die Größen φ_i^L wie linkshändige Antiquarks transformieren.

Wir können nun die chirale Lagrange-Dichte hinschreiben, die die Diquark-Matrizen D_R , D_L und die Quarkonium-Matrix Φ enthält:

$$\mathcal{L}_{tq-\Phi} = -\frac{c_1}{G_0} G \text{Tr} \left(D_R \Phi^t D_L^\dagger \Phi + D_L^* \Phi^* D_R^\dagger \Phi^\dagger \right) + \frac{c_2}{G_0} G \text{Tr} \left(D_R D_R^\dagger \Phi^\dagger \Phi + D_L D_L^\dagger \Phi \Phi^\dagger \right). \quad (143)$$

Diese Terme stellen die Large- N_c dominanten Beiträge dar. Die Lagrange-Dichte (143) ist auch invariant unter P , C und $U(1)_A$ Transformationen. Das Dilaton-Feld G wurde auch eingeführt, damit die Dilatationssymmetrie erhalten bleibt. Die zwei Kopplungskonstanten $\frac{c_1}{G_0}$ und $\frac{c_2}{G_0}$ sind dimensionslos. Die Gl. (137) folgt aus Gl. (143), wenn nur skalare Tetraquarks berücksichtigt werden und $G = G_0$ gesetzt wird.

Die Anwesenheit von zwei unterschiedlichen Diquarks führt zu vier Tetraquark-Multipletts: zwei skalare Multipletts, $\varphi_i^\dagger \varphi_j$ ($= \mathcal{S}^{[4q]}$) und $\tilde{\varphi}_i^\dagger \tilde{\varphi}_j$, und zwei pseudoskalare Multipletts, $\varphi_i^\dagger \tilde{\varphi}_j$ und $\tilde{\varphi}_i^\dagger \varphi_j$.

Im chiralen Limes sind skalare und pseudoskalare Diquarks entartet. Durch spontane Symmetriebrechung und durch Wechselwirkungen, die auf Instantonen basieren, wird eine starke Anziehung im skalaren Sektor und eine Abstoßung im pseudoskalaren Sektor erzeugt [40], siehe auch die Gitter-Studie in Ref. [41]. Das wiederum heißt, dass das pseudoskalare Diquark von Gl. (138) keine wesentliche Rolle für die Spektroskopie der Mesonen im Niederenergiebereich darstellt. (Das pseudoskalare Diquark ist in dieser Hinsicht ein wichtiger Zwischenschritt, der aber nach der Konstruktion der Lagrange-Dichte wieder entfernt werden kann). Aus diesen Gründen wird nur das Multiplett $\mathcal{S}^{[4q]} = \varphi_i^\dagger \varphi_j$ in Betracht gezogen. Die anderen drei Multipletts mögen zwar existieren, sind aber wesentlich schwerer und/oder zu breit, um gemessen zu werden.

Aus der Lagrange-Dichte (143) können also die Terme mit $\mathcal{S}_{ij}^{[4q]}$ isoliert werden, wobei die anderen für unsere Zwecke vernachlässigt werden können. Man bekommt:

$$\mathcal{L}_{tq-\Phi} = -\frac{c_1}{G_0} G \mathcal{S}_{ij}^{[4q]} \text{Tr} \left(A^j \Phi^t A^i \Phi + A^j A^i \Phi^\dagger \Phi \right) + \frac{c_2}{G_0} G \mathcal{S}_{ij}^{[4q]} \text{Tr} \left(A^j A^i \Phi^\dagger \Phi + A^j A^i \Phi \Phi^\dagger \right) + \dots \quad (144)$$

Ein ähnlicher Term soll auch eingeführt werden, der die Wechselwirkung der skalaren Tetraquarks mit den (Axial)Vektormesonen beschreibt:

$$\mathcal{L}_{tq-AV} = -\frac{c_1^V}{G_0} G \text{Tr} \left(D_R R_\mu^t D_R^\dagger R^\mu + D_L L_\mu^t D_L^\dagger L^\mu \right) + \frac{c_2^V}{G_0} G \text{Tr} \left(D_R D_R^\dagger R_\mu^\dagger R^\mu + D_L D_L^\dagger L_\mu^\dagger L^\mu \right), \quad (145)$$

wobei wieder Dilatationssymmetrie, chirale Invarianz und CPT berücksichtigt wurden. Die zwei Terme sind dominant in der Large- N_c Entwicklung. Wenn nur das leichte skalare Tetraquark-Nonett mitgenommen wird, bekommt man die folgende Wechselwirkung zwischen $\mathcal{S}^{[4q]}$ und (Axial)Vektormesonen:

$$\mathcal{L}_{tq-AV} = -\frac{c_1^V}{G_0} G \mathcal{S}_{ij}^{[4q]} \text{Tr} \left(A^j R_\mu^t A^i R^\mu + A^j L_\mu^t A^i L^\mu \right) + \frac{c_2^V}{G_0} G \mathcal{S}_{ij}^{[4q]} \text{Tr} \left(A^j A^i R_\mu^\dagger R^\mu + A^j A^i L_\mu^\dagger L^\mu \right) + \dots \quad (146)$$

Es fehlt nun der Term, der den “dynamischen” Teil der Tetraquarks beschreibt:

$$\begin{aligned} \mathcal{L}_{tq-quad} &= \text{‘dyn-part’} + G^2 \left\{ \text{Tr}[D_R D_R^\dagger D_R D_R^\dagger] + \text{Tr}[D_L D_L^\dagger D_L D_L^\dagger] \right\} + \dots \\ &= \text{Tr} \left[(\partial^\mu \mathcal{S}^{[4q]})^+ (\partial^\mu \mathcal{S}^{[4q]})^2 - e G^2 \mathcal{S}^{[4q]} \mathcal{S}^{[4q]} \right] + \dots \end{aligned} \quad (147)$$

Die gesamte Lagrange-Dichte $\mathcal{L}_{\bar{q}q+ tq}$ für Quarkonia und Tetraquarks ist die Summe der verschiedenen Terme:

$$\mathcal{L}_{\bar{q}q+ tq} = \mathcal{L}_{mes} + \mathcal{L}_{tq-quad} + \mathcal{L}_{tq-\Phi} + \mathcal{L}_{tq-AV} + \dots \quad (148)$$

wobei die Punkte explizite Symmetriebrechungsterme und weitere Large- N_c unterdrückte Terme symbolisieren.

Die Fälle $N_f = 2$, $N_f = 3$ und die physikalischen Implikationen werden in den folgenden Unterabschnitten beschrieben.

4.5 $N_f = 2$

Die physikalischen Eigenschaften des Modells können im Fall $N_f = 2$ auf eine einfache Art erläutert werden. Wir beschränken uns auf die Freiheitsgrade σ und $\vec{\pi}$ (das heißt: $\Phi = \sigma t^0 + i\vec{\pi} \cdot \vec{t}$; die (Axial)Vektormesonen werden in diesem vereinfachten Fall ausgelassen). Das Potential lautet

$$V = V_{dil}(G) + aG^2(\sigma^2 + \vec{\pi}^2)^2 + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 - \sigma h_0 + \frac{1}{2}m_\chi^2 \frac{G^2}{G_0^2} \chi^2 - g \frac{G}{G_0} \chi (\sigma^2 + \vec{\pi}^2) , \quad (149)$$

wobei $g = c_1 + c_2$, $\lambda = \lambda_1 + \lambda_2$ und $c = 0$.

Wie bekannt, ist das Minimum für $G_0 \neq 0$ aufgrund der Brechung der Dilatationssymmetrie realisiert. Wenn $a < 0$ erhält auch σ einen nicht verschwindenden Vakuumerwartungswert σ_0 , $\sigma_0 \propto G_0 \neq 0$. Aus den zwei entstehenden Termen $\frac{1}{2}m_\chi^2 \chi^2 + g \frac{G}{G_0} \chi \sigma_0^2$ entwickelt auch χ einen ‘vev’:

$$\chi_0 \propto g \frac{\sigma_0^2}{m_\chi^2} \propto g \frac{G_0^2}{m_\chi^2} .$$

Das bedeutet, dass ein Tetraquark-Kondensat im Rahmen des Modells entstanden ist.

Außerdem führt die Verschiebung der Felder $\sigma \rightarrow \sigma_0 + \sigma$, $\chi \rightarrow \chi + \chi_0$ und $G \rightarrow G_0 + G$ zu Mischungen zwischen den skalaren Feldern:

- G - σ Glueball-Quarkonium-Mischung, die schon in Abschnitt 3 erwähnt wurde.
- χ - σ Tetraquark-Quarkonium-Mischung.
- χ - G Tetraquark-Glueball-Mischung.

Es ist deutlich, dass schon dieses vereinfachte Potential im $N_f = 2$ Fall ein ziemlich schwieriges Mischungsproblem generiert, das mit Hilfe einer $O(3)$ -Rotation lösbar ist (obwohl das explizit noch nicht gemacht wurde).

Im Limes $m_G \rightarrow \infty$ ist der Glueball eingefroren und es bleibt nur die χ - σ Mischung. Das ist der Anfangspunkt für eine Studie der Rolle des Tetraquarks bei endlicher Temperatur T und wird ausführlich im Abschnitt 7 präsentiert.

4.6 $N_f = 3$

Die Situation ist qualitativ dem vorherigen Fall ähnlich, obwohl eine größere Anzahl von Feldern auftritt: es gibt ein (pseudo)skalares Quarkonium-Nonett, ein skalares Tetraquark-Nonett und einen Glueball.

Im $I = 1$ Sektor gibt es eine Mischung zwischen zwei a_0 Zuständen: das Tetraquark $a_0^0[4q]$ (von $\mathcal{S}^{[4q]}$ in Gl. (134)-(135)) und das Quarkonium $a_0^0[\bar{q}q]$ (aus $\mathcal{S}$, siehe Tabelle 2). Diese zwei Felder mischen und generieren die physikalischen Resonanzen $a_0(980)$ und $a_0(1450)$, wobei $a_0(980)$ überwiegend Tetraquark und $a_0(1450)$ überwiegend Quarkonium ist. Die entsprechende Gleichung ist

$$\begin{pmatrix} a_0^0(980) \\ a_0^0(1470) \end{pmatrix} = B \begin{pmatrix} a_0^0[4q] \\ a_0^0[\bar{q}q] \end{pmatrix} , \quad B = \begin{pmatrix} \cos \theta_{a_0} & \sin \theta_{a_0} \\ -\sin \theta_{a_0} & \cos \theta_{a_0} \end{pmatrix} . \quad (150)$$

Die numerische Studie in Ref. [7] führt zum Resultat

$$\sin^2 \theta_{a_0} = 0.03-0.10 ,$$

was eine relativ kleine Beimischung bedeutet.

Eine ähnliche Situation gilt im kaonischen $I = 1/2$ Sektor: hier mischen das Tetraquark $k[4q]$ und das Quarkonium $K_0[\bar{q}q]$, die dann die physikalischen Resonanzen $k \equiv K_0(700)$ und $K_0(1450)$ generieren. Der Mischungswinkel θ_k ist auch in diesem Fall klein: $\sin^2 \theta_k \simeq 0.03$.

Die Studie im isoskalaren Sektor wird in diesem Kontext wesentlich schwieriger, da es im Modell fünf skalar-isoskalare Felder gibt: die Tetraquark-Felder $\chi = \sigma_B[4q]$ und $f_B[4q]$, der Glueball G und die Quarkonium-Felder $\sigma = \sigma[\bar{q}q]$ und $f_B[\bar{q}q]$, die die Resonanzen $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ generieren. Jedes von den fünf Feldern hat ein nicht verschwindendes Kondensat. Wie genau die Mischung stattfindet, ist Teil von moderner hadronischen Forschung [42]. Eine Studie dieses Systems innerhalb des hier präsentierten Modells wurde noch nicht unternommen, stellt jedoch ein sehr interessantes und wichtiges Thema für künftige Forschungen dar.

4.7 Large- N_c und Tetraquarks

Bisher wurde das Large- N_c Verhalten der Tetraquarks noch nicht untersucht. Zu diesem Zweck soll man zuerst klären, was man unter einem Tetraquark in einer Large- N_c Welt versteht. Ein Tetraquark-Objekt, das aus zwei Quarks und zwei Antiquarks besteht, überlebt nicht im Large- N_c Limes. Das wurde schon in den bahnbrechenden Artikeln von 't Hooft [12] und Witten [13] klargemacht: solche Diquark-Antidiquark-Objekte gibt es in diesem Limes nicht. Anstatt eines solchen Tetraquarks gibt es im Large- N_c zwei übliche Quark-Antiquark-Mesonen.

Es gibt aber ein anderes Objekt, das im Large- N_c Limes betrachtet werden kann: unsere Diskussion über Tetraquarks beruht auf den guten Diquarks. Die Frage ist nun, was ein gutes Diquark für beliebiges N_c ist. Die Antwort ist leicht: ein ‘gutes Diquark’ wird für $N_c > 3$ zu einem Objekt mit $N_c - 1$ Quarks in einer antisymmetrischen Farbwellenfunktion:

$$d_{a_1} = \varepsilon_{a_1 a_2 a_3 \dots a_{N_c}} q^{a_2} q^{a_3} \dots q^{a_{N_c}} \text{ mit } a_2, \dots, a_{N_c} = 1, \dots, N_c . \quad (151)$$

Das bedeutet, dass ein ‘Tetraquark’ für $N_c > 3$ ein gebundener Zustand aus einem $(N_c - 1)$ -Quark-Objekt und einem $(N_c - 1)$ -Antiquark-Objekt ist:

$$\chi = \sum_{a_1=1}^{N_c} d_{a_1}^\dagger d_{a_1} . \quad (152)$$

Dieses Objekt ist auch als Dibaryonium bekannt [13]. In dieser Hinsicht hat das verallgemeinerte Tetraquark einen wohldefinierten im Large- N_c Limes. Die Masse skaliert wie

$$M_\chi \propto 2(N_c - 1) \sim N_c . \quad (153)$$

Das bedeutet, dass der Parameter $e = \frac{m_\chi^2}{G_0^2}$ in Gl. (147) wie N_c^0 skaliert.

Jetzt studieren wir den Zerfall von χ in zwei Quark-Antiquark-Mesonen. Damit das Dibaryonium zerfällt, muss eine Vernichtung von $N_c - 3$ Quark-Antiquark-Paaren stattfinden. Sei p die Wahrscheinlichkeit einer solcher Vernichtung. Für $N_c > 3$ ist dann die Wahrscheinlichkeit, zwei Quark-Antiquark-Mesonen zu erzeugen, proportional zu p^{N_c-3} , wobei $p < 1$. Das wiederum heißt, dass die Amplitude des Zerfalls wie

$$p^{(N_c-3)/2} \propto e^{-N_c} \text{ für } N_c \gg 3 \quad (154)$$

skaliert. Der entsprechende Term der Lagrange-Dichte lautet z.B. $g\chi\vec{\pi}^2$, was bedeutet:

$$g \propto e^{-N_c}. \quad (155)$$

Im Bezug auf die ursprünglichen Konstanten c_1, c_2, c_1^V, c_2^V gilt (unabhängig von N_f):

$$e \propto N_c^0 \rightarrow M_\chi \propto N_c, \quad (156)$$

$$c_1, c_1^V \propto e^{-N_c}, \quad (157)$$

$$c_2, c_2^V \propto \frac{e^{-N_c}}{N_c}. \quad (158)$$

Der Zerfall $\chi \rightarrow \pi\pi$ skaliert dann wie e^{-N_c}/N_c , d.h. er wird sehr klein für ein großes N_c . Das gibt uns Auskunft über das Large- N_c Verhalten der Mischungsparameter:

$$G\text{-}\sigma : z_{G\sigma} G\sigma \rightarrow z_{G\sigma} \propto a G_0 \sigma_0 \propto N_c^{-1/2},$$

$$\chi\text{-}\sigma : z_{\chi\sigma} \chi\sigma \rightarrow z_{\chi\sigma} \propto g \sigma_0 \propto N_c^{1/2} e^{-N_c},$$

$$\chi\text{-}G : z_{\chi G} \chi G \rightarrow z_{\chi G} \propto g \propto e^{-N_c}.$$

Diese Beziehungen zeigen, dass die Tetraquark-Beimischungen schnell verschwinden. Das ist zu erwarten, da die große Anzahl von Quarks in einem verallgemeinerten Tetraquark solche Mischungsprozesse unterdrückt.

5 Die Konstruktion des Modells: Baryon Sektor

5.1 Einleitung

Die Baryonen sind ein zentraler Bestandteil der Natur. Die wichtigsten Baryonen sind die Nukleonen, d.h. die Protonen und die Neutronen: sie bilden die Kerne der Atome, die die Basis für komplexe Strukturen darstellen. Die Nukleonen können im Rahmen eines chiralen Modells berücksichtigt werden. Die Wechselwirkungen der Nukleonen mit den Mesonen und die Erzeugung eines Nukleon-Nukleon-Potentials werden in diesem Schema beschrieben.

Auch der Ursprung der Nukleonenmasse wird im Rahmen des chiralen Modells untersucht. Es ist bekannt, dass die (nackten) Quarkmassen nur geringfügig zur Nukleonenmasse beitragen. Das Phänomen der Massenerzeugung erfolgt durch verschiedene Kondensate: durch das chirale Kondensat, durch das Gluon-Kondensat und womöglich auch durch das Tetraquark-Kondensat. Alle drei Kondensate spielen in diesem Prozess eine potentiell wichtige Rolle. Die Erzeugung der Nukleonenmasse ist letztendlich für die ‘‘Stofflichkeit’’ der Materie zuständig: nämlich, $\sim 95\%$ der Masse der sichtbaren Materie im Universum kann auf den Prozess der Formation der Nukleonenmasse zurückgeführt werden⁸. Wie genau das im Rahmen eines chiralen Modells passiert, wird in diesem Abschnitt erläutert.

Die dazugehörige Frage, ob das Nukleon einen chiralen Partner hat, wird auch behandelt. In diesem Zusammenhang können zwei Szenarios diskutiert werden: die sogenannte triviale Zuordnung (‘naive assignment’) und die sogenannte Spiegelzuordnung (‘mirror assignment’). Die Generierung der Nukleonenmasse hängt stark von der Zuordnung ab, die tatsächlich realisiert wird.

5.2 Die baryonische Lagrange-Dichte in der trivialen und gespiegelten Zuordnung

5.2.1 Allgemeine Beschreibung

Im klassischen linearen Sigmamodell wird die Nukleonenmasse fast ausschließlich (bis auf einen kleinen Beitrag der nackten Quarkmassen) durch die spontane Symmetriebrechung erzeugt, die zur Beziehung

$$m_N \sim \phi \sim f_\pi \quad (159)$$

führt. Das chirale Kondensat ϕ kann in Verbindung mit dem Quark-Kondensat $\langle \bar{q}q \rangle$ gebracht werden: eine einfache dimensionale Studie führt zu $\phi \simeq \Lambda_{QCD}^{-2} \langle \bar{q}q \rangle$, wobei Λ_{QCD} die schon besprochene QCD-Yang-Mills-Energieskala darstellt. In der Tat kann man durch QCD-Summenregeln ein ähnliches Verhältnis zwischen m_N und $\langle \bar{q}q \rangle$ bekommen: die sogenannte Ioffe Formel [44] lautet

$$m_N \sim \frac{-4\pi^2}{\Lambda_B^2} \langle \bar{q}q \rangle, \quad (160)$$

wobei Λ_B ein energiebehafteter Parameter von der Ordnung ~ 1 GeV ist.

Dennoch können nicht nur das Quark-Kondensat, sondern auch andere Kondensate wie das Gluon- und das Tetraquark-Kondensat zur Nukleonenmasse m_N beitragen [45]. Es steht noch nicht fest, wieviel diese Kondensate zur Nukleonenmasse beitragen. Die Möglichkeit, diese weiteren Kondensate innerhalb eines chiralen Modells zu studieren, wurde dank der Arbeit von Ref. [46] ermöglicht, wo die Spiegelzuordnung im linearen Sigmamodell zuerst diskutiert wurde. Neben dem Isodublett-Nukleonfeld N , mit $N^t = (p, n)$, wobei p das Proton und n das Neutron symbolisiert, wird auch sein chiraler Partner

⁸Der sogenannte Nukleon-Sigma-Term beschreibt den Anteil der Nukleonenmasse, der aus den expliziten nicht-verschwindenden Quarkmassen stammt: er beträgt 30-70 MeV, d.h. etwa 3-8% der gesamten Nukleonenmasse [43]. Der Baryonenanteil ist der dominierende Beitrag zur Masse der sichtbaren Materie im Universum.

N^* (das mit der Resonanz $N(1535)$ identifiziert werden kann) betrachtet. In diesem Fall ist es möglich, einen expliziten Massenterm in der Lagrange-Dichte einzuführen, der einen dimensionsbehafteten Parameter m_0 enthält.

Um diese Überlegungen genau zu diskutieren, werden zuerst zwei Nukleonfelder Ψ_1 und Ψ_2 eingeführt, wobei Ψ_1 positive Parität und Ψ_2 negative Parität besitzt. Wir beschränken uns in diesem Abschnitt auf den Fall $N_f = 2$. Das bedeutet, dass die Nukleonfelder Ψ_1 und Ψ_2 zwei Isospin-Dubletts sind.

5.2.2 Naive Zuordnung

In der naiven Zuordnung transformieren Ψ_1 und Ψ_2 auf dieselbe Weise unter einer $SU(2)_R \times SU(2)_L$ Transformation:

$$\Psi_{1R} \longrightarrow U_R \Psi_{1R}, \quad \Psi_{1L} \longrightarrow U_L \Psi_{1L}, \quad \Psi_{2R} \longrightarrow U_R \Psi_{2R}, \quad \Psi_{2L} \longrightarrow U_L \Psi_{2L} \quad . \quad (161)$$

Die folgende Lagrange-Dichte ist dann chiral-invariant:

$$\begin{aligned} \mathcal{L}_{naive} = & \bar{\Psi}_{1L} i \gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i \gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i \gamma_\mu D_{2L}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i \gamma_\mu D_{2R}^\mu \Psi_{2R} \\ & - \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi^\dagger \Psi_{1L}) - \hat{g}_2 (\bar{\Psi}_{2L} \Phi \Psi_{2R} + \bar{\Psi}_{2R} \Phi^\dagger \Psi_{2L}) \quad . \end{aligned} \quad (162)$$

Die Größen

$$D_{1R}^\mu = \partial^\mu - i c_1 R^\mu, \quad D_{1L}^\mu = \partial^\mu - i c_1 L^\mu$$

und

$$D_{2R}^\mu = \partial^\mu - i c_2 R^\mu, \quad D_{2L}^\mu = \partial^\mu - i c_2 L^\mu$$

sind kovariante Ableitungen, die auf die Nukleonfelder wirken und die dimensionslosen Kopplungen c_1 und c_2 enthalten. Die Felder $\Phi = (\sigma + i\eta)t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}$ und $R^\mu = (\omega^\mu - f_1^\mu)t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu) \cdot \vec{t}$ und $L^\mu = (\omega^\mu + f_1^\mu)t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu) \cdot \vec{t}$ wurden in Abschnitt 3 eingeführt. Die entsprechenden Kopplungskonstanten $\hat{g}_1$ und $\hat{g}_2$ sind auch dimensionslos.

5.2.3 Spiegelzuordnung

In der Spiegelzuordnung transformieren Ψ_1 und Ψ_2 wie folgt

$$\Psi_{1R} \longrightarrow U_R \Psi_{1R}, \quad \Psi_{1L} \longrightarrow U_L \Psi_{1L}, \quad \Psi_{2R} \longrightarrow U_L \Psi_{2R}, \quad \Psi_{2L} \longrightarrow U_R \Psi_{2L} \quad . \quad (163)$$

Das Feld Ψ_1 transformiert genau wie vorher. Das Feld Ψ_2 transformiert hingegen ‘gespiegelt’ unter einer chiralen Transformation. Diese Spiegeltransformation erlaubt einen weiteren Term in der Lagrange-Dichte, der in diesem Fall lautet

$$\begin{aligned} \mathcal{L}_{mirror} = & \bar{\Psi}_{1L} i \gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i \gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i \gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i \gamma_\mu D_{2L}^\mu \Psi_{2R} \\ & - \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi^\dagger \Psi_{1L}) - \hat{g}_2 (\bar{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \bar{\Psi}_{2R} \Phi \Psi_{2L}) + \mathcal{L}_{mass} \end{aligned} \quad (164)$$

wobei

$$\mathcal{L}_{mass} = -m_0 (\bar{\Psi}_{1L} \Psi_{2R} - \bar{\Psi}_{1R} \Psi_{2L} - \bar{\Psi}_{2L} \Psi_{1R} + \bar{\Psi}_{2R} \Psi_{1L}) \quad . \quad (165)$$

Der zusätzliche Term $\mathcal{L}_{mass}$ enthält den parameter m_0 und spielt eine wichtige Rolle in der Erzeugung der Nukleonenmasse. Der Parameter m_0 trägt die Dimension Energie, und deswegen ist der Term $\mathcal{L}_{mass}$ nicht dilatationsinvariant. Es ist einfach, diesen Term mit Hilfe des Glueball-Feldes und des Tetraquark-Feldes dilatationsinvariant zu machen, siehe Abschnitt 5.4.2.

5.3 Erzeugung der Nukleonenmasse

5.3.1 Erzeugung der Masse in der trivialen Zuordnung (nur durch spontane Symmetriebrechung)

Wenn die Baryonen unter Gl. (161) transformieren, ist die baryonische Lagrange-Dichte durch Gl. (162) beschrieben. Die Felder Ψ_1 und Ψ_2 können somit mit dem Nukleon N und mit der Resonanz $N^* \equiv N(1535)$ identifiziert werden ($\Psi_1 = N$ und $\Psi_2 = N^*$)⁹.

Wie schon im Mesonsektor diskutiert, gilt $\phi = \sigma_0 = Zf_\pi$, wobei $Z \approx 1.67$ [4]. Die Konsequenz der Verschiebung $\sigma \rightarrow \sigma + \phi$ führt zur Erzeugung der Nukleonenmassen:

$$\mathcal{L}_{\text{bar},\sigma} = -\frac{\hat{g}_1}{2}\bar{\Psi}_1(\sigma + \phi)\Psi_1 - \frac{\hat{g}_2}{2}\bar{\Psi}_2(\sigma + \phi)\Psi_2 .$$

Es ist deutlich, dass die Massen von N und N^* in diesem Fall durch die einfachen Beziehungen

$$m_N = \frac{\hat{g}_1}{2}\phi \text{ und } m_{N^*} = \frac{\hat{g}_2}{2}\phi \quad (166)$$

ausgedrückt werden. Mit den experimentellen Werten $m_N = 939$ MeV und $m_{N^*} = 1535$ MeV [1] (und $\phi = Zf_\pi = 154.3$ MeV) bekommt man

$$\hat{g}_1 = 12.2 \text{ und } \hat{g}_2 = 20 . \quad (167)$$

Beide Massen sind einfache Funktionen von ϕ , siehe Gl. (166), und verschwinden im Limes $\phi \rightarrow 0$.

Es ist aber schon an dieser Stelle notwendig zu betonen, dass die naive Zuordnung die Pion-Nukleon-Streulängen nicht beschreiben kann. Die Streulänge $a_0^{(+)}$ erweist sich um zwei Größenordnungen zu groß [49]. Dieses Resultat gilt für jede natürliche Wahl der Parameter des Modells. Aus diesem Grund wird unsere Aufmerksamkeit auf die Spiegelzuordnung und ihre Implikationen fokussiert. In Abweichung von der naiven Zuordnung ist eine korrekte Beschreibung der Phänomenologie im Rahmen der Spiegelzuordnung möglich.

5.3.2 Erzeugung der Masse in der Spiegelzuordnung

Wenn die Baryonfelder gespiegelt transformieren, siehe Gl. (163), ist die Lagrange-Dichte in Gl. (164) chiralinvariant. Der Term $\mathcal{L}_{\text{mass}}$ verursacht eine Mischung zwischen den Feldern Ψ_1 und Ψ_2 . Die physikalischen Felder N und N^* , die dem Nukleon und dem chiralen Partner entsprechen, entstehen durch die Diagonalisierung der entsprechenden Massenmatrix:

$$\begin{pmatrix} N \\ N^* \end{pmatrix} = \frac{1}{\sqrt{2 \cosh \delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} . \quad (168)$$

Die Massen von N und N^* lauten:

$$m_{N,N^*} = \sqrt{m_0^2 + \left(\frac{\hat{g}_1 + \hat{g}_2}{4}\right)^2 \phi^2} \pm \frac{(\hat{g}_1 - \hat{g}_2)\phi}{4} . \quad (169)$$

In diesem Szenario trägt nicht nur das chirale Kondensat ϕ zur Entstehung der Masse bei. Auch der m_0 -Term kommt hinzu. Das chirale Kondensat ist hingegen für die Massendifferenz zwischen m_N und m_{N^*} verantwortlich. Für $\phi \rightarrow 0$ entarten nämlich die Massen, $m_N = m_{N^*} = m_0$.

Der Parameter δ in Gl. (168) kann als Funktion von m_0 und den physikalischen Massen m_N , m_{N^*} ausgedrückt werden:

$$\cosh \delta = \frac{m_N + m_{N^*}}{2m_0} . \quad (170)$$

⁹Ein Mischungsterm ist allerdings möglich. Man kann aber zeigen, dass die Entmischung zu einer vollständigen Trennung der zwei physikalischen Feldern führt, siehe Details in Ref. [9, 47]. Aus diesem Grund ist es erlaubt, den Mischungsterm auszulassen.

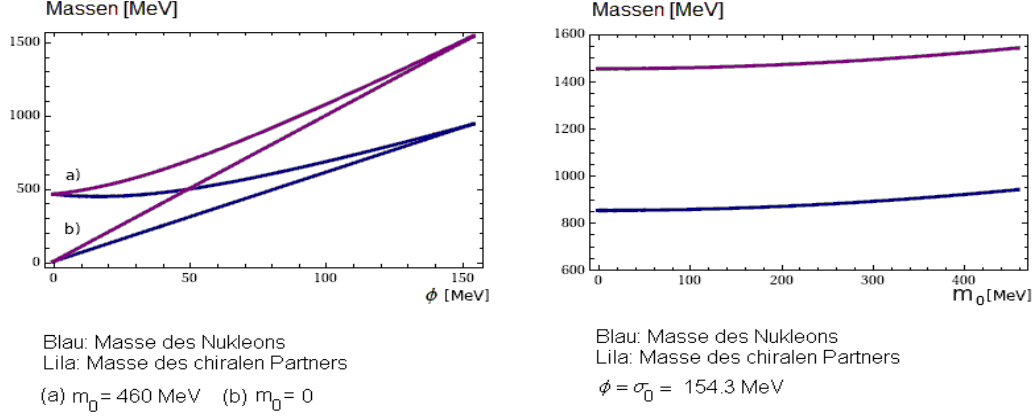


Abbildung 6: Links: Massen des Nukleons (blau) und des chiralen Partners (lila) als Funktion des chiralen Kondensats ϕ für (a) den realistischen Wert $m_0 = 460$ MeV von Gl. (171) und (b) für den Grenzfall $m_0 = 0$, der dem naiven Fall entspricht. Rechts: Massen des Nukleons (blau) und des chiralen Partners (lila) als Funktion des Massenparameters m_0 für den physikalischen Wert des chiralen Kondensats $\phi = Zf_\pi$.

Für $\delta \rightarrow \infty$ ($m_0 \rightarrow 0$) verschwindet die Mischung der Felder: $\Psi_1 = N$ und $\Psi_2 = N^*$. In diesem Fall entkoppeln die zwei Felder und die Situation ist dem naiven Fall ähnlich.

Aus der Form von Gl. (169) wird es deutlich, dass die Nukleonenmasse nicht als Summe von zwei Beiträgen in der Form $m_N = m_0 + c\phi$ geschrieben werden kann. Das bedeutet, dass m_0 nicht als linearer Beitrag zur Masse interpretiert werden sollte. (Diese Linearisierung ist nur dann möglich, wenn einer der zwei Beiträge dominiert). Die Resultate in Ref. [9] zeigen, dass beide Größen nicht vernachlässigbar sind. Die Parameter m_0 , $\hat{g}_1$ und $\hat{g}_2$ und ihre Fehler wurden in Ref. [9] mit Hilfe der experimentellen Werte [1]

$$\Gamma_{N^* \rightarrow N\pi}^{\text{exp}} = 65.7 \pm 23.6 \text{ MeV}, g_A^{N(\text{exp})} = 1.267 \pm 0.004,$$

und der auf Gitter berechneten axialen Kopplungskonstante des Partners [48]

$$g_A^{N^*(\text{latt})} = 0.2 \pm 0.3$$

berechnet:

$$m_0 = 460 \pm 136 \text{ MeV}, \hat{g}_1 = 11.0 \pm 1.5, \hat{g}_2 = 18.8 \pm 2.4. \quad (171)$$

Es ist wichtig zu betonen, dass m_0 größer ist als in der ursprünglichen Arbeit von Ref. [46]. Diese Tatsache zeigt ein Zusammenspiel verschiedener Kondensate zur Formation der Nukleonenmasse.

Zur Erläuterung der Massen legen wir die folgenden zwei Fälle dar:

a) Der Parameter m_0 wird bei 460 MeV festgehalten, und das chirale Kondensat ϕ wird variiert. Die entsprechenden Massen werden in Abb. 6 gezeichnet. Wenn ϕ von 0 bis zum physikalischen Wert Zf_π wächst, variiert die Nukleonenmasse zwischen $m_0 = 460$ MeV und 939 MeV. Es wird somit gezeigt, dass die Masse nicht verschwindet wenn $\phi \rightarrow 0$.

b) Das chirale Kondensat ϕ wird bei Zf_π konstant gehalten, und der Parameter m_0 wird variiert. Die entsprechenden Baryonenmassen werden als Funktion von m_0 in Abb. 6 gezeichnet. Wegen Gl. (169) variiert m_N nur wenig mit wachsendem m_0 . Im Limes $m_0 \rightarrow 0$ stammt die Nukleonenmasse nur

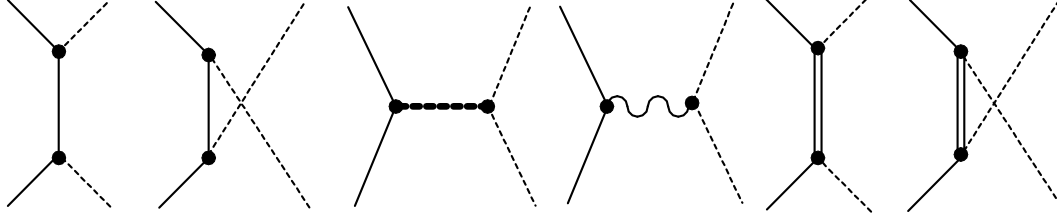


Abbildung 7: Feynman-Diagramme für die πN -Streuung. Durchgezogene Linie: das Nukleon. Gestrichelte Linie: das Pion. Dicke gestrichelte Linie: das σ -Feld. Schlangenlinie: das ρ -Meson. Die doppelte Linie: der chirale Partner des Nukleons N^* .

vom chiralen Kondensat: $m_N = \hat{g}_1 \phi / 2 \simeq 850$ MeV; in demselben Limes ist die Masse des Partners $m_{N^*} = \hat{g}_2 \phi / 2 \simeq 1450$ MeV. Von diesem Standpunkt verursacht der Wert $m_0 = 460$ MeV eine Zunahme der Masse von nur 100 MeV. Diese Eigenschaften folgen aus der nichtlinearen Natur der Gleichung für die Nukleonenmasse. Es ist interessant zu beobachten, dass die Ioffe-Formel Gl. (160) immer noch qualitativ gilt, obwohl der Wert von m_0 nicht vernachlässigbar ist.

5.4 Weitere Resultate und Implikationen der Spiegelzuordnung

5.4.1 Phänomenologie

Wir beschreiben einige relevante phänomenologische Größen, die mit dem Modell berechnet werden.

πN -Streuung

Pion-Nukleon-Streuprozesse, siehe Abb. 7, werden im Rahmen des Modells beschrieben. Das theoretische Resultat für die Streulänge lautet $a_0^{(-)} = (6.04 \pm 0.63) 10^{-4}$ MeV $^{-1}$ und ist in sehr gutem Einklang mit dem Experiment, $a_0^{(-)\text{exp}} = (6.4 \pm 0.1) 10^{-4}$ MeV $^{-1}$. Die theoretische Bestimmung der Streulänge $a_0^{(+)}$ ist komplizierter, da sie von dem Wert der σ -Masse abhängt. Das ist die einzige Größe im Bereich der Baryonen, die vom skalaren mesonischen Sektor abhängt, siehe die genaue Diskussion in Ref. [9]. Dort wird gezeigt, dass auch in diesem Fall eine Übereinstimmung für eine natürliche Wahl der Parameter möglich ist.

Zerfallsbreite $N^* \rightarrow N\pi$

Das theoretische Resultat für die Zerfallsbreite $N^* \rightarrow N\eta$ lautet $\Gamma_{N^* \rightarrow N\eta} = 10.9 \pm 3.8$ MeV. Dieser Wert ist eindeutig zu klein im Vergleich zum Experiment: $\Gamma_{N^* \rightarrow N\eta}^{\text{exp}} = 78.7 \pm 24.3$ MeV. Eine Verallgemeinerung zu $N_f = 3$ und die Berücksichtigung OZI-unterdrückter Prozesse wäre in dieser Hinsicht wichtig, um eine bessere Übereinstimmung zu erhalten.

Alternative Zuordnung für den Partner: $N^* = N(1650)$

Die alternative Zuordnung für den Partner $N^* = N(1650)$ wurde auch getestet. Ähnliche Resultate gelten für die πN -Streuung. Dieses Szenario steht aber in wesentlich besserem Einklang mit dem experimentellen Wert des Zerfalls $N^* \rightarrow N\eta$: $\Gamma_{N^* \rightarrow N\eta} = 18.3 \pm 8.5$ MeV. Dieser Wert soll mit dem Experimentalwert $\Gamma_{N^* \equiv N(1650) \rightarrow N\eta}^{\text{exp}} = 10.7 \pm 6.7$ MeV verglichen werden. In diesem Zusammenhang sollte eine alternative Interpretation der Resonanz $N(1535)$ gesucht werden. Ein mögliches erweitertes Mischungsszenario wurde in Ref. [9] vorgeschlagen: zwei Baryonfelder mit Parität $+1$ und zwei mit Parität -1 werden eingeführt. Ihre Mischung generiert die Resonanzen N , $N(1440)$ (Roper), und

$N(1535)$, $N(1650)$. Die experimentell große Zerfallsbreite von $N(1535) \rightarrow N\eta$ könnte als Konsequenz einer konstruktiven Interferenz auftreten.

Eine hypothetische Resonanz

Ein weiterer, leichter Kandidat als chiraler Partner des Nukleons wurde in Ref. [50] vorgeschlagen: die Masse dieser hypothetischen Resonanz wäre $M_{N^*} \simeq 1200$ MeV und die Zerfallsbreite wäre sehr groß, $\Gamma_{N^*} \gtrsim 700 - 800$ MeV. Eine solche Breite würde erklären, warum dieser hypothetische Zustand nicht gemessen wurde. Der Grund dieser Annahme beruht auf Resultaten bei endlicher Dichte (wie z.B. die Berechnung der Kompressibilität). Dennoch kann mit Hilfe unseres Modells dieser hypothetische Zustand ausgeschlossen werden: die Streulängen wären um zwei Größenordnungen falsch.

5.4.2 Die Rolle der Gluon- und Tetraquark-Kondensate

Der Term $\mathcal{L}_{\text{mass}}$ ist nicht invariant unter einer Dilatationstransformation, da der Parameter m_0 dimensionsbehaftet ist. Das ist nicht im Einklang mit der Diskussion im Abschnitt 3.3.4: im chiralen Limes muss die einzige Brechung dieser Symmetrie aus dem Eichsektor stammen.

Es ist aber nicht schwierig, den Term so zu modifizieren, dass die Dilatationssymmetrie wiederhergestellt ist. Zu diesem Zweck werden die zwei chiralinvarianten Felder G und χ benutzt: G ist der skalare Glueball und $\chi \equiv \frac{1}{2}[\bar{u}, \bar{d}][u, d]$ das Tetraquark. (Für $N_f = 2$ gibt es nur ein skalar-isoskalares Tetraquark, siehe Abschnitt 4.)

Der Massenterm $\mathcal{L}_{\text{mass}}$ wird wie folgt modifiziert:

$$\mathcal{L}_{\text{mass}} = (\alpha\chi + \beta G)(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}) , \quad (172)$$

wobei die Konstanten α und β dimensionslos sind.

Wenn beide Skalarfelder um den Vakuumerwartungswert verschoben werden, $G \rightarrow G_0 + G$ und $\chi \rightarrow \chi_0 + \chi$, findet man wieder Gl. (165) durch folgende Identifikation:

$$m_0 = \alpha\chi_0 + \beta G_0 , \quad (173)$$

wobei χ_0 und G_0 das Tetraquark-Kondensat und das Gluon-Kondensat sind. Auf diese Art wird der Parameter m_0 als Summe von zwei Beiträgen geschrieben, die diesen zwei weiteren Kondensaten entsprechen.

5.4.3 Die Rolle des Tetraquarks für die Nukleon-Nukleon-Wechselwirkung

Üblicherweise wird die Nukleon-Nukleon-Wechselwirkung durch Austausch von Quark-Antiquark-Mesonen erklärt. Zwei Nukleonen vertauschen jeweils ein Quark: als Zwischenzustand bildet sich ein Quark-Antiquark-Meson. Auf dem hadronischen Niveau entspricht dieser Prozess dem Austausch eines leichten Quarkonium-Mesons, wie π , ω und σ . Dieser Austausch wird in unserem Modell durch die Nukleon-Meson Wechselwirkungsterme beschrieben (wobei das σ -Meson dem schweren Zustand $f_0(1370)$ entspricht).

Der Term $\mathcal{L}_{\text{mass}}$ in Gl. (172) beschreibt aber weitere Wechselwirkungen: das Nukleon-Nukleon-Potential hängt auch von dem Austausch eines Tetraquarks χ und eines Glueballs G ab. Da der Glueball relativ schwer ist (~ 1.5 GeV), wird er in dieser Diskussion ausgelassen. Das Tetraquark kann hingegen leicht sein (~ 600 MeV) und kann somit ein wichtiges Austauscheteilchen für die Nukleon-Nukleon-Wechselwirkung sein.

Der Austausch eines Tetraquark-Mesons wird in Abb. 8 gezeigt: das Diquark eines Nukleons wird mit dem Diquark des anderen Nukleons getauscht. Als Zwischenzustand bildet sich ein Diquark-Antidiquark-Zustand, d.h. ein Tetraquark. Dass dieser Austausch (potentiell) sehr wichtig ist, liegt

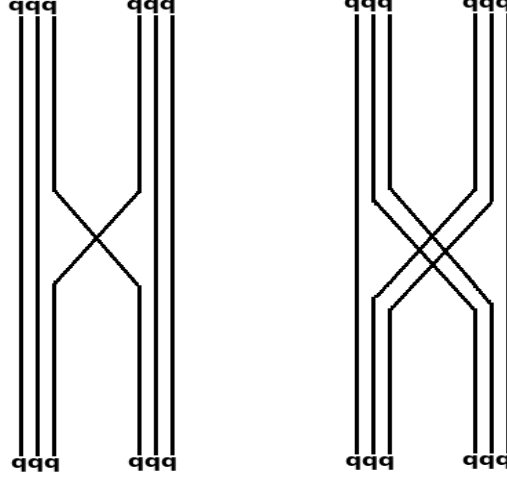


Abbildung 8: Links: bildliche Darstellung des Beitrags eines Quarkonium-Mesons zum NN Potential. Rechts: bildliche Darstellung des Beitrags eines Tetrquark-Mesons zum NN Potential.

in der Tatsache, dass ein (gutes) Diquark eine primäre Rolle für das Verständnis eines Nukleons spielt. Da das Diquark stark gebunden ist, kann man, wie viele Studien gezeigt haben, ein Nukleon als Diquark-Quark-Objekt auffassen. Es liegt jetzt nahe zu erwarten, dass ein Diquark-Austausch, der einem Tetraquark als Zwischenzustand entspricht, das Nukleon-Nukleon-Potential beeinflusst.

Es ist eigentlich bekannt, dass die Stabilität der Kerne auf die Anziehungskraft eines leichten skalaren Mesons zurückzuführen ist. Die Masse lautet ungefähr 600 MeV, die der Resonanz $f_0(600)$ entspricht. Üblicherweise wurde dieser skalare Zustand mit dem Quark-Antiquark σ -Meson identifiziert. Wie aber in dieser Arbeit mehrmals diskutiert wurde, führt diese Identifikation zu Problemen. Die Rolle des leichten skalaren Zustands kann aber von dem Tetraquark übernommen werden, $\chi = f_0(600)$, wie schon in den Abschnitten 3 und 4 motiviert wurde. Das ist im Einklang mit der gesamten Vakuumphänomenologie.

Diese Diskussion zeigt, dass ein leichtes Tetraquark dafür zuständig sein kann, dass die Kerne überhaupt existieren. Das ist eine interessante Behauptung, die weitere Untersuchungen benötigt. Eine Studie des Systems bei endlicher Dichte ist in dieser Hinsicht notwendig. Vorläufige Resultate zeigen tatsächlich ein interessantes Zusammenspiel von Tetraquark und chiralen Kondensaten. Auch die Beziehung zu der sogenannten ‘Quarkyonic Phase’ [51] (confinierte, aber chiralsymmetrische Phase) stellt einen Ausblick dieser Arbeit dar.

5.4.4 Baryonen im Large- N_c Limes

Baryonen in der Large- N_c Entwicklung sind ein reiches theoretisches Forschungsgebiet, das in Ref. [13] eingeleitet wurde. Hier beschränken wir uns auf die Darstellung der folgenden Skalierungen der Parameter:

$$\widehat{g}_1, \widehat{g}_2 \propto N_c^{1/2}, \alpha, \beta \propto N_c^0. \quad (174)$$

Es folgt, dass die Nukleonenmasse -wie erwartet- wie N_c skaliert. (Der Tetraquark-Term $\alpha\chi_0$ geht schnell gegen Null, weil $\chi_0 \propto e^{-N_c}$.)

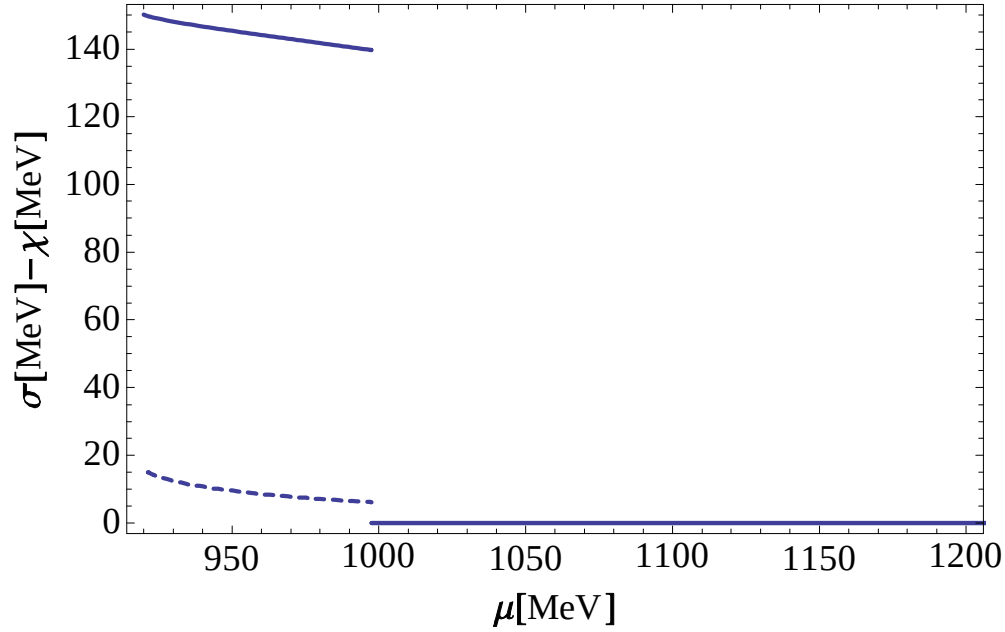


Abbildung 9: Das Quark-Antiquark-Kondensat σ_0 und das Tetraquark-Kondensat χ_0 werden als Funktion von μ geplottet. Ein Phasenübergang erster Ordnung findet bei $\mu \simeq 1$ GeV statt.

5.4.5 Weitere Ausblicke für die Vakuum-Physik

- Δ -Resonanz: es ist bekannt, dass die Δ -Resonanz stark an πN koppelt. Ihre Einführung in das chirale Modell würde die theoretische Studie vollständiger machen, vor allem was die Rechnung der πN -Streuprozesse betrifft.
- $N_f = 3$: anders als im mesonischen Sektor ist hier die Verallgemeinerung zu $N_f = 3$ nicht trivial. Auch in diesem Fall werden keine weiteren Parameter benötigt. Die Baryonfelder treten als Oktett auf, was ein Umschreiben der Lagrange-Dichte mit Hilfe der Spuren auch im Baryon-Sektor impliziert. Bis auf die technischen Schwierigkeiten gibt es kein konzeptuelles Hindernis, eine solche Studie durchzuführen.
- Nukleon-Nukleon Streuprozesse und Dilepton-Produktion. Im Rahmen des Modells können solche Prozesse berechnet werden. Wegen der experimentellen Relevanz dieser Prozesse (siehe z.B. Ref. [52] und Referenzen darin), ist eine solche Studie, die auf einem chiralen Modell basiert ist, interessant.

5.5 Das Modell bei nicht verschwindender Dichte

Das Modell bei nicht verschwindender Dichte ρ wurde in Ref. [10] studiert. Der Term in Gl. (172) für $\beta = 0$ wurde dafür verwendet: das bedeutet, dass der Massenparameter $m_0 = \alpha\chi_0$ ausschließlich vom Tetraquark-Kondensat χ_0 generiert wird. Das Diagramm in Abb. 8 wird also im Rahmen einer Mean-Field-Näherung untersucht, um die Rolle eines zusätzlichen Tetraquark-Feldes bei $\rho > 0$ zu untersuchen.

Folgende Resultate werden hier zusammengefasst:

- Es ist möglich, im Rahmen des hier präsentierten Modells die Kernmaterie korrekt zu beschreiben. Dabei wird der Wert des Parameters $m_0 = \alpha\chi_0 = 460 \pm 136$ MeV, der in Vakuum bestimmt wurde, benutzt. In diesem Zusammenhang es ist wichtig anzumerken, dass ein Sigma-Modell in der trivialen Zuordnung die Sättigung der Kernmaterie nicht beschreiben kann. Wenn hingegen die Spiegelzuordnung ohne Tetraquark benutzt wird, können die Eigenschaften der Kernmaterie (Sättigung und

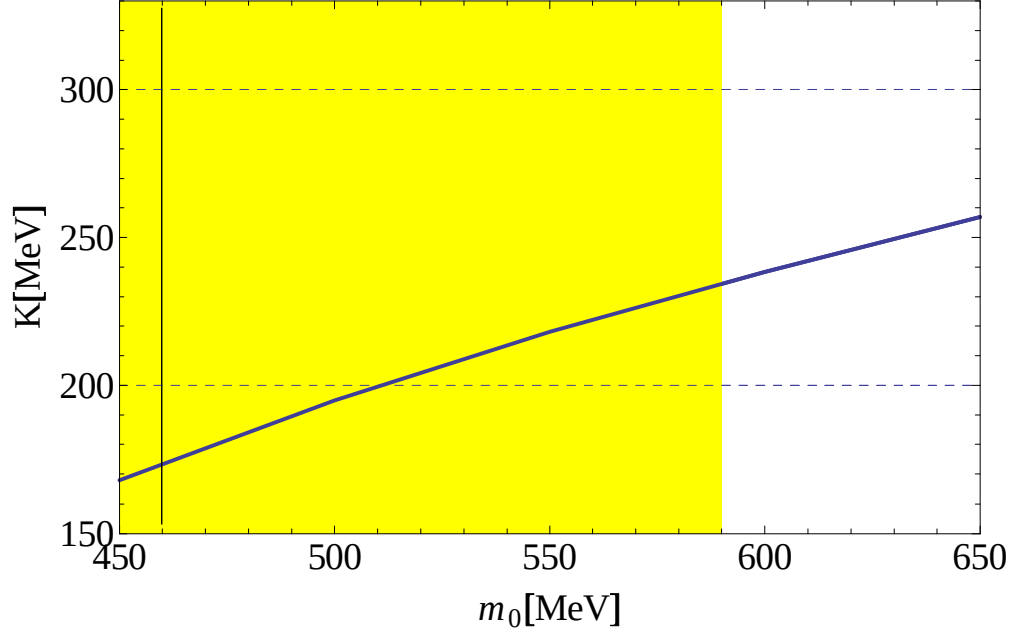


Abbildung 10: Die Kompressibilität der Kernmaterie wird als Funktion von m_0 geplottet. Der Gelber Band zeigt die Werte von m_0 , die im Einklang mit der Vakuumsphysik sind. Die zwei horizontalen gestrichelten Linien zeigen den experimentell bestimmten Wert der Kompressibilität: $200 \text{ MeV} \lesssim K \lesssim 300 \text{ MeV}$. Die korrekte Beschreibung der Kompressibilität ist sichtbar.

Kompressibilität) für den großen Wert $m_0 \simeq 800 \text{ MeV}$ korrekt berechnet werden [50]. Dieser große Wert von m_0 führt aber zu Schwierigkeiten in der phänomenologischen Beschreibung von Vakuumsgrößen. In unserem Modell, in dem das Tetraquark hinzugefügt wurde, ist eine korrekte Beschreibung von Größen im Vakuum und bei nicht verschwindender Dichte gleichzeitig möglich.

- Der Verlauf des Quark-Antiquark-Kondensats und des Tetraquark-Kondensats in Abhängigkeit des chemischen Potentials μ wird in Abb. 9 präsentiert. Ein Phasentübergang erster Ordnung findet bei $\mu \simeq 1 \text{ GeV}$ statt. Das entstprechende Intervall der Dichte variiert zwischen 3-10 ρ_0 , siehe Details in Ref. [10]. Bei dem Phasentübergang sind beide Kondensate unstetig.
- Die Kompressibilität K der Kernmaterie wird in Abb. 10 als Funktion von m_0 geplottet. Für $m_0 = 500 \text{ MeV}$ lautet sie $K \simeq 200 \text{ MeV}$, der im Einklang mit dem Experiment ist ($200 \text{ MeV} \lesssim K \lesssim 300 \text{ MeV}$ [53]). Es ist bemerkenswert, dass die Kompressibilität und die Pion-Nukleon-Streulängen kompatibel sind.
- Wenn das Tetraquark-Feld der leichtesten skalaren Resonanz $f_0(600)$ entspricht, gibt es eine kuriose Konsequenz: die Kernmaterie scheint nur für $N_c = 3$ zu existieren. Schon für $N_c = 4$ wäre die Attraktion durch das Tetraquark zu schwach, um die Sättigung der Kernmaterie zu ermöglichen [54].

6 Das Tetraquark bei nicht verschwindender Temperatur

6.1 Einleitung

Die Studie eines chiralen Modells bei nicht verschwindender Temperatur ist ein wichtiges Thema der Hadronenphysik der letzten 30 Jahre. Viele Erkenntnisse wurden aus diesen Studien gewonnen: die Phänomenologie des chiralen Phasentüberganges, der Verlauf der Massen als Funktion von T , die Wiederherstellung der chiralen Symmetrie sind einige der meistuntersuchten thermodynamischen Eigenschaften.

Dennoch ist eine Kritik an solchen Modellen angebracht. Sowohl die hadronischen Modelle als auch dem Quark-Bild basierte auf Modelle (wie das NJL-Modell und moderne Generalisierungen, wie das PNJL-Modell) enthalten eine leichte skalare Quark-Antiquark Resonanz, die in der Regel mit $f_0(600)$ identifiziert wird. Das entsprechende Feld kondensiert und beschreibt das berühmte chirale Kondensat. Folgendes Problem ist aber nicht zu übersehen: wenn die Resonanz $f_0(600)$ *nicht* einem Quarkonium entspricht, enthalten viele Studien bei $T \neq 0$ von Anfang an eine falsche Identifizierung. Wie im Lauf dieser Arbeit diskutiert wurde, ist es die Resonanz $f_0(1370)$, die als (dominanter) Quark-Antiquark/Zustand beschrieben wird. Dennoch kann die leichte Resonanz $f_0(600)$ auch als Tetraquark eine wichtige Rolle bei endlichem T spielen.

In diesem Abschnitt werden die Resultate der ersten Studie in dieser Richtung präsentiert [11]. Das einfache Modell von Abschnitt 4.5 wird dafür benutzt. Natürlich ist ein solches Modell noch nicht realitätsgetreu genug, um die Natur vollständig zu beschreiben. Dennoch enthält dieses Modell schon wichtige Freiheitsgrade, um die Phänomenologie bei $T \neq 0$ untersuchen zu können. Diese einfache Wahl zeigt, dass ein leichtes zusätzliches Tetraquark-Feld die Physik des Phasentübergangs beeinflussen kann. Außerdem wird das Quarkonium/Tetraquark/Mischungsphänomen bei $T > 0$ sogar intensiver als bei $T = 0$.

6.2 Einfaches Modell bei $T \neq 0$

Wir arbeiten mit dem Modell von Gl. (149) im Limes $m_G \rightarrow \infty$ (eingefrorener Glueball): das Pion $\vec{\pi}$, das nackte Quarkonium $\sigma \equiv \bar{n}n \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ und das nackte Tetraquark $\chi \equiv \frac{1}{2}[u, d][\bar{u}, \bar{d}]$ sind die Freiheitsgrade. Das Potential lautet:

$$V = \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - F^2)^2 - \varepsilon \sigma + \frac{1}{2} M_\chi^2 \chi^2 - g \chi (\sigma^2 + \vec{\pi}^2) . \quad (175)$$

Das Minimum des Potentials lautet (bis zur Ordnung $O(\varepsilon)$):

$$\sigma_0 \simeq \frac{F}{\sqrt{1 - 2g^2/(\lambda M_\chi^2)}} + \frac{\varepsilon}{2\lambda F^2}, \quad \chi_0 = \frac{g}{M_\chi^2} \sigma_0^2, \quad \vec{\pi}_0 = 0. \quad (176)$$

Wie üblich gilt: $\sigma_0 = f_\pi = 92.4$ MeV. Das Tetraquark-Kondensat χ_0 ist proportional zu σ_0^2 , was bedeutet, dass das Tetraquark-Kondensat von dem chiralen Kondensat induziert wird.

Man verschiebt die Felder um die vev's, $\sigma \rightarrow \sigma + \sigma_0$ und $\chi \rightarrow \chi + \chi_0$, und entwickelt:

$$V = \frac{1}{2} (\chi, \sigma) \begin{pmatrix} M_\chi^2 & -2g\sigma_0 \\ -2g\sigma_0 & M_\varphi^2 \end{pmatrix} \begin{pmatrix} \chi \\ \sigma \end{pmatrix} + \frac{1}{2} M_\pi^2 \vec{\pi}^2 + \dots , \quad (177)$$

wobei

$$M_\sigma^2 = \sigma_0^2 \left(3\lambda - \frac{2g^2}{M_\chi^2} \right) - \lambda F^2, \quad M_\pi^2 = \frac{\varepsilon}{\sigma_0} . \quad (178)$$

Da die Matrix in Gl. (177) nicht diagonal ist, sind die Felder σ und χ keine Masseneigenzustände von V . Das ist ein expliziter Spezialfall der Mischungsprozesse, die schon in Abschnitt 4.5 allgemein

erläutert wurden. Hier berechnen wir auch die Entmischung der Felder: man führt die Eigenzustände (H, S) , die im Rahmen dieses Modells mit den Resonanzen $f_0(600)$ und $f_0(1370)$ identifiziert werden, als $SO(2)$ -Rotation der nackten Felder σ und χ ein:

$$\begin{pmatrix} H = f_0(600) \\ S = f_0(1370) \end{pmatrix} = \begin{pmatrix} \cos \theta_0 & \sin \theta_0 \\ -\sin \theta_0 & \cos \theta_0 \end{pmatrix} \begin{pmatrix} \chi \\ \sigma \end{pmatrix} ,$$

$$\theta_0 = \frac{1}{2} \arctan \frac{4g\sigma_0}{M_\sigma^2 - M_\chi^2} . \quad (179)$$

Die physikalischen Massen lauten

$$M_H^2 = M_\chi^2 \cos^2 \theta_0 + M_\sigma^2 \sin^2 \theta_0 - 2g\sigma_0 \sin(2\theta_0) , \quad (180)$$

$$M_S^2 = M_\sigma^2 \cos^2 \theta_0 + M_\chi^2 \sin^2 \theta_0 + 2g\sigma_0 \sin(2\theta_0) . \quad (181)$$

Wie in Abschnitt 4 erklärt, gilt die Massenordnung

$$M_S > M_\sigma > M_\chi > M_H .$$

Der Zustand $H \equiv f_0(600)$ ist dann überwiegend ein Tetraquark, und $S \equiv f_0(1370)$ ist überwiegend ein Quarkonium.

Im Limes $g \rightarrow 0$ entkoppelt das Feld χ : ein einfaches lineares Sigma-Modell für die Felder $\vec{\pi}$ und σ und eine freies Tetraquark-Feld χ bleiben übrig. Der Mischungswinkel θ_0 und das Tetraquark-Kondensat verschwinden. Es ist insofern klar, dass der Parameter g die neuen Eigenschaften des Modells beschreibt. Je größer g ist, desto intensiver die Mischung und das Zusammenspiel von Quarkonium und Tetraquark.

6.3 Resultate und Diskussion

Um die Physik bei $T \neq 0$ zu untersuchen, wird der CJT-Formalismus [55] in der Hartree-Fock-Näherung benutzt [55, 56]. Die Massen der Resonanzen werden T -abhängige Funktionen: $M_S(T)$, $M_H(T)$, und $M_\pi(T)$.

Auch der Mischungswinkel, das chirale Kondensat und das Tetraquark-Kondensat entwickeln eine T -Abhängigkeit: $\theta_0 \rightarrow \theta(T)$, $\sigma_0 \rightarrow \sigma_0(T)$ und $\chi_0 \rightarrow \chi_0(T)$. Natürlich müssen die Werte bei $T = 0$ reproduziert werden ($i = \pi, \sigma, \chi$):

$$M_i(T=0) = M_i , \theta(T=0) = \theta_0 , \sigma_0(T=0) = \sigma_0 , \chi_0(T=0) = \chi_0 . \quad (182)$$

Im Limes $g \rightarrow 0$ entkoppelt das Tetraquark: S ist ein reines Quarkonium und H ein reines Tetraquark. Das Tetraquark spielt in diesem Limes keine Rolle für den Phasenübergang. Der Phasenübergang ist ein Crossover für $M_S \leq 0.95$ GeV und von erster Ordnung für $M_S > 0.95$ GeV. Dieses Resultat ist bekannt, siehe z.B. Ref. [57]. Ein schwerer chiraler Partner des Pions (> 1 GeV) in einem linearen σ -Modell führt zu einem Phasenübergang erster Ordnung, was nicht im Einklang mit QCD-Gitter-Simulationen ist [58].

Die Einführung des Tetraquarks mit $g > 0$ hat aber einen nicht vernachlässigbaren Einfluss auf die Phänomenologie, wie in Abb. 11 gezeigt wird. Im linken Kasten von Abb. 11 wird die Ordnung des Phasenübergangs als Funktion der Parameter g und M_S für die Wahl $M_H = 0.4$ GeV gezeigt. Im rechten Kasten von Abb. 11 wird der Verlauf des chiralen Kondensats für die Parameterwahl $M_S = 1.0$ GeV und $M_H = 0.4$ GeV veranschaulicht: die verschiedenen g -Werte entsprechen den schwarzen Punkten im linken Kasten.

Es ist leicht zu erkennen, dass: (i) je größer g ist, desto kleiner wird T_c : $T_c = 250$ MeV für $g \rightarrow 0$ und $T_c \simeq 200$ MeV für $g = 2.0$ GeV; (ii) die Ordnung des Phasenüberganges wird beeinflusst: für

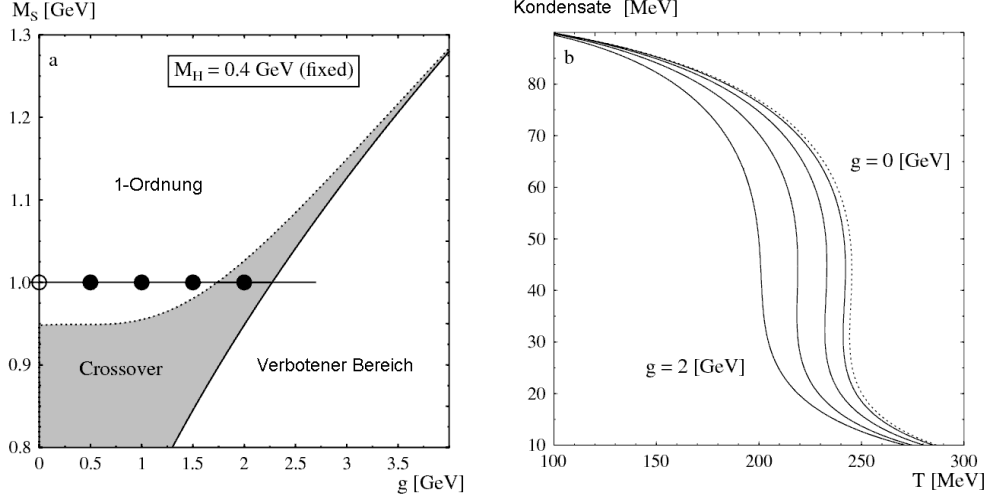


Abbildung 11: Links: Ordnung des Phasenüberganges in Abhängigkeit von den Parametern g und M_S . ($M_H = 0.4 \text{ GeV}$). Rechts: Verlauf des chiralen Kondensats ϕ als Funktion von T für verschiedene g -Werte. Die Kurven entsprechen den schwarzen Punkten im linken Kasten.

wachsende g -Werte wird der Phasenübergang erster Ordnung zuerst geschwächt, und für g groß genug ist ein Crossover vorhanden. Das bedeutet, dass die Einkopplung des Tetraquarks in das Modell ($g > 0$) zu einer Verkleinerung der kritischen Temperatur und eventuell auch einem Crossover führen kann. Es ist nützlich, eine ausführliche Studie eines Crossover-Falles zu präsentieren. Die Vakuummassen werden bei den (üblichen) Werten $M_S = 1.2 \text{ GeV}$ und $M_H = 0.4 \text{ GeV}$ festgehalten [1]. Die Kopplungskonstante g wird dann bei $g = 3.4 \text{ GeV}$ fixiert, damit im Einklang mit Lattice Simulationen ein Crossover stattfindet. Außerdem werden die Masse $M_\pi = 139 \text{ MeV}$ und die Zerfallskonstante $\sigma_0 = f_\pi = 92.4 \text{ MeV}$ benutzt. Dadurch können alle Parameter des Modells festgelegt werden: $\lambda = 52.85$, $M_\chi = 0.96 \text{ GeV}$ und $F = 64.2 \text{ MeV}$.

Der Verlauf der Kondensate wird im linken Kasten von Abb. 12 gezeigt. Bei $T_c = 180 \text{ MeV}$ fällt das Quark-Kondensat $\sigma_0(T)$ schnell ab und die chirale Symmetrie wird wiederhergestellt. Für $T < T_c$ zeigt das Tetraquark-Kondensat $\chi_0(T)$ einen qualitativ ähnlichen Verlauf wie $\sigma_0(T)$, aber für $T > T_c$ wächst die Funktion $\chi_0(T)$. Das ist eine ziemlich überraschende Eigenschaft. Eine weitere Studie dieses Verlaufs wäre sicherlich interessant. Die Frage, ob ein solches wachsendes Tetraquark-Kondensat einige thermodynamische Observablen beeinflussen kann, ist offen.

Der Verlauf der Massen wird im rechten Kasten von Abb. 12 gezeigt: $M_S(T)$ fällt zuerst langsam ab, aber bei $T_s \simeq 160 \text{ MeV}$ findet eine unstetige Verkleinerung statt. Umgekehrt zeigt $M_H(T)$ bei T_s eine unstetige Erhöhung. Es ist in diesem Zusammenhang wichtig, die folgende Definition klarzustellen: S ist der Zustand, der einen Quark-Antiquark-Anteil größer als 50% besitzt, und H ist der Zustand, der einen Tetraquark-Anteil größer als 50% besitzt. Für $T < T_s$ ist $S(H)$ der schwere (leichte) Zustand, für $T > T_s$ umgekehrt. Man kann grob sagen, dass bei $T = T_c$ die Rollen des Tetraquarks und des Quarkoniums vertauscht werden.

Der Mischungswinkel $\theta(T)$ wird in Abb. 13 gezeigt. Es ist bemerkenswert, dass die Mischung mit der Temperatur T wächst: das ist eine allgemeine Eigenschaft, die nicht von den numerischen Details

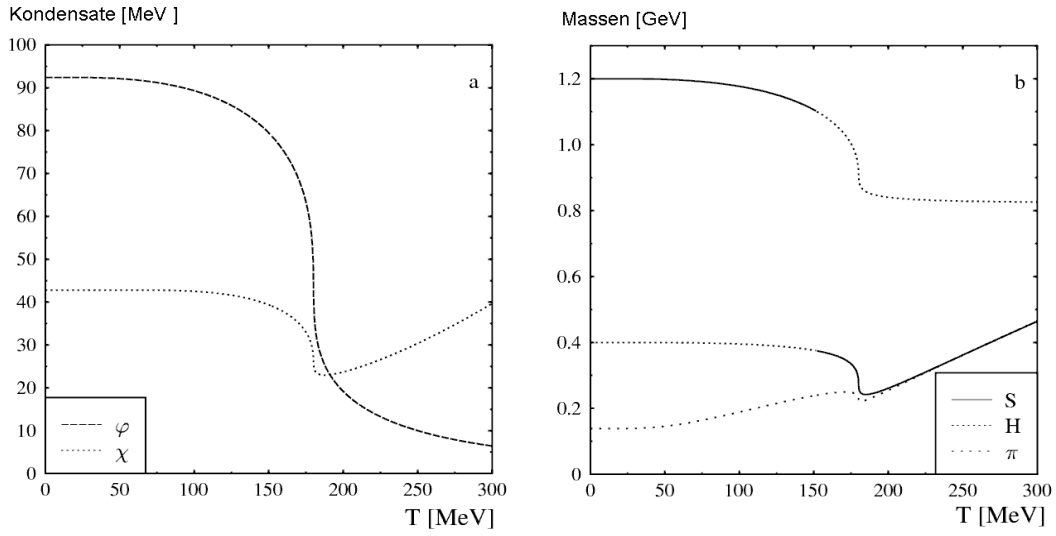


Abbildung 12: Links: das chirale Kondensat und das Tetraquark-Kondensat werden als Funktion von T dargestellt. Rechts: die Massen der drei Zustände π , S und H werden gezeigt.

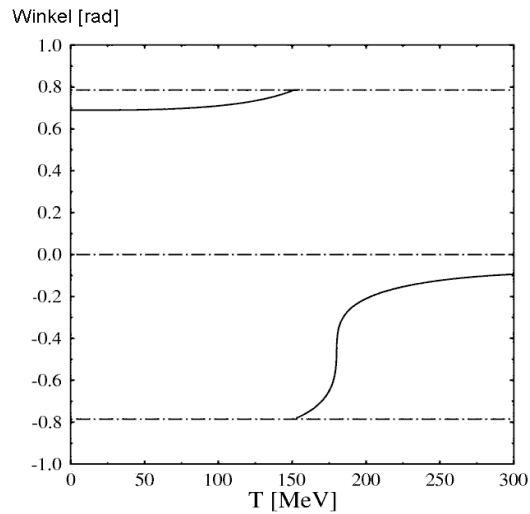


Abbildung 13: Der Mischungswinkel $\theta(T)$ wird gezeigt. Bei $T = T_s$ erreicht θ den Wert von $\pi/4$, der einer maximalen Mischung entspricht.

abhängt. Bei $T = T_s$ ist die Mischung maximal:

$$|\theta(T = T_s)| = \pi/4 .$$

Das erlaubt auch, eine präzise mathematische Definition der Temperatur T_s zu formulieren. Die Funktion $\theta(T)$ ist bei T_s unstetig: sie springt von dem Wert $\pi/4$ zu dem Wert $-\pi/4$,

$$\lim_{T \rightarrow T_s} \theta(T) = \mp \pi/4 .$$

Für $T = T_s$ bestehen beide Zustände S und H aus 50% Quarkonium und 50% Tetraquark.

Es wurde somit gezeigt, dass die Interpretation von $f_0(600)$ als (überwiegend) Tetraquark die Thermodynamik beeinflusst. Weitere Studien sind in diesem Gebiet notwendig: man sollte die schon besprochenen skalaren Resonanzen $f_0(980)$, $f_0(1500)$ und $f_0(1710)$ einbauen und ein vollständiges Szenario untersuchen ($N_f = 3 + \text{Glueball}$). Außerdem, (Axial)Vektormesonen sollten auch berücksichtigt werden. Da sie eine so wichtige Rolle für die Vakuumphysik spielen, ist es notwendig, sie für eine vollständige Studie der thermodynamischen Eigenschaften in Betracht zu ziehen. Das ist eine wichtige Fragestellung für künftige Projekte.

7 Zusammenfassung und Ausblick

In dieser Arbeit wurde die Konstruktion eines hadronischen Modells der QCD mit linearer Realisierung der chiralen Symmetrie präsentiert.

Zuerst ist unser Blickwinkel ganz allgemein gewesen: anhand der Large- N_c Entwicklung wurden grundlegende Eigenschaften eines (beliebigen) hadronischen Modells diskutiert. Auch die moderne Problematik der sogenannten ‘dynamisch generierten’ Resonanzen wurde kritisch analysiert; es wurde somit festgestellt, warum Quark-Antiquark, Gluebälle und auch Tetraquarks nicht als dynamisch generiert interpretiert werden sollen. Die Benutzung eines einfachen Modells hat uns ebenfalls erlaubt, die tückischen Aspekte solcher Studien offensichtlich zu machen.

Als zweiter Schritt wurden die Symmetrien der QCD aufgelistet und zusammengefasst. Die besondere Rolle der Dilatationssymmetrie wurde betont. Das hat uns erlaubt, ein chirales Modell zu erstellen, das nur Terme mit Dimension vier enthält. Ein wichtiger Aspekt dieser Arbeit ist, dass die (Axial)Vektormesonen von Anfang an berücksichtigt wurden: die Motivation dafür ist, dass diese Resonanzen sehr wichtig für die Vakuumphysik sind. Unter anderem beeinflussen sie auch (pseudo)skalare Prozesse, wie. z.B. den Zerfall $\sigma \rightarrow \pi\pi$.

Der Fall $N_f = 2$ wurde diskutiert und die wichtige Rolle der skalaren Resonanzen klar gemacht: das Bild, in dem der chirale Partner des Pions die Resonanz $f_0(1370)$ ist und nicht -wie sonst in verschiedenen Arbeiten angenommen- $f_0(600)$ ist, wird bevorzugt. Die Resonanz $f_0(600)$, zusammen mit $f_0(980)$, $a_0(980)$ und $k(800)$, kann erfolgreich als Tetraquark interpretiert werden.

Die explizite Einführung der Tetraquarks in das chirale Modell wurde durchgeführt. Die guten skalaren und pseudoskalaren Diquarks und das daraus entstehende skalare Tetraquark-Multiplett wurden beschrieben. Im Fall $N_f = 2$ existiert nur ein Tetraquark. Diese Tatsache hat uns erlaubt, ein einfaches Modell für die Tetraquark-Quarkonium-Glueball-Mischung zu konstruieren. Der Fall $N_f = 3$ ist hingegen realistischer und kann einen wichtigen Aspekt der Natur beschreiben: ein ganzes Tetraquark-Nonett existiert, das die leichten Resonanzen $f_0(600)$, $f_0(980)$, $a_0(980)$ und $k(800)$ erklären kann. Sowohl die Massenordnung als auch die Zerfallseigenschaften dieser physikalischen Resonanzen sind eine natürliche Konsequenz des Tetraquark-Szenarios. Als letzter Schritt der Diskussion über Tetraquarks wurde die Besonderheit ihres Large- N_c Limes diskutiert: es wurde gezeigt, dass die korrekte Verallgemeinerung eines Tetraquarks nicht ein Vier-Quark-Objekt ist, sondern ein ‘Dibaryonium’: das ist ein Objekt, das aus $N_c - 1$ Quarks und $N_c - 1$ Antiquarks besteht. Das Large- N_c Verhalten kann unter dieser Perspektive leicht bestimmt werden.

Die Baryonenfelder sind ein zentraler Bestandteil der Hadronenphysik. Aus diesem Grund wurden das Nukleonfeld und sein chiraler Partner in das Modell eingeführt. Ein wichtiger Teil dieser Studie ist die Frage über den Ursprung der Nukleonenmasse: m_N ist eine (nichtlineare) Funktion von den Kondensaten: dem Gluon-Kondensat, dem chiralen (Quark-Antiquark) Kondensat und dem Tetraquark-Kondensat. Das Resultat der Studie zeigt, dass ein Zusammenspiel all dieser Größen notwendig ist, um die Nukleonenmasse zu verstehen.

Als letztes Thema dieser Arbeit wurde eine Studie der Thermodynamik mit dem ‘neuen’ Tetraquark-Freiheitsgrad durchgeführt: ein leichtes Tetraquark kann die thermodynamischen Eigenschaften beeinflussen. Das ist zwar eine wichtige Erkenntnis, aber solche Studien müssen mit realistischeren Modellen (mit allen relevanten Freiheitsgraden) wiederholt werden. Die Fortsetzung dieser Arbeit bei endlicher Temperatur und Dichte stellt einen wichtigen Ausblick dar, der zur Beschreibung und zum theoretischen Verständnis von modernen hoch-energetischen Experimenten dienen soll.

Verschiedene Ausblicke wurden im Lauf dieser Arbeit präsentiert. Sie können wie folgt klassifiziert werden: (i) $N_f > 2$. (ii) Skalare Glueball und Tetraquarks. (iii) Pseudoskalare Glueball. (iv) Weitere

Prozesse im Standardmodell. (v) Nicht verschwindende Temperatur und Dichte. Wir fassen zuletzt diese verschiedenen Forschungsvorhaben zusammen.

(i) Das chirale Modell wurde für eine beliebige Flavoranzahl N_f präsentiert. Explizite Rechnungen wurden aber für den Fall $N_f = 2$ durchgeführt. Die Verallgemeinerung zu $N_f = 3$ stellt ein künftiges Projekt dar. Nur zwei zusätzliche Parameter werden im Fall $N_f = 3$ im Vergleich zu $N_f = 2$ gebraucht. Es gibt jedoch eine wesentlich größere Anzahl von experimentellen Werten, die benutzt werden können. Auch der Fall $N_f = 4$ kann untersucht werden: einige Charmonium-Zustände und ihre Zerfälle (wie z.B. der J/ψ Zerfall) können somit beschrieben werden. Das würde auch eine Verknüpfung der hadronischen Physik der leichten Quarks mit schweren Quarks ermöglichen.

(ii) Das Dilaton-Feld hat von Anfang an eine wichtige Rolle für die Konstruktion des Modells gespielt. Die Symmetrie unter Dilatationstransformationen und ihre anomale Brechung sind ein fundamentales Phänomen der QCD. Die explizite Rechnung der Eigenschaften des entsprechenden Teilchens, des Glueballs, im Rahmen des Modells mit $N_f = 3$ und die Berechnung der Mischung mit Quarkonia ist ein vielversprechendes Projekt. Außerdem sollten auch die leichten Tetraquarks berücksichtigt werden: daraus folgen interessante Mischungsphänomene im leichten skalaren Sektor, die Quarkonia, Tetraquarks und den Glueball betreffen.

(iii) Der pseudoskalare Glueball kann im Rahmen des Modells beschrieben werden. Die Kopplung an die skalaren und pseudoskalaren Mesonen erfolgt durch die axiale Anomalie. Gitter-Rechnungen vorhersagen eine Masse des pseudoskalaren Glueballs von ungefähr 2.6 GeV [14]. In diesem Fall kann eine solche hypothetische Resonanz im künftigen PANDA-Projekt bei FAIR in der GSI/Darmstadt experimentell untersucht werden [36].

(iv) Weitere Prozesse des Standard-Modells können studiert werden: Nukleon-Nukleon Streuprozesse und die daraus entstehende Produktion von Dileptonen können gerechnet werden. Außerdem ermöglicht die Einkopplung der schwachen Eichbosonen schwache Prozesse, die Hadronen enthalten, zu berechnen. Darüber hinaus kann auch das Higgs-Feld und seine Kopplung an Hadronen berücksichtigt werden.

(v) Weitere Studien bei nicht verschwindender Dichte und Temperatur sind notwendig. Die Analyse in den Abschnitten 5 und 6 stellen nur eine erste Studie in dieser Richtung dar. Die potentiell wichtige Rolle des Tetraquarks, siehe Abb. 6, benötigt weitere Untersuchungen sowohl im Vakuum als auch bei endlicher Dichte: die Möglichkeit, dass ein Tetraquark-Feld letztendlich für die Stabilität der Kerne zuständig ist, ist faszinierend. Es werden jedoch weitere Tests gebraucht, um die eigentliche Rolle dieses Feldes für die Hadronenphysik festzulegen.

Danksagung

Ich möchte allen meinen Mitarbeiterinnen und Mitarbeitern für die erfolgreiche Zusammenarbeit in den vergangenen Jahren herzlich danken.

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TEIL II

Veröffentlichte Arbeiten

Liste der Veröffentlichungen,
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- 1) [Francesco Giacosa](#),
Dynamical generation and dynamical reconstruction.
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- 2) [Francesco Giacosa](#), [Giuseppe Pagliara](#),
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- 3) [Denis Parganlija](#), [Francesco Giacosa](#), [Dirk H. Rischke](#),
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- 4) [Stanislaus Janowski](#), [Denis Parganlija](#), [Francesco Giacosa](#), [Dirk H. Rischke](#)
The Glueball in a Chiral Linear Sigma Model with Vector Mesons.
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- 5) [Francesco Giacosa](#),
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- 6) [Francesco Giacosa](#),
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- 7) [Francesco Giacosa](#), [Giuseppe Pagliara](#),
Decay of light scalar mesons into vector-photon and into pseudoscalar mesons.
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- 8) [Susanna Gallas](#), [Francesco Giacosa](#), , [Dirk H. Rischke](#),
Vacuum phenomenology of the chiral partner of the nucleon in a linear sigma model with vector mesons,
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- 9) [Susanna Gallas](#), [Francesco Giacosa](#), [Giuseppe Pagliara](#)
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- 10) [Achim Heinz](#), [Stefan Struber](#), [Francesco Giacosa](#), [Dirk H. Rischke](#),
Role of the tetraquark in the chiral phase transition.
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Dynamical generation and dynamical reconstruction

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A definition of “dynamical generation,” a hotly debated topic at present, is proposed and its implications are discussed. This definition, in turn, leads to a method allowing one to distinguish, in principle, tetraquark and molecular states. The different concept of “dynamical reconstruction” is also introduced and applies to the generation of preexisting mesons (quark-antiquark, glueballs, ...) via unitarization methods applied to low-energy effective Lagrangians. Large- N_c arguments play an important role in all these investigations. A simple toy model with two scalar fields is introduced to elucidate these concepts. The large- N_c behavior of the parameters is chosen in order that the two scalar fields behave as quark-antiquark mesons. When the heavier field is integrated out, one is left with an effective Lagrangian with the lighter field only. A unitarization method applied to the latter allows one to “reconstruct” the heavier “quarkoniumlike” field, which was previously integrated out. It is shown that a Bethe-Salpeter analysis is capable of reproducing the preformed quark-antiquark state, and that the corresponding large- N_c behavior can be brought in agreement with the expected large- N_c limit; this is a subtle and interesting issue on its own. However, when only the lowest term of the effective Lagrangian is retained, the large- N_c limit of the reconstructed state is not reproduced: Instead of the correct large- N_c quarkonium limit, it fades out as a molecular state would do. Implications of these results are presented: It is proposed that axial-vector, tensor, and (some) scalar mesons just above 1 GeV, obtained via the Bethe-Salpeter approach from the corresponding low-energy, effective Lagrangian in which only the lowest term is kept, are quarkonia states, in agreement with the constituent quark model, although they might fade away as molecular states in the large- N_c limit.

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I. INTRODUCTION

A central topic of past and modern hadron physics is the determination of the wave function of resonances in terms of quark and gluon degrees of freedom, both in the baryon and meson sectors and both for light and heavy quarks (for reviews see Refs. [1,2]). In the mesonic sector, beyond conventional quark-antiquark ($\bar{q}q$) mesons, one has glueball states, multiquark states such as tetraquarks, and “dynamically generated resonances,” most notably molecular states. Indeed, different authors used the term “dynamical generation” in rather different contexts. In the works of Refs. [3–6] a dynamically generated state is regarded as a resonance obtained via unitarization methods from a low-energy Lagrangian; in Refs. [7–9] it is considered as a state which does not follow the quark-antiquark pattern in large- N_c expansion. In Refs. [10–13] the concept of “additional, companion poles” as dynamically generated states is introduced, while in Refs. [14–16] a dynamically generated resonance is regarded as a loosely bound molecular state. These various definitions are not mutually exclusive and describe different points of view of the problem.

In this article (Secs. II and III) a definition for a dynamically generated state is proposed and its implications, also in connection with the aforementioned works, are presented. This definition, in turn, leads to a method allowing

us to distinguish, in principle, tetraquark and molecular states, although they are both four-quark states. The concept of “dynamical reconstruction” is then introduced and discussed: It applies to resonances which are obtained from low-energy effective Lagrangians via unitarization methods, but still correspond to “fundamental” (not dynamically generated) $\bar{q}q$, glueball, or multiquark states. In this context, the study of the large- N_c behavior of these resonance constitutes a useful tool to discuss their nature. At the end of Sec. III some general thoughts about the form of an effective theory of hadrons valid up to 2 GeV are also presented.

In Sec. IV the attention is focused on a simple toy model, in which only two scalar fields are considered. The parameters of the toy model are chosen in such a way that both fields behave as quarkoniumlike states in the large- N_c limit. The heavier state is first integrated out in order to obtain an effective low-energy Lagrangian in the toy world and then is reobtained via a Bethe-Salpeter (BS) study applied to the low-energy Lagrangian. It is shown that this is (at least in some cases) possible and that the corresponding large- N_c behavior can be brought in agreement with the expected large- N_c limit; this is a subtle and interesting issue on its own. However, when only the lowest term of the effective, low-energy Lagrangian is retained, the large- N_c limit of the reconstructed state is not

reproduced: Instead of the correct large- N_c quarkonium limit (which must hold by construction), it fades out as a molecular state would do. Implications of these results are presented: It is proposed that axial-vector, tensor, and (some) scalar mesons just above 1 GeV obtained via the BS approach from the corresponding low-energy, effective Lagrangian, in which only the lowest term is kept, are (reconstructed) quark-antiquark fields, in agreement with the constituent quark model, although they might fade away as molecular states in the large- N_c limit. Finally, in Sec. V the conclusions are presented.

II. DYNAMICAL GENERATION

Consider a physical system which is correctly and completely described in the energy range $0 \leq E \leq E_{\max}$ by a quantum field theory in which N fields $\phi_1, \phi_2, \dots, \phi_N$, their masses, and interactions are encoded in a Lagrangian $\mathcal{L} = \mathcal{L}(\phi_i, E_{\max})$. Each Lagrangian has such an $E_{\max}$ beyond which it cannot be trusted. In particular, we shall refer to $E_{\max}$ in the following sense: All the masses M_i of the states ϕ_i and the energy transfer in a two-body scattering $\phi_i + \phi_j \rightarrow \phi_i + \phi_j$ should be smaller than $E_{\max}$ ($M_i < E_{\max}$ and, in the s channel, $\sqrt{s} \leq E_{\max}$).

Moreover, we also assume that (i) the theory is not confining, and thus to each field ϕ_i there is a corresponding, measurable resonance (at least one with zero width); (ii) if not renormalizable, an appropriate regularization shall be specified.

A resonance R , emerging in the system described by $\mathcal{L}$, is said to be dynamically generated if it does *not* correspond to any of the original fields $\phi_1, \phi_2, \dots, \phi_N \subset \mathcal{L}$ in the Lagrangian and if its mass M_R lies below $E_{\max}$ ($M_R \leq E_{\max}$).

The last requirement $M_R \leq E_{\max}$ is natural because the state R can be regarded as an additional, dynamically generated resonance only if it belongs to the energy range in which the theory is valid. This simple consideration plays an important role in the following discussion. Clearly, the dynamically generated state R emerges via interactions of the original resonances ϕ_i . When switching them off, R must disappear. For this reason a dynamically generated mesonic resonance in QCD fades out in the large- N_c limit, which corresponds to a decreasing interaction strength of mesons; see below for more details.

Some examples and comments are in order:

- (a) $\mathcal{L} = \mathcal{L}_{\text{QED}}$, in which the electron and the photon fields are the basic fields. This theory is valid up to a very large $E_{\max}$ (grand unified theory scale). Positronium states are molecular, electron-positron bound states. They appear as poles close to the real axis just below the threshold $2m_e$, but slightly shifted due to their nonzero decay widths into photons. Clearly, positronium states are dynamically generated according to the given definition and should not be included in the original QED

Lagrangian; otherwise they would be double counted. Note that the number of positronium states is infinite.

- (b) In Lagrangians describing nucleon-nucleon interaction via meson exchange (ω , ρ , π , and σ), a bound state close to threshold, called the deuteron, emerges via Yukawa interactions; see, for instance, Ref. [17] and references therein. The deuteron is a dynamically generated molecular state. In this case, when lowering the interaction strength below a critical value [by reducing the coupling and/or increasing the mass of the exchanged particle(s)], the bound state disappears. In fact, the number of molecular states which can be obtained via a Yukawa potential is finite [18], and eventually zero if the attraction is too weak. In such models the deuteron should not be included in the original Lagrangian in order to avoid double counting [19].
- (c) $\mathcal{L} = \mathcal{L}_{\text{F}}$, the Fermi theory of the weak interaction, in which the neutrino and electron fields interact via a local, quartic interaction. This theory is valid up to $E_{\max} \ll M_W$, where W is the boson mediator of the weak force. As already mentioned in Ref. [2], the linear rise of the $\bar{\nu}_e e^-$ cross section—as calculated from $\mathcal{L}_{\text{F}}$ —shows a loss of unitarity at high energy. Unitarization applied to $\mathcal{L}_{\text{F}}$ implies that a resonance well above $E_{\max}$ exists, and this resonance is exactly the W meson. However, with $M_W > E_{\max}$ one cannot state in the framework of the Fermi theory if the W meson is dynamically generated or not. A straightforward way to answer this question is with the knowledge of the corresponding theory, valid up to an energy $E_{\max} > M_W$. Of course, this theory is known: It is the electroweak theory described by the Lagrangian $\mathcal{L}_{\text{EW}}$, which is part of the standard model [20] and is valid up to a very high energy (grand unified theory scale). In the framework of $\mathcal{L}_{\text{EW}}$, the neutrino, the electron, and the W meson are all elementary fields. One can then conclude that the W meson is not a dynamically generated state. Indeed, $\mathcal{L}_{\text{F}}$ can be seen as the result of integrating out the W field from the electroweak Lagrangian $\mathcal{L}_{\text{EW}}$. Unitarization arguments applied to the Fermi Lagrangian $\mathcal{L}_{\text{F}}$ allow us, in a sense, to dynamically reconstruct the W , which is already present as a fundamental field in $\mathcal{L}_{\text{EW}}$.
- (d) It is important to discuss in more depth and to formalize the issue raised in the previous example. To this end let us consider the Lagrangian $\mathcal{L} = \mathcal{L}(\phi_i, E_{\max})$ as the low-energy limit of a Lagrangian $\mathcal{L}' = \mathcal{L}'(\phi_i, \varphi_k, E'_{\max})$ valid up to an energy $E'_{\max} > E_{\max}$. Beyond the fields ϕ_i , $\mathcal{L}'$ depends also on the fields φ_k , which are heavier than $E_{\max}$. Formally, when integrating out the fields φ_k from $\mathcal{L}'$, one obtains $\mathcal{L}$.

In general, a unitarization scheme uses the information encoded in a low-energy effective Lagrangian and the principle of unitarity in quantum field theories, in order to deduce the existence and some properties of resonances beyond the limit of validity of the theory itself. When applying a unitarization scheme to the Lagrangian $\mathcal{L}$, an energy window between $E_{\max}$ and a new energy scale $E_U > E_{\max}$ —which depends on the details of the unitarization—becomes (partially) accessible. For our purposes, we assume that $E_U \lesssim E'_{\max}$.

Let R be a resonance with mass $E_{\max} < M_R < E_U$ obtained from $\mathcal{L}$ via a unitarization approach. Is this resonance R dynamically generated or not? A straightforward way to answer this question would be with the knowledge of $\mathcal{L}'$. If R corresponds to one of the fields, φ_k is not dynamically generated and vice versa. However, if $\mathcal{L}'$ is not known, it is not possible to answer this question at the level of the unitarized version of $\mathcal{L}$ only.

In conclusion, although the unitarization approach opens a window between $E_{\max}$ and E_U and the existence of resonances in this range can be inferred from the unitarized Lagrangian $\mathcal{L}$ only, the knowledge of the latter is still not complete [21]. If $\mathcal{L}'$ is unknown, some other kind of additional information is required to deduce the nature of R . In the framework of low-energy QCD, this additional information can be provided by large- N_c arguments; see Sec. III C.

- (e) Let us consider a scalar field $\varphi = \varphi(t, x)$ in a 1 + 1-dimensional world (t, x) subject to the potential $V(\varphi) = \frac{\lambda}{4!}(\varphi^2 - F^2)^2$. We assume that this theory is valid up to high energies. When expanding around one of the two minima $\varphi = \pm F$, the mass of φ is found to be $m = \lambda F^2/3$. In addition, this theory also admits a soliton with mass $M = \frac{2m^3}{\lambda}$, which is large if λ is small [22]. In this example the solitonic state with mass M can be regarded as a dynamically generated state.
- (f) Mixing can take place among two “fundamental fields,” ϕ_i and ϕ_k : Two physical resonances arise as an admixture of these two fields. One is predominantly ϕ_i and the other predominantly ϕ_k . Also, mixing can take place among a dynamically generated resonance R and one (or more) of the ϕ_i . It decreases when the interaction is lowered (large- N_c limit in the mesonic world): One state reduces to the original, preexisting resonance and the other disappears. In conclusion, mixing surely represents a source of technical complication which renders the identification of states (much) more difficult, but it does *not* change the number of states and the meaning of the previous discussion.

III. APPLICATION TO MESONS

A. Effective hadron theory up to 2 GeV

Let us turn to the hadronic world below 2 GeV. The basic ingredients of each low-energy hadronic Lagrangian are quark-antiquark mesons and three-quark baryons. In the framework of our formalism, we shall consider each quark-antiquark (3-quark) state as a fundamental state, which is described by a corresponding field in the hadronic Lagrangian (as long as its mass is below an upper energy $E_{\max}$).

Let us formalize this point in the mesonic sector as follows. Consider the correct, effective theory describing mesons up to $E_{\max} \simeq 2$ GeV given by

$$\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c), \quad (1)$$

where N_c is the number of colors. Its precise form is, unfortunately, unknown. In fact, because confinement has not yet been analytically solved, it is not possible to derive $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c)$ from the QCD Lagrangian. In the limit $N_c \rightarrow \infty$ the effective Lagrangian $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c)$ is expected to be more simple. Although even in this limit a mathematical derivation is not possible, it is known that it must primarily consist of noninteracting quark-antiquark states. In fact, their masses scale as N_c^0 and their decay widths as N_c^{-1} , respectively [23–26]. Considering that the $\bar{q}q$ mass scales as N_c^0 , these states shall also be clearly present when going from the large- N_c limit to the physical world $N_c = 3$. The next-expected states which are present in the large- N_c limit are glueballs, i.e. bound states of pure gluonic nature. Their masses also scale as N_c^0 and the decay widths as N_c^{-2} . They thus are also expected to be present in the real world for $N_c = 3$. In particular, the lightest glueball is a scalar field which is strongly related to the trace anomaly, i.e. the breaking of the classical dilatation invariance of the QCD Lagrangian (see also Sec. III D). In addition to quark-antiquark and glueball states, hybrid states also survive in the large- N_c limit [26]. They constitute an interesting subject of meson spectroscopy (see Ref. [27] and references therein), but will not be considered in the following discussion. All these states are therefore “preexisting” and not dynamically generated states of the mesonic Lagrangian under consideration.

An intermediate comment is devoted to baryon states: They have a linearly increasing mass with N_c ($M \sim N_c$), which exceeds $E_{\max}$ for a large enough N_c , and are therefore not present in the large- N_c limit of $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c)$. Thus, although they appear in the $N_c = 3$ world, they are not present in the effective Lagrangian because of the way in which the limit is constructed. If, instead, we construct the large- N_c limit as $\mathcal{L}_{\text{eff}}^{\text{had}}(\frac{N_c}{3} E_{\max}, N_c \rightarrow \infty)$ [in such a way that at $N_c = 3$ it coincides with Eq. (1)], baryons are well-defined states as proven originally in Ref. [24]. For simplicity, baryons will not be discussed in this paper but,

together with hybrid states, should be included in a more complete treatment.

The reason why the value $E_{\max} \simeq 2$ GeV is chosen is that all the resonances under study in this work are lighter than 2 GeV. Thus, they either correspond to a field in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c = 3)$ or arise as additional resonances (i.e. dynamically generated) via interaction of preexisting states of $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c = 3)$. The full knowledge of the Lagrangian $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max} \simeq 2 \text{ GeV}, N_c = 3)$ would allow one to see if a resonance lighter than 2 GeV is dynamically generated or not by simply looking at it. Clearly, if one were interested in a resonance whose mass is heavier than 2 GeV, then $E_{\max}$ should be increased. Moreover, one expects to find below 2 GeV all the relevant ground-state mesons in the channel $J^{\text{PC}} = 0^{-+}, 0^{++}, 1^{-+}, 1^{++}, 2^{++}$. Thus, $\mathcal{L}_{\text{eff}}^{\text{had}}(2 \text{ GeV}, 3)$ may be described by an effective Lagrangian which exhibits linear realization of chiral symmetry and its spontaneous breakdown; see Sec. III D for a closer discussion.

Most of the mesonic resonances listed in the Particle Data Group paper [28] can be immediately associated to a corresponding, underlying quark-antiquark state. Yet, the question of whether some resonances of Ref. [28] are not $\bar{q}q$ is interesting and at the basis of many studies. In the mesonic sector, two alternative possibilities are well known:

- (i) Molecular states: These are bound states of two distinct quark-antiquark mesons. They correspond to the example of the positronium [example (a) in Sec. II]. Just as the positronium states are not included in the QED Lagrangian, hadronic molecular states should *not* be included directly in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c)$. They arise upon meson-meson interactions in the $N_c = 3$ physical world; see the general discussion above. However, they inevitably fade out in the large- N_c limit because the interaction of an n -leg meson vertex decreases as $N_c^{-(n-2)/2}$. This is therefore a clear example of dynamically generated states within a mesonic system (see also the next subsection for a closer description of physical candidates below 1 GeV).
- (ii) Tetraquark states: These consist of two distinct, colored “bumps,” in contrast to a molecular state, which is made of two colorless, quark-antiquark bumps [29]. Loops of $\bar{q}q$ mesons, corresponding to the interaction of two colorless states, cannot generate the color distribution of a tetraquark. If present at $N_c = 3$, they shall be included directly in the effective Lagrangian $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c = 3)$ and, in view of the given definition, should not be regarded as dynamically generated states.

A second, slightly different way to see it is the following: Let us imagine constructing the Lagrangian $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c)$. We first put in quark-antiquark and glueball states, that is, those configurations which surely

survive in the large- N_c limit and correspond to non-dynamically generated states: $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c) = \mathcal{L}_{\text{eff}}^{\bar{q}q+\text{glueballs}}(E_{\max}, N_c)$. Then the question is the following: Does this Lagrangian describe the physical world for $N_c = 3$? (Note that loops shall be taken into account and dynamically generated states can eventually emerge out of this Lagrangian.) If the answer is positive, no multiquark states are needed. If the answer is negative, the basic Lagrangian shall be extended to include, from the very beginning, multiquark states, most notably tetraquark states: $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, 3) = \mathcal{L}_{\text{eff}}^{\bar{q}q+\text{glueballs}}(E_{\max}, 3) + \mathcal{L}_{\text{eff}}^{\text{multiquark}}(E_{\max}, 3)$.

A third approach to the problem is via large- N_c arguments. In Refs. [24,25] it has been shown that a tetraquark state also vanishes in the large- N_c limit. However, for $N_c = 3$ the most prominent and potentially relevant for spectroscopy is the “good” diquark, which is antisymmetric in color space: $d_a = \varepsilon_{abc} q^b q^c$ (with $a, b, c = 1, 2, 3$) [30]. The tetraquark is the composition of a good diquark and a good antidiquark: $d_a^\dagger d_a$. The extension to N_c of a good diquark is the antisymmetric configuration $d_{a_1} = \varepsilon_{a_1 a_2 a_3 \dots a_{N_c}} q^{a_2} q^{a_3} \dots q^{a_{N_c}}$ with $a_1, \dots, a_{N_c} = 1, \dots, N_c$, which constitutes $(N_c - 1)$ quarks. Thus, the generalization of the tetraquark to the N_c world is not a diquark-antidiquark object, but rather the state $\chi = \sum_{a_1=1}^{N_c} d_{a_1}^\dagger d_{a_1}$ which is made of $(N_c - 1)$ quarks and $(N_c - 1)$ antiquarks; see also the discussion in Ref. [31]. It is the dibaryonium already described in Ref. [24] which has a well-defined large- N_c limit: Its mass scales as $M_\chi \propto 2(N_c - 1)$ and decays into a baryon and an antibaryon. The state χ , while not present in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c \rightarrow \infty)$ because its mass overshoots $E_{\max}$, appears in $\mathcal{L}_{\text{eff}}^{\text{had}}(N_c E_{\max}, N_c \rightarrow \infty)$ in which the baryons also survive: This is contrary to a dynamically generated state, which also disappears in this case.

As a result of our discussion, tetraquark states and molecular states, although both formally four-quark states, are crucially different: The former are “elementary” and should be directly included in the effective Lagrangian $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c = 3)$; the latter can emerge as dynamically generated resonances. We now turn to the particular case of the light scalar mesons, where all these concepts play an important role.

B. Light scalar mesons

One of the fundamental questions of low-energy QCD concerns the nature of the lightest scalar states $\sigma \equiv f_0(600)$, $k \equiv k(800)$, $f_0 \equiv f_0(980)$, and $a_0 \equiv a_0(980)$. Shall these states be included from the very beginning in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c)$? If yes, they correspond to quark-antiquark or tetraquark nonets (one of them can also be related to a light scalar glueball). If not, they shall be regarded as dynamically generated states. The main point of the following subsection is to discuss previous works about light scalar mesons in connection with the proposed definition of

dynamical generation. In fact, it is easy to classify previous works into two classes (not dynamically generated and dynamically generated), thus allowing us to order different works of the last three decades in a clear way. We first review works in which scalar states are not dynamically generated and then works in which they are dynamically generated.

First we discuss the light scalar states that are not dynamically generated and should be directly included in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c)$.

- (i) In the quark-antiquark picture these light scalar states form the nonet of chiral partners of pseudo-scalar mesons. Their flavor wave functions read $\sigma \approx \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$, $f_0 \approx \bar{s}s$, $a_0^+ \equiv u\bar{d}$, $k^+ \equiv u\bar{s}$. At the microscopic level this is the prediction of the NJL model [32], where a σ mass of about $2m^*$ is obtained and where $m^* \sim 300$ MeV is the constituent quark mass. This is usually the picture adopted in linear sigma models at zero [33] and at nonzero density and temperature [34]. However, this assignment encounters a series of problems: It can hardly explain the mass degeneracy of a_0 and f_0 , the strong coupling of a_0 to kaon-kaons, and the large mass difference with the other p -wave nonets of tensor and axial-vector mesons [35]; it is at odds with large- N_c studies (see below) and with recent lattice works [36].
- (ii) In the tetraquark picture, first proposed by Jaffe [37] and revisited in Refs. [38–40], $\sigma \approx \frac{1}{2}[\bar{u}, \bar{d}][u, d]$, $f_0 \approx \frac{1}{2\sqrt{2}}([\bar{u}, \bar{s}][u, s] + [\bar{d}, \bar{s}][d, s])$, $a_0^+ \equiv \frac{1}{2}[\bar{d}, \bar{s}] \times [u, d]$, $k^+ \equiv \frac{1}{2}[\bar{d}, \bar{s}][u, d]$, where $[\cdot, \cdot]$ stands for an antisymmetric configuration in flavor space (which, together with the already-mentioned antisymmetric configuration in color space, also implies an s -wave and spinless structure of the diquarks and of the tetraquarks). Degeneracy of a_0 and f_0 is a natural consequence. A good phenomenology of decays can be obtained if the next-to-leading order contribution in the large- N_c expansion is also taken into account [40] and/or if instanton induced terms are included [41]. Linear sigma models with an additional nonet of scalar states can be constructed [42–44]. The quark-antiquark states lie above 1 GeV [45] and mix with the scalar glueball whose mass is placed at ~ 1.7 GeV by lattice QCD calculations [46]. This reversed scenario directly affects the physics of chiral restoration at nonzero temperature [47].
- (iii) Different assignments, in which the glueball state also shows up below 1 GeV, have been proposed; see Refs. [2,48,49] and references therein.
- (iv) In all these assignments the very existence of the scalar mesons is due to some preformed compact bare fields entering in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c = 3)$. By removing the corresponding bare resonances from $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c = 3)$, they disappear. Dressing via

meson-meson loops, such as $\pi\pi$ for σ , $K\pi$ for k , and KK for a_0 and f_0 , surely takes place. In particular, due to the intensity in these channels and the closeness to thresholds, they can cause a strong distortion and affect the properties of the scalar states [50]. However, the important point is that in all these scenarios mesonic loops represent a further complication of light scalars, but are not the reason for their existence.

- (v) As discussed in Sec. III A, nondynamically generated scalar states survive in the large- N_c limit, although in a different way according to quarkonium, glueball, or tetraquark interpretations.

Next we discuss the light scalar states that are dynamically generated and should not be included in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c)$.

- (i) In Ref. [14] the σ pole arises as a broad enhancement due to the inclusion of ρ mesons in the t -channel isoscalar $\pi\pi$ scattering. In this case the σ is dynamically generated and arises because of a Yukawa-like interaction due to ρ meson exchange (pretty much as the deuteron described above, but above threshold). When reducing the $\rho\pi\pi$ coupling $g_{\rho\pi\pi}$ (which, in the large- N_c limit, scales as $1/\sqrt{N_c}$), the σ fades out. Alternatively, the limit $M_\rho \rightarrow \infty$ also implies a disappearance of the σ enhancement.
- (ii) Similar conclusions for the $f_0(980)$ and $a_0(980)$ mesons, described as molecular $\bar{K}K$ bound states just below threshold, have been obtained in Refs. [15]. In particular, in Ref. [16] the origin of these states is directly related to a one-meson-exchange potential. Within all these approaches the $a_0(980)$ and the $f_0(980)$ are dynamically generated.
- (iii) In the model of Ref. [10] the a_0 state also arises as an additional, dynamically generated state, but in a different way. Scalar and pseudoscalar quark-antiquark mesons are the original states. A bare scalar state with a mass of 1.6 GeV is the original, quark-antiquark “seed.” When loops of pseudoscalar mesons are switched on, the mass is slightly lowered and the state is identified with $a_0(1450)$. In addition, a second state, arising in this model as a further zero of the real part of the denominator of the propagator, is identified with the $a_0(980)$ meson: It is dynamically generated and disappears in the large- N_c limit, where only the original quark-antiquark seed survives. More generally, we refer to [11–13] for the emergence of additional, companion poles not originally present as preexisting states in the starting Lagrangian. In particular, in Ref. [13] the conventional scalar quark-antiquark states, calculated within a harmonic oscillator confining potential, lie above 1 GeV. When meson loops are switched on, a *complete* second nonet

of dynamically generated states below 1 GeV emerges. Note, in all these studies the validity of the employed theories lies well above 1 GeV, so that the definition of dynamical generation given in Sec. II holds for the light scalars.

- (iv) Note, in (i) and (ii) the emergence of states is due to t -channel forces. This is not the case in (iii). However, a common point is that the light scalar states disappear in the large- N_c limit.

It is clear that the situation concerning light scalars is by far not understood. We wish, however, to stress once more that there is a crucial difference among the two outlined options in relation to the Lagrangian $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c)$. Note also that in this subsection we only discussed works for which it is possible to immediately conclude if the scalar mesons are dynamically generated or not according to the definition given in Sec. II. Unitarization methods were not discussed here; in fact, when the latter are applied, care is needed. This is the subject of the next subsection.

C. Low-energy Lagrangians, unitarization, and dynamical reconstruction

The Lagrangian $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c)$ with $E_{\text{max}} \simeq 2$ GeV induces the breakdown of chiral symmetry $SU_A(N_f)$, where N_f is the number of light flavors. There are therefore $N_f^2 - 1$ Goldstone bosons: the pion triplet for $N_f = 2$, in addition to four kaonic states and the η meson for $N_f = 3$.

If we integrate out all the fields in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c)$ besides the three light pions, we obtain the Lagrangian of chiral perturbation theory (see Ref. [51] and references therein) for $N_f = 2$:

$$\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c) \rightarrow \mathcal{L}_{\chi\text{PT}}(E_{\chi\text{PT}}, N_c), \quad (2)$$

where $E_{\chi\text{PT}}$ should be smaller than the mass of the first resonance heavier than the pions (~ 400 MeV). $\mathcal{L}_{\chi\text{PT}}(E_{\chi\text{PT}}, N_c)$ is recast in an expansion of the pion momentum $O(p^{2n})$, and for each n there is a certain number of low-energy coupling constants, which, in principle, could be calculated from $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c)$, if it were known. Since this is not the case, they are directly determined by experimental data. [Similar properties hold when the kaons and the η are retained in $\mathcal{L}_{\chi\text{PT}}(E_{\chi\text{PT}}, N_c)$].

For instance, the vector isotriplet ρ meson is predicted by a large variety of approaches (such as quark models) to be a preexisting 1^- quark-antiquark field. In this sense it is a fundamental field appearing in $\mathcal{L}_{\text{eff}}^{\text{had}}(2 \text{ GeV}, 3)$, which is integrated out (together with other fields) to obtain $\mathcal{L}_{\chi\text{PT}}(E_{\chi\text{PT}}, 3)$. However, the ρ meson spectral function cannot be obtained from chiral perturbation theory unless a unitarization scheme is employed [3,4,7,8]. As an example, via the inverse amplitude method (IAM) unitarization scheme applied to $\mathcal{L}_{\chi\text{PT}}(E_{\chi\text{PT}}, N_c = 3)$ [3], a window between the original energy $E_{\chi\text{PT}}$ and $4\pi f_\pi \sim 1$ GeV is opened: Resonances with masses in this window, such as

the ρ meson, can be described within unitarized χPT . As discussed in point (d) of Sec. II, the very last question of whether the ρ meson is dynamically generated or not *cannot* be answered at the level of unitarized χPT . One still does not know if ρ corresponds to a basic, preexisting field entering in $\mathcal{L}_{\text{eff}}^{\text{had}}(2 \text{ GeV}, N_c)$ or not.

Some additional information is needed. In the interesting and important case of large- N_c studies of unitarized χPT , the required additional knowledge is provided by the large- N_c scaling of the low-energy constants: It has been shown in Ref. [7] that the ρ mass scales as N_c^0 and the width as N_c^{-1} , and thus the ρ meson should be considered as a fundamental (not dynamically generated) quark-antiquark field, which shall be directly included in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c = 3)$. We also refer to the analytic results of [8], where the large- N_c limit is evident.

Let us turn to the lightest scalar-isoscalar resonance $\sigma \equiv f_0(600)$ as obtained from (unitarized) χPT . In Refs. [52,53] precise determinations of the σ pole are obtained, but—as stated in Ref. [52]—it is difficult to understand its properties in terms of quarks and gluons. In Ref. [7] a study of the σ pole within the IAM scheme in the large- N_c limit has been performed: A result which is at odds with a predominantly quarkonium, or glueball, interpretation of the σ meson has been obtained. The mass is not constant and the width does not decrease. However, even at this stage one still cannot say if the σ is dynamically generated or reconstructed in relation to $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c = 3)$ because it is hard to distinguish the molecular and the tetraquark assignments in the large- N_c limit (see discussion above).

Recently, the Bethe-Salpeter unitarization approach has been used to generate various axial-vector [5,9], tensor, and scalar mesons above 1 GeV [6]. The starting points are low-energy Lagrangians for the vector-pseudoscalar (such as $\rho\pi$) and vector-vector (such as $\rho\rho$) interactions. These Lagrangians are also, in principle, derivable by integrating out heavier fields from the complete $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c = 3)$. For instance, in the $\rho\pi$ axial-vector channel the $a_1(1260)$ meson is obtained, and in the $\rho\rho$ tensor and scalar channels the states $f_2(1270)$ and $f_0(1370)$ are found. Are these dynamically generated states of molecular type? The answer is, not necessarily. In fact, the masses of the obtained states lie *above* the energy limit of the low-energy effective theories out of which they are derived. Even for these states the possibility of dynamical reconstruction—just as for the ρ meson described above—is not excluded: In this scenario, these resonances above 1 GeV are intrinsic, preexisting quark-antiquark or glueball (multiquark states are improbable here) fields of $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c = 3)$. While first integrated out to obtain the low-energy Lagrangians, unitarization methods applied to the latter allow us to reconstruct them. In the next section a toy model is presented, in which this mechanism is explicitly shown: Although a state obtained via the BS equation looks like a molecule, it still

can represent a fundamental, preexisting quark-antiquark (or glueball) state.

As discussed in the summary of the PDG compilation [54] and in Refs. [1,2] (and references therein), the tensor resonances $f_2(1275)$, $f_2(1525)$, $a_2(1320)$, and $K_2(1430)$ represent a nonet of quark-antiquark states. The ideal mixing, the very well measured strong and electromagnetic decay rates [55], the masses, and the mass splitting are all in excellent agreement with the quark-antiquark assignment. In this case they are fundamental (intrinsic) fields of $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c = 3)$, which can be dynamically reconstructed (rather than generated) via unitarization scheme (s) applied to low-energy Lagrangians.

Although experimentally and theoretically more involved, the same can hold in the axial-vector channel: The resonances $f_1(1285)$, $f_1(1510)$, $a_1(1260)$, and $K_1(1270)$ are in good agreement with the low-lying 1^{++} quark-antiquark assignment. Even more complicated is the situation in the scalar channel: The low-lying quark-antiquark states mix with the scalar glueball [45]. Also in this case, however, the possibility of dynamical reconstruction rather than generation is upheld.

If dynamical reconstruction takes place, there is no conflict between the quark model assignment of Ref. [54] and the above-mentioned recent studies. Note, also, that dynamical reconstruction is in agreement with the discussion of Ref. [56].

D. A simplification of the Lagrangian $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c)$

The Lagrangian $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c)$ with $E_{\text{max}} \sim 2$ GeV has been a key ingredient throughout the present discussion, but it has not been made explicit because it is unknown. An improved knowledge of (at least parts of) $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c)$ would surely represent progress in understanding the low-energy hadron system. As a last step we discuss which properties it might have. Clearly, the Lagrangian must reflect the symmetries of QCD, most notably spontaneous breakdown of chiral symmetry. The (pseudo)scalar meson matrix Φ , the (axial-)vector and tensor mesons, and the scalar glueball are its basic building blocks. Moreover, if additional nondynamically generated scalar states such as tetraquarks exist, they shall also be included. We thus have a complicated, general σ -model Lagrangian with many terms, in which operators of all orders can enter, because renormalization is not a property that an effective hadronic Lagrangian should necessarily have. The question is if it is possible to obtain a (relatively) simple form from this complicated picture; see also [42–44,57].

A possibility to substantially simplify the situation is via dilation invariance; let us consider the (pseudo)scalar meson matrix Φ , which transforms as $\Phi \rightarrow R\Phi L^\dagger$ [$R, L \in SU(3)$] under chiral transformation and the dilaton field G , subject to the potential [58] $V_G = \propto G^4(\log G/\Lambda_G + 1/4)$, where Λ_G is a dimensional parameter of the order of Λ_{QCD} (the glueball emerges upon

shifting G around the minimum of its potential $G_0 \sim \Lambda_G$). Consider the Lagrangian $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c) = T - V$, where T is the dynamical part and $V = V[G, \Phi, \dots]$ is the potential describing masses and interactions of the fields [dots refer to other degrees of freedom, such as (axial-)vector ones]. We assume that (i) in the chiral limit the only term in V which breaks dilation invariance—and thus mimics the trace anomaly of QCD—is encoded in V_G (via the dimensional parameter Λ_G), and that (ii) the potential V is finite for any finite value of the fields. As a consequence of (i), only operators of order (exactly) 4 can be included. They have the form $G^2 \text{Tr}[\Phi^\dagger \Phi]$, $\text{Tr}[\Phi^\dagger \Phi \Phi^\dagger \Phi]$, $\text{Tr}[\Phi^\dagger \Phi]^2, \dots$. As a consequence of (ii), a huge set of operators are not admitted. In fact, an operator of the kind $G^{-2} \text{Tr}[\partial_\mu \Phi^\dagger \partial^\mu \Phi]^2$ is excluded because, although of dimension 4, it blows up for $G \rightarrow 0$. In this way we are left with a sizably smaller number of terms, even smaller than what renormalizability alone would impose [59]. Work along this direction, including (axial-)vector degrees of freedom, is ongoing [60] and can constitute an important source of information for spectroscopy and for future developments at nonzero temperature and densities, where in the chirally restored phase a degeneration of chiral partners is manifest.

In conclusion, a way to implement these ideas and use the definition of dynamical generation can be sketched as follows: After writing a general chirally symmetric Lagrangian up to fourth order including the glueball and the quark-antiquark fields as basic states, one should attempt, without further inclusion of any other state, to describe physical processes up to ~ 2 GeV, as pion-pion scattering, decay widths, etc. In doing this one should of course include loops. If, for instance, we start with a basic scalar-isoscalar quark-antiquark field above 1 GeV, do we correctly reproduce the resonance $f_0(600)$ when solving the Bethe-Salpeter channel in the $\pi\pi$ sector below 1 GeV? If the answer is positive, the latter resonance is dynamically generated and there is no need for any other additional state. If, albeit including loops, the attraction among pions turns out to be too weak to generate the resonance $f_0(600)$, we conclude that it is necessary to enlarge our model by explicitly introducing a field which describes it. As argued previously, this field can be identified as a tetraquark state. Whether or not this ambitious program will lead to a successful result is a matter of future research.

IV. A TOY MODEL FOR DYNAMICAL RECONSTRUCTION

A. Definitions and general discussion

In this section we start from a toy Lagrangian, in which two mesons, φ and S , interact. A large- N_c dependence is introduced in such a way that both fields behave as quarkonium states. Then, the field S , which is taken to be heavier than φ , is integrated out and a low-energy Lagrangian with the field φ only is obtained. A Bethe-Salpeter study is

applied to the latter Lagrangian: The question is if the original state S , which was previously integrated out, can be reobtained in this way. The answer is generally positive; however, care is needed concerning the large- N_c limit. In a straight BS approach the quarkoniumlike large- N_c limit of the state S cannot be reproduced. However, as it shall be shown, within a modified BS approach the large- N_c limit can be correctly obtained.

The toy Lagrangian [61,62] consisting of the two fields φ (with mass m) and S (with bare mass $M_0 > 2m$) reads

$$\mathcal{L}_{\text{toy}}(E_{\text{max}}, N_c) = -\frac{1}{2}\varphi(\square + m^2)\varphi - \frac{1}{2}S(\square + M_0^2)S + gS\varphi^2, \quad (3)$$

which we assume to be valid up to $E_{\text{max}} \gg M_0$. The N_c dependence of the effective Lagrangian is encoded in g only: $g = g(N_c) = g_0\sqrt{3/N_c}$. In this way both masses behave like N_c^0 and the decay amplitude for $S \rightarrow 2\varphi$ scales as $1/\sqrt{N_c}$, just as if φ and S were quarkonia states. $\mathcal{L}_{\text{toy}}(E_{\text{max}}, N_c = 3)$ is the analogue of $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c = 3)$ in a simplified toy world. For definiteness we refer to values in GeV: $m = 0.3$, $M_0 = 1$, g_0 will be varied between 1.5 and 5.

The propagator of the field S is modified via φ -meson loops and takes the form (at the resummed one-loop level; see Fig. 1)

$$\Delta = i[p^2 - M_0^2 + (\sqrt{2}g)^2 \Sigma_\Lambda(p^2)]^{-1} \quad (4)$$

where $\Sigma_\Lambda(p^2)$ is the one-loop contribution, which is regu-

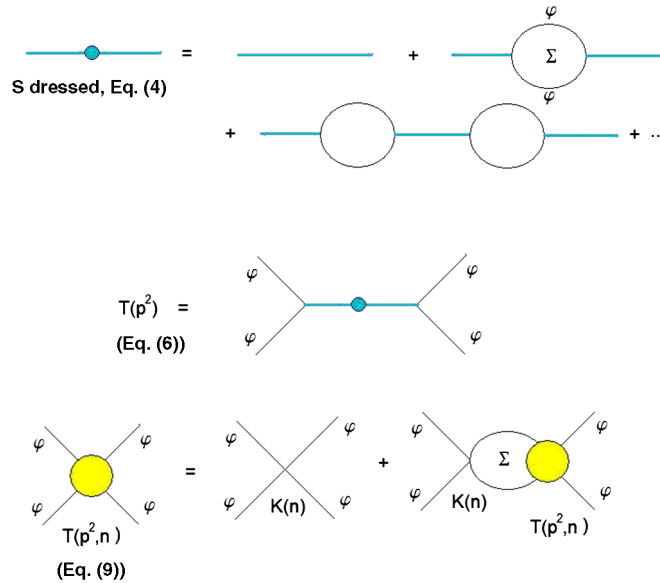


FIG. 1 (color online). Equations (4), (6), and (9) are depicted: In Eq. (4) the S resonance is dressed via loops of φ mesons. In Eq. (6) the T matrix is represented by an exchange of a dressed S meson. Finally, Eq. (9) represents a BS equation applied to the quartic terms of the Lagrangian $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$, in which the T matrix appears both on the left- and the right-hand sides of the equation.

larized via a 3D sharp cutoff Λ [63]. The dressed mass can be defined via the zero of the real part of Δ^{-1} , i.e.

$$M^2 - M_0^2 + (\sqrt{2}g)^2 \text{Re}\Sigma_\Lambda(M^2) = 0. \quad (5)$$

In the large- N_c limit $M \rightarrow M_0$. This is true whichever definition of the mass of the resonance is chosen. For finite N_c one has, in general, $M < M_0$ due to the loop corrections (see Ref. [61] for details). The T matrix for $\varphi\varphi$ scattering in the s channel upon one-loop resummation is depicted in Fig. 1 and reads [64]

$$T(p^2) = i(\sqrt{2}g)^2 \Delta = \frac{1}{-K^{-1} + \Sigma_\Lambda(p^2)}, \quad (6)$$

$$K = \frac{(\sqrt{2}g)^2}{M_0^2 - p^2}.$$

Note, the present focus is on the one-particle pole of the S resonance and its corresponding enhancement in the T matrix. Thus, for simplicity, we limit the study of two-body scattering to the exchange of one (dressed) meson S . Other diagrams, such as the exchange of two (or more) S mesons, are not considered here (see, for instance, Ref. [65]). A more refined and complete approach should also include the dressing of the φ propagator and of the $S\varphi^2$ vertex. All these complications, while important in a realistic treatment, can be neglected at this illustrative level.

We now turn to the development of a low-energy Lagrangian which involves only the light meson field φ . We assume that g_0 is not too large so that for $N_c = 3$ the mass M lies above the threshold $2m$. In this way one can integrate out the field S from $\mathcal{L}_{\text{toy}}$ and obtain a low-energy (le) effective Lagrangian for the $\varphi\varphi$ interaction valid up to $E_{\text{le}} \lesssim 2m < M_0$ [66]:

$$\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c) = -\frac{1}{2}\varphi(\square + m^2)\varphi + V, \quad V = \sum_{k=0}^{\infty} V^{(k)}, \quad (7)$$

$$V^{(k)} = L^{(k)} \varphi^2 (-\square)^k \varphi^2, \quad L^{(k)} = \frac{g^2}{2M_0^{2+2k}}. \quad (8)$$

The Lagrangian $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ contains only quartic terms of the kind φ^4 , $\varphi^2 \square \varphi^2$, $\dots$. $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ is the analogue of chiral perturbation theory or, more generally, of a low-energy Lagrangian in this simplified system. The fact that we know explicitly the form of $\mathcal{L}_{\text{toy}}(E_{\text{max}}, N_c)$ allows us to calculate the “low-energy constants” $L^{(k)}$ of Eq. (8). If the precise expression of $\mathcal{L}_{\text{toy}}(E_{\text{max}}, N_c)$ were unknown, then $L^{(k)}$ would also be unknown. Note, each $L^{(k)}$ scales as N_c^{-1} .

BS-inspired unitarization, way I.—As a first exercise let us consider the low-energy Lagrangian $\mathcal{L}_{\text{le}}$ up to a certain order n by approximating the potential to $V(n) = \sum_{k=0}^n V^{(k)}$. By performing a Bethe-Salpeter study with

this approximate potential (see Fig. 1), we obtain the following T matrix:

$$T(p^2, n) = -K(n) + K(n)\Sigma_\Lambda(p^2)T(p^2, n), \quad (9)$$

$$T(p^2, n) = \frac{1}{-K(n)^{-1} + \Sigma_\Lambda(p^2)},$$

$$K(n) = \frac{(\sqrt{2}g)^2}{M_0^2} \sum_{k=0}^n \left(\frac{p^2}{M_0^2}\right)^k, \quad (10)$$

where $K(n)$ is the bare tree-level amplitude corresponding to the sum of all the quartic terms up to order n .

Clearly, $T(p^2, n)$ is an approximate form of $T(p^2)$ of Eq. (6). The larger n , the better the approximation. Formally one has $\lim_{n \rightarrow \infty} T(p^2, n) = T(p^2)$. What we are doing is a dynamical reconstruction of the state S via a Bethe-Salpeter analysis applied to the low-energy

Lagrangian $\mathcal{L}_{\text{le}}$: We reobtain the state S which has been previously integrated out.

Let us keep n fixed and perform a large- N_c study of $T(p^2, n)$. Do we obtain the correct result, that is, $M = M_0$? The answer is no. In fact, in the large- N_c limit $K(n)$ scales as $1/N_c$ due to the dependence encoded in g , while $\Sigma_\Lambda(p^2)$ scales as N_c^0 (we assume that the cutoff does not scale with N_c [67]). In the large- N_c limit we obtain $T(p^2, n) \simeq -K(n)$. But $K(n)$ is a polynomial in p^2 and, for any finite n , does not admit poles for finite p^2 , but only for $p^2 \rightarrow \infty$. Thus, we find the incorrect result that in the large- N_c limit the mass of the dynamically reconstructed state is infinity. This is shown in Fig. 2 for a particular numerical choice.

Although our analysis has been applied to a simple toy model, the form of Eq. (9) is general. One has a polynomial form for $K(n)$ as function of p^2 and a mesonic loop $\Sigma_\Lambda(p^2)$ which is independent of N_c . Complications due to different quantum numbers do not alter the conclusion. We also note

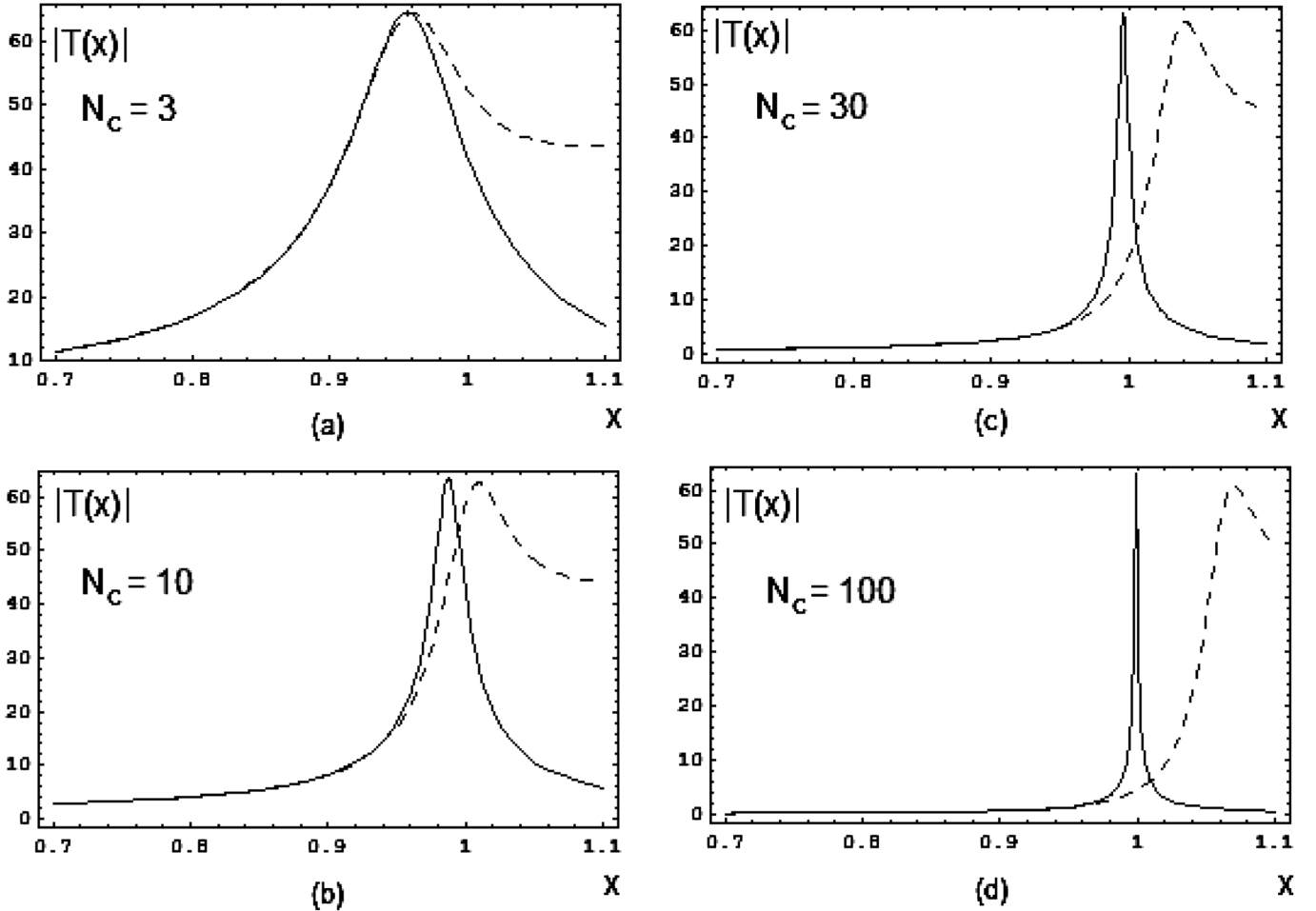


FIG. 2. Solid line: Absolute value of the full solution of the T matrix $|T(x)|$ with $x = \sqrt{p^2}$, Eq. (6). Dashed line: the approximate solution for $n = 10$, $|T(x, 10)|$, Eq. (10). The values (in GeV) $g_0 = \Lambda = 1.5$, $m = 0.3$, and $M_0 = 1$ are used. The dressed mass reads $M = 0.96$. The agreement is very good up to 1 GeV for $N_c = 3$ [panel (a)]. When increasing N_c the full solution is centered on M_0 and becomes narrower, as it should. On the contrary, the peak of the approximate solution increases and the width is only slightly affected by it. The approximate solution does not have the correct large- N_c expected behavior.

that these results are in agreement with the discussion of Ref. [68], where the scalar σ meson is first integrated out and then reconstructed in the framework of the linear sigma model.

IAM-inspired unitarization.—If we, instead, apply the IAM unitarization scheme to the $n = 1$ approximate form, we would obtain the correct result in the large- N_c expansion. In fact, in this case one schematically has (neglecting t and u channels)

$$T_{\text{IAM}} \simeq T_2(T_2 - T_4 - iT_2\sigma T_2)^{-1}T_2, \quad (11)$$

where $\sigma = \sqrt{\frac{p^2}{4} - m^2}$ in our notation. Since $T_2 = \frac{(\sqrt{2}g)^2}{M_0^2}$ and $T_4 = -\frac{(\sqrt{2}g)^2}{M_0^2}p^2$, one finds

$$T_{\text{IAM}} \simeq (\sqrt{2}g)^2(M_0^2 - p^2 - i(\sqrt{2}g)^2\sigma)^{-1}, \quad (12)$$

which represents a valid approximation of the full T matrix if g is not too large ($M \simeq M_0$). It is straightforward to see that the IAM approximation delivers the correct large- N_c result, namely, $M \rightarrow M_0$ and a width decreasing as $1/N_c$. Clearly one could repeat this study for increasing n , finding a better and better approximation of T .

BS-inspired unitarization, way 2.—Contrary to the BS-inspired unitarization described above (way 1), it is possible to follow a different BS-inspired approach which is in agreement with the large- N_c limit. For simplicity we discuss it in the explicit case $n = 1$ [69]. One has

$$K(1) = \frac{(\sqrt{2}g)^2}{M_0^2} \left(1 + \frac{p^2}{M_0^2}\right). \quad (13)$$

Now, instead of plugging $K(1)^{-1}$ directly into Eq. (10), we first invert it, obtaining the approximate form $K(1)_{\text{way 2}}^{-1}$ valid up to order $O(p^4/M_0^4)$:

$$K(1)_{\text{way 2}}^{-1} = \frac{M_0^2}{(\sqrt{2}g)^2} \left(1 - \frac{p^2}{M_0^2} + \dots\right). \quad (14)$$

The next step is to write the T matrix in terms of $K(1)_{\text{way 2}}^{-1}$:

$$T(p^2, 1)_{\text{way 2}} = \frac{1}{-K(1)_{\text{way 2}}^{-1} + \Sigma_\Lambda(p^2)} \quad (15)$$

$$\simeq \frac{(\sqrt{2}g)^2}{p^2 - M_0^2 + (\sqrt{2}g)^2 \Sigma_\Lambda(p^2)}. \quad (16)$$

Thus, this new approximate form derived from the BS equation is now in agreement with the large- N_c limit and is equivalent to the IAM-inspired unitarization approach described above. This shows an important fact in this discussion: It is not the BS method which fails in BS-way 1, but rather the adopted perturbative expansion. We could, as well, develop a second IAM-inspired unitarization which fails to reproduce the correct large- N_c results and that is equivalent to BS-way 1. From this perspective we can rearrange the unitarizations as “large- N_c correct”

(BS-way 2 and IAM) and “large- N_c violating” (BS-way 1 and IAM-way 2). The reason why we associate the names BS or IAM to the different unitarizations is simply due to the way the equations settle down in the different cases. It offers a simple mnemonic to their development.

By studying the large- N_c limit one can see more closely the relations between the two described BS unitarizations: In the case $n = 1$ and in the large- N_c limit, the T matrix in the first BS form reads $T_{\text{way 1}} \simeq -K(1) = -\frac{(\sqrt{2}g)^2}{M_0^2} \times (1 + \frac{p^2}{M_0^2})$, which obviously has no pole. In the second BS unitarization one has in the large- N_c limit $T_{\text{way 2}} \simeq -K(1)_{\text{way 2}} \simeq -\frac{(\sqrt{2}g)^2}{M_0^2} (1 - \frac{p^2}{M_0^2})^{-1}$, and the correct pole $p^2 = M_0^2$ is recovered.

It is, however, important to notice that—just as in the IAM case—at least two terms in the expansion of the amplitude K are necessary to perform this second BS unitarization. This is the reason why it cannot be applied in the case studied in the next subsection (Sec. IV B), where only the lowest term of the amplitude is kept.

B. BS equation with the lowest term only

In most studies employing the BS analysis, only the lowest term of the effective low-energy Lagrangian is kept. Within the present toy model it is not possible to reconstruct a resonance with mass $M > 2m$ with only the lowest term ($n = 0$) [70]. However, a simple modification of the model which allows for such a study is possible:

$$\mathcal{L}_{\text{toy}}^{\text{new}}(E_{\text{max}}, N_c) = \mathcal{L}_{\text{toy}}(E_{\text{max}}, N_c) + \frac{g^2}{2M_0^2} \varphi^4. \quad (17)$$

In this way an extra repulsion (whose quartic form is assumed to be valid up to E_{max}) has been introduced. The T matrix takes the form

$$T(p^2) = \frac{1}{-K^{-1} + \Sigma_\Lambda(p^2)}, \quad K = \frac{(\sqrt{2}g)^2}{M_0^2 - p^2} - \frac{(\sqrt{2}g)^2}{M_0^2}. \quad (18)$$

When deriving the low-energy Lagrangian, everything goes as before, but the $k = 0$ term is now absent:

$$V(n) = \sum_{k=1}^n V^{(k)}, \quad V^{(k)} = L^{(k)} \varphi^2(-\square)^k \varphi^2.$$

Note, in this case the $\varphi\varphi$ scattering vanishes at low momenta and in the chiral limit $m \rightarrow 0$ (just as the $\pi\pi$ scattering does in reality).

A study of the case $n = 1$ (corresponding to the first term only in the expansion) is now possible. We consider the following situation: Let the original Lagrangian $\mathcal{L}_{\text{toy}}^{\text{new}}$ of Eq. (17) be unknown. The low-energy potential at the lowest order reads $V \simeq V^{(1)} = L^{(1)} \varphi^2(-\square) \varphi^2$, but the low-energy coefficient $L^{(1)}$ is also unknown. Moreover,

from low-energy information only, one does not know the value of the cutoff Λ to be employed in mesonic loops: A new cutoff $\tilde{\Lambda}$, not necessarily equal to the original Λ , is also introduced as a free parameter. From the perspective of low-energy phenomenology, one writes down the following approximate form for the T matrix, which depends on two “free parameters,” $L^{(1)}$ and $\tilde{\Lambda}$:

$$\tilde{T}(p^2) = T(p^2, 1) = \frac{1}{-\tilde{K}^{-1} + \Sigma_{\tilde{\Lambda}}(p^2)}, \quad (19)$$

$$\tilde{K} = K(1) = 4L^{(1)}p^2.$$

The question is if it is possible to vary $L^{(1)}$ and $\tilde{\Lambda}$ in such a way that the approximate T matrix $\tilde{T}(p^2)$ reproduces the “full” result $T(p^2)$ of Eq. (6) between, say, $2m = 0.6$ GeV and 1.3 GeV for $N_c = 3$.

The answer is that this is generally possible, but the results for $L^{(1)}$ and $\tilde{\Lambda}$ vary drastically with the coupling constant g_0 in the original Lagrangian. In particular, if g_0 is small, a good fit implies a very large and unnatural value of $\tilde{\Lambda}$ [Fig. 3(a)]. For instance, for $g_0 = g(N_c = 3) = 1.5$ GeV the mass $M = 0.95$ GeV is only slightly shifted from the bare mass $M_0 = 1$ GeV. In this case the approxi-

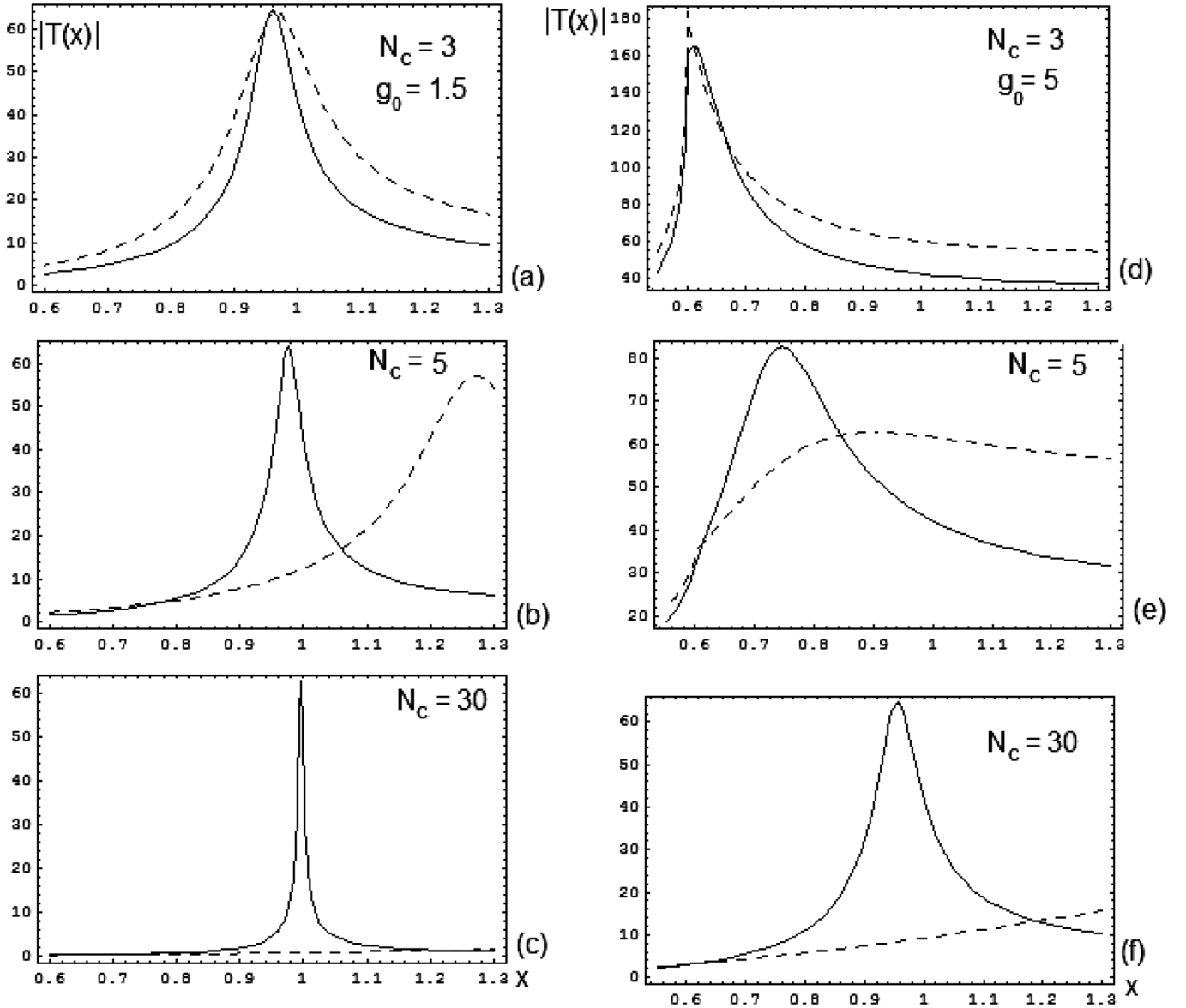


FIG. 3. Full $|T(x)|$ [solid line, Eq. (18) with $\Lambda = 1.5$ GeV] and approximate $|\tilde{T}(x)|$ [dashed line, Eq. (19)] in the cases $g_0 = 1.5$ GeV (left column) and $g_0 = 5$ GeV (right column) for different values of N_c . The values of $L^{(1)}$ and $\tilde{\Lambda}$, which determine the approximate dashed curve, are determined by fitting the approximate form to the full one in the $N_c = 3$ cases. One has $\tilde{\Lambda} \approx 15000\Lambda$ in the left column, and $\tilde{\Lambda} = \Lambda$ in the right column. As soon as N_c is increased, the approximate solution quickly fades out, while the real solution approaches M_0 where it becomes more and more peaked.

mate form $|\tilde{T}(p^2)|$ reproduces $|T(p^2)|$ only if $\tilde{\Lambda} \sim 10^4 \Lambda$ (astronomically high and seemingly unnatural from the perspective of the low-energy theory).

The situation changes completely if g_0 is large: It is possible to find a satisfactory description in which $\tilde{\Lambda} \sim \Lambda$. For instance, for $g_0 = 5$ GeV one has $M = 0.65$ GeV and the approximate $|\tilde{T}(p^2)|$ reproduces well $|T(p^2)|$ for $\tilde{\Lambda} = \Lambda$ [Fig. 3(d)].

In the first case the failure of the dynamical reconstruction with a meaningful value of the cutoff $\tilde{\Lambda}$ is due to the quantitative inappropriate behavior of the Bethe-Salpeter approach when only the first term is kept. In the second case a rather satisfactory description is possible for a meaningful value of the cutoff. In light of the results of the low-energy Lagrangian only, one could also propose the interpretation that the obtained state S is dynamically generated, and shall be regarded as a $\varphi\varphi$ molecular state. This is *not*, however, the correct interpretation in the present example. We know, in fact, that this state corresponds—by construction—to the original, preexisting, quarkoniumlike state S .

In both cases, as soon as we increase the number of colors, the approximate T matrix $|\tilde{T}(p^2)|$ and the full $|T(p^2)|$ show a completely different behavior [Figs. 3(b), 3(c), 3(e), and 3(f)]: While the peak of $|T(p^2)|$ approaches $M_0 = 1$ GeV and becomes narrower according to the correct large- N_c limit of the S meson, the dynamically reconstructed state fades out, because of the *incorrect* behavior of BS unitarization with large N_c . This is clearly visible from the interaction term $V^{(1)} = L^{(1)}\varphi^2(-\square)\varphi^2$, because $L^{(1)}$ scales as N_c^{-1} . However, although the interaction term disappears with large N_c , the state S is still the original quark-antiquark state. This example shows that the reconstruction of the state S is not possible in the large- N_c limit, but this does not mean that S is a dynamically generated state of molecular type. Note, this is just a subcase of the previous general discussion on large- N_c dependence: The fact that only one term is kept generates a much faster “fading out” of the reconstructed state; compare Figs. 2 and 3.

In the previous subsection it was shown that—while a straight application of the BS equation is at odds with the large- N_c limit—a second BS unitarization allows for a correct description of the large- N_c limit. The second BS unitarization is not, however, applicable in the present case. In fact, *at least* two terms in the expansion of $K(n)$ are needed to follow it. If only the lowest term is kept, as done here with the term $n = 1$ in Eq. (19), this is no longer feasible. This is similar to the fact that the IAM method also needs at least two terms in the expansion of the amplitude K in order to be applicable [71].

C. Analogy with the real world

The original toy Lagrangian $\mathcal{L}_{\text{toy}}(E_{\text{max}}, N_c)$ of Eq. (3) is assumed to be valid up to an energy $E_{\text{max}} \gg M_0$. The

corresponding low-energy Lagrangian $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ of Eq. (7)—obtained by integrating out the S field—is valid up to an energy $E_{\text{le}} \ll M_0$. When unitarizing $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$, one can enlarge the validity of the low-energy theory up to M_0 and then infer the existence of the resonance S with mass $M < M_0$. However, if no other input is known, the nature of the state S cannot be further studied; see the general discussion of point (d) in Sec. II.

This situation is similar to example (c) in Sec. II: The Fermi Lagrangian $\mathcal{L}_F$ alone does not allow one to deduce the nature of the W meson, even if the existence of the latter is inferred by unitarization arguments applied to $\mathcal{L}_F$. It is also similar to the cases studied in Sec. III C: When a resonance is obtained by unitarizing a low-energy mesonic Lagrangian, (at first) no statement about its nature can be made.

Further information is needed: In the case of the W meson, the full electroweak Lagrangian $\mathcal{L}_{\text{EW}}$ is known and leads to the straightforward conclusion that the W meson is not a dynamically generated state, but a fundamental field of the standard model. In the framework of the toy model, this corresponds to the knowledge of the “full Lagrangian” $\mathcal{L}_{\text{toy}}(E_{\text{max}}, N_c)$ of Eq. (3). The “quarkonium” nature of S can then be easily deduced.

In the case of low-energy mesonic theories discussed in Sec. III C, the full hadronic Lagrangian is not known. The only additional knowledge is the large- N_c scaling of the low-energy constants of the low-energy Lagrangian(s). In the framework of the toy model, this corresponds to the knowledge of the low-energy Lagrangian $\mathcal{L}_{\text{le}}(E_{\text{le}}, N_c)$ of Eq. (7) (up to a certain n) together with the scaling of the quantities $L^{(k)}$ in Eq. (8). The latter additional knowledge can lead to the correct conclusions about the nature of the S meson, although—as discussed in Sec. IV A—care is needed when the BS method is chosen.

Moreover, as further studied in Sec. IV B, when only the *lowest term* of the low-energy Lagrangian is kept, it is not possible to reproduce the correct large- N_c behavior of the resonance S . Although the “dynamical reconstruction” of the state S is possible, the state S “looks like” a molecular state which fades out in the large- N_c limit. This, however, is not true: In fact, we know from the very beginning that the state S corresponds “by construction” to a quark-antiquark state.

Although this discussion is based on toy models and the real world is much more complicate than this, the same qualitative picture can hold in low-energy QCD. In fact, the use of the BS equation in the literature is often limited to the lowest term of a low-energy Lagrangians for $\pi\pi$, $\rho\rho$, ... interactions. In our view, such low-energy Lagrangians emerge upon integrating out all the heavier fields in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\text{max}}, N_c = 3)$, in which tensor, axial-vector, and scalar quark-antiquark fields must exist below 2 GeV. Then, the use of the BS equation, similarly to the dynamical reconstruction of S in this simple example, leads to the

dynamical reconstruction of the axial-vector, tensor, and scalar mesons above 1 GeV: They are preexisting, quark-antiquark states, which are reobtained from low-energy Lagrangians via unitarization methods. Future unitarization studies, involving the leading and the next-to-leading terms in the effective Lagrangians, may shed light on this point.

V. CONCLUSIONS AND OUTLOOK

In this work we studied the issue of dynamical generation, both in a general context and in low-energy QCD. A dynamically generated resonance has been defined as a state which does not correspond to any of the fields of the original Lagrangian describing the system up to a certain maximal energy $E_{\max}$, provided that its mass lies below this maximal energy. This discussion also offered us the possibility to distinguish, in principle, tetraquark from mesonic molecular states in low-energy QCD: While the former are fundamental and shall be included as bare fields in the (yet-unknown) hadronic Lagrangian $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c = 3)$, this is not the case for the latter. Note, $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c = 3)$ represents the complete hadron theory valid up to $E_{\max} \approx 2$ GeV.

In the application to the hadronic world, we also discussed dynamical reconstruction of resonances: These are resonances which are obtained via unitarization methods from low-energy effective Lagrangians, but still represent fundamental fields (such as quark-antiquark states) in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c = 3)$. Note, the low-energy effective Lagrangians can be seen as a result of integrating out heavier (quarkonia, glueballs, etc.) fields representing intrinsic, fundamental states in $\mathcal{L}_{\text{eff}}^{\text{had}}(E_{\max}, N_c = 3)$. In the scenario of dynamical reconstruction, one reconstructs these heavier resonances by unitarizing the appropriate low-energy Lagrangian.

Within a simple toy model, these issues have been examined. This model consists of two fields, φ and S , with the latter being heavier and with a nonzero decay width into $\varphi\varphi$. We introduced a large- N_c dependence

which mimics that of quarkonium states in QCD. The field S was first integrated out and the emerging low-energy interaction Lagrangian involving only the field φ was derived. Out of this, the state S was dynamically reconstructed.

In order to do this, we have used a unitarization inspired by the Bethe-Salpeter equation, and we have shown that the original, quarkoniumlike large- N_c behavior of S cannot be reproduced if only the lowest term in the effective Lagrangian is kept. (Note, when more terms are kept this problem can be easily solved and the large- N_c result is correct also within the BS approach. The problem is not the latter but the adopted perturbative expansion; see Sec. IV A). We then proposed that a similar, although more complicated, dynamical reconstruction mechanism takes place for tensor, axial-vector, and scalar mesons above 1 GeV: These resonances, studied in recent works, can be interpreted as fundamental quark-antiquark states, which are reobtained when unitarizing low-energy effective Lagrangians. In this scenario there is no conflict between the “old” quark model assignments and recent developments, because they would both represent a dual description of the same, preexisting quark-antiquark resonances. This interpretation, although not yet conclusive, represents a possibility which deserves further study.

Dynamical reconstruction can also hold for light scalar mesons below 1 GeV, if they form a quarkonium (quite improbable) or a tetraquark nonet. The situation in this case is, as discussed in the text, still unclear. In this work we limited the study to the light mesonic sector, but the present discussion about dynamical generation/reconstruction can also hold, with due changes, in the baryon and the heavy quark sectors.

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Spectral functions of scalar mesons

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In this work we study the spectral functions of scalar mesons in one- and two-channel cases by using nonlocal interaction Lagrangian(s). When the propagators satisfy the Källen-Lehman representation, a normalized spectral function is obtained, allowing one to take into account finite-width effects in the evaluation of decay rates. In the one-channel case, suitable to the light σ and k mesons, the spectral function can deviate consistently from a Breit-Wigner shape. In the two-channel case with one subthreshold channel, the evaluated spectral function is well approximated by a Flatté distribution; when applying the study to the $a_0(980)$ and $f_0(980)$ mesons, the tree-level forbidden KK decay is analyzed.

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I. INTRODUCTION

The scalar mesons below 2 GeV have been a center of debate for many years [1–4]. More states than expected from the quark-antiquark assignment are reported in Particle Data Group (PDG) [5], leading to the introduction of a scalar glueball [6], tetraquark states [7], and mesonic molecules [8]. In particular, the scalar resonances below 1 GeV have appealing characteristics, such as the reversed level ordering of masses, expected from tetraquark states [3,7,9–11]. In turn, this scenario implies that quarkonia lie between 1 and 2 GeV. A complication in the analysis of scalar states is mixing: between 1 and 2 GeV a quarkonia-glueball mixing in the isoscalar sector is considered, for instance, in Ref. [12]. Mixing among tetraquark states below 1 GeV and quarkonia above 2 GeV is studied in Refs. [13–15], where, however, the results do not coincide: while a large mixing is found in Ref. [14], a negligible mixing is the outcome of Ref. [15]. It should be stressed that different interpretations of scalar state are possible: a nonet of scalar quarkonia is settled below 1 GeV in Ref. [16] in agreement with the linear σ model and the Nambu Jona-Lasinio (NJL) model, while in Ref. [17] a broad glueball, to be identified with $f_0(600)$, is proposed. We refer the reader to Refs. [1–3,10] for a discussion of arguments in favor of and against the outlined assignments.

Studies on scalar mesons have been extensively performed by using chiral perturbation theory [18], where a scalar resonance at about 440 MeV is inferred out of pion-pion scattering. A full nonet of molecularlike scalar states is generated in the unitarized chiral perturbation theory of Ref. [19]. In particular, Pelaez [20] studied the large- N_c dependence of the light scalar resonances, finding that they do not scale as quarkonia but agree with a molecular or tetraquark composition (see, however, also the discussions in Ref. [21]).

In the present paper, we concentrate on an important aspect of light scalar resonances, namely, the form of their spectral functions, in a simple theoretical context. In this study, relevant to both quarkonium and tetraquark assignment of light scalars [22], effects arising from loops of pseudoscalar mesons are considered: this leads to parametrizations of spectral functions beyond the (usually employed) Breit-Wigner and

Flatté distributions and allow us to include finite-width effects in the evaluation of decay rates. In particular, we consider the following physical scenarios: (i) the case of a broad scalar resonance, strongly coupled to one decay channel, such as the $\sigma \equiv f_0(600)$ in the pion-pion decay mode, for which the spectral function can deviate substantially from the Breit-Wigner form; (ii) the case of two channels, one of which is subthreshold for the mass and thus forbidden at the tree level, as the $\bar{K}K$ decay mode for the resonances $f_0(980)$ and $a_0(980)$. In the latter case, a comparison with the usually employed Flatté distribution is performed.

A crucial aspect of our study is to consider a nonlocal interaction Lagrangian, which implies a ultraviolet regularization and directly affects the real and imaginary part of the mesonic loop. In a phenomenological perspective, it is reasonable that the mesonic states in the loop cannot have indefinitely high virtual momenta which are naturally limited due to the finite range of the meson-meson interaction. We also show that the dependence on the chosen cutoff function, specifying the delocalized interaction, and on the specific value of the cutoff is mild. We also compare the use of a nonlocal Lagrangian with other approaches investigated in the literature.

The key quantity of the discussion is the propagator of scalar resonances dressed by mesonic loops in one or more channels. When the Källen-Lehmann representation is satisfied, as verified at the one-loop level in the case of light scalar mesons for large ranges of parameters [22], the spectral function (proportional to the imaginary part of the propagator) is correctly normalized and is interpreted as a “mass distribution” for the scalar state. A general definition of the decay of a scalar state, which involves the obtained mass distribution and does not require a study of the properties of the propagator in the complex plane, is then possible. In this way, one takes into account in a consistent fashion finite-width effects for the decay, hence allowing one to study deviations from the usually employed tree-level formula for decay rates. Furthermore, the fulfillment of the Källen-Lehmann representation offers a criterion to delimit the validity of our one-loop study: as soon as violations appear (generally for large coupling constants) the obtained spectral functions are no longer usable.

To render the paper easily understandable and self-contained, we start in Sec. II with the one-channel case by recalling the basic definitions and properties, then we apply the study to the scalar sigma σ and kaon k resonances: the corresponding spectral function shows consistent deviations from the usual Breit-Wigner one. In Sec. III, we turn to the two-channel case, with particular attention to the resonances $a_0(980)$ and $f_0(980)$, their decay rates and spectral functions in comparison with the Flatté distribution [23,24]. Implications of the results in view of a nonet of tetraquark states below 1 GeV is discussed. In Sec. IV, we derive our conclusions, emphasizing as in Ref. [22] that the use of propagators fulfilling the Källen-Lehmann representation, which implies normalized distributions and a correct definition of decay rates, should be preferable both in theoretical and experimental work.

II. SCALAR SPECTRAL FUNCTION: ONE-CHANNEL CASE

A. Definitions and properties

We consider the scalar fields S and φ described by the Lagrangian

$$\mathcal{L}_S^1 = \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M_0^2 S^2 + \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2 \varphi^2 + gS\varphi^2. \quad (1)$$

In the limit $g = 0$, the propagator of the field S reads

$$\Delta_S(p) = \frac{1}{p^2 - M_0^2 + i\varepsilon}, \quad g = 0. \quad (2)$$

We intend to study the modification to $\Delta_S(p)$ when $g \neq 0$, which arises by considering the loop diagram of Fig. 1, and how this contribution affects the decay mechanism $S \rightarrow \varphi\varphi$. We recall that at the tree level, the decay width reads

$$\Gamma_{S\varphi\varphi}^{l-1}(M_0) = \frac{p_{S\varphi\varphi}}{8\pi M_0^2} [g_{S\varphi\varphi}]^2 \theta(M_0 - 2m),$$

$$p_{S\varphi\varphi} = \sqrt{\frac{M_0^2}{4} - m^2}, \quad g_{S\varphi\varphi} = \sqrt{2}g, \quad (3)$$

where $\theta(x)$ is the step function and the factor $\sqrt{2}$ in the amplitude $g_{S\rightarrow\varphi\varphi}$ takes into account that the final state consists of two identical particles. In general, the symbol p_{SAB} is

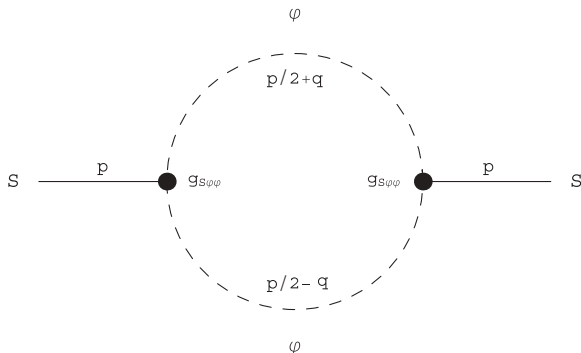


FIG. 1. Mesonic loop.

understood to be the expression

$$p_{\text{SAB}} = \frac{1}{2M_S} \sqrt{M_S^4 + (M_A^2 - M_B^2)^2 - 2(M_A^2 + M_B^2)M_S^2}, \quad (4)$$

i.e., the momentum of the outgoing particle(s).

At tree level, the particle S is treated as stable. However, the very fact that the decay $\Gamma_{S\varphi\varphi}^{l-1} \neq 0$ for $M_0 > 2m$ means that S is not stable and cannot be considered as an asymptotic state of the Lagrangian $\mathcal{L}_S^1$. The tree-level expression $\Gamma_{S\varphi\varphi}^{l-1}$ is only valid in the limit $g \rightarrow 0$. The evaluation of the loop of Fig. 1 offers a way to define and interpret the decay $S \rightarrow \varphi\varphi$ as we describe in the following. The modified propagator of S is obtained by (re)summing the loop diagrams of Fig. 1, that is,

$$\Delta_S(p^2) = \frac{1}{p^2 - M_0^2 + g_{S\varphi\varphi}^2 \Sigma(p^2) + i\varepsilon}, \quad (5)$$

where the self-energy $\Sigma(p^2)$ reads

$$\Sigma(p^2) = -i \int \frac{d^4 q}{(2\pi)^4} \times \frac{1}{[(q+p/2)^2 - m^2 + i\varepsilon][(q-p/2)^2 - m^2 + i\varepsilon]}. \quad (6)$$

The integral defining $\Sigma(p^2)$ is, as known, logarithmic divergent. Our intention is to consider the Lagrangian $\mathcal{L}_S^1$ as an effective low-energy description of the fields S and φ , and not as a fundamental theory valid up to indefinitely high mass scales. We do not apply the renormalization scheme to $\mathcal{L}_S^1$, but we introduce a regularization function $f_\Lambda(q)$ which depends on a cutoff scale Λ for the large momenta. The self-energy $\Sigma(p^2)$ is then modified to

$$\Sigma(p^2) = -i \int \frac{d^4 q}{(2\pi)^4} \times \frac{f_\Lambda^2(q^0, \vec{q})}{[(q+p/2)^2 - m^2 + i\varepsilon][(q-p/2)^2 - m^2 + i\varepsilon]}. \quad (7)$$

When choosing $f_\Lambda(q) = f_\Lambda(q^2)$, the covariance of the theory is preserved, otherwise it is lost. Indeed, in many calculations related to mesonic loops, a regularization of the kind $f_\Lambda(q) = f_\Lambda(\vec{q}^2)$ is chosen, which leads to simple expressions for the self-energy contribution but breaks covariance explicitly, and thus this regularization is strictly valid only in the rest frame of the decaying particle. In particular, the three-dimensional cutoff $f_\Lambda(\vec{q}^2) = \theta(\Lambda^2 - \vec{q}^2)$ is often used. Appendix A provides a closer analysis of the self-energy $\Sigma(p^2)$ and presents the case of unequal masses circulating in the loop. The interaction strength among light mesons is suppressed for distances larger than $l \sim 0.5\text{--}1$ fm: in this particular physical example, it is then natural to implement a cutoff $\Lambda \sim 1/l$, which varies between 1 and 2 GeV.

The cutoff function $f_\Lambda(q)$ is not present in the Lagrangian $\mathcal{L}_S^1$ of Eq. (1). In this sense, the Lagrangian is incomplete because it does not specify how to cut the high momenta. One can take into account $f_\Lambda(q)$ already at the Lagrangian level by

rendering the interaction term nonlocal:

$$(\mathcal{L}_S^1)_{\text{int}} = gS\varphi^2 \rightarrow gS \int d^4y \varphi(x+y/2) \times \varphi(x-y/2)\Phi(y). \quad (8)$$

The Feynman rule for the three-leg vertex is modified as

$$ig \rightarrow igf_\Lambda \left(\frac{q_1 - q_2}{2} \right), \quad f_\Lambda(q) = \int d^4y \Phi(y) e^{-iyq}, \quad (9)$$

where q_1 and q_2 are the momenta of the two particles φ . The function $f_\Lambda(q)$ enters directly into the expression of all amplitudes. In particular, the self-energy contribution of Eq. (7) is now obtained by application of the (modified) Feynman rules to the loop diagram of Fig. 1. Indeed, the delocalization of the interaction term also induces a change of the tree-level result for the decay, which becomes [for the case $f_\Lambda(q) = f_\Lambda(\vec{q}^2)$]

$$\Gamma_{S\varphi\varphi}^{t-1}(M_0) = \frac{P_{S\varphi\varphi}}{8\pi M_0^2} [g_{S\varphi\varphi} f_\Lambda(\vec{q}^2 = p_{S\varphi\varphi}^2)]^2 \theta(M_0 - 2m),$$

$$g_{S\varphi\varphi} = \sqrt{2}g, \quad (10)$$

that is, the function $f_\Lambda(\vec{q}^2)$ is explicitly present in the tree-level decay expression and can be interpreted as a phenomenological form factor.¹

If a step function is used, the local tree-level expression of Eq. (3) is recovered, provided that the cutoff Λ is large enough. In this work, we use the following cutoff function:

$$f_\Lambda(q) = f_\Lambda(\vec{q}^2) = (1 + \vec{q}^2/\Lambda^2)^{-1}. \quad (11)$$

With this choice, the Fourier transform $\Phi(y)$, see Eq. (9), takes the form $\delta(y^0) \exp[-|\vec{y}| \Lambda]/|\vec{y}|$, thus decreasing rapidly for increasing distance of the two interacting mesons φ . The interaction range l is of the order Λ^{-1} , as discussed above, based on general dimensional grounds. At each step of the forthcoming study, we employed also different forms of $f_\Lambda(q)$, finding that the dependence on the precise form of $f_\Lambda(q)$ affects only slightly the results. Notice that in Ref. [25] a similar equation to Eq. (10) [where $s = \vec{q}^2$ and $f_\Lambda = G(s)$ in the notation of Ref. [25]] represents the starting point of the analysis. The function $G(s)$ in the above-cited works is taken to be a Gaussian, Λ is of the order of 1 GeV. The present approach shows the link between such a form factor $f_\Lambda = G(s)$ and a nonlocal Lagrangian. However, we do not concentrate as in Ref. [25] on scattering amplitudes but on spectral functions and decay widths. At the same time, we do not relate the imaginary and real parts of the propagator via the Källen-Lehmann dispersion relation, but we evaluate them independently, and subsequently we check numerically if it is satisfied, as detailed in the following discussion.

Let us now turn to the self-energy $\Sigma(p^2)$. A general property for $\Sigma(p^2)$ follows from the optical theorem

$$I(x) = g_{S\varphi\varphi}^2 \text{Im}[\Sigma(x = \sqrt{p^2})] = x \Gamma_{S\varphi\varphi}^{t-1}(x). \quad (12)$$

¹For a covariant vertex function $f_\Lambda(q)$ the decay amplitude takes the form $[g_{S\varphi\varphi} f_\Lambda(q^0 = 0, \mathbf{q}^2 = p_{S\varphi\varphi}^2)]$.

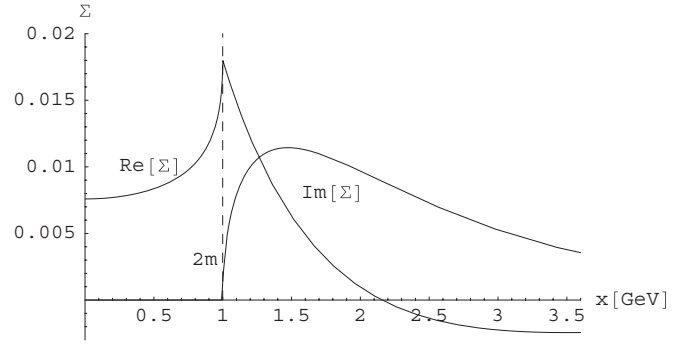


FIG. 2. Real and imaginary parts of the mesonic loop for $m = 0.5$ GeV and $\Lambda = 1.5$ GeV.

The imaginary part of the self-energy diagram is zero for $0 < x < 2m$ and nonzero starting at threshold. The real part

$$R(x) = g_{S\varphi\varphi}^2 \text{Re}[\Sigma(x = \sqrt{p^2})] \quad (13)$$

is nonzero below and above threshold. In Fig. 2 the functions $R(x)$ and $I(x)$ are plotted using Eq. (11). A particular choice is made for the parameters $m = 0.5$ GeV and $\Lambda = 1.5$ GeV (of the order of physical cases studied later). Anyway, the plotted functions are qualitatively similar for large ranges of parameters. As noticeable, $R(x)$ is continuous but not derivable in $x = 2m$. It has a cusp at $x = 2m$: the left derivative is $+\infty$, while the right derivative is finite and negative.

In terms of the two functions $R(x)$ and $I(x)$, the propagator of Eq. (2) reads

$$\Delta_S(x) = \frac{1}{x^2 - M_0^2 + R(x) + iI(x) + i\varepsilon}. \quad (14)$$

We define the (Breit-Wigner) mass M for the scalar field S as the solution of the equation

$$M^2 - M_0^2 + R(M) = 0. \quad (15)$$

When the function $R(M)$ is positive, which is usually the physical case (Fig. 2), the dressed mass M is smaller than the bare mass M_0 , showing that the quantum fluctuations tend to lower it.

We now turn to the spectral function $d_S(x)$ of the scalar field S related to the imaginary part of the propagator as

$$d_S(x) = \frac{2x}{\pi} \left| \lim_{\varepsilon \rightarrow 0} \text{Im}[\Delta_S(x)] \right|. \quad (16)$$

In the limit $g \rightarrow 0$, we obtain the desired spectral function $d_S(x) = \delta(x - M_0)$. The normalization of $d_S(x)$ holds for each g , i.e.,

$$\int_0^\infty d_S(x) dx = 1. \quad (17)$$

The latter equation is a consequence of the Källen-Lehmann representation

$$\Delta_S(x) = \int_0^\infty dy \frac{2y}{\pi} \frac{-\text{Im}[\Delta_S(y)]}{y^2 - x^2 + i\varepsilon} \quad (18)$$

when taking the limit $x \rightarrow \infty$. Equations (17) and (18) hold in general for the full propagator. In our case, we check

numerically the validity of the normalization condition (17) at the one-loop level of Fig. 1. We find that it is fulfilled to a high level of accuracy for large ranges of parameters, see also the discussion in Ref. [22] and in the next section.

Let us consider $d_S(x)$ in the two interesting cases $M < 2m$ and $M > 2m$. If $M < 2m$, Eq. (16) becomes

$$d_S(x) = Z\delta(x - M)\theta(2m - x) + \frac{2x}{\pi} \frac{I(x)}{(x^2 - M_0^2 + R(x))^2 + I(x)^2}, \quad (19)$$

where

$$Z = \left[1 + \frac{1}{2M} \left(\frac{dR}{dx} \right)_{x=M} \right]^{-1}. \quad (20)$$

When $M < 2m$, the constant Z is usually reabsorbed into the definition of the wave function renormalization, hence recovering the free propagator properly normalized as $(p^2 - M^2 + i\varepsilon)^{-1}$, corresponding to $d_S(x) = \delta(x - M)$ for $x < 2m$ as in the free case $g = 0$. Thus, we still have a stable particle with dressed mass M instead of M_0 . Notice that $0 < Z < 1$ because $R'(M)$ is a positive number: the quantity $(1 - Z)$ can be interpreted as the amount of virtual clouds of 2φ contributing to the wave function.

If $M > 2m$, the spectral function reads

$$d_S(x) = \frac{2x}{\pi} \frac{I(x)}{(x^2 - M_0^2 + R(x))^2 + I(x)^2}. \quad (21)$$

No δ functions are present, but typically a peaked distribution $d_S(x)$ is obtained, corresponding to a physical resonance. The mass M is not the maximum of the $d_S(x)$, although in general it is very close to it. Consistent deviations can appear when M is close to threshold and for a large coupling constant; see next subsection for a more detailed discussion of this point. Notice moreover that $d_S(x)$ is zero for $x < 2m$.

We plot the typical behavior of the spectral function in both cases $M < 2m$ and $M > 2m$ in Fig. 3. We used the values $M = 0.9$ and $M = 1.3$ GeV corresponding to the two cases below and above the threshold, $m = 0.5$ GeV as before, and $g_{S\phi\phi} = 3$ GeV. The value of Z , for the subthreshold case,

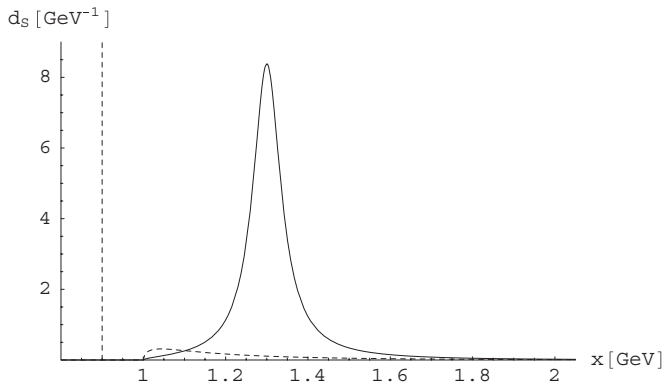


FIG. 3. Spectral functions in the cases $M < 2m$ (dashed line) and $M > 2m$ (continuous line). The coupling constant is $g_{S\phi\phi} = 3$ GeV, the two mass values chosen are $M = 0.9$ and $M = 1.3$ GeV, and $m = 0.5$ GeV.

is ~ 0.9 , and we have numerically verified that the spectral functions are normalized in both cases.

When $M > 2m$, the function $d_S(x)$ can be interpreted as the mass distribution of the resonance; see also Appendix B for an intuitive discussion about this point. We then define the decay rate for the process $S \rightarrow \varphi\varphi$ by implementing the distribution $d_S(x)$, and thus including finite width effects, as

$$\Gamma_{S\varphi\varphi} = \int_0^\infty dx d_S(x) \Gamma_{S\varphi\varphi}^{t-1}(x). \quad (22)$$

This formula reduces to the tree-level amplitude $\Gamma_{S\varphi\varphi}^{t-1}(M_0)$ of Eq. (10) in the limit of small g :

$$\Gamma_{S\varphi\varphi}^{t-1}(M_0) \simeq \Gamma_{S\varphi\varphi} \quad \text{for } g \rightarrow 0. \quad (23)$$

Notice that in this limit, $M \rightarrow M_0$. However, even for finite g , when $M \neq M_0$, the formula $\Gamma_{S\varphi\varphi} \simeq \Gamma_{S\varphi\varphi}^{t-1}(M)$ offers a first approximation to the decay width of the state as long as the distribution is peaked, i.e., the scalar state S is not too broad. Notice that Eq. (22) does not in general coincide with the standard width obtained by complex analysis search of pole positions, see Ref. [26]. In fact, in our study, only the real valued x variable is considered.

The definition Eq. (22) for the decay $S \rightarrow \varphi\varphi$ is thus a generalization of the tree-level result of Eq. (10) and takes automatically into account that the state S has a finite width parametrized by the mass distribution $d_S(x)$, which naturally arises by considering the self-energy of the scalar propagator. Notice that the real part of the propagator is necessary in order for Eq. (17) to hold: its neglect would spoil the correct normalization.

Evaluating the real and imaginary parts at $x = M$ and neglecting their x dependence, the distribution (21) is approximated by

$$d_S^{\text{bw}}(x) \simeq \frac{2M}{\pi} \frac{I(M)}{(x^2 - M^2)^2 + (I(M))^2}, \quad (24)$$

which is the relativistic Breit-Wigner distribution for the resonance S , usually employed in theoretical and experimental studies. However, the distribution $d_S^{\text{bw}}(x)$ neglects the real part of the loop diagram, and consequently the normalization of Eq. (17) does not hold, implying that $d_S^{\text{bw}}(x)$ has to be normalized by hand. At the same time, the mass M does not coincide with the maximum of $d_S(x)$. Thus, we insist on that the usage of automatically normalized distribution emerging from propagators fulfilling Källén-Lehmann should be preferable.

B. Application to the light scalar mesons σ and k

An interesting example for the one-channel case is the decay of the scalar meson $\sigma \equiv f_0(600)$. As reported by the PDG [5], experimental data are affected by large uncertainties both for the value of the mass, $M_\sigma = 0.4\text{--}1.2$ GeV, and the value of the Breit-Wigner width, $\Gamma_\sigma = 0.6\text{--}1$ GeV. The dominant channel, which we will consider here, is the decay into two pions, for which $M > 2m$.

By applying the formulas of Sec. II A, we show in the left panel of Fig. 4 the spectral functions $d_\sigma(x)$ of the σ resonance

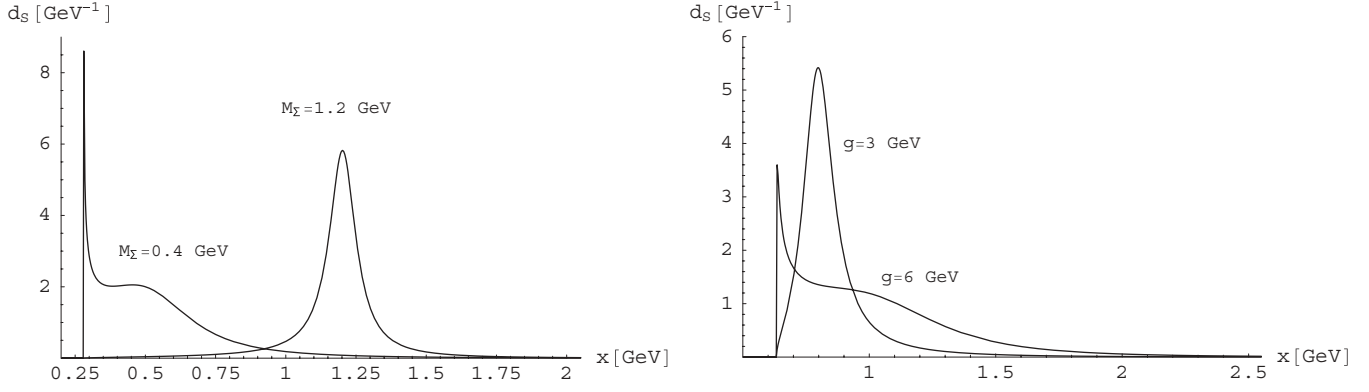


FIG. 4. Spectral functions of σ and k . Left panel: $\Lambda = 1.5$ GeV, $g = 3$ GeV, $M_\sigma = 0.4$ GeV with a correspondent $\Gamma = 0.242$ GeV, and $M_\sigma = 1.2$ GeV with a correspondent $\Gamma = 0.113$ GeV. Right panel: $M_k = 0.8$ GeV, $\Lambda = 1.5$ GeV, $g = 3$ GeV with a correspondent $\Gamma = 0.112$ GeV, and $g = 6$ GeV with a correspondent $\Gamma = 0.382$ GeV.

for the two boundary cases of PDG, namely, $M_\sigma = 0.4$ and $M_\sigma = 1.2$ GeV, for the coupling constant $g_{\sigma\pi\pi} = 3$ GeV.

The spectral function assumes different shapes for different values of the mass. While for $M_\sigma = 1.2$, far from the threshold, the spectral function has a regular Breit-Wigner-like form, in the case $M_\sigma = 0.4$ GeV, a distorted shape, with a narrow peak just above threshold, is visible.² The employed value of the coupling constant, $g_{\sigma\pi\pi} = 3$ GeV, serves as illustration and actually corresponds to a somewhat too narrow width. The increase of $g_{\sigma\pi\pi}$ leads, however, outside the range of validity of the normalization of Eq. (17) at the one-loop level; see below.

The description of the scalar kaonic resonance k follows the same line [27]. As shown in the right panel of Fig. 4, a strong deviation from the Breit-Wigner form is obtained when $g_{k\pi K} = 6$ GeV, corresponding to $\Gamma_k = 0.38$ GeV, while a less distorted shape is found for $g_{k\pi K} = 3$ GeV, for which $\Gamma_k = 0.11$ GeV.

At this point, a short discussion on the definition of the mass of an unstable particle is needed. In the Breit-Wigner scheme, the mass of the particle is the value corresponding to the maximum of the spectral function, and it is one parameter of the distribution (the second one is of course the width). In our scheme, using the spectral functions coming from the loop evaluation this is no longer the case: the mass M defined in Eq. (15) is again a parameter of the distribution (together with the coupling constant g), but it does not coincide with the maximum of the distribution. For $M_\sigma = 1.2$ GeV (far from threshold), the maximum of the spectral function occurs at 1.202 GeV, thus only slightly shifted from the mass; however, when $M_\sigma = 0.4$ GeV, the maximum of the spectral function (apart from the threshold enhancement peak) occurs at sizably larger values with respect to the mass, here 0.454 GeV.³ (Notice

also that in the latter case, the bare mass M_0 is 0.614 GeV, thus implying a strong influence of the pion loop to the σ mass, see also Appendix A for a comparison of different “masses”.) Indeed, although the mass M being the zero of the real part of the inverse propagator, see Eq. (15), is referred to as a Breit-Wigner mass [25], the best fit to the full spectral function $d_\sigma(x)$ by using a Breit-Wigner form is obtained for a Breit-Wigner mass M_{BW} coinciding with the maximum of the distribution. It is also remarkable that in some cases, the spectral function does not have a maximum, apart from the threshold enhancement peak, as we can see for the k meson in the right panel of Fig. 4 for $g_{k\pi K} = 6$ GeV.

We now study closer the decay process $\sigma \equiv f_0(600) \rightarrow \pi\pi$ using Eq. (22) which implements the spectral function $d_\sigma(x)$. In Fig. 5, we compare the full and tree-level decay rates for different values of the cutoff and of the coupling constants (a mass $M_\sigma = 600$ MeV is used).

As expected, there is not a strong dependence on the choice of the cutoff. Moreover, the results obtained with our formulas are well in agreement with the tree-level results for small values of g (since the spectral function tends to a δ function). Nonnegligible differences instead occur for the larger values of g . This is due to a different analytic dependence of the two

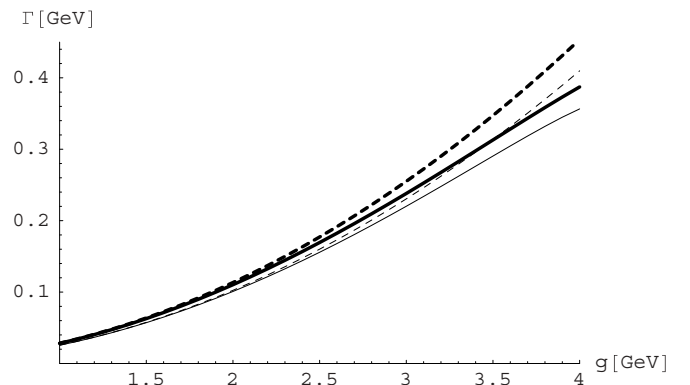


FIG. 5. Full (solid) and tree-level (dashed) decay rates $\sigma \rightarrow \pi\pi$ as functions of the coupling constant. The cases $\Lambda = 1$ and $\Lambda = 2$ GeV correspond to thin and thick lines, respectively.

²In the limit $M_\sigma \rightarrow 2m_\pi$ the spectral function $d_s(x) \sim 1/I(x) \rightarrow \infty$ for $x \rightarrow 2m_\pi$ due to the threshold enhancement.

³Recent theoretical works [18] find a σ mass at around 450 MeV, thus not far from threshold in the lower side of the PDG data. This is indeed the case of an irregular form for the spectral function of this resonance, for which care is needed.

corresponding formulas from the coupling constant: while the tree-level expression depends quadratically on g , in the loop formula g appears also in the distribution $d_S(x)$.

The limit of the validity of the employed one-loop level analysis is an important aspect which deserves further discussion. Namely, when the coupling constant is too large and the mass is not far from threshold, the normalization condition of Eq. (17) is lost; see also the corresponding discussion in Ref. [22]. This fact means that higher orders must be taken into account to satisfy the Källen-Lehmann representation and thus to recover the correct normalization of Eq. (17). At the same time, the violation of the normalization is a valid criterion to establish the limit of our study. For this reason in Fig. 5 we stop the plot at $g_{\sigma\pi\pi} = 4$ GeV (corresponding to $\Gamma_\sigma \sim 400$ MeV); in fact, larger values imply $\int_0^\infty d_S(x) dx < 1$. At this point, the full decay width [using Eq. (22)] is already ~ 60 MeV smaller than the tree-level counterpart. A decay width of about 400 MeV is on the low side for the σ (see Ref. [21]). In Ref. [18], a width 150 MeV larger is obtained. A study beyond the one-loop level would then be necessary to evaluate the spectral function for larger coupling (i.e., larger width) and represents a possible outlook of the present work. Surely the overestimation of the tree-level formula keeps growing for increasing interaction strengths. Similar considerations hold for the k meson.

III. SCALAR SPECTRAL FUNCTION: TWO-CHANNEL CASE

A. Definitions and properties

We now consider two channels for the scalar resonance S described by the Lagrangian density ($m_2 > m_1$):

$$\begin{aligned} \mathcal{L}_S^2 = & \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M_0^2 S^2 + \frac{1}{2}(\partial_\mu \varphi_1)^2 - \frac{1}{2}m_1^2 \varphi_1^2 \\ & + \frac{1}{2}(\partial_\mu \varphi_2)^2 - \frac{1}{2}m_2^2 \varphi_2^2 + g_1 S \varphi_1^2 + g_2 S \varphi_2^2. \end{aligned} \quad (25)$$

The processes $S \rightarrow \varphi_1 \varphi_1$ and $S \rightarrow \varphi_2 \varphi_2$ correspond to the tree-level decay rates

$$\begin{aligned} \Gamma_{S\varphi_1\varphi_1}^{t-1}(M_0) &= \frac{P_{S\varphi_1\varphi_1}}{8\pi M_0^2} [g_{S\varphi_1\varphi_1}]^2 \theta(M_0 - 2m_1), \\ g_{S\varphi_1\varphi_1} &= \sqrt{2}g_1, \end{aligned} \quad (26)$$

$$\begin{aligned} \Gamma_{S\varphi_2\varphi_2}^{t-1}(M_0) &= \frac{P_{S\varphi_2\varphi_2}}{8\pi M_0^2} [g_{S\varphi_2\varphi_2}]^2 \theta(M_0 - 2m_2), \\ g_{S\varphi_2\varphi_2} &= \sqrt{2}g_2. \end{aligned} \quad (27)$$

The propagator is modified by loops of φ_1 and φ_2 , denoted as $\Sigma_1(p^2)$ and $\Sigma_2(p^2)$ and given by Eq. (7) for $m = m_1$ and $m = m_2$, respectively. A delocalization of the interaction, via a vertex function $\Phi(y)$ and the corresponding Fourier transform $f_\Lambda(q) = \int d^4y \Phi(y) e^{-iyq}$, is then introduced as in Eq. (8) for both channels in order to regularize the self-energy contributions. As a consequence, the tree-level results are modified as

$$\begin{aligned} \Gamma_{S\varphi_1\varphi_1}^{t-1}(M_0) &= \frac{P_{S\varphi_1\varphi_1}}{8\pi M_0^2} [g_{S\varphi_1\varphi_1} f_\Lambda(\vec{q}^2 = p_{S\varphi_1\varphi_1}^2)]^2 \\ &\times \theta(M_0 - 2m_1), \end{aligned} \quad (28)$$

$$\begin{aligned} \Gamma_{S\varphi_2\varphi_2}^{t-1}(M_0) &= \frac{P_{S\varphi_2\varphi_2}}{8\pi M_0^2} [g_{S\varphi_2\varphi_2} f_\Lambda(\vec{q}^2 = p_{S\varphi_2\varphi_2}^2)]^2 \\ &\times \theta(M_0 - 2m_2), \end{aligned} \quad (29)$$

and the propagator as

$$\Delta_S(x) = \frac{1}{x^2 - M_0^2 + R(x) + iI(x) + i\varepsilon}. \quad (30)$$

where

$$\begin{aligned} R(x) &= g_{S\varphi_1\varphi_1}^2 \text{Re}[\Sigma_1(x = \sqrt{p^2})] \\ &+ g_{S\varphi_2\varphi_2}^2 \text{Re}[\Sigma_2(x = \sqrt{p^2})], \end{aligned} \quad (31)$$

and

$$\begin{aligned} I(x) &= g_{S\varphi_1\varphi_1}^2 \text{Im}[\Sigma_1(x = \sqrt{p^2})] \\ &+ g_{S\varphi_2\varphi_2}^2 \text{Im}[\Sigma_2(x = \sqrt{p^2})] \end{aligned} \quad (32)$$

$$= x \Gamma_{S\varphi_1\varphi_1}^{t-1}(x) + x \Gamma_{S\varphi_2\varphi_2}^{t-1}(x). \quad (33)$$

In the last equation, the optical theorem was used. The mass M of the state S is given by $M^2 - M_0^2 + R(M) = 0$. Again, we have two cases:

- (i) $M < 2m_1$: the distribution $d_S(x)$ takes the form $Z\delta(x - M)$ for $x < 2m_1$. The discussion is similar to the one-channel case; Eq. (19) is still valid. At threshold $2m_1$, the continuum starts.
- (ii) $M > 2m_1$: as in Eq. (21), the distribution is

$$d_S(x) = \frac{2x}{\pi} \frac{I(x)}{(x^2 - M_0^2 + R(x))^2 + I(x)^2}. \quad (34)$$

It vanishes for $M < 2m_1$. At $x = 2m_2$, the second channel opens.

In case (ii), we have a resonant state. The decay rates into the two channels $S \rightarrow \varphi_1 \varphi_1$ and $S \rightarrow \varphi_2 \varphi_2$ are given by the integrals

$$\begin{aligned} \Gamma_{S\varphi_1\varphi_1} &= \int_0^\infty dx d_S(x) \Gamma_{S\varphi_1\varphi_1}^{t-1}(x), \\ \Gamma_{S\varphi_2\varphi_2} &= \int_0^\infty dx d_S(x) \Gamma_{S\varphi_2\varphi_2}^{t-1}(x). \end{aligned} \quad (35)$$

A particularly interesting case takes place when $2m_1 < M < 2m_2$. While the tree-level result for $S \rightarrow \varphi_2 \varphi_2$ vanishes, we find that $\Gamma_{S\varphi_2\varphi_2}$ is not zero. In this case, the tree-level approximation is absolutely not applicable: the particle S does decay in virtue of the high-mass tail of its distribution. A physical example is well known: the resonances $f_0(980)$ and $a_0(980)$ have a nonzero decay rate into $\bar{K}K$, although their masses are below the threshold $2m_K$. Clearly, a sizable decay rate $\Gamma_{S\varphi_2\varphi_2}$ is obtained only when M is close to threshold. A generalization to the present definitions to N channels is straightforward [22].

When applying the decay formulas (36) it is, however, important to verify numerically that the normalization of the distribution $d_S(x)$ holds; in fact, as discussed in Sec. II B, the formalism is self-consistent in this case.

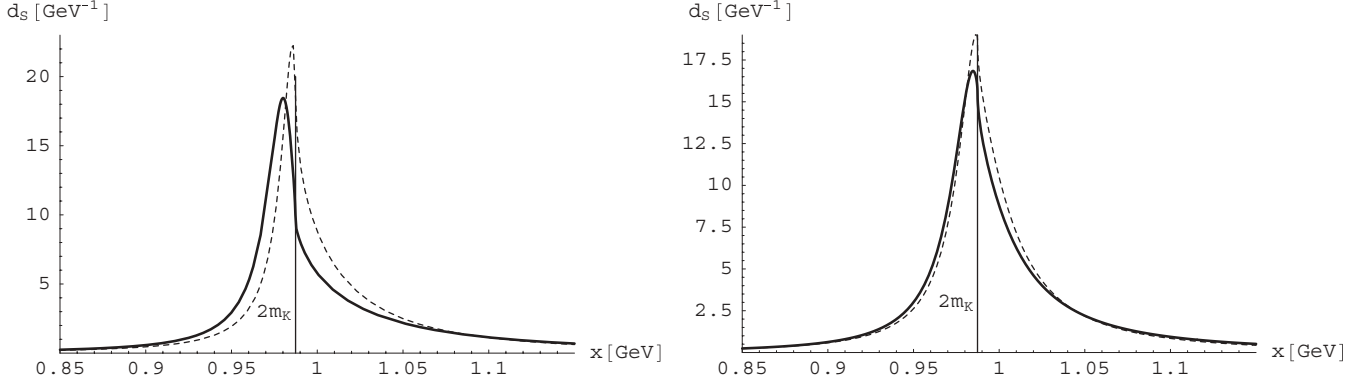


FIG. 6. Left panel: spectral function of f_0 within our formalism (solid line) and using the Flatté distribution (dashed line). Parameters are: $\Lambda = 1.5$ GeV, $g_{f_0 K \bar{K}} = 3$ GeV with a correspondent total decay rate $\Gamma = 0.058$ GeV. Right panel: same as left panel, but for spectral function of a_0 , and $\Gamma = 0.048$ GeV.

B. Application to $a_0(980)$ and $f_0(980)$

In this section, we study the spectral functions of the scalar mesons $a_0 \equiv a_0(980)$ and $f_0 \equiv f_0(980)$, whose masses are $M_{a_0} = 984.7 \pm 1.2$ MeV and $M_{f_0} = 980 \pm 10$ MeV [5]. For both resonances, two decays have been observed: $a_0 \rightarrow \pi\eta$, $a_0 \rightarrow K\bar{K}$ and $f_0 \rightarrow \pi\pi$, $f_0 \rightarrow K\bar{K}$. Notice that both masses are below the threshold of kaon-antikaon production, $M_{a_0, f_0} < 2M_K = 987.3$ MeV, thus the decay of both resonances in $K\bar{K}$ vanishes at tree-level, while experimentally it was seen for both a_0 and f_0 states.

For definiteness, we use the ratios obtained in the experimental analysis [4], i.e.,

$$\begin{aligned} \frac{g_{f_0 K \bar{K}}^2}{g_{f_0 \pi \pi}^2} &= 4.21 \pm 0.46, & \frac{g_{f_0 K \bar{K}}^2}{g_{a_0 K \bar{K}}^2} &= 2.15 \pm 0.40, \\ \frac{g_{a_0 \pi \eta}^2}{g_{a_0 K \bar{K}}^2} &= 0.75 \pm 0.11, \end{aligned} \quad (36)$$

thereby leaving us with only one free parameter, chosen to be $g_{f_0 K \bar{K}}$. Although experimental uncertainties are still large, the results of Eq. (36) are qualitatively similar to those of

various studies, see Ref. [24] and references therein, pointing to a large $K\bar{K}$ coupling for both resonances with a particular enhancement for f_0 (see Refs. [4,10,11,15] and references therein for spectroscopic interpretations).

For the typical value $g_{f_0 K \bar{K}} = 3$ GeV, we report in Fig. 6 the spectral functions of a_0 and f_0 . There is a large probability, $\sim 50\%$, in both cases, that these two mesons have a mass larger than the threshold of production $2m_K$, and therefore the tree-level forbidden decay occurs. In the same figure, we compare our distribution with the Flatté one [23,24], which is usually employed for the a_0 and f_0 mesons. At variance from our distribution, the Flatté distribution must be normalized by hand. The two distributions are quite similar, except that for f_0 the values of the mass corresponding to the maximum of the distributions are slightly different. This is due to the strong coupling of f_0 to kaons, and, as already argued by Achasov [22], the meson loop distributions coincide with the Flatté ones only in the limit of weak coupling.

In Fig. 7, we show the decay rates $f_0 \rightarrow \pi\pi$, $f_0 \rightarrow K\bar{K}$ and $a_0 \rightarrow \pi\eta$, $a_0 \rightarrow K\bar{K}$ as function of $g_{f_0 K \bar{K}}$. The dashed areas in both plots correspond to the total decay rate of f_0 and a_0 as indicated by the PDG (notice, however, that

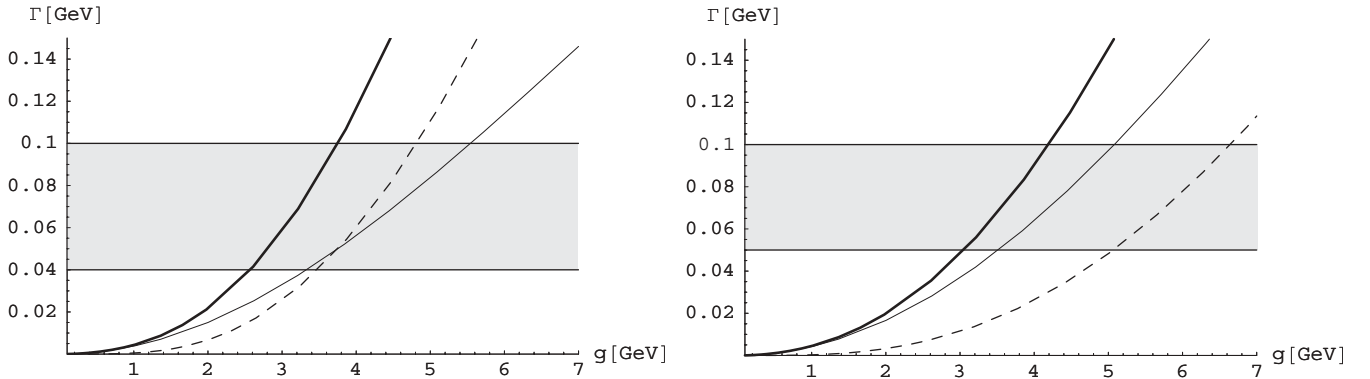


FIG. 7. Left panel: decay rates of f_0 as functions of the coupling constant g . The thin solid line corresponds to the decay into two pions, the dashed line to the decay into two kaons, and the thick solid line is the sum of the two decay rates. Right panel: decay rates of a_0 as functions of the coupling constant g . The thin solid line corresponds to the decay into pion and η , the dashed line to the decay into two kaons, and the thick solid line is the sum of the two decay rates.

PDG specified that the real width could be larger). For our theoretical total decay rates to agree with the measured ones, $g_{f_0 K \bar{K}}$ has to lie between 3 and 4 GeV. The outgoing branching ratio $\Gamma_{a_0 K \bar{K}} / \Gamma_{a_0 \pi \eta}$ is ~ 0.3 – 0.4 , which is larger than the PDG average of 0.183 ± 0.024 ; while the obtained ratio $\Gamma_{f_0 \pi \pi} / (\Gamma_{f_0 \pi \pi} + \Gamma_{f_0 K \bar{K}})$ is ~ 0.48 – 0.56 , which is in qualitative agreement with the (not bold) results listed by the PDG. Notice furthermore that the f_0 meson typically has a larger width than the a_0 meson, in agreement with Ref. [24].

We finally comment on a possible tetraquark unified interpretation of the light scalar mesons as presented in Ref. [11]. A too small decay constant $g_{f_0 K \bar{K}}$ would also imply by far too narrow σ and k mesons (related by Clebsch-Gordon coefficients [11]), thus against a tetraquark nonet. On the contrary, $g_{f_0 K \bar{K}}$ between 3 and 4 GeV is in agreement with a tetraquark nonet below 1 GeV, although problems, such as a too narrow k , persist, see discussions in [11,15]. Such a strong coupling in the $K \bar{K}$ channel implies that a virtual cloud of kaon-antikaon pairs plays an important role, in particular for the f_0 resonance. A heuristic indicator of the mesonic cloud can be given by the quantity

$$Z_{\bar{K}K} = \left[1 + \frac{1}{2M_S} \left(\frac{dR_{S\bar{K}K}}{dx} \right)_{x=M_S} \right]^{-1},$$

where $R_{S\bar{K}K} = g_{S\bar{K}K}^2 \text{Re}[\Sigma_{KK}(x)]$ (with $S = f_0, a_0$) refers to the kaonic loop only. As discussed in Sec. II, in the subthreshold case (which applies to the kaonic channel here) the quantity $(1 - Z)$ varies between 0 and 1 and measures the mesonic cloud dressing of the original bare resonance S . In the f_0 case, by using Eq. (36) together with $g_{f_0 K \bar{K}} = 3$ GeV, one finds $(1 - Z_{\bar{K}K}) = 0.38$, hence implying a 38% of kaonic cloud. This number increases for increasing coupling strength $g_{f_0 K \bar{K}}$. This discussion confirms the interpretation put forward in Ref. [2], where the light scalar mesons possess a tetraquark core but are dressed by kaonic clouds. We also refer the reader to Ref. [28] for a related study of the a_0 and f_0 mesons that employs the so-called compositeness condition introduced by Weinberg in Ref. [29].

IV. SUMMARY AND CONCLUSIONS

In this work we studied the spectral functions of scalar mesons in one- and two-channel cases suitable for the description of light scalar mesons below 1 GeV. We have computed, by using nonlocal interaction Lagrangians with nonderivative couplings, the propagators of scalar mesons at the one-loop level. They satisfy for large ranges of parameters the Källén-Lehmann representation, therefore implying normalized spectral functions. In this way a correct definition of decay amplitudes, weighted over the spectral function, is

formulated: the finite-width effects are automatically taken into account. The resulting decay rates are smaller than the tree-level ones with increasing mismatch for increasing interaction strength. On the other hand, a subthreshold tree-level forbidden decay, such as the $K \bar{K}$ mode for $a_0(980)$ and $f_0(980)$, becomes large.

The resulting spectral functions for the σ and k mesons may deviate consistently from the Breit-Wigner form. The Flatté distribution, although it approximates to a good level of accuracy the $a_0(980)$ and $f_0(980)$ spectral functions, emerges as a small-coupling limit of our more general spectral function.

As stressed by Achasov [22], it is important to use distributions satisfying Källén-Lehmann representations in experimental and theoretical studies. We thus believe that the use of distributions obtained from quantum field theoretical models fulfilling the correct normalization requirements can be helpful in correctly disentangling the nature of the scalar states. Future studies with derivative couplings, mixing effects, such as in the recent work of Ref. [30], and ϕ decays represent a possible interesting outlook.

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APPENDIX A: LOOP CONTRIBUTIONS

Here we report basic formulas for the loop diagram of Eq. (7) drawn in Fig. 1 for the vertex function $f_\Lambda(q) = f_\Lambda(\vec{q}^2)$. By evaluating the residua, one obtains the one-dimensional integral

$$\Sigma(x^2 = p^2) = \frac{1}{2\pi^2} \int_0^\infty dw \frac{w^2 f_\Lambda^2(w)}{\sqrt{w^2 + m^2} (4(w^2 + m^2) - x^2)}, \quad (\text{A1})$$

which can be easily evaluated numerically for each well-behaved $f_\Lambda(w)$. We recall that within our conventions $f_\Lambda(0) = 1$ and that $w = |\vec{k}|$; Eq. (A1) refers to a three-dimensional vertex function. In Ref. [22], the form $f_\Lambda(w) = \theta(\Lambda - w)$ is used and the limit $\Lambda \rightarrow \infty$ is taken. As described in the text, we did not follow this procedure; instead, we used a definite form(s) for the vertex function $f_\Lambda(w)$. As remarked in the text, we performed the calculations also with different forms for $f_\Lambda(w)$ (different power form and exponential functions): the precise form of the cutoff function does not affect the physical picture.

When the scalar state S couples to two particles of masses m_1 and m_2 , the loop contribution is modified as

$$\Sigma(x) = \frac{1}{4\pi^2} \int_0^\infty dw \frac{w^2 (\sqrt{w^2 + m_1^2} + \sqrt{w^2 + m_2^2}) f_\Lambda^2(w)}{\sqrt{w^2 + m_1^2} \sqrt{w^2 + m_2^2} [(\sqrt{w^2 + m_1^2} + \sqrt{w^2 + m_2^2})^2 - x^2]}. \quad (\text{A2})$$

TABLE I. Comparison of masses.

M	M_0	$M_{\max}$	$\langle M \rangle$
0.4	0.61	0.45	0.54
0.6	0.71	0.62	0.65
0.8	0.86	0.81	0.82
1	1.03	1.00	1.01

The choice $f_\Lambda(w) = (1 + w^2/\Lambda^2)^{-1}$ with $\Lambda = 1\text{--}2$ GeV has been used in this work.

In relation to the mass definition of Sec. II B, we report and compare in Table I the mass M defined in Eq. (15), the bare mass M_0 , the maximum $M_{\max}$ of the distribution $d_{S=\sigma}(x)$, and the average mass $\langle M \rangle = \int_0^\infty dx x d_S(x)$. We use $m = m_\pi$, $g_{\sigma\pi\pi} = 3$ GeV, and $\Lambda = 1.5$ GeV.

As expected, the larger the mass, the smaller the differences among the various masslike quantities.

APPENDIX B: SPECTRAL FUNCTION AS MASS DISTRIBUTION: AN INTUITIVE DISCUSSION

We present an intuitive argument for the correctness of interpretation of the spectral function $d_S(x)$ as the “mass distribution” of the state S . To this end, we introduce two scalar fields A and B , the first massless and the second with $M_B > M_S$, and write down the interaction Lagrangian

$$L = cBAS + gS\varphi^2. \quad (\text{B1})$$

[for the following discussion, the “delocalization” of Eq. (8) is not important]. We suppose that the interaction strength c is small enough to allow a tree-level analysis for the decay of the state B . The term $cBAS$ generates the decay process $B \rightarrow AS$, which reads (at tree-level)

$$\Gamma_{BAS}^{t-1}(M_B) = \frac{P_{BAS}}{8\pi M_B^2} [c]^2. \quad (\text{B2})$$

However, when $g \neq 0$ the state S decays into $\varphi\varphi$; that is, the state S is not an asymptotic state. Physically, we observe a tree-body decay $B \rightarrow A\varphi\varphi$, whose decay-rate reads

$$\Gamma_{BA\varphi\varphi}^{t-1}(M_B) = \int_0^{M_B} \Gamma_{BAS}^{t-1}(M_B) d_S(x) dx. \quad (\text{B3})$$

The tree-body decay is decomposed into two steps: $B \rightarrow AS$ and $S \rightarrow \varphi\varphi$. The quantity $\Gamma_{BAS}^{t-1}(M_B)$ represents the probability for $B \rightarrow AS$ (at a given mass x for the state S) and $d_S(x) dx$ is the corresponding weight, i.e., the probability that the resonance S has a mass between x and $x + dx$. In this example, $d_S(x)$ emerges naturally as a mass distribution, correctly normalized, for the scalar state S . Furthermore, notice that in virtue of the limit $d_S(x) = \delta(M - M_S)$ for $g \rightarrow 0$, one has

$$\Gamma_{BA\varphi\varphi}^{t-1}(M_B) = \Gamma_{BAS}^{t-1}(M_B) \quad \text{for } g \rightarrow 0. \quad (\text{B4})$$

In fact, if g is very small, the state S is long lived and Eq. (B2) is recovered. The present analysis also shows that studies on the tree-body decay of the ϕ meson can be consistent only if propagators satisfying the Källen-Lehmann representation are used.

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Vacuum properties of mesons in a linear sigma model with vector mesons and global chiral invariance

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We present a two-flavor linear sigma model with global chiral symmetry and vector and axial-vector mesons. We calculate $\pi\pi$ scattering lengths and the decay widths of scalar, vector, and axial-vector mesons. It is demonstrated that vector and axial-vector meson degrees of freedom play an important role in these low-energy processes and that a reasonable theoretical description requires globally chirally invariant terms other than the vector-meson mass term. An important question for meson vacuum phenomenology is the quark content of the physical scalar $f_0(600)$ and $a_0(980)$ mesons. We investigate this question by assigning the quark-antiquark σ and a_0 states of our model with these physical mesons. We show via a detailed comparison with experimental data that this scenario can describe all vacuum properties studied here except for the decay width of the σ , which turns out to be too small. We also study the alternative assignment $f_0(1370)$ and $a_0(1450)$ for the scalar mesons. In this case the decay width agrees with the experimental value, but the $\pi\pi$ scattering length a_0^0 is too small. This indicates the necessity to extend our model by additional scalar degrees of freedom.

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I. INTRODUCTION

The fundamental theory of strong interactions, quantum chromodynamics (QCD), possesses an exact $SU(3)_c$ local gauge symmetry (the color symmetry) and an approximate global $U(N_f)_R \times U(N_f)_L$ symmetry for N_f massless quark flavors (the chiral symmetry). For sufficiently low temperature and density quarks and gluons are confined into colorless hadrons (i.e., $SU(3)_c$ invariant configurations). Thus, it is the chiral symmetry which predominantly determines hadronic interactions in the low-energy region.

Effective field theories which contain hadrons as degrees of freedom rather than quarks and gluons have been developed along two lines which differ in the way in which chiral symmetry is realized: linear [1] and nonlinear [2]. In the nonlinear realization, the so-called nonlinear sigma model, the scalar states are integrated out, leaving the pseudoscalar states as the only degrees of the freedom. On the other hand, in the linear representation of the symmetry, the so-called linear sigma model, both the scalar and pseudoscalar degrees of freedom are present.

In this work, we consider the linear representation of chiral symmetry. An exactly linearly realized chiral symmetry implies that the QCD eigenstates come in degenerate pairs, the so-called chiral partners. Chiral partners have the same quantum numbers with the exception of parity and G-parity—for example, the scalar states sigma and pion and the vector states ρ and a_1 , respectively, are chiral partners. Experimental data in vacuum and at sufficiently low temperatures and densities of matter, however, show that the mass degeneracy is lifted, because the chiral $U(N_f)_R \times U(N_f)_L \equiv U(1)_V \times U(1)_A \times SU(N_f)_V \times SU(N_f)_A$ symmetry is broken in two ways: explicitly and spontaneously.

Because of the $U(1)_A$ anomaly [3], the $U(N_f)_R \times U(N_f)_L$ symmetry is broken explicitly by quantum effects to $U(1)_V \times SU(N_f)_V \times SU(N_f)_A$. In the case of small but nonzero degenerate quark masses, the latter is explicitly broken to $U(N_f)_V$. If the quark masses are not degenerate, the $U(N_f)_V$ symmetry is furthermore explicitly broken to $U(1)_V$, corresponding to baryon number conservation. QCD also possesses discrete symmetries such as the charge conjugation (C), parity (P) and time reversal (T) symmetry (CPT), which are to a very good precision separately conserved by strong interactions. This fact offers further constraints in the construction of effective models of QCD. [A review of a possible, although small, CP violation in strong interactions may be found, e.g., in Ref. [4].]

In addition to the explicit breaking of axial symmetry $SU(N_f)_A$ due to nonzero quark masses, the latter symmetry is also spontaneously broken in vacuum by the nonvanishing expectation value of the quark condensate: $\langle \bar{q}q \rangle = \langle \bar{q}_R q_L + \bar{q}_L q_R \rangle \neq 0$ [5]. This symmetry breaking mechanism leads to the emergence of $N_f^2 - 1$ pseudoscalar Goldstone bosons, as well as of massive scalar states representing the chiral partners of the Goldstone bosons. For $N_f = 2$, the three lightest mesonic states, the pions, are identified with these Goldstone bosons of QCD. Their nonvanishing mass arises due to the explicit breaking of the chiral symmetry, rendering them pseudo-Goldstone bosons.

In this paper we study an $N_f = 2$ linear sigma model which contains scalar ($\sigma, \tilde{a}_0$) and pseudoscalar ($\eta, \tilde{\pi}$), and in addition also vector ($\omega, \tilde{\rho}$) and axial-vector ($f_1, \tilde{a}_1$) degrees of freedom. Usually, such models are constructed under the requirement of local chiral invariance

$U(N_f)_R \times U(N_f)_L$, with the exception of the vector-meson mass term which renders the local symmetry a global one [6,7]. In a slight abuse of terminology, we will refer to these models as locally chirally invariant models in the following. A study of the QCD phase transition and its critical temperature T_c within such a model can be found, e.g., in Ref. [8].

However, as shown in Refs. [6,7,9–11], the locally invariant linear sigma model fails to simultaneously describe meson decay widths and pion-pion scattering lengths in vacuum. As outlined in Ref. [10], there are at least two ways to solve this issue. One way is to utilize a model in which the (up to the vector-meson mass term) local invariance of the theory is retained while higher-order terms are added to the Lagrangian [6,7,9]. The second way which is pursued here is the following: we construct a linear sigma model with global chiral invariance containing all terms up to naive scaling dimension four [12]. The global invariance allows for additional terms to appear in our Lagrangian in comparison to the locally invariant case presented, e.g., in Ref. [8]. We remark that, introducing a dilaton field, one can argue [13,14] that chirally invariant terms of higher order than scaling dimension four should be absent.

In Ref. [11], we have presented a first study of meson decays and pion-pion scattering lengths in vacuum in the framework of the globally invariant linear sigma model. We have distinguished two different assignments for the scalar fields $\sigma = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$ and $a_0^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$: (i) they may be identified with $f_0(600)$ and $a_0(980)$ which are members of a nonet that in addition consists of $f_0(980)$ and $\kappa(800)$; (ii) they may be identified with $f_0(1370)$ and $a_0(1450)$ which are members of a decuplet that in addition consists of $f_0(1500)$, $f_0(1710)$, and $K_0(1430)$, where the additional scalar-isoscalar state emerges from the admixture of a glueball field [15]. In the following, we will refer to assignment (i) as Scenario I, and to assignment (ii) as Scenario II. In the latter, scalar mesons below 1 GeV are not (predominantly) quark-antiquark states. Their spectroscopic wave functions might contain a dominant tetraquark or mesonic molecular contribution [16]. The correct assignment of the scalar quark-antiquark fields of the model to physical resonances is not only important as a contribution to the ongoing debate about the nature of these resonances, but it is also vital for a study of the properties of

hadrons at nonzero temperature and density, where the chiral partner of the pion plays a crucial role [17].

It is important to stress that the theoretical σ and a_0 fields entering the linear sigma model describe pure quark-antiquark states, just as all the other fields (η , $\vec{\pi}$, ω , $\vec{\rho}$, f_1 , $\vec{a}_1$). This property can be easily proven by using well-known large- N_c results [18]: the mass and the decay widths of both σ and a_0 fields scale in the model as N_c^0 and N_c^{-1} , respectively.

In this paper we first investigate the consequences of Scenario I on various decay widths and pion-pion scattering lengths. This assignment is disfavored because a consistent description of all experimental data cannot be achieved. To reach this conclusion, vector and axial-vector degrees of freedoms play an important role. On the one hand, their decays (such as $\rho \rightarrow \pi\pi$ and $a_1 \rightarrow \pi\gamma$) and the role of the ρ meson in $\pi\pi$ scattering provide strong constraints, on the other hand they affect, indirectly but sizably, some decay channels, such as $\sigma \rightarrow \pi\pi$. We then present a study of Scenario II. Although the latter is not yet conclusive because additional scalar fields (glueball, tetraquark) are not yet taken into account, our preliminary results for the decay widths (albeit not for the scattering length a_0^0) are consistent with the data.

The paper is organized as follows: in Sec. II, we present the Lagrangian of our model and discuss the parameters which are known to very good precision and thus do not enter the fit of the decay widths and the scattering lengths. In Sec. III, we present the formulas for the decay widths and the pion-pion scattering lengths which will be used to fit the remaining parameters and to compare the results to experimental data. This fit and comparison are discussed in Sec. IV, both for Scenario I and Scenario II. In Sec. V we summarize our results in the conclusions and give an outlook to future work. In the Appendix, we show the explicit form of our Lagrangian in terms of the meson fields.

II. THE LINEAR SIGMA MODEL WITH GLOBAL CHIRAL SYMMETRY

A. The Lagrangian

The Lagrangian of the globally invariant linear sigma model with $U(2)_R \times U(2)_L$ symmetry for $N_f = 2$ reads [6,7,11,19]:

$$\begin{aligned}
\mathcal{L} = & \text{Tr}[(D^\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 - \frac{1}{4} \text{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] \\
& + \frac{m_1^2}{2} \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + \text{Tr}[H(\Phi + \Phi^\dagger)] + c(\det \Phi + \det \Phi^\dagger) - 2ig_2(\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& - 2g_3[\text{Tr}\{(\partial_\mu L_\nu - ieA_\mu[t^3, L_\nu] + \partial_\nu L_\mu - ieA_\nu[t^3, L_\mu])\{L^\mu, L^\nu\}\} + \text{Tr}\{(\partial_\mu R_\nu - ieA_\mu[t^3, R_\nu] + \partial_\nu R_\mu \\
& - ieA_\nu[t^3, R_\mu])\{R^\mu, R^\nu\}\}] + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + h_2 \text{Tr}[|\Phi R^\mu|^2 + |L^\mu \Phi|^2] + 2h_3 \text{Tr}(\Phi R_\mu \Phi^\dagger L^\mu) \\
& + g_4\{\text{Tr}[L^\mu L^\nu L_\mu L_\nu] + \text{Tr}[R^\mu R^\nu R_\mu R_\nu]\} + g_5\{\text{Tr}[L^\mu L_\mu L^\nu L_\nu] + \text{Tr}[R^\mu R_\mu R^\nu R_\nu]\} + g_6 \text{Tr}[R^\mu R_\mu] \text{Tr}[L^\nu L_\nu] \\
& + g_7\{\text{Tr}[L^\mu L_\mu] \text{Tr}[L^\nu L_\nu] + \text{Tr}[R^\mu R_\mu] \text{Tr}[R^\nu R_\nu]\}.
\end{aligned} \tag{1}$$

Note that the locally chirally invariant linear sigma model emerges from the globally invariant Lagrangian (1) by setting $h_1 = h_2 = h_3 = g_3 = 0$, $g_2 = g_4 = g_5 = g_6 = g_7 \equiv g$.

In Eq. (1),

$$\Phi = (\sigma + i\eta_N)t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t} \quad (2)$$

contains scalar and pseudoscalar mesons, where t^0 , $\vec{t}$ are the generators of $U(2)$ in the fundamental representation and η_N denotes the nonstrange content of the η meson. Vector and axial-vector mesons are contained in the left-handed and right-handed vector fields:

$$L^\mu = (\omega^\mu + f_1^\mu)t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu) \cdot \vec{t}, \quad (3a)$$

$$R^\mu = (\omega^\mu - f_1^\mu)t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu) \cdot \vec{t}, \quad (3b)$$

respectively. The covariant derivative

$$D^\mu \Phi = \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA^\mu[t^3, \Phi] \quad (4)$$

couples scalar and pseudoscalar degrees of freedom to vector and axial-vector ones as well as to the electromagnetic field A^μ . Note that local chiral invariance requires $g_1 \equiv g$. The left-handed and right-handed field strength tensors,

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu[t^3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu[t^3, L^\mu]\}, \quad (5a)$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu[t^3, R^\nu] - \{\partial^\nu R^\mu - ieA^\nu[t^3, R^\mu]\}, \quad (5b)$$

respectively, couple vector and axial-vector mesons to the electromagnetic field A^μ . Explicit breaking of the global symmetry is described by the term $\text{Tr}[H(\Phi + \Phi^\dagger)] \equiv h_0\sigma$ ($h_0 = \text{const}$). The chiral anomaly is described by the term $c(\det\Phi + \det\Phi^\dagger)$ [3]. The model has been extended to include the nucleon field and its putative chiral partner; for details, see Refs. [13,20].

In the pseudoscalar and (axial-)vector sectors the identification of mesons with particles listed in Ref. [21] is straightforward, as already indicated in Eqs. (2), (3a), and (3b): the fields $\vec{\pi}$ and η_N correspond to the pion and the $SU(2)$ counterpart of the η meson, $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$, with a mass of about 700 MeV. This value can be obtained by "unmixing" the physical η and η' mesons, which also contain $\bar{s}s$ contributions. The fields ω^μ and $\vec{\rho}^\mu$ represent the $\omega(782)$ and $\rho(770)$ vector mesons, respectively, while the fields f_1^μ and $\vec{a}_1^\mu$ represent the $f_1(1285)$ and $a_1(1260)$ axial-vector mesons, respectively. (In principle, the physical ω and f_1 states also contain $\bar{s}s$ contributions, however their admixture is negligibly small.) Unfortunately, the identification of the σ and $\vec{a}_0$ fields is controversial, the possibilities being the pairs $\{f_0(600), a_0(980)\}$ and $\{f_0(1370), a_0(1450)\}$. As mentioned in the Introduction, we will refer to these two assignments as Scenarios I and

II, respectively. We discuss the implications of these two scenarios in the following.

The inclusion of (axial-)vector mesons in effective models of QCD has been done also in other ways than the one presented here. Vector and axial-vector mesons have been included in chiral perturbation theory in Ref. [22]. While the mathematical expressions for the interaction terms turn out to be similar to our results, in our linear approach the number of parameters is smaller. In Ref. [23], the so-called hidden gauge formalism is used to introduce vector mesons, and subsequently axial-vector mesons, into a chiral Lagrangian with a nonlinear realization of chiral symmetry. In this case, the number of parameters is smaller. This approach is closely related to the locally chirally invariant models [6,7] (also called massive Yang-Mills approaches), which have been discussed in the Introduction as a motivation for the present work. We refer also to Ref. [24], where a comparative analysis of effective chiral Lagrangians for spin-1 mesons is presented.

One may raise the question whether vector-meson dominance (VMD) is still respected in the globally invariant linear sigma model (1). As outlined in Ref. [25], there are two ways to realize VMD in a linear sigma model. The standard version of VMD was introduced by Sakurai [26] and considers vector mesons as Yang-Mills gauge fields [27]. The gauge symmetry is explicitly broken by the vector-meson masses. Another realization of VMD was first explored by Lurie [28] whose theory contained a Lagrangian which was globally invariant. It is interesting to note that Lurie's Lagrangian contained direct couplings of the photon to pions and ρ mesons, as well as a ρ - π coupling. It was shown in Ref. [25] that the two representations of VMD are equivalent if the ρ - π coupling $g_{\rho\pi\pi}$ equals the photon- ρ coupling g_ρ (the so-called "universal limit"). It was also shown that, if the underlying theory is globally invariant, the pion form factor at threshold $F_\pi(q^2 = 0) = 1$ for *any* value of the above mentioned couplings. On the other hand, in Sakurai's theory $F_\pi(q^2 = 0) \neq 1$ unless one demands $g_{\rho\pi\pi} = g_\rho$, or other parameters are adjusted in such a way that $F_\pi(q^2 = 0) = 1$. In other words, for *any* globally invariant model, and thus also for ours, one has the liberty of choosing different values for the photon- ρ and ρ - π couplings, without violating VMD.

B. Tree-level masses

The Lagrangian (1) contains 16 parameters. However, the parameters g_k with $k = 3, \dots, 7$ are not relevant for the results presented here so that the number of undetermined parameters decreases to 11:

$$m_0, \lambda_1, \lambda_2, m_1, g_1, g_2, c, h_0, h_1, h_2, h_3. \quad (6)$$

The squared tree-level masses of the mesons in our model contain a contribution arising from spontaneous symmetry breaking, proportional to ϕ^2 . The value ϕ is the vacuum expectation value of the σ field and coincides with the

minimum of the potential that follows from Eq. (1). The σ field is the only field with the quantum numbers of the vacuum, $J^{PC} = 0^{++}$, i.e., the condensation of which does not lead to the breaking of parity, charge conjugation, and Lorentz invariance. The potential for the σ field reads explicitly

$$V(\sigma) = \frac{1}{2}(m_0^2 - c)\sigma^2 + \frac{1}{4}\left(\lambda_1 + \frac{\lambda_2}{2}\right)\sigma^4 - h_0\sigma, \quad (7)$$

and its minimum is determined by

$$0 = \left(\frac{dV}{d\sigma}\right)_{\sigma=\phi} = \left[m_0^2 - c + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi^2\right]\phi - h_0. \quad (8)$$

Spontaneous symmetry breaking corresponds to the case when the potential $V(\phi)$ assumes its minimum for a non-vanishing value $\sigma = \phi \neq 0$. In order to determine the fluctuation of the σ field around the new vacuum, one shifts it by its vacuum expectation value $\phi \neq 0$, $\sigma \rightarrow \sigma + \phi$. The shift leads also to η_N - f_1 and $\tilde{\pi}$ - $\tilde{a}_1$ mixing terms and thus to nondiagonal elements in the scattering matrix. These terms are removed from the Lagrangian by shifting the f_1 and $\tilde{a}_1$ fields as follows:

$$\begin{aligned} f_1^\mu &\rightarrow f_1^\mu + Z w \partial^\mu \eta_N, & \tilde{a}_1^\mu &\rightarrow \tilde{a}_1^\mu + Z w \partial^\mu \tilde{\pi}, \\ \eta_N &\rightarrow Z \eta_N, & \tilde{\pi} &\rightarrow Z \tilde{\pi}, \end{aligned} \quad (9)$$

where we defined the quantities

$$w := \frac{g_1 \phi}{m_{a_1}^2}, \quad Z := \left(1 - \frac{g_1^2 \phi^2}{m_{a_1}^2}\right)^{-1/2}. \quad (10)$$

Note that the field renormalization of η_N and $\tilde{\pi}$ guarantees the canonical normalization of the kinetic terms. This is necessary in order to interpret the Fourier components of the properly normalized one-meson states as creation or annihilation operators [6]. Note also that the ρ and ω masses as well as the f_1 and a_1 masses are degenerate in the globally as well as in the locally invariant model. Once the shift $\sigma \rightarrow \sigma + \phi$ and the transformations (9) have been performed, the mass terms of the mesons in the Lagrangian (1) read

$$m_\sigma^2 = m_0^2 - c + 3\left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi^2, \quad (11)$$

$$m_{\eta_N}^2 = Z^2 \left[m_0^2 + c + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi^2 \right] = m_\pi^2 + 2cZ^2, \quad (12)$$

$$m_{a_0}^2 = m_0^2 + c + \left(\lambda_1 + 3\frac{\lambda_2}{2}\right)\phi^2, \quad (13)$$

$$m_\pi^2 = Z^2 \left[m_0^2 - c + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi^2 \right] \stackrel{(8)}{=} \frac{Z^2 h_0}{\phi}, \quad (14)$$

$$m_\omega^2 = m_\rho^2 = m_1^2 + \frac{\phi^2}{2}(h_1 + h_2 + h_3), \quad (15)$$

$$m_{f_1}^2 = m_{a_1}^2 = m_1^2 + g_1^2 \phi^2 + \frac{\phi^2}{2}(h_1 + h_2 - h_3). \quad (16)$$

In the Appendix, we show the Lagrangian in the form when all shifts have been explicitly performed. From Eqs. (15) and (16), we obtain

$$m_{a_1}^2 = m_\rho^2 + g_1^2 \phi^2 - h_3 \phi^2. \quad (17)$$

The pion decay constant, f_π is determined from the axial current,

$$J_{A_\mu}^a = \frac{\phi}{Z} \partial_\mu \pi^a + \dots \equiv f_\pi \partial_\mu \pi^a + \dots \rightarrow \phi = Z f_\pi. \quad (18)$$

The large- N_c dependence of the parameters is given by

$$\begin{aligned} g_1, g_2 &\propto N_c^{-1/2}, & \lambda_2, h_2, h_3, c &\propto N_c^{-1}, \\ \lambda_1, h_1 &\propto N_c^{-2}, & m_0^2, m_1^2 &\propto N_c^0, & h_0 &\propto N_c^{1/2}. \end{aligned} \quad (19)$$

We remind the reader that a vertex of n quark-antiquark mesons scales as $N_c^{-(n-2)/2}$. As a consequence, the parameters g_1, g_2 scale as $N_c^{-1/2}$, because they are associated with a three-point vertex of quark-antiquark vector fields (of the kind ρ^3). Similarly, the parameters λ_2, h_2, h_3 scale as N_c^{-1} , because they are associated with quartic terms such as π^4 and $\pi^2 \rho^2$. The parameter c is suppressed by a factor N_c although it enters quadratic masslike terms. This is due to the fact that the axial anomaly is suppressed in the large- N_c limit. As is evident from Eq. (12), the η_N meson would also be a Goldstone boson for $N_c \rightarrow \infty$. The parameters λ_1, h_1 also describe quartic interactions, but are further suppressed by a factor $1/N_c$ because of the trace structure of the corresponding terms in the Lagrangian. The quantities m_0^2, m_1^2 are mass terms and therefore scale as N_c^0 . Then the pion decay constant f_π scales as $N_c^{1/2}$. The quantity h_0 scales as $N_c^{1/2}$ in order that m_π scales as N_c^0 as expected. Note that without any assumptions about the σ, a_0 , and f_1, a_1 fields, we immediately obtain that their masses scale as N_c^0 and their decay widths as N_c^{-1} , as we shall see in the following section. Therefore, they must also correspond to quark-antiquark degrees of freedom.

There are, however, also approaches to the phenomenology of low-lying axial-vector mesons, such as the one in Ref. [29], where the Bethe-Salpeter equation is used to unitarize the scattering of vector and pseudoscalar mesons. Here, the Bethe-Salpeter kernel is given by the lowest-order effective Lagrangian. This leads to the dynamical generation of resonances, one of which has a pole mass of 1011 MeV and is consequently assigned to the $a_1(1260)$ meson. This unitarized approach is used in Ref. [30] to study the large- N_c behavior of the dynamically generated resonances, with the conclusion that the $a_1(1260)$ resonance is not a genuine quark-antiquark state.

However, it was shown in Ref. [14] that, while unitarizing the chiral Lagrangian by means of a Bethe-Salpeter study allows one to find poles in the complex plane and

identify them with physical resonances, it does not necessarily allow one to make a conclusion about the structure of those resonances in the large- N_c limit. In order to be able to draw correct conclusions, a Bethe-Salpeter study requires at least one additional term of higher-order not included in the Lagrangian of Refs. [29,30]. Alternatively, the Inverse amplitude method of Ref. [31] can be used.

A very similar approach to the one in Refs. [29,30] was also used in Ref. [32] where a very good fit to the τ decay data from the ALEPH Collaboration [33] was obtained by fine-tuning the subtraction point of a loop diagram. Note, however, that detuning the subtraction point by 5% will spoil the agreement with experimental data. Alternately, these data may be described by approaches with the $a_1(1260)$ meson as an explicit degree of freedom, such as the one in Ref. [12], where $a_1(1260)$ is a quark-antiquark state and where the experimental $a_1(1260)$ spectral function is fitted very well. In Ref. [12], $m_{a_1(1260)} \simeq 1150$ MeV and a full width $\Gamma_{a_1(1260)} \simeq 410$ MeV are obtained. Note that our results, as will be shown later, give very good results on the $a_1(1260)$ phenomenology, for example, in the $a_1(1260) \rightarrow \pi\gamma$ and $a_1(1260) \rightarrow \rho\pi$ decay channels, see Sec. IVA 3.

For the following discussion, it is interesting to note that the ρ meson mass can be split into two contributions:

$$m_\rho^2 = m_1^2 + \frac{\phi^2}{2}(h_1 + h_2 + h_3). \quad (20)$$

Without further assumptions, it is not possible to relate the quantity m_1^2 to microscopic condensates of QCD. However, invoking dilatation invariance, the term $\frac{m_1^2}{2} \text{Tr}[(L^\mu)^2 + (R^\mu)^2]$ in Eq. (1) arises from a term $a \frac{G^2}{2} \text{Tr}[(L^\mu)^2 + (R^\mu)^2]$ where G is the dilatation field and a a dimensionless constant. Upon shifting the dilatation field by $G \rightarrow G_0 + G$, with G_0 being the gluon condensate, one obtains the term in our Lagrangian upon identifying $m_1^2 = aG_0^2$. Thus, the quantity m_ρ^2 in Eq. (20) is expressed as a sum of a term which is directly proportional to the gluon condensate G_0 , and a term which is directly proportional to the chiral condensate ϕ^2 .

We shall require that none of the two contributions be negative: in fact, a negative $m_1^2 = aG_0^2$ would imply that the system is unstable when $\phi \rightarrow 0$; a negative $\frac{\phi^2}{2}(h_1 + h_2 + h_3)$ would imply that spontaneous chiral symmetry breaking decreases the ρ mass. This is clearly unnatural because the breaking of chiral symmetry generates a sizable effective mass for the light quarks, which is expected to positively contribute to the meson masses. This positive contribution is a feature of all known models (such as the Nambu–Jona-Lasinio model and constituent quark approaches). Indeed, in an important class of hadronic models (see Ref. [34] and references therein) the only and obviously positive contribution to the ρ mass is proportional to ϕ^2 (i.e., $m_1 = 0$).

In the vacuum, the very occurrence of chiral symmetry breaking can be also traced back to the interaction with the dilaton field: in fact, the quantity $-m_0^2 \text{Tr}(\Phi^\dagger \Phi)$, where $m_0^2 < 0$, arises from a dilatation-invariant interaction term of the form $bG^2 \text{Tr}(\Phi^\dagger \Phi)$ upon the identification $m_0^2 = bG_0^2$. This property also implies that the chiral condensate ϕ is proportional to the gluon condensate G_0 , $\phi \sim G_0$. This means that the vacuum expression in Eq. (20) can be rewritten in the form $m_\rho^2 \sim \phi^2$, which resembles the Kawarabayashi-Suzuki-Fayyazuddin-Riazuddin relation [35]. However, the quantities G_0 are ϕ may vary independently from each other at nonzero temperature and density, thus generating a nontrivial behavior of m_ρ^2 .

C. Equivalent set of parameters

Instead of the 11 parameters in Eq. (6), it is technically simpler to use the following, equivalent set of 11 parameters in the expressions for the physical quantities:

$$m_\pi, m_\sigma, m_{a_0}, m_{\eta_N}, m_\rho, m_{a_1}, Z, \phi, g_2, h_1, h_2. \quad (21)$$

The quantities m_π, m_ρ, m_{a_1} are taken as the mean values for the masses of the π, ρ , and a_1 meson, respectively, as given by the PDG [21]: $m_\pi = 139.57$ MeV, $m_\rho = 775.49$ MeV, and $m_{a_1} = 1230$ MeV. While m_π and m_ρ are measured to very good precision, this is not the case for m_{a_1} . The mass value given above is referred to as an “educated guess” by the PDG [21]. Therefore, we shall also consider a smaller value, as suggested, e.g., by the results of Ref. [12]. We shall see that, although the overall picture remains qualitatively unchanged, the description of the decay width of a_1 into $\rho\pi$ can be substantially improved.

As outlined in Ref. [11], the mass of the η_N meson can be calculated using the mixing of strange and nonstrange contributions in the physical fields η and $\eta'(958)$:

$$\begin{aligned} \eta &= \eta_N \cos \varphi + \eta_S \sin \varphi, \\ \eta' &= -\eta_N \sin \varphi + \eta_S \cos \varphi, \end{aligned} \quad (22)$$

where η_S denotes a pure $\bar{s}s$ state and $\varphi \simeq -36^\circ$ [36]. In this way, we obtain the value $m_{\eta_N} = 716$ MeV. Given the well-known uncertainty of the value of φ , one could also consider other values, e.g., $\varphi = -41.4^\circ$, as published by the KLOE Collaboration [37]. In this case, $m_{\eta_N} = 755$ MeV. The variation of the η_N mass does not change the results significantly.

The quantities ϕ and Z are linked to the pion decay constant as $\phi/Z = f_\pi = 92.4$ MeV. Therefore, the following six quantities remain as free parameters:

$$m_\sigma, m_{a_0}, Z, g_2, h_1, h_2. \quad (23)$$

The masses m_σ and m_{a_0} depend on the scenario adopted for the scalar mesons.

At the end of this subsection we report three useful formulas which link the parameters g_1 , h_3 , and m_1 of the original set (6) to the second set of parameters (21) [see also Eq. (10)]:

$$g_1 = g_1(Z) = \frac{m_{a_1}}{Z f_\pi} \sqrt{1 - \frac{1}{Z^2}}, \quad (24)$$

$$h_3 = h_3(Z) = \frac{m_{a_1}^2}{Z^2 f_\pi^2} \left(\frac{m_\rho^2}{m_{a_1}^2} - \frac{1}{Z^2} \right), \quad (25)$$

$$\begin{aligned} m_1^2 &= m_1^2(Z, h_1, h_2) \\ &= \frac{1}{2} [m_\rho^2 + m_{a_1}^2 - Z^2 f_\pi^2 (g_1^2 + h_1 + h_2)]. \end{aligned} \quad (26)$$

III. DECAY WIDTHS AND $\pi\pi$ SCATTERING LENGTHS

In this section, we quote the formulas for the decay widths and the $\pi\pi$ scattering lengths and specify their dependence on the parameters m_σ , m_{a_0} , Z , g_2 , h_1 , and h_2 . Using the scaling behavior (19) we obtain that all strong decays and scattering lengths scale as N_c^{-1} , as expected.

For future use we introduce the momentum function

$$\begin{aligned} k(m_a, m_b, m_c) &= \frac{1}{2m_a} \\ &\times \sqrt{m_a^4 - 2m_a^2(m_b^2 + m_c^2) + (m_b^2 - m_c^2)^2} \\ &\times \theta(m_a - m_b - m_c). \end{aligned} \quad (27)$$

In the decay process $a \rightarrow b + c$, with masses m_a , m_b , m_c , respectively, the quantity $k(m_a, m_b, m_c)$ represents the modulus of the three-momentum of the outgoing particles b and c in the rest frame of the decaying particle a . The theta function ensures that the decay width vanishes below threshold.

A. The $\rho \rightarrow \pi\pi$ decay width

The decay width for $\rho \rightarrow \pi\pi$ reads

$$\begin{aligned} \Gamma_{\rho \rightarrow \pi\pi}(Z, g_2) &= \frac{m_\rho^5}{48\pi m_{a_1}^4} \\ &\times \left[1 - \left(\frac{2m_\pi}{m_\rho} \right)^2 \right]^{3/2} \left[g_1 Z^2 + (1 - Z^2) \frac{g_2}{2} \right]^2. \end{aligned} \quad (28)$$

The experimental value is $\Gamma_{\rho \rightarrow \pi\pi}^{(\text{exp})} = (149.1 \pm 0.8) \text{ MeV}$ [21]. The small experimental error can be neglected and the central value is used as a further constraint allowing us to fix the parameter g_2 as function of Z :

$$g_2 = g_2(Z) = \frac{2}{Z^2 - 1} \left(g_1 Z^2 \pm \frac{4m_{a_1}^2}{m_\rho} \sqrt{\frac{3\pi\Gamma_{\rho \rightarrow \pi\pi}^{(\text{exp})}}{(m_\rho^2 - 4m_\pi^2)^{3/2}}} \right). \quad (29)$$

Note that all input values in Eq. (29) are experimentally known [21]. The parameter $g_1 = g_1(Z)$ is fixed via Eq. (24).

As apparent from Eq. (29), two solutions for g_2 are obtained. The solution with the positive sign in front of the square root may be neglected because it leads to unphysically large values for the $a_1 \rightarrow \rho\pi$ decay width, which is another quantity predicted by our study that also depends on g_2 [see Eq. (40)]. For example, the value $Z = 1.6$ (see below) would lead to $g_2 \cong 40$ which in turn would give $\Gamma_{a_1 \rightarrow \rho\pi} \cong 14 \text{ GeV}$ —clearly an unphysically large value. Therefore, we will take the solution for g_2 with the negative sign in front of the square root. In this case, reasonable values for both g_2 (see Sec. IV A 1) and $\Gamma_{a_1 \rightarrow \rho\pi}$ (see Sec. IV A 3) are obtained.

B. The $f_1 \rightarrow a_0\pi$ decay width

The decay width $f_1 \rightarrow a_0\pi$ reads

$$\begin{aligned} \Gamma_{f_1 \rightarrow a_0\pi}(m_{a_0}, Z, h_2) &= \frac{g_1^2 Z^2}{2\pi} \frac{k^3(m_{f_1}, m_{a_0}, m_\pi)}{m_{f_1}^2 m_{a_1}^4} \\ &\times \left[m_\rho^2 - \frac{1}{2}(h_2 + h_3)\phi^2 \right]^2. \end{aligned} \quad (30)$$

There is a subtle point to comment on here. When the quark-antiquark a_0 state of our model is identified as the $a_0(980)$ meson of the PDG compilation (Scenario I), then this decay width can be used to fix the parameter h_2 as function of Z , $h_2 \equiv h_2(Z)$, by using the corresponding experimental value $\Gamma_{f_1 \rightarrow a_0\pi}^{(\text{exp})} = (8.748 \pm 2.097) \text{ MeV}$ [21].

$$h_2 = h_2(Z)$$

$$\begin{aligned} &= \frac{2}{\phi^2} \left(m_\rho^2 - \frac{h_3}{2}\phi^2 \pm \frac{m_{f_1} m_{a_0}^2}{g_1 Z} \sqrt{\frac{2\pi\Gamma_{f_1 \rightarrow a_0\pi}^{(\text{exp})}}{k^3(m_{f_1}, m_{a_0}, m_\pi)}} \right). \end{aligned} \quad (31)$$

Again, there are two solutions, just as in the case of the parameter g_2 . How strongly the somewhat uncertain experimental value of $\Gamma_{f_1 \rightarrow a_0\pi}$ influences the possible values of h_2 , depends on the choice of the sign in front of the square root in Eq. (31). Varying $\Gamma_{f_1 \rightarrow a_0\pi}$ within its experimental range of uncertainty changes the value of h_2 by an average of 25% if the negative sign is chosen, but the same variation of $\Gamma_{f_1 \rightarrow a_0\pi}$ changes h_2 by an average of only 6% if the positive sign is considered. This is due to the fact that the solution with the positive square root sign yields larger values of $h_2 \sim 80$, while the solution with the negative sign leads to $h_2 \sim 20$. The absolute change of h_2 is the same in both cases. Our calculations have shown that using the

negative sign in front of the square root yields a too small value of the η - η' mixing angle $\varphi \cong -9^\circ$. This follows by inserting h_2 into Eq. (33) so that it is removed as a degree of freedom (i.e., replaced by Z) and calculating the mixing angle φ from Eq. (32) using the experimental value of the $a_0 \rightarrow \eta\pi$ decay amplitude from Ref. [38]. For this reason, we only use the positive sign in front of the square root in Eq. (31), i.e., the constraint leading to higher values of h_2 . Then $\varphi \cong -41.8^\circ$ is obtained, in very good agreement with the central value quoted by the KLOE Collaboration [37], $\varphi \cong -41.4^\circ$ (see also Sec. IV A 1).

It may be interesting to note that only the (disregarded) lower value of h_2 leads to the expected behavior of the parameter h_1 which [according to Eq. (19)] should be large- N_c suppressed: the lower value of h_2 yields $h_1 = 1.8$ whereas the higher value of h_2 yields $h_1 = -68$ (see Sec. IV A 1).

Note that if the quark-antiquark a_0 meson of our model is identified as the $a_0(1450)$ meson of the PDG compilation (Scenario II) then the described procedure of replacing h_2 by Z using Eq. (31) is no longer applicable because the decay $f_1 \rightarrow a_0\pi$ is kinematically not allowed and its counterpart $a_0 \rightarrow f_1\pi$ has not been measured.

C. The $a_0 \rightarrow \eta\pi$ and $a_0 \rightarrow \eta'\pi$ decay amplitudes

Our $N_f = 2$ Lagrangian contains the unphysical field η_N . However, by making use of Eq. (22) and invoking the Okubo-Zweig-Iizuka rule, it is possible to calculate the decay amplitude for the physical process $a_0 \rightarrow \eta\pi$ as

$$A_{a_0\eta\pi} = \cos\varphi A_{a_0\eta_N\pi}. \quad (32)$$

From Eq. (1) the formula for the decay amplitude containing the nonstrange η_N field is

$$A_{a_0\eta_N\pi}(m_{a_0}, Z, h_2) = \frac{1}{Zf_\pi} \left\{ m_{\eta_N}^2 - m_{a_0}^2 + \left(1 - \frac{1}{Z^2}\right) \times \left[1 - \frac{1}{2} \frac{Z^2\phi^2}{m_{a_1}^2} (h_2 - h_3)\right] \times (m_{a_0}^2 - m_\pi^2 - m_{\eta'}^2) \right\}. \quad (33)$$

Note that Eq. (33) contains the unmixed mass m_{η_N} which enters when expressing the coupling constants in terms of the parameters (21), as well as the physical mass $m_\eta = 547.8$ MeV. The latter arises because the derivative couplings in the Lagrangian lead to the appearance of scalar invariants formed from the four-momenta of the particles emerging from the decay, which can be expressed in terms of the physical (invariant) masses.

The decay width $\Gamma_{a_0 \rightarrow \eta\pi}$ follows from Eq. (32) by including a phase space factor:

$$\Gamma_{a_0 \rightarrow \eta\pi}(m_{a_0}, Z, h_2) = \frac{k(m_{a_0}, m_\eta, m_\pi)}{8\pi m_{a_0}^2} \times [A_{a_0\eta\pi}(m_{a_0}, Z, h_2)]^2. \quad (34)$$

In the case of Scenario I, in which $a_0 \equiv a_0(980)$, we shall compare the decay amplitude $A_{a_0\eta\pi}$, Eq. (32), with the corresponding experimental value deduced from Crystal Barrel data: $A_{a_0\eta\pi}^{(\text{exp})} = (3330 \pm 150)$ MeV [38]. This is preferable to the use of the decay width quoted by the PDG [21] for $a_0(980)$, which refers to the mean peak width, an unreliable quantity due to the closeness of the kaon-kaon threshold.

In the case of Scenario II, in which $a_0 \equiv a_0(1450)$, it is also possible to calculate the decay width $a_0(1450) \rightarrow \eta'\pi$, using the Okubo-Zweig-Iizuka rule. The amplitude $A_{a_0\eta'\pi}(m_{a_0}, Z, h_2)$ is obtained following the same steps as in the previous case, Eq. (33):

$$A_{a_0\eta'\pi}(m_{a_0}, Z, h_2) = -\frac{\sin\varphi}{Zf_\pi} \left\{ m_{\eta_N}^2 - m_{a_0}^2 + \left(1 - \frac{1}{Z^2}\right) \times \left[1 - \frac{1}{2} \frac{Z^2\phi^2}{m_{a_1}^2} (h_2 - h_3)\right] \times (m_{a_0}^2 - m_\pi^2 - m_{\eta'}^2) \right\}, \quad (35)$$

where the difference compared to Eqs. (32) and (33) is the prefactor $-\sin\varphi$ and the physical η' mass $m_{\eta'} = 958$ MeV. The corresponding decay width reads

$$\Gamma_{a_0(1450) \rightarrow \eta'\pi}(m_{a_0}, Z, h_2) = \frac{k(m_{a_0}, m_{\eta'}, m_\pi)}{8\pi m_{a_0}^2} \times [A_{a_0\eta'\pi}(m_{a_0}, Z, h_2)]^2. \quad (36)$$

D. The $a_1 \rightarrow \pi\gamma$ decay width

We obtain the following formula for the $a_1 \rightarrow \pi\gamma$ decay width:

$$\Gamma_{a_1 \rightarrow \pi\gamma}(Z) = \frac{e^2}{96\pi} (Z^2 - 1) m_{a_1} \left[1 - \left(\frac{m_\pi}{m_{a_1}}\right)^2\right]^3. \quad (37)$$

Note that the $a_1 \rightarrow \pi\gamma$ decay width depends only on the renormalization constant Z . In fact, it is generated via the a_1 - π mixing and vanishes in the limit $Z \rightarrow 1$. (A similar mechanism for this decay is described in Ref. [22].) The fact that we include photons following the second realization of VMD described in Ref. [25] renders this process possible in our model. Using $\Gamma_{a_1 \rightarrow \pi\gamma}^{(\text{exp})} = (0.640 \pm 0.246)$ MeV [21], one obtains $Z = 1.67 \pm 0.2$. Unfortunately, the experimental error for the quantity $\Gamma_{a_1 \rightarrow \pi\gamma}$ is large. Given that almost all quantities of interest depend very strongly on Z , a better experimental knowledge of this

decay would be useful to constrain Z . In the study of Scenario I this decay width will be part of a χ^2 analysis, but still represents the main constraint for Z .

E. The $\sigma \rightarrow \pi\pi$ decay width

We obtain the following formula:

$$\begin{aligned} \Gamma_{\sigma \rightarrow \pi\pi}(m_\sigma, Z, h_1, h_2) \\ = \frac{3}{32\pi m_\sigma} \sqrt{1 - \left(\frac{2m_\pi}{m_\sigma}\right)^2} \\ \times \left\{ \frac{m_\sigma^2 - m_\pi^2}{Zf_\pi} - \frac{g_1^2 Z^3 f_\pi}{m_{a_1}^4} \left[m_\rho^2 - \frac{\phi^2}{2} (h_1 + h_2 + h_3) \right] \right. \\ \left. \times (m_\sigma^2 - 2m_\pi^2) \right\}^2. \end{aligned} \quad (38)$$

It is apparent from Eqs. (19) that the sigma decay width decreases as the number of colors N_c increases. Thus, the sigma field in our model is a $\bar{q}q$ state [31]. In Scenario I we have assigned the σ field as $f_0(600)$, correspondingly we are working with the assumption that $f_0(600)$ [as well as $a_0(980)$] is a $\bar{q}q$ state. In Scenario II, the same assumption is valid for the $f_0(1370)$ and $a_0(1450)$ states.

Note that in Eq. (38) the first term in braces arises from the scalar $\sigma\pi\pi$ vertex, while the second term comes from the coupling of the σ to the a_1 , which becomes a derivatively coupled pion after the shift (9). Because of the different signs, these two terms interfere destructively. As the decay width of a light σ meson into two pions can be very well reproduced in the linear sigma model without vector mesons (corresponding to the case $g_1 \rightarrow 0$), this interference prevents obtaining a reasonable value for this decay width in the present model with vector mesons, see Sec. IVA 2. This problem does not occur for a heavy σ meson, see Sec. IV B 3 and Ref. [39].

F. The $a_1 \rightarrow \sigma\pi$ decay width

The formula for the decay width reads

$$\begin{aligned} \Gamma_{a_1 \rightarrow \sigma\pi}(m_\sigma, Z, h_1, h_2) \\ = \frac{k^3(m_{a_1}, m_\sigma, m_\pi)}{6\pi m_{a_1}^6} g_1^2 Z^2 \left[m_\rho^2 - \frac{\phi^2}{2} (h_1 + h_2 + h_3) \right]^2. \end{aligned} \quad (39)$$

G. The $a_1 \rightarrow \rho\pi$ decay width

Let P be the four-momentum of the a_1 meson, K_1 the four-momentum of the ρ meson and K_2 the four-momentum of the pion. Then the following formula for the $a_1 \rightarrow \rho\pi$ decay width is obtained:

$$\begin{aligned} \Gamma_{a_1 \rightarrow \rho\pi}(Z) = \frac{k(m_{a_1}, m_\rho, m_\pi)}{12\pi m_{a_1}^2} \left[(h_{\mu\nu})^2 - \frac{(h_{\mu\nu} K_1^\nu)^2}{m_\rho^2} \right. \\ \left. - \frac{(h_{\mu\nu} P^\mu)^2}{m_{a_1}^2} + \frac{(h_{\mu\nu} P^\mu K_1^\nu)^2}{m_\rho^2 m_{a_1}^2} \right], \end{aligned} \quad (40)$$

where $h_{\mu\nu}$ is the vertex following from the relevant part of the Lagrangian (1) that reads

$$\begin{aligned} h_{\mu\nu} = Z^2 f_\pi \left\{ (g_1^2 - h_3) g_{\mu\nu} + \frac{g_1 g_2}{m_{a_1}^2} \right. \\ \left. \times [K_{1\mu} K_{2\nu} + K_{2\mu} P_\nu - K_2 \cdot (K_1 + P) g_{\mu\nu}] \right\} \end{aligned} \quad (41)$$

and

$$\begin{aligned} K_1 \cdot K_2 &= \frac{m_{a_1}^2 - m_\rho^2 - m_\pi^2}{2}, \\ P \cdot K_2 &= m_{a_1} E_\pi \equiv m_{a_1} \sqrt{k^2 + m_\pi^2}. \end{aligned}$$

Thus, we have

$$\begin{aligned} h_{\mu\nu}^2 &= Z^4 f_\pi^2 \left\{ 4(g_1^2 - h_3)^2 + \frac{g_1^2 g_2^2}{m_{a_1}^4} [m_{a_1}^4 + m_\pi^4 + m_\rho^4 + m_\pi^2 m_\rho^2 + m_{a_1}^2 (m_\pi^2 - 2m_\rho^2) + 3(m_{a_1}^2 - m_\rho^2 - m_\pi^2) m_{a_1} E_\pi] \right. \\ &\quad \left. - 3 \frac{g_1 g_2 (g_1^2 - h_3)}{m_{a_1}^2} (m_{a_1}^2 - m_\rho^2 - m_\pi^2 + 2m_{a_1} E_\pi) \right\}, \\ (h_{\mu\nu} K_1^\nu)^2 &= Z^4 f_\pi^2 \left\{ (g_1^2 - h_3)^2 m_\rho^2 + \frac{g_1^2 g_2^2}{4m_{a_1}^4} [(m_\pi^2 - m_\rho^2)^2 (m_\pi^2 + m_\rho^2 - 2m_{a_1}^2) + (m_\pi^2 + m_\rho^2) m_{a_1}^4 \right. \\ &\quad \left. - 4(m_{a_1}^2 - m_\rho^2 - m_\pi^2) m_{a_1}^2 E_\rho E_\pi] + \frac{g_1 g_2 (g_1^2 - h_3)}{m_{a_1}^2} [(m_{a_1}^2 - m_\pi^2) m_{a_1} E_\rho - 2m_{a_1} m_\rho^2 \left(E_\pi + \frac{E_\rho}{2} \right)] \right\}, \\ (h_{\mu\nu} P^\mu)^2 &= Z^4 f_\pi^2 \left\{ (g_1^2 - h_3)^2 m_{a_1}^2 + \frac{g_1^2 g_2^2}{4m_{a_1}^4} [(m_{a_1}^2 - m_\pi^2)^2 (m_{a_1}^2 + m_\pi^2 - 2m_\rho^2) + (m_\pi^2 + m_{a_1}^2) m_\rho^4 \right. \\ &\quad \left. - 4(m_{a_1}^2 - m_\rho^2 - m_\pi^2) m_{a_1}^2 E_\rho E_\pi] + \frac{g_1 g_2 (g_1^2 - h_3)}{m_{a_1}^2} [2m_{a_1}^2 E_\rho E_\pi - (m_{a_1}^2 - m_\rho^2 - m_\pi^2) m_{a_1}^2] \right\}, \\ (h_{\mu\nu} P^\mu K_1^\nu)^2 &= (g_1^2 - h_3)^2 Z^4 f_\pi^2 m_{a_1}^2 E_\rho^2. \end{aligned}$$

H. The tree-level scattering lengths

The partial wave decomposition [40] leads to the following formula for the s-wave $I = 0$ pion-pion scattering length a_0^0 (in units of m_π^{-1}):

$$\begin{aligned} a_0^0(Z, m_\sigma, h_1) = & \frac{1}{4\pi} \left(2g_1^2 Z^4 \frac{m_\pi^2}{m_{a_1}^4} \left\{ m_\rho^2 + \frac{\phi^2}{16} [12g_1^2 - 2(h_1 + h_2) - 14h_3] \right\} - \frac{5}{8} \frac{Z^2 m_\sigma^2 - m_\pi^2}{f_\pi^2} \right. \\ & - \frac{3}{2} \left\{ g_1^2 Z^2 \phi \frac{m_\pi^2}{m_{a_1}^4} \left[2m_{a_1}^2 + m_\rho^2 - \frac{\phi^2}{2} (h_1 + h_2 + h_3) \right] - \frac{Z^2 m_\sigma^2 - m_\pi^2}{2\phi} \right\}^2 \frac{1}{4m_\pi^2 - m_\sigma^2} \\ & \left. + \left\{ g_1^2 Z^2 \phi \frac{m_\pi^2}{m_{a_1}^4} \left[m_\rho^2 - \frac{\phi^2}{2} (h_1 + h_2 + h_3) \right] + \frac{Z^2 m_\sigma^2 - m_\pi^2}{2\phi} \right\}^2 \frac{1}{m_\sigma^2} \right). \end{aligned} \quad (42)$$

We use the value $a_0^{0(\text{exp})} = 0.218 \pm 0.020$ in accordance with the 2003 and 2004 data from the NA48/2 Collaboration [41].

An analogous calculation leads to the s-wave $I = 2$ pion-pion scattering length a_0^2 :

$$\begin{aligned} a_0^2(Z, m_\sigma, h_1) = & -\frac{1}{4\pi} \left(\frac{Z^2 m_\sigma^2 - m_\pi^2}{4f_\pi^2} + g_1^2 Z^4 \frac{m_\pi^2}{m_{a_1}^4} \left[m_\rho^2 - \frac{\phi^2}{2} (h_1 + h_2 + h_3) \right] \right. \\ & \left. - \left\{ g_1^2 Z^2 \phi \frac{m_\pi^2}{m_{a_1}^4} \left[m_\rho^2 - \frac{\phi^2}{2} (h_1 + h_2 + h_3) \right] + \frac{Z^2 m_\sigma^2 - m_\pi^2}{2\phi} \right\}^2 \frac{1}{m_\sigma^2} \right). \end{aligned} \quad (43)$$

The experimental result for a_0^2 from the NA48/2 Collaboration is $a_0^{2(\text{exp})} = -0.0457 \pm 0.0125$ [41]. Note that the $\pi\pi$ scattering lengths were also studied away from threshold in Ref. [42], in a model quite similar to ours.

IV. STUDY OF DIFFERENT SCENARIOS FOR THE STRUCTURE OF SCALAR MESONS

In this section we discuss two different interpretations of the scalar mesons. The following subsection describes the results obtained when $f_0(600)$ and $a_0(980)$ are interpreted as scalar quarkonia (Scenario I). Then we discuss the results obtained when $f_0(1370)$ and $a_0(1450)$ are interpreted as scalar quarkonia (Scenario II).

A. Scenario I: light scalar quarkonia

1. Fit procedure

As a first step we utilize the central value of the experimental result $\Gamma_{\rho \rightarrow \pi\pi}^{(\text{exp})} = 149.1$ MeV [21] in order to express the parameter g_2 as a function of Z via Eq. (29). Moreover, we fix the mass $m_{a_0} = 0.98$ GeV [21] and we also use the central value $\Gamma_{f_1 \rightarrow a_0 \pi}(Z, h_2) = 8.748$ MeV to express h_2 as a function of Z . The results are practically unaffected by the 6% uncertainty in h_2 originating from the uncertainty in $\Gamma_{f_1 \rightarrow a_0 \pi}$, see Eq. (31).

As a result, the set of free parameters in Eq. (23) is further reduced to three parameters:

$$Z, m_\sigma, h_1. \quad (44)$$

Note that in this scenario the field σ is identified with the resonance $f_0(600)$, but the experimental uncertainty on its

mass is so large that it does not allow us to fix m_σ . We therefore keep m_σ as a free parameter.

We now determine the parameters Z , h_1 , and m_σ using known data on the $a_1 \rightarrow \pi\gamma$ decay width (37) and on the $\pi\pi$ scattering lengths a_0^0 and a_0^2 reported in Eqs. (42) and (43). This is a system of three equations with three variables and can be solved uniquely. We make use of the χ^2 method in order to determine not only the central values for our parameters but also their error intervals:

$$\begin{aligned} \chi^2(Z, m_\sigma, h_1) = & \left(\frac{\Gamma_{a_1 \rightarrow \pi\gamma}(Z) - \Gamma_{a_1 \rightarrow \pi\gamma}^{(\text{exp})}}{\Delta \Gamma_{\text{decay}}^{(\text{exp})}} \right)^2 \\ & + \sum_{i \in \{0,2\}} \left(\frac{a_0^i(Z, m_\sigma, h_1) - a_0^{i(\text{exp})}}{\Delta a_0^{i(\text{exp})}} \right)^2. \end{aligned} \quad (45)$$

The errors for the model parameters are calculated as the square roots of the diagonal elements of the inverted Hessian matrix obtained from $\chi^2(Z, m_\sigma, h_1)$. The minimal value is obtained for $\chi^2 = 0$, as expected given that the parameters are determined from a uniquely solvable system of equations. The values of the parameters are as follows:

$$\begin{aligned} Z &= 1.67 \pm 0.2, & m_\sigma &= (332 \pm 456) \text{ MeV}, \\ h_1 &= -68 \pm 338. \end{aligned} \quad (46)$$

Clearly, the error intervals for m_σ and h_1 are very large. Fortunately, it is possible to constrain the h_1 error interval as follows. As described at the end of Sec. IIB, the ρ mass squared contains two contributions—the bare mass term m_1^2 and the quark condensate contribution ($\sim \phi^2$). The contribution of the quark condensate is special for the

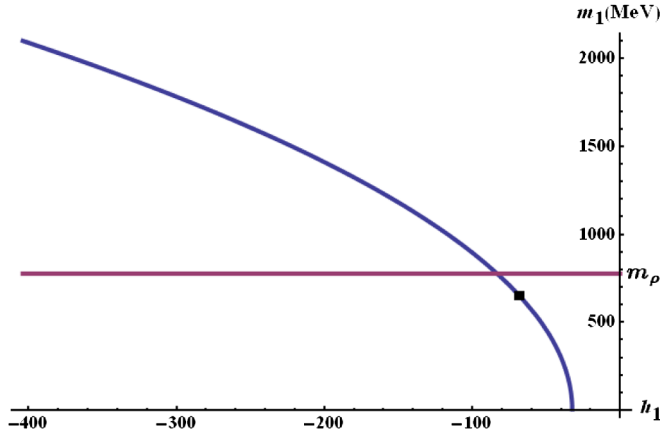


FIG. 1 (color online). m_1 as function of h_1 , constrained at the central value of $Z = 1.67$. The black dot marks the position of central values $h_1 = -68$ and $m_1 = 652$ MeV.

globally invariant sigma model; in the locally invariant model m_ρ is always equal to m_1 [8]. Each of these contributions should have at most the value of 775.49 MeV ($= m_\rho$) because otherwise either the bare mass or the quark condensate contribution to the rho mass would be negative, which appears to be unphysical. A plot of the function $m_1 = m_1(Z, h_1, h_2(Z))$, see Eq. (26), for the central values of $Z = 1.67$ and $\Gamma_{f_1 \rightarrow a_0 \pi}^{(\text{exp})} = 8.748$ MeV is shown in Fig. 1.

Note that varying the value of $\Gamma_{f_1 \rightarrow a_0 \pi}^{(\text{exp})}$ within its experimental boundaries would only very slightly change h_1 by ± 4 and this parameter is thus unaffected by the experimental error for $\Gamma_{f_1 \rightarrow a_0 \pi}^{(\text{exp})}$. If the value of m_1 was known exactly, then Eq. (26) would allow us to constrain h_1 via Z . However, given that at this point we can only state that $0 \leq m_1 \leq m_\rho$, for each Z one may consider all values of h_1 between two boundaries, one obtained from the condition $m_1(Z, h_1, h_2(Z)) \equiv 0$ and another obtained from the con-

dition $m_1(Z, h_1, h_2(Z)) \equiv m_\rho$. For example, using the central value of $Z = 1.67$, we obtain $-83 \leq h_1 \leq -32$. The lower boundary follows from $m_1 \equiv m_\rho$ and the upper boundary from $m_1 \equiv 0$, see Fig. 1. Note that the central value $h_1 = -68$ from Eq. (46) corresponds to $m_1 = 652$ MeV. If the minimal value of $Z = 1.47$ is used, then $h_1 = -112$ is obtained from $m_1 \equiv m_\rho$ and $h_1 = -46$ from $m_1 \equiv 0$. Thus, $-112 \leq h_1 \leq -46$ for $Z = 1.47$. Analogously, $-64 \leq h_1 \leq -24$ is obtained for the maximal value $Z = 1.87$.

Clearly, each lower boundary for h_1 is equivalent to $m_1 \equiv m_\rho$ and each upper boundary for h_1 is equivalent to $m_1 \equiv 0$. Thus, in the following we will only state the values of Z and m_1 ; h_1 can always be calculated using Eq. (26). In this way, the dependence of our results on m_1 and thus on the origin of the ρ mass will be exhibited.

The value of m_σ can be constrained in a way similar to h_1 using the scattering length a_0^0 ; the scattering length a_0^2 possesses a rather large error interval making it unsuitable to constrain m_σ . Figure 2 shows the different values for a_0^0 and a_0^2 depending on the choice of Z and m_1 .

It is obvious that the value of a_0^0 is only consistent with the NA48/2 value [41], if m_σ is in the interval [288, 477] MeV, i.e., $m_\sigma = 332_{-44}^{+145}$ MeV. This value for m_σ follows if the parameters Z and m_1 are varied within the allowed boundaries. If we only consider the a_0^0 curve that is obtained for the central values of Z and m_1 , a much more constrained value of $m_\sigma = 332_{-13}^{+24}$ MeV follows from Fig. 2. We will be working with the broader interval of m_σ . Even then, constraining m_1 to the interval $[0, m_\rho]$, the error bars for m_σ are reduced by at least a factor of 3 in comparison to the result (46) following from the χ^2 calculation.

We summarize our results for the parameters Z and m_σ :

$$Z = 1.67 \pm 0.2, \quad m_\sigma = 332_{-44}^{+145} \text{ MeV}.$$

The central values of all parameters of the original set (6)

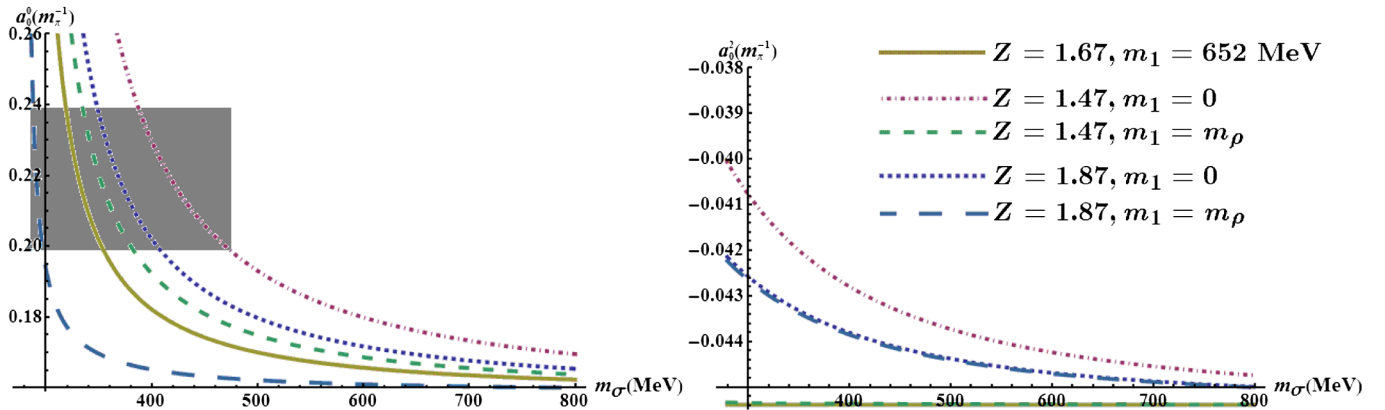


FIG. 2 (color online). Scattering lengths a_0^0 and a_0^2 as function of m_σ (the shaded band corresponds to the NA48/2 value of a_0^0 ; no error interval is shown for a_0^2 due to interval size [41]).

TABLE I. Central values of parameters for Scenario I.

Parameter	m_σ	h_1	h_2	h_3
Value	332 MeV	-68	80	2.4
Parameter	g_1	g_2	m_0	m_1
Value	6.4	3.1	210 MeV	652^{+123}_{-652} MeV
Parameter	λ_1	λ_2	c	h_0
Value	-14	33	88 744 MeV ²	1×10^6 MeV ³

are given in Table I. They follow from the χ^2 fit (m_σ, h_1), via decay width constraints (h_2, g_2), and from Eqs. (11)–(16), (24), and (25). The central values of Z , m_σ , and h_1 , Eq. (46), have been used to calculate all other parameters. We neglect the errors, apart from those of m_1 , which in this scenario vary in a large range.

Note that the values of a_0^2 depend strongly on the choice of the parameters Z and m_1 . Whereas for the central values of Z and m_1 this scattering length is constant and has the value $a_0^2 = -0.0454$, its value increases if Z and m_1 are considered at their respective boundaries, see Fig. 2.

The value of Z alone allows us to calculate certain decay widths in the model. For example, as a consistency check we obtain $\Gamma_{a_1 \rightarrow \pi\gamma} = 0.640^{+0.261}_{-0.231}$ MeV which is in good agreement with the experimental result. Also, given that the $a_0 \rightarrow \eta_N \pi$ decay amplitude only depends on Z , it is possible to calculate the value of this amplitude, Eq. (33). For $Z = 1.67$, we obtain $A_{a_0 \rightarrow \eta\pi} = 3939$ MeV for the decay amplitude $a_0 \rightarrow \eta\pi$ involving the physical η field if the η - η' mixing angle of $\varphi = -36^\circ$ [36] is taken. The Crystal Barrel data [38] read $A_{a_0 \rightarrow \eta\pi}^{(\text{exp})} = 3330$ MeV and hence there is an approximate discrepancy of 20%. If the KLOE Collaboration [37] value of $\varphi = -41.4^\circ$ is considered, then the value of $A_{a_0 \rightarrow \eta\pi} = 3373$ MeV follows—in perfect agreement with the Crystal Barrel value. From this we conclude that this scenario prefers a relatively large value of the η - η' mixing angle. In fact, if we use the Crystal Barrel value $A_{a_0 \rightarrow \eta\pi}^{(\text{exp})} = 3330$ MeV as input, we would predict $\varphi = -41.8^\circ$ for the central value of Z as well as $\varphi = -42.3^\circ$ and $\varphi = -41.6^\circ$ for the highest and lowest values of Z , respectively, i.e., $\varphi = -41.8^{+0.2^\circ}_{-0.5^\circ}$. This is in excellent agreement with the KLOE Collaboration result $\varphi = -41.4^\circ \pm 0.5^\circ$ but also with the results from approaches using the Bethe-Salpeter formalism, such as the one in Ref. [43].

2. The decay $\sigma \rightarrow \pi\pi$

The sigma decay width $\Gamma_{\sigma \rightarrow \pi\pi}$ depends on all three parameters Z , m_1 (originally h_1), and m_σ . In Fig. 3, we show the dependence of this decay width on the sigma mass for fixed values of Z and m_1 , varying the latter within their respective boundaries.

Generally, the values that we obtain are too small when compared to the PDG data [21] and to other calculations of the sigma meson decay width, such as the one performed

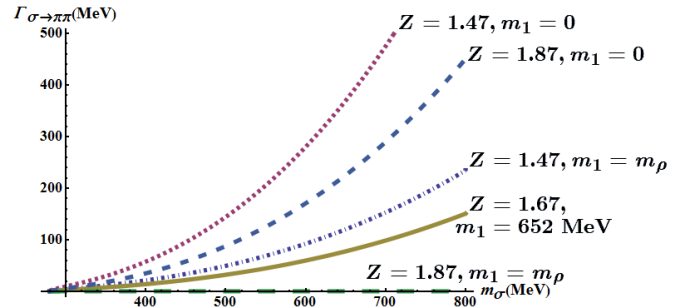


FIG. 3 (color online). $\Gamma_{\sigma \rightarrow \pi\pi}$ as function of m_σ for different values of Z and m_1 . The PDG [21] notes $\Gamma_\sigma = (600\text{--}1000)$ MeV; the results from the chiral perturbation theory suggest $\Gamma_\sigma = 544$ MeV [44] and $\Gamma_\sigma = 510$ MeV [45].

by Leutwyler *et al.* [44] who found $\Gamma_{\sigma \rightarrow \pi\pi}/2 = 272^{+9}_{-12.5}$ MeV and Peláez *et al.* [45] who found $\Gamma_{\sigma \rightarrow \pi\pi}/2 = (255 \pm 16)$ MeV. The largest values for the decay width that we were able to obtain within our model are for the case when Z is as small as possible, $Z = 1.47$, and $m_1 = 0$, i.e., when the ρ mass is solely generated by the quark condensate. As seen above, for this case the scattering lengths allow a maximum value $m_\sigma = 477$ MeV, for which $\Gamma_{\sigma \rightarrow \pi\pi} \cong 145$ MeV. In all other cases, the decay width is even smaller. However, as will be discussed in Sec. IVA 3, the case $m_1 = 0$ leads to the unphysically small value $\Gamma_{a_1 \rightarrow \sigma\pi} \cong 0$ and should therefore not be taken too seriously. As apparent from Fig. 2, excluding small values of m_1 would require smaller values for m_σ in order to be consistent with the scattering lengths. According to Fig. 3, however, this in turn leads to even smaller values for the decay width.

Hence, we conclude that the isoscalar meson in our model cannot be $f_0(600)$, thus excluding that this resonance is predominantly a $\bar{q}q$ state and the chiral partner of the pion. Then the interpretation of the isospin-one state $a_0(980)$ as a (predominantly) quarkonium state is also excluded. The only choice is to consider Scenario II, see Sec. IV B, i.e., to interpret the scalar states above 1 GeV, $f_0(1370)$ and $a_0(1450)$, as being predominantly quarkonia. If the decay width of $f_0(1370)$ can be described by the model, this would be a very strong indication that these higher-lying states can be indeed interpreted as (predominantly) $\bar{q}q$ states. Note that very similar results about the nature of the light scalar mesons were also found using different approaches: from an analysis of the meson behavior in the large- N_c limit in Refs. [31,46] as well as from lattice studies, such as those in Ref. [47].

We remark that the cause for preventing a reasonable fit of the light sigma decay width is the interference term arising from the vector mesons in Eq. (38). In the unphysical case without vector-meson degrees of freedom, a simultaneous fit of the decay width and the scattering lengths is possible [39].

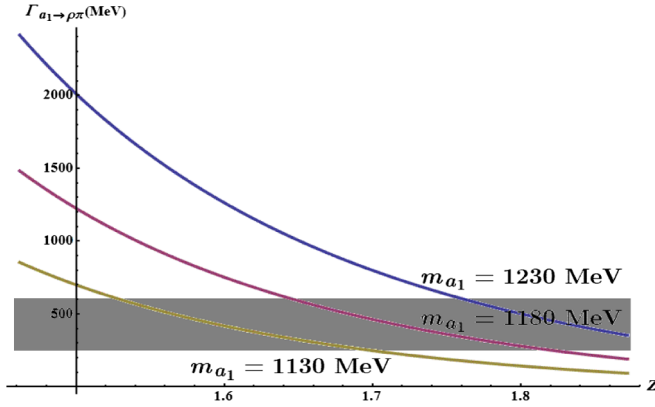


FIG. 4 (color online). $\Gamma_{a_1 \rightarrow \rho\pi}$ for different values of m_{a_1} . The shaded area corresponds to the possible values of $\Gamma_{a_1 \rightarrow \rho\pi}$ as stated by the PDG.

3. Decays of the a_1 meson

We first consider the decay width $\Gamma_{a_1 \rightarrow \rho\pi}$. For a given m_{a_1} , this decay width depends only on Z . The PDG quotes a rather large band of values, $\Gamma_{a_1 \rightarrow \rho\pi}^{(\text{exp})} = (250\text{--}600)$ MeV. For $m_{a_1} = 1230$ MeV, our fit of meson properties yields $Z = 1.67 \pm 0.2$. The ensuing region is shown as shaded area in Fig. 4. For $m_{a_1} = 1230$ MeV, $\Gamma_{a_1 \rightarrow \rho\pi}$ decreases from 2.4 GeV to 353 MeV, if Z varies from 1.47 to 1.87.

We also observe from Fig. 4 that the range of values for Z , which give values for $\Gamma_{a_1 \rightarrow \rho\pi}$ consistent with the experimental error band, becomes larger if one considers smaller masses for the a_1 meson. We have taken $m_{a_1} = 1180$ MeV and $m_{a_1} = 1130$ MeV, the latter being similar to the values used in Refs. [12,48]. Repeating our calculations, we obtain a new range of possible values for Z , $Z \simeq 1.69 \pm 0.2$ for $m_{a_1} = 1180$ MeV and $Z \simeq 1.71 \pm 0.2$ for $m_{a_1} = 1130$ MeV. For the respective central values of Z we then compute $\Gamma_{a_1 \rightarrow \rho\pi}^{m_{a_1}=1180 \text{ MeV}} = 483$ MeV ($Z_{m_{a_1}=1180 \text{ MeV}} = 1.69$) and $\Gamma_{a_1 \rightarrow \rho\pi}^{m_{a_1}=1130 \text{ MeV}} = 226$ MeV ($Z_{m_{a_1}=1130 \text{ MeV}} = 1.71$), in good agreement with experimental data. All other results remain valid when m_{a_1} is decreased by about 100 MeV. Most notably, the $f_0(600)$ decay width remains too small.

We also consider the $a_1 \rightarrow \sigma\pi$ decay width. Experimental data on this decay channel [21] are inconclusive. The value $\Gamma_{a_1 \rightarrow \sigma\pi} = 56$ MeV is obtained for the central values of Z , m_1 , m_σ , and $\Gamma_{f_1 \rightarrow a_0\pi}$ (which was used to constrain h_2 via Z). Taking the limit $m_1 = 0$ pulls the value of $\Gamma_{a_1 \rightarrow \sigma\pi}$ down to practically zero, regardless whether $Z = Z_{\min}$ or $Z = Z_{\max}$. This is an indication that the $m_1 = 0$ limit, where m_ρ is completely generated from the quark condensate, cannot be physical. Note that the case $Z = Z_{\max} = 1.87$ and $m_1 \equiv m_\rho$, i.e., where the quark condensate contribution to the ρ mass vanishes, leads to a rather large value of $\Gamma_{a_1 \rightarrow \sigma\pi}$, e.g., for the central value of $m_\sigma = 332$ MeV the value of $\Gamma_{a_1 \rightarrow \sigma\pi} = 120$ MeV follows.

Interestingly, this picture persists even if lower values of m_{a_1} are considered. Improving experimental data for this decay channel would allow us to further constrain our parameters.

4. The case of isospin-exact scattering lengths

So far, the values of the scattering lengths used in our fit, $a_0^0 = 0.218 \pm 0.020$ and $a_0^2 = -0.0457 \pm 0.0125$ [41], account for the small explicit breaking of isospin symmetry due to the difference of the up and down quark masses. However, in our model the isospin symmetry is exact. Thus, one should rather use the isospin-exact values $a_0^{0(I)} = 0.244 \pm 0.020$ and $a_0^{2(I)} = -0.0385 \pm 0.0125$ [49]. In this section we will briefly show that the conclusions reached so far remain qualitatively unchanged if the isospin-exact values for the scattering lengths are considered.

Performing the χ^2 fit, Eq. (45), with $\Gamma_{a_1 \rightarrow \pi\gamma}$, $a_0^{0(I)}$ and $a_0^{2(I)}$ as experimental input yields $Z = 1.67 \pm 0.2$ —unchanged in comparison with the previous case (Z is largely determined by $\Gamma_{a_1 \rightarrow \pi\gamma}$ which is the same in both χ^2 calculations), $h_1 = -116 \pm 70$, and $m_\sigma = (284 \pm 16)$ MeV. Note that in this case the errors are much smaller than previously. The reason is that the mean value of m_σ is almost on top of the two-pion decay threshold and thus leads to an artificially small error band. For such small values of m_σ the decay width $\Gamma_{\sigma \rightarrow \pi\pi}$ is at least an order of magnitude smaller than the physical value, but even for values of m_σ up to 500 MeV (not supported by our error analysis) the decay width never exceeds 150 MeV, see Fig. 3.

B. Scenario II: Scalar quarkonia above 1 GeV

1. General discussion

A possible way to resolve the problem of the unphysically small two-pion decay width of the sigma meson is to identify the fields σ and a_0 of the model with the resonances $f_0(1370)$ and $a_0(1450)$, respectively. Thus, the scalar quarkonium states are assigned to the energy region above 1 GeV. In the following we investigate the consequences of this assignment. However, the analysis cannot be conclusive for various reasons:

- (i) The glueball field is missing. Many studies find that its role in the mass region at about 1.5 GeV is crucial, since it mixes with the other scalar resonances.
- (ii) The light scalar mesons below 1 GeV, such as $f_0(600)$ and $a_0(980)$, are not included as elementary fields in our model. The question is if they can be dynamically generated from the pseudoscalar fields already present in our model by solving a Bethe-Salpeter equation. If not, they should be introduced as additional elementary fields from the very beginning (see also the discussion in Ref. [14]).
- (iii) Because of absence of the resonance $f_0(600)$, the $\pi\pi$ scattering length a_0^0 cannot be correctly de-

scribed at tree-level: whereas a_0^2 stays always within the experimental error band, a_0^0 clearly requires a light scalar meson for a proper description of experimental data because a large value of m_σ drives this quantity to the Weinberg limit (≈ 0.159) which is outside the experimental error band.

Despite these drawbacks, we turn to a quantitative analysis of this scenario.

2. Decay of the $a_0(1450)$ meson

As in Scenario I, the parameter g_2 can be expressed as a function of Z by using the $\rho \rightarrow \pi\pi$ decay width (28). However, the parameter h_2 can no longer be fixed by the $f_1 \rightarrow a_0\pi$ decay width: the a_0 meson is now identified with the $a_0(1450)$ resonance listed in Ref. [21], with a central mass of $m_{a_0} = 1474$ MeV, and thus f_1 is too light to decay into a_0 and π . One would be able to determine h_2 from the (energetically allowed) decay $a_0(1450) \rightarrow f_1\pi$, but the corresponding decay width is not experimentally known.

Instead of performing a global fit, it is more convenient to proceed step by step and calculate the parameters Z , h_1 , h_2 explicitly. We vary $m_\sigma \equiv m_{f_0(1370)}$ within the experimentally known error band [21] and check if our result for $\Gamma_{f_0(1370) \rightarrow \pi\pi}$ is in agreement with experimental data.

We first determine Z from $a_1 \rightarrow \pi\gamma$, Eq. (37), and obtain $Z = 1.67 \pm 0.21$. We then immediately conclude that the $a_1 \rightarrow \rho\pi$ decay width, Eq. (40), will remain the same as in Scenario I because this decay width depends on Z (which is virtually the same in both scenarios) and g_2 [which is fixed via $\Gamma_{\rho \rightarrow \pi\pi}$, Eq. (29), in both scenarios].

The parameter h_1 , being large- N_c suppressed, will be set to zero in the present study. We then only have to determine the parameter h_2 . This is done by fitting the total decay width of the $a_0(1450)$ meson to its experimental value [21],

$$\begin{aligned} \Gamma_{a_0(1450)}(Z, h_2) &= \Gamma_{a_0 \rightarrow \pi\eta} + \Gamma_{a_0 \rightarrow \pi\eta'} + \Gamma_{a_0 \rightarrow KK} \\ &\quad + \Gamma_{a_0 \rightarrow \omega\pi\pi} \\ &\equiv \Gamma_{a_0(1450)}^{(\text{exp})} = (265 \pm 13) \text{ MeV}. \end{aligned} \quad (47)$$

Although kaons have not been included into the calculations, we can easily evaluate the decay into KK by using flavor symmetry

$$\Gamma_{a_0(1450) \rightarrow KK}(Z, h_2) = 2 \frac{k(m_{a_0}, m_K, m_K)}{8\pi m_{a_0}^2} [A_{a_0 KK}(Z, h_2)]^2, \quad (48)$$

$$\begin{aligned} A_{a_0 KK}(Z, h_2) &= \frac{1}{2Zf_\pi} \left\{ m_{\eta_N}^2 - m_{a_0}^2 + \left(1 - \frac{1}{Z^2}\right) \right. \\ &\quad \times \left[1 - \frac{1}{2} \frac{Z^2 \phi^2}{m_{a_1}^2} (h_2 - h_3) \right] (m_{a_0}^2 - 2m_K^2) \Big\}. \end{aligned} \quad (49)$$

The remaining, experimentally poorly known decay width $\Gamma_{a_0(1450) \rightarrow \omega\pi\pi}$ can be calculated from the sequential decay $a_0 \rightarrow \omega\rho \rightarrow \omega\pi\pi$. Note that the first decay step requires the ρ to be slightly below its mass-shell, since $m_{a_0} < m_\rho + m_\omega$. We denote the off-shell mass of the ρ meson by x . From the Lagrangian (1) we obtain the following formula for the $a_0 \rightarrow \omega\rho$ decay width:

$$\begin{aligned} \Gamma_{a_0(1450) \rightarrow \omega\rho}(x) &= \frac{k(m_{a_0}, m_\omega, x)}{8\pi m_{a_0}^2} (h_2 + h_3)^2 Z^2 f_\pi^2 \\ &\quad \times \left[3 - \frac{x^2}{m_\rho^2} + \frac{(m_{a_0}^2 - x^2 - m_\omega^2)^2}{4m_\omega^2 m_\rho^2} \right]. \end{aligned} \quad (50)$$

The full decay width $\Gamma_{a_0(1450) \rightarrow \omega\pi\pi}$ is then obtained from the following equation:

$$\Gamma_{a_0(1450) \rightarrow \omega\pi\pi} = \int_0^\infty dx \Gamma_{a_0 \rightarrow \omega\rho}(x) d_\rho(x), \quad (51)$$

where $d_\rho(x)$ is the mass distribution of the ρ meson, which is taken to be of relativistic Breit-Wigner form:

$$d_\rho(x) = N \frac{x^2 \Gamma_{\rho \rightarrow \pi\pi}^{(\text{exp})}}{(x^2 - m_\rho^2)^2 + (x \Gamma_{\rho \rightarrow \pi\pi}^{(\text{exp})})^2} \theta(x - 2m_\pi), \quad (52)$$

where $\Gamma_{\rho \rightarrow \pi\pi}^{(\text{exp})} = 149.1$ MeV and $m_\rho = 775.49$ MeV [21]. (In general, one should use the theoretical quantity $\Gamma_{\rho \rightarrow \pi\pi}$, which is itself a function of x , instead of $\Gamma_{\rho \rightarrow \pi\pi}^{(\text{exp})}$, see for instance Refs. [50,51] and references therein. This is, however, numerically irrelevant in the following.) The normalization constant N is chosen such that

$$\int_0^\infty dx d_\rho(x) = 1, \quad (53)$$

in agreement with the interpretation of $dx d_\rho(x)$ as the probability that the off-shell ρ meson has a mass between x and $x + dx$.

Inserting Eqs. (34), (36), (48), and (51) into Eq. (47), we can express h_2 as a function of Z , analogously to Eq. (29) where g_2 was expressed as a function of Z . Similar to that case, we obtain two bands for h_2 , $-115 \leq h_2 \leq -20$ and $-25 \leq h_2 \leq 10$, the width of the bands corresponding to the uncertainty in determining Z , $Z = 1.67 \pm 0.21$. Both bands for h_2 remain practically unchanged if the 5% experimental uncertainty of $\Gamma_{a_0(1450)}^{(\text{exp})}$ is taken into account and thus we only use the mean value 265 MeV in the following. Since h_1 is assumed to be zero, Eq. (26) allows to express m_1 as a function of Z , $m_1 = m_1(Z, h_1 = 0, h_2(Z))$ (we neglect the experimental uncertainties of m_ρ , m_{a_1} , and f_π). The result is shown in Fig. 5. The first band of (lower) h_2 values should be discarded because it leads to $m_1 > m_\rho$. The second set of (higher) values leads to $m_1 < m_\rho$ only if the lower boundary for Z is 1.60 rather than 1.46. Thus, we shall use the set of larger h_2 values and take the constraint $m_1 < m_\rho$ into account by restricting the

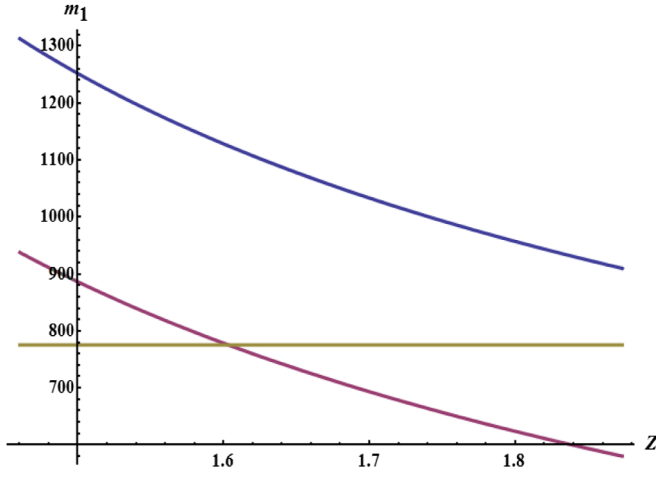


FIG. 5 (color online). Dependence of m_1 on Z . The upper curve corresponds to the lower set of h_2 values and the lower curve to the higher set of h_2 values. The horizontal line corresponds to m_ρ .

values for Z to the range $Z = 1.67^{+0.21}_{-0.07}$. As can be seen from Fig. 5, this sets a lower boundary for the value of m_1 , $m_1 \geq 580$ MeV. Thus, in this scenario we obtain $m_1 = 720^{+55}_{-140}$ MeV.

The values for the other parameters can be found in Table II (only central values are shown with the exception of m_1 where the corresponding uncertainties are stated as well).

Note that $\lambda_1 \ll \lambda_2$, in agreement with the expectations from the large- N_c limit, Eq. (19). The value of $m_1 = 720$ MeV is sizable and constitutes a dominant contribution to the ρ mass. This implies that nonquark contributions, for instance a gluon condensate, play a decisive role in the ρ mass generation.

As a final step, we study the ratios $\Gamma_{a_0(1450) \rightarrow \eta' \pi} / \Gamma_{a_0(1450) \rightarrow \eta \pi}$ and $\Gamma_{a_0(1450) \rightarrow K \bar{K}} / \Gamma_{a_0(1450) \rightarrow \eta \pi}$. Their experimental values read [21]

$$\begin{aligned} \frac{\Gamma_{a_0(1450) \rightarrow \eta' \pi}^{(\text{exp})}}{\Gamma_{a_0(1450) \rightarrow \eta \pi}^{(\text{exp})}} &= 0.35 \pm 0.16; \\ \frac{\Gamma_{a_0(1450) \rightarrow K \bar{K}}^{(\text{exp})}}{\Gamma_{a_0(1450) \rightarrow \eta \pi}^{(\text{exp})}} &= 0.88 \pm 0.23. \end{aligned} \quad (54)$$

TABLE II. Central values of the parameters for Scenario II.

Parameter	h_1	h_2	h_3	g_1
Value	0	4.7	2.4	6.4
Parameter	g_2	m_0^2	m_1	λ_1
Value	3.1	$-811\,987 \text{ MeV}^2$	$720^{+55}_{-140} \text{ MeV}$	-3.6
Parameter	λ_2	c	h_0	
Value	84	$88\,747 \text{ MeV}^2$	$1 \times 10^6 \text{ MeV}^3$	

Using the central value $Z = 1.67$ and $\varphi = -36^\circ$ for the η - η' mixing angle, we obtain $\Gamma_{a_0(1450) \rightarrow \eta' \pi} / \Gamma_{a_0(1450) \rightarrow \eta \pi} = 1.0$ and $\Gamma_{a_0(1450) \rightarrow K \bar{K}} / \Gamma_{a_0(1450) \rightarrow \eta \pi} = 0.96$. The latter is in very good agreement with the experiment, the former a factor of 2 larger. Note, however, that according to Eqs. (32) and (35) the value of the ratio $\Gamma_{a_0(1450) \rightarrow \eta' \pi} / \Gamma_{a_0(1450) \rightarrow \eta \pi}$ is proportional to $\sin^2 \varphi / \cos^2 \varphi$. If a lower value of the angle is considered, e.g., $\varphi = -30^\circ$, then we obtain $\Gamma_{a_0(1450) \rightarrow \eta' \pi} / \Gamma_{a_0(1450) \rightarrow \eta \pi} = 0.58$ for the central value of Z and the central value of $\Gamma_{a_0(1450)}$ in Eq. (47). Taking $Z = Z_{\text{max}}$ and the upper boundary $\Gamma_{a_0(1450)}^{(\text{exp})} = 278 \text{ MeV}$ results in $\Gamma_{a_0(1450) \rightarrow \eta' \pi} / \Gamma_{a_0(1450) \rightarrow \eta \pi} = 0.48$, i.e., in agreement with the experimental value. Therefore, our results in this scenario favor a smaller value of φ than the one suggested by the KLOE Collaboration [37].

It is possible to calculate the decay width $\Gamma_{a_0(1450) \rightarrow \omega \pi \pi}$ using Eq. (51). We have obtained a very small value $\Gamma_{a_0(1450) \rightarrow \omega \pi \pi} = 0.1 \text{ MeV}$. From Eq. (34) we obtain $\Gamma_{a_0(1450) \rightarrow \eta \pi} = 89.5 \text{ MeV}$, such that the ratio $\Gamma_{a_0(1450) \rightarrow \omega \pi \pi} / \Gamma_{a_0(1450) \rightarrow \eta \pi} = 0.0012$, in contrast to the results of Ref. [52].

3. Decay of the $f_0(1370)$ meson

It is now possible to calculate the width for the $f_0(1370) \rightarrow \pi \pi$ decay using Eq. (38). The decay width depends on the $f_0(1370)$ mass, Z , h_1 , and h_2 which is expressed via Z using Eq. (47). The values of the latter three are listed in Table II. In Fig. 6, we show the decay width as a function of the mass of $f_0(1370)$.

Assuming that the two-pion decay dominates the total decay width, we observe a good agreement with the experimental values if $m_{f_0(1370)} \lesssim 1380 \text{ MeV}$. Other contributions to the decay width are likely to reduce this upper bound on $m_{f_0(1370)}$ somewhat. Nevertheless, the correspondence with the experiment is a lot better in this scenario where we have identified $f_0(1370)$ rather than $f_0(600)$ as the (predominantly) isoscalar $\bar{q}q$ state. Note that this result

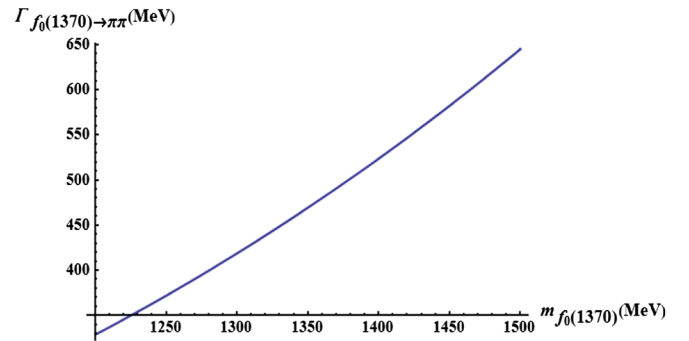


FIG. 6 (color online). Dependence of the $f_0(1370)$ decay width on $m_{f_0(1370)}$. The experimental value of the width is expected to be in the range (1200–1500) MeV [21].

has been obtained using the decay width of the $a_0(1450)$ meson (in order to express h_2 via Z), which is also assumed to be a scalar $\bar{q}q$ state in this scenario.

It is remarkable that vector mesons are crucial to obtain realistic values for the decay width of $f_0(1370)$: without vector mesons, the decay width is ~ 10 GeV and thus much too large. This is why Scenario II has not been considered in the standard linear sigma model.

The four-body decay $f_0(1370) \rightarrow 4\pi$ can also be studied. Similarly to the $a_0(1450) \rightarrow \omega\rho$ decay, we view $f_0(1370) \rightarrow 4\pi$ as a sequential decay of the form $f_0(1370) \rightarrow \rho\rho \rightarrow 4\pi$. The Lagrangian (1) leads to

$$\begin{aligned} \Gamma_{f_0(1370) \rightarrow \rho\rho}(x_1, x_2) &= \frac{3}{16\pi} \frac{k(m_{f_0}, x_1, x_2)}{m_{f_0}^2} (h_1 + h_2 + h_3)^2 Z^2 f_\pi^2 \\ &\times \left[4 - \frac{x_1^2 + x_2^2}{m_\rho^2} + \frac{(m_{f_0}^2 - x_1^2 - x_2^2)^2}{4m_\rho^4} \right], \end{aligned} \quad (55)$$

where x_1 and x_2 are the off-shell masses of the ρ mesons. The decay width $\Gamma_{f_0 \rightarrow 4\pi}$ is then given by

$$\Gamma_{f_0(1370) \rightarrow 4\pi} = \int_0^\infty \int_0^\infty dx_1 dx_2 \Gamma_{f_0(1370) \rightarrow \rho\rho}(x_1, x_2) d_\rho(x_1) \times d_\rho(x_2),$$

with $\Gamma_{f_0(1370) \rightarrow \rho\rho}(x_1, x_2)$ from Eq. (55) and $d_\rho(x)$ from Eq. (52).

Using the previous values for the parameters we obtain that the $\rho\rho$ contribution for the decay is small: $\Gamma_{f_0(1370) \rightarrow \rho\rho \rightarrow 4\pi} \simeq 10 \pm 10$ MeV. (The error comes from varying Z between 1.6 and 1.88.) Reference [53] quotes 54 MeV for the total 4π decay width. Since Ref. [54] ascertains that about 26% of the total 4π decay width originates from the $\rho\rho$ decay channel, our result is consistent with these findings.

V. CONCLUSIONS AND OUTLOOK

We have presented a linear sigma model with vector mesons and global chiral invariance. The motivation for considering global invariance rather than the standard Sakurai model with local chiral invariance (with the exception of the vector-meson mass term) was that the latter fails to describe some important low-energy meson decay processes correctly, most notably the two-pion decay width of the ρ meson [10]. This Lagrangian describes mesons as pure quarkonium states. As shown in Sec. IV A, the resulting low-energy phenomenology is in general in good agreement with experimental data—with one exception: the model fails to correctly describe the $f_0(600) \rightarrow \pi\pi$ decay width. This led us to conclude that $f_0(600)$ and $a_0(980)$ cannot be predominantly $\bar{q}q$ states.

Assigning the scalar fields σ and a_0 of the model to the $f_0(1370)$ and $a_0(1450)$ resonances, respectively, improves the results for the decay widths considerably. We have

obtained $\Gamma_{f_0(1370) \rightarrow \pi\pi} \simeq 300\text{--}500$ MeV for $m_{f_0(1370)} = 1200\text{--}1400$ MeV (see Fig. 6). Thus, the scenario in which the scalar states above 1 GeV, $f_0(1370)$ and $a_0(1450)$, are considered to be (predominantly) $\bar{q}q$ states appears to be favored over the assignment in which $f_0(600)$ and $a_0(980)$ are considered (predominantly) $\bar{q}q$ states. However, a more detailed study of this scenario is necessary, because a glueball state with the same quantum numbers mixes with the quarkonium states. This allows to include the experimentally well-known resonance $f_0(1500)$ into the study.

Of course, interpreting $f_0(1370)$ and $a_0(1450)$ as $\bar{q}q$ states leads to question about the nature of $f_0(600)$ and $a_0(980)$. Their presence is necessary for the correct description of $\pi\pi$ scattering lengths that differ from experiment for too large values of the isoscalar mass (see Sec. IV A 1). We distinguish two possibilities: (i) They can arise as (quasi-)molecular states. This is possible if the attraction in the $\pi\pi$ and KK channels is large enough. In order to prove this, one should solve the corresponding Bethe-Salpeter equation in the framework of Scenario II. In this case $f_0(600)$ and $a_0(980)$ can be classified as genuinely dynamically generated states and should not appear in the Lagrangian, see the discussion in Ref. [14]. If, however, the attraction is not sufficient to generate the two resonances $f_0(600)$ and $a_0(980)$ we are led to the alternative possibility that (ii) these two scalar states must be incorporated into the model as additional tetraquark states. In this case they shall appear from the very beginning in the Lagrangian and should not be considered as dynamically generated states. Of course, the isoscalar tetraquark, quarkonium, and glueball will mix to produce $f_0(600)$, $f_0(1370)$, and $f_0(1500)$, and the isovector tetraquark and quarkonium will mix to produce $a_0(980)$ and $a_0(1450)$.

The issue of restoration of chiral symmetry at nonzero temperature and density is one of the fundamental questions of modern hadron and nuclear physics, see, e.g., Refs. [8,17]. Linear sigma models constitute an effective approach to study chiral symmetry restoration because they contain from the onset not only pseudoscalar and vector mesons, but also their chiral partners with which they become degenerate once the chiral symmetry has been restored. Once vacuum phenomenology is reasonably well reproduced within our model, we also plan to apply it to studies of chiral symmetry restoration at nonzero temperatures and densities.

Another important check of the model is the description of the ALEPH data for the decay of the τ lepton into two and three pions [55]. In this way, we will have a better constraint on the parameters of the model, e.g., the value for the a_1 mass.

An extension of the model to $N_f = 3$ can be performed [56]; with the exception of the strange quark condensate, no further free parameters will arise in this extension.

However, much more data are available for the strange mesons, which constitute an important test for the validity of our approach.

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APPENDIX: THE FULL LAGRANGIAN

This is the final form of the Lagrangian (1) that is obtained after the shifts (9) and the renormalization of the pseudoscalar wave functions; $\rho^{\mu\nu} \equiv \partial^\mu \rho^\nu - \partial^\nu \rho^\mu$; $a_1^{\mu\nu} \equiv \partial^\mu a_1^\nu - \partial^\nu a_1^\mu$; $(\vec{A})_3$ marks the third component of the vector $\vec{A}$. Note that the term $\mathcal{L}_4$ contains the (axial-) vector four-point vertices [the terms $\sim g_{4,5,6,7}$ in the Lagrangian (1)]. We do not give the explicit form of $\mathcal{L}_4$ because it is not relevant for the results that are presented in this paper.

$$\begin{aligned}
\mathcal{L} = & \frac{1}{2}(\partial^\mu \sigma + g_1 Z \vec{\pi} \cdot \vec{a}_1^\mu + g_1 w Z^2 \partial^\mu \vec{\pi} \cdot \vec{\pi} + g_1 Z \eta f_1^\mu + g_1 w Z^2 \eta \partial^\mu \eta)^2 - \frac{1}{2} \left[m_0^2 - c + 3 \left(\lambda_1 + \frac{\lambda_2}{2} \right) \phi^2 \right] \sigma^2 \\
& + \frac{1}{2} (Z \partial^\mu \vec{\pi} + g_1 Z \vec{\rho}^\mu \times \vec{\pi} - g_1 f_1^\mu \vec{a}_0 - g_1 w Z \partial^\mu \eta \vec{a}_0 - g_1 \sigma \vec{a}_1^\mu - g_1 w Z \sigma \partial^\mu \vec{\pi})^2 \\
& + \frac{1}{2} (Z \partial^\mu \eta - g_1 \sigma f_1^\mu - g_1 w Z \sigma \partial^\mu \eta - g_1 \vec{a}_1^\mu \cdot \vec{a}_0 - g_1 w Z \partial^\mu \vec{\pi} \cdot \vec{a}_0)^2 - \frac{1}{2} \left[m_0^2 - c + \left(\lambda_1 + \frac{\lambda_2}{2} \right) \phi^2 \right] Z^2 \vec{\pi}^2 \\
& - \frac{1}{2} \left[m_0^2 + c + \left(\lambda_1 + \frac{\lambda_2}{2} \right) \phi^2 \right] Z^2 \eta^2 + \frac{1}{2} [\partial^\mu \vec{a}_0 + g_1 \vec{\rho}^\mu \times \vec{a}_0 + g_1 Z f_1^\mu \vec{\pi} + g_1 w Z^2 \vec{\pi} \partial^\mu \eta \\
& + g_1 Z \eta \vec{a}_1^\mu + g_1 w Z^2 \eta \partial^\mu \vec{\pi}]^2 - \frac{1}{2} \left[m_0^2 + c + \left(\lambda_1 + \frac{3}{2} \lambda_2 \right) \phi^2 \right] \vec{a}_0^2 \\
& - \frac{\lambda_2}{2} [(\sigma \vec{a}_0 + Z^2 \eta \vec{\pi})^2 + Z^2 \vec{a}_0^2 \vec{\pi}^2 - Z^2 (\vec{a}_0 \cdot \vec{\pi})^2] - \frac{1}{4} \left(\lambda_1 + \frac{\lambda_2}{2} \right) (\sigma^2 + \vec{a}_0^2 + Z^2 \eta^2 + Z^2 \vec{\pi}^2)^2 \\
& - \left(\lambda_1 + \frac{\lambda_2}{2} \right) \phi \sigma (\sigma^2 + \vec{a}_0^2 + Z^2 \eta^2 + Z^2 \vec{\pi}^2) - \lambda_2 \phi \vec{a}_0 \cdot (\sigma \vec{a}_0 + Z^2 \eta \vec{\pi}) - \frac{1}{4} (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu)^2 + \frac{m_1^2}{2} (\omega^\mu)^2 \\
& - \frac{1}{4} [\partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu + g_2 \vec{\rho}^\mu \times \vec{\rho}^\nu + g_2 \vec{a}_1^\mu \times \vec{a}_1^\nu + g_2 w Z \partial^\mu \vec{\pi} \times \vec{a}_1^\nu + g_2 w Z \vec{a}_1^\mu \times \partial^\nu \vec{\pi} + g_2 w^2 Z^2 (\partial^\mu \vec{\pi}) \times (\partial^\nu \vec{\pi})]^2 \\
\\
& + \frac{m_1^2}{2} (\vec{\rho}^\mu)^2 - \frac{1}{4} (\partial^\mu f_1^\nu - \partial^\nu f_1^\mu)^2 + \frac{m_1^2 + g_1^2 \phi^2}{2} (f_1^\mu)^2 + \frac{m_1^2 + g_1^2 \phi^2}{2} (\vec{a}_1^\mu)^2 \\
& - \frac{1}{4} [\partial^\mu \vec{a}_1^\nu - \partial^\nu \vec{a}_1^\mu + g_2 \vec{\rho}^\mu \times \vec{a}_1^\nu + g_2 w Z \vec{\rho}^\mu \times \partial^\nu \vec{\pi} + g_2 \vec{a}_1^\mu \times \vec{\rho}^\nu + g_2 w Z (\partial^\mu \vec{\pi}) \times \vec{\rho}^\nu]^2 \\
& - g_1^2 \phi \vec{a}_{1\mu} \cdot [\vec{\rho}^\mu \times Z \vec{\pi} - f_1^\mu \vec{a}_0 - w Z \vec{a}_0 \partial^\mu \eta] - g_1^2 w Z \phi \partial_\mu \vec{\pi} \cdot [Z \vec{\rho}^\mu \times \vec{\pi} - f_1^\mu \vec{a}_0 - w Z \partial^\mu \eta \vec{a}_0] \\
& + g_1^2 \phi f_{1\mu} (\vec{a}_1^\mu \cdot \vec{a}_0 + w Z \partial^\mu \vec{\pi} \cdot \vec{a}_0) + g_1^2 w Z \phi \partial_\mu \eta (\vec{a}_1^\mu \cdot \vec{a}_0 + w Z \partial^\mu \vec{\pi} \cdot \vec{a}_0) \\
& + g_1^2 \phi \sigma [(f_1^\mu)^2 + 2 w Z f_{1\mu} \partial^\mu \eta + w^2 Z^2 (\partial^\mu \eta)^2] \\
& + g_1^2 \phi \sigma [(\vec{a}_1^\mu)^2 + 2 w Z \vec{a}_{1\mu} \cdot \partial^\mu \vec{\pi} + w^2 Z^2 (\partial^\mu \vec{\pi})^2] - \frac{1}{2} \frac{g_1^2 \phi^2}{m_{a_1}^2} Z^2 (\partial_\mu \eta \partial^\mu \eta + \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi})
\end{aligned}$$

$$\begin{aligned}
& + eA_\mu \{(\vec{a}_0 \times \partial^\mu \vec{a}_0)_3 + Z^2(\vec{\pi} \times \partial^\mu \vec{\pi})_3 - 4(\vec{\rho}^{\mu\nu} \times \vec{\rho}_\nu)_3 - 4[(\vec{a}_{1\nu} + Zw\partial_\nu \vec{\pi}) \times \vec{a}_1^{\mu\nu}]_3 \\
& + g_1\{2Z(f_1^\mu + Zw\partial^\mu \eta)(\vec{a}_0 \times \vec{\pi})_3 + Z(\sigma + \phi)[(\vec{a}_1^\mu + Zw\partial^\mu \vec{\pi}) \times \vec{\pi}]_3 + \eta[\vec{a}_0 \times (\vec{a}_1^\mu + Zw\partial^\mu \vec{\pi})]_3 \\
& - a_0^3(a_0^\mu \rho^{\mu 1} + a_0^\nu \rho^{\mu 2}) - Z^2\pi^3(\pi^1 \rho^{\mu 1} + \pi^2 \rho^{\mu 2}) + \rho^{\mu 3}[(a_0^1)^2 + (a_0^2)^2 + Z^2(\pi^1)^2 + Z^2(\pi^2)^2]\} \\
& + 4g_2\{[\vec{\rho}_\nu^2 + \vec{a}_{1\nu}^2 + Z^2w^2(\partial_\nu \vec{\pi})^2 + Zw\vec{a}_{1\nu} \cdot \partial^\nu \vec{\pi}]\rho^{\mu 3} + 2\vec{\rho}_\nu \cdot (\vec{a}_1^\nu + Zw\partial^\nu \vec{\pi}) \times (a_1^{\mu 3} + Zw\partial^\mu \pi^3) \\
& - (\vec{\rho}_\nu \cdot \vec{\rho}^\mu + \vec{a}_{1\nu} \cdot \vec{a}_1^\mu + Zw\vec{a}_{1\nu} \cdot \partial^\mu \vec{\pi} + Zw\vec{a}_1^\mu \cdot \partial_\nu \vec{\pi} + Z^2w^2\partial_\nu \vec{\pi} \cdot \partial^\mu \vec{\pi})\rho^{\nu 3} \\
& - (\vec{\rho}^\mu \cdot \vec{a}_{1\nu} + \vec{a}_1^\mu \cdot \vec{\rho}_\nu + Zw\vec{\rho}^\mu \cdot \partial_\nu \vec{\pi} + Zw\vec{\rho}_\nu \cdot \partial^\mu \vec{\pi})a_1^{\nu 3}\} + \frac{e^2}{2}A_\mu A^\mu[(a_0^1)^2 + (a_0^2)^2 + Z^2(\pi^1)^2 \\
& + Z^2(\pi^2)^2 + 4(\rho^{\nu 1})^2 + 4(\rho^{\nu 2})^2 + 4(a_{1\nu}^1 + Zw\partial_\nu \pi^1)^2 + 4(a_{1\nu}^2 + Zw\partial_\nu \pi^2)^2] \\
& - 2e^2A_\mu A_\nu[\rho^{\mu 1}\rho^{\nu 1} + \rho^{\mu 2}\rho^{\nu 2} + (a_1^{\mu 1} + Zw\partial^\mu \pi^1)(a_1^{\nu 1} + Zw\partial^\nu \pi^1) + (a_1^{\mu 2} + Zw\partial^\mu \pi^2)(a_1^{\nu 2} + Zw\partial^\nu \pi^2)] \\
& + \mathcal{L}_{h_{1,2,3}} + \mathcal{L}_{g_3} + \mathcal{L}_4
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{h_{1,2,3}} = & \left(\frac{h_1}{4} + \frac{h_2}{4} + \frac{h_3}{4}\right)(\sigma^2 + 2\phi\sigma + Z^2\eta^2 + \vec{a}_0^2 + Z^2\vec{\pi}^2)(\omega_\mu^2 + \vec{\rho}_\mu^2) + \left(\frac{h_1}{4} + \frac{h_2}{4} - \frac{h_3}{4}\right) \\
& \times (\sigma^2 + 2\phi\sigma + Z^2\eta^2 + \vec{a}_0^2 + Z^2\vec{\pi}^2)\{f_{1\mu}^2 + \vec{a}_{1\mu}^2 + Z^2w^2[(\partial_\mu \eta)^2 + (\partial_\mu \vec{\pi})^2] + 2Zw(f_{1\mu}\partial^\mu \eta + \vec{a}_{1\mu} \cdot \partial^\mu \vec{\pi})\} \\
& + \left(\frac{h_1}{4} + \frac{h_2}{4} + \frac{h_3}{4}\right)\phi^2(\omega_\mu^2 + \vec{\rho}_\mu^2) + \left(\frac{h_1}{4} + \frac{h_2}{4} - \frac{h_3}{4}\right)\phi^2(f_{1\mu}^2 + \vec{a}_{1\mu}^2) + (h_2 + h_3)\omega_\mu[(\sigma + \phi)\vec{a}_0 + Z^2\eta\vec{\pi}] \cdot \vec{\rho}^\mu \\
& + (h_2 - h_3)[(\sigma + \phi)\vec{a}_0 + Z^2\eta\vec{\pi}] \cdot [f_{1\mu}\vec{a}_1^\mu + Zw(\vec{a}_{1\mu}\partial^\mu \eta + f_{1\mu}\partial^\mu \vec{\pi}) + Z^2w^2(\partial_\mu \eta)(\partial^\mu \vec{\pi})] \\
& + (h_2 + h_3)Z(\vec{a}_0 \times \vec{\pi}) \cdot (\omega_\mu\vec{a}_1^\mu + Zw\omega_\mu\partial^\mu \vec{\pi}) + (h_2 - h_3)Z(\vec{a}_0 \times \vec{\pi}) \cdot (f_{1\mu}\vec{\rho}^\mu + Zw\vec{\rho}_\mu\partial^\mu \eta) \\
& + h_3Z[\eta\vec{a}_0 - (\sigma + \phi)\vec{\pi}] \cdot [\vec{\rho}_\mu \times (\vec{a}_1^\mu + Zw\partial^\mu \vec{\pi})] - \frac{h_3}{2}\{(\vec{a}_0 \times \vec{\rho}^\mu)^2 - [\vec{a}_0 \times (\vec{a}_1^\mu + Zw\partial^\mu \vec{\pi})]^2 \\
& + Z^2(\vec{\pi} \times \vec{\rho}^\mu)^2 - Z^2[\vec{\pi} \times (\vec{a}_1^\mu + Zw\partial^\mu \vec{\pi})]^2\}
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{g_3} = & -4g_3\{\partial_\mu \omega_\nu[\omega^\mu \omega^\nu + f_1^\mu f_1^\nu + \vec{\rho}^\mu \cdot \vec{\rho}^\nu + \vec{a}_1^\mu \cdot \vec{a}_1^\nu + Zw(f_1^\mu \partial^\nu \eta + f_1^\nu \partial^\mu \eta + \vec{a}_1^\mu \cdot \partial^\nu \vec{\pi} + \vec{a}_1^\nu \cdot \partial^\mu \vec{\pi}) \\
& + Z^2w^2(\partial^\mu \eta \partial^\nu \eta + \partial^\mu \vec{\pi} \cdot \partial^\nu \vec{\pi})] + (\partial_\mu f_{1\nu} + Zw\partial_\mu \partial_\nu \eta)[\omega^\mu f_1^\nu + \omega^\nu f_1^\mu + \vec{\rho}^\mu \cdot \vec{a}_1^\nu + \vec{\rho}^\nu \cdot \vec{a}_1^\mu \\
& + Zw(\omega^\mu \partial^\nu \eta + \omega^\nu \partial^\mu \eta + \vec{\rho}^\mu \cdot \partial^\nu \vec{\pi} + \vec{\rho}^\nu \cdot \partial^\mu \vec{\pi})] + \partial_\mu \vec{\rho}_\nu \cdot [\omega^\mu \vec{\rho}^\nu + \omega^\nu \vec{\rho}^\mu + f_1^\mu \vec{a}_1^\nu + f_1^\nu \vec{a}_1^\mu \\
& + Zw(\vec{a}_1^\mu \partial^\nu \eta + \vec{a}_1^\nu \partial^\mu \eta + f_1^\mu \partial^\nu \vec{\pi} + f_1^\nu \partial^\mu \vec{\pi}) + Z^2w^2(\partial^\mu \eta \partial^\nu \vec{\pi} + \partial^\nu \eta \partial^\mu \vec{\pi})] + (\partial_\mu \vec{a}_{1\nu} + Zw\partial_\mu \partial_\nu \vec{\pi}) \\
& \cdot [f_1^\mu \vec{\rho}^\nu + f_1^\nu \vec{\rho}^\mu + \omega^\mu \vec{a}_1^\nu + \omega^\nu \vec{a}_1^\mu + Zw(\vec{\rho}^\mu \partial^\nu \eta + \vec{\rho}^\nu \partial^\mu \eta + \omega^\mu \partial^\nu \vec{\pi} + \omega^\nu \partial^\mu \vec{\pi})]\} \\
& + 4eg_3A_\mu\{\omega_\nu[(\vec{\rho}^\mu \times \vec{\rho}^\nu)_3 + (\vec{a}_1^\mu \times \vec{a}_1^\nu)_3 + Zw(\partial^\mu \vec{\pi} \times \vec{a}_1^\nu)_3 + Zw(\vec{a}_1^\mu \times \partial^\nu \vec{\pi})_3 + Z^2w^2(\partial^\mu \vec{\pi} \times \partial^\nu \vec{\pi})_3] \\
& + (f_{1\nu} + Zw\partial_\nu \eta)[(\vec{\rho}^\mu \times \vec{a}_1^\nu)_3 + (\vec{a}_1^\mu \times \vec{\rho}^\nu)_3 + Zw(\vec{\rho}^\mu \times \partial^\nu \vec{\pi})_3 + Zw(\partial^\mu \vec{\pi} \times \vec{\rho}^\nu)_3]\}
\end{aligned}$$

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Glueball in a chiral linear sigma model with vector mesons

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We present a two-flavor linear sigma model with global chiral symmetry and (axial-)vector mesons as well as an additional glueball degree of freedom. We study the structure of the well-established scalar resonances $f_0(1370)$ and $f_0(1500)$: by a fit to experimentally known decay widths we find that $f_0(1370)$ is predominantly a $\bar{q}q$ state and $f_0(1500)$ is predominantly a glueball state. The overall phenomenology of these two resonances can be well described. Other assignments for our mixed quarkonium-glueball states are also tested, but turn out to be in worse agreement with the phenomenology. As a by-product of our analysis, the gluon condensate is determined.

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I. INTRODUCTION

Glueballs, the bound states of the gauge bosons of QCD, the gluons, are naturally expected in QCD due to the non-Abelian nature of the theory: the gluons interact strongly with themselves and thus they can bind and form colorless states, analogously to what occurs in the quark sector. The existence of glueballs has been studied in the framework of the effective bag model for QCD already four decades ago [1] and it has been further investigated in a variety of approaches [2]. Numerical calculations of the Yang-Mills sector of QCD also find a full glueball spectrum in which the scalar glueball is the lightest state [3].

Glueballs can mix with quarkonium ($\bar{q}q$) states with the same quantum numbers. This makes the experimental search for glueballs more complicated, because physical resonances emerge as mixed states. The scalar sector $J^{PC} = 0^{++}$ has been investigated in many works in the past. The resonance $f_0(1500)$ is relatively narrow when compared to other scalar-isoscalar states: for this reason it has been considered as a convincing candidate for a glueball state. Mixing scenarios in which two quark-antiquark isoscalar states $\bar{n}n$ and $\bar{s}s$ and one scalar glueball gg mix and generate the physical resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ have been discussed in Refs. [4,5].

The aim of this work is to extend the linear chiral model of Refs. [6,7] by including the dilaton/glueball field. The first attempt to incorporate a glueball into a linear sigma model was performed long ago in Ref. [8]. The novel features of the present study are the following: (i) The glueball is introduced as a dilaton field within a theoretical framework where not only scalar and pseudoscalar mesons, but also vector and axial-vector mesons, are present from the very beginning. This fact allows also for a calculation of decays into vector mesons. The model is explicitly evaluated for the case of $N_f = 2$, for which only one scalar-isoscalar quarkonium state exists: $\sigma \equiv \bar{n}n$ which mixes with the glueball. The two emerging mixed states are assigned to the resonances $f_0(1370)$ which is, in

accordance with Ref. [6], predominantly a $\bar{q}q$ state, and with $f_0(1500)$, which is predominantly a glueball state. (ii) We consequently test—to our knowledge for the first time—this mixing scenario above 1 GeV in the framework of a chiral model.

The model under consideration is built in accordance with the symmetries of the QCD Lagrangian. It possesses the known degrees of freedom of low-energy QCD [(pseudo)scalar and (axial-)vector mesons] as well as the same global chiral invariance. Another feature of the QCD Lagrangian is scale (or dilatation) invariance $x^\mu \rightarrow \lambda^{-1}x^\mu$ (where x^μ is a Minkowski-space coordinate and λ the scale parameter of the conformal group). It is realized at the classical level but broken at the quantum level due to the loop corrections in the Yang-Mills sector (scale anomaly). In this work the breaking of scale invariance is implemented at tree-level by means of a dilaton field (representing a glueball) with the usual logarithmic dilaton potential [8]. However, all the other interaction terms (with the exception of the chiral anomaly) are dilatation-invariant in the chiral limit.

Having constructed the Lagrangian of the effective model, we calculate the masses of the pure $\bar{q}q$ and glueball states in the $J^{PC} = 0^{++}$ channel, study their mixing and calculate the decay widths of the mixed states. Although we work with $N_f = 2$, the use of flavor symmetry enables us to calculate the decay widths of the scalar resonances into kaons and into both the η and η' mesons which contain the s -quark in their flavor wave functions. After the study of the already mentioned scenario where $f_0(1370)$ and $f_0(1500)$ are predominantly quarkonium and glueball, respectively, we also test the alternative scenario in which the resonance $f_0(1710)$ is predominantly glueball and scenarios in which $f_0(600)$ is predominantly quarkonium. They, however, lead to inconsistencies when compared to the present data and are therefore regarded as less favorable.

This paper is organized as follows. In Sec. II, the Lagrangian of the linear sigma model with (axial-)vector

and glueball degrees of freedom is constructed. In Sec. III, we discuss the results for the masses of the quarkonium-glueball mixed states and their decay widths. In Sec. IV, we present our conclusions.

II. THE MODEL

The Yang-Mills (YM) sector of QCD (QCD without quarks) is classically invariant under dilatations. This symmetry is, however, broken at the quantum level. The divergence of the corresponding current is the trace of the energy-momentum tensor $T_{YM}^{\mu\nu}$ of the YM Lagrangian

$$(T_{YM})_\mu^\mu = \frac{\beta(g)}{4g} G_{\mu\nu}^a G^{a,\mu\nu} \neq 0, \quad (1)$$

where $G_{\mu\nu}^a$ is the field-strength tensor of the gluon fields, $g = g(\mu)$ is the renormalized coupling constant at the scale μ , and the β -function is given by $\beta(g) = \partial g / \partial \ln \mu$. At the one-loop level $\beta(g) = -bg^3$ with $b = 11N_c/(48\pi^2)$. This implies $g^2(\mu) = [2b \ln(\mu/\Lambda_{YM})]^{-1}$, where $\Lambda_{YM} \simeq 200$ MeV is the Yang-Mills scale. A finite energy scale thus emerges in a theory which is classically invariant under dilatation (dimensional transmutation). The expectation value of the trace anomaly does not vanish and represents the so-called gluon condensate:

$$\langle T_{YM,\mu}^\mu \rangle = -\frac{11N_c}{48} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a,\mu\nu} \right\rangle = -\frac{11N_c}{48} C^4, \quad (2)$$

where

$$C^4 \simeq (300\text{--}600 \text{ MeV})^4. \quad (3)$$

The numerical values have been obtained through QCD sum rules (lower range of the interval) [9] and lattice simulations (higher range of the interval) [10].

At the composite level, one can build an effective theory of the YM sector of QCD by introducing a scalar dilaton

field G which describes the trace anomaly. The dilaton Lagrangian reads [8]

$$\mathcal{L}_{\text{dil}} = \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4} \frac{m_G^2}{\Lambda^2} \left(G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right). \quad (4)$$

The minimum G_0 of the dilaton potential is realized for $G_0 = \Lambda$. Upon shifting $G \rightarrow G_0 + G$ a particle with mass m_G emerges, which is interpreted as the scalar glueball. The numerical value has been evaluated in Lattice QCD and reads $m_G \sim 1.5$ GeV [3]. The logarithmic term of the potential explicitly breaks the invariance under a dilatation transformation. The divergence of the corresponding current reads $\partial_\mu J_{\text{dil}}^\mu = T_{\text{dil},\mu}^\mu = -\frac{1}{4} m_G^2 \Lambda^2$. This can be compared with the analogous quantity in Eq. (2), which implies $\Lambda = \sqrt{11} C^2 / (2m_G)$.

QCD with quarks is also classically invariant under dilatation transformations in the limit of zero quark masses (chiral limit). The scale of all hadronic phenomena is given by the previously introduced energy scale Λ_{YM} . This fact holds true also when the small but nonzero values of the quark masses are considered. In order to describe these properties in a hadronic model we now extend the linear sigma model with $U(N_f)_R \times U(N_f)_L$ of Refs. [6,11–13] by including the dilaton. To this end, the following criteria are applied [14]: (i) With the exception of the chiral anomaly, the parameter Λ from Eq. (4), which comes from the Yang-Mills sector of the theory in accordance with QCD, is the only dimensionful parameter of the Lagrangian in the chiral limit. (ii) The Lagrangian is required to be finite for every finite value of the gluon condensate G_0 . This, in turn, also assures that no singular terms arise in the limit $G_0 \rightarrow 0$. In accordance with the requirements (i) and (ii) only terms with dimension exactly equal to 4 are allowed in the chiral limit.

The hadronic Lagrangian obeying these requirements reads

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{dil}} + \text{Tr} \left[(D^\mu \Phi)^\dagger (D_\mu \Phi) - m_0^2 \left(\frac{G}{G_0} \right)^2 \Phi^\dagger \Phi - \lambda_2 (\Phi^\dagger \Phi)^2 \right] - \lambda_1 (\text{Tr}[\Phi^\dagger \Phi])^2 + c [\det(\Phi^\dagger) + \det(\Phi)] \\ & + \text{Tr}[H(\Phi^\dagger + \Phi)] - \frac{1}{4} \text{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2} \left(\frac{G}{G_0} \right)^2 \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + \frac{h_1}{2} \text{Tr}[\Phi^\dagger \Phi] \text{Tr}[L_\mu L^\mu + R_\mu R^\mu] \\ & + h_2 \text{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3 \text{Tr}[\Phi R_\mu \Phi^\dagger L^\mu] + \dots, \end{aligned} \quad (5)$$

where Φ denotes the $N_f \times N_f$ (pseudo)scalar multiplet and L^μ and R^μ the left- and right-handed vector multiplets, respectively. The dots represent further terms which do not affect the processes studied in this work.

In the particular case of $N_f = 2$ studied in this manuscript, one has $\Phi = (\sigma + i\eta_N)t^0 + (\tilde{a}_0 + i\tilde{\pi}) \cdot \vec{t}$ (our eta meson η_N contains only nonstrange degrees of freedom), $L^\mu = (\omega^\mu + f_1^\mu)t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu) \cdot \vec{t}$ and

$R^\mu = (\omega^\mu - f_1^\mu)t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu) \cdot \vec{t}$; $t^0, \vec{t}$ are the generators of $U(2)$. Moreover, $D^\mu \Phi = \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu)$, $L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu$, and $R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$.

The explicit breaking of the global chiral symmetry is described by the term $\text{Tr}[H(\Phi + \Phi^\dagger)] \equiv h\sigma$ ($h = \text{const.} \sim m_q^2$), which allows us to take into account the nonvanishing value m_q of the quark mass. This term contains the dimensionful parameter h with

$[h] = [\text{energy}^3]$ and also explicitly breaks the dilatation invariance, just as the quark masses do in the underlying QCD Lagrangian. Finally, the chiral anomaly is described by the term $c(\det\Phi + \det\Phi^\dagger)$ [15]. For $N_f = 2$, the parameter c carries the dimension $[\text{energy}^2]$ and represents a further breaking of dilatation invariance. This term arises from instantons which are also a property of the Yang-Mills sector of QCD.

The identification of the fields of the models with the resonances listed in Ref. [16] is straightforward in the pseudoscalar and (axial-)vector sectors: the fields $\vec{\pi}$ and η_N correspond to the pion and the $SU(2)$ counterpart of the η meson, respectively, $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$, with a mass of about 700 MeV. This value can be obtained by “unmixing” the physical η and η' mesons which also contain $\bar{s}s$ contributions. The fields ω^μ and $\vec{\rho}^\mu$ represent the $\omega(782)$ and $\rho(770)$ vector mesons, respectively, while the fields f_1^μ and $\vec{a}_1^\mu$ represent the $f_1(1285)$ and $a_1(1260)$ axial-vector mesons, respectively. (In principle, the physical ω and f_1 states also contain $\bar{s}s$ contributions but their admixture is small.) As shown in Ref. [6], the σ field should be interpreted as a predominantly $\bar{q}q$ state because its decay width decreases as $1/N_c$ in the limit of a large number of colors. The σ and G fields mix: the physical fields σ' and G' are obtained through an $SO(2)$ rotation, as we shall show in the following. Then the first and most natural assignment is $\{\sigma', G'\} = \{f_0(1370), f_0(1500)\}$, see Sec. III A. Note that the $\vec{a}_0$ state is assigned to the physical $a_0(1450)$ resonance in accordance with results of Ref. [6]. Other assignments for $\{\sigma', G'\}$ will be also tested in Sec. III B and III C and turn out to be less favorable.

In order to study the nonvanishing vacuum expectation values (VEV's) of the two $J^{PC} = 0^{++}$ scalar-isoscalar fields of the model σ and G , we set all the other fields in Eq. (5) to zero and obtain:

$$\mathcal{L}_{\sigma G} = \mathcal{L}_{\text{dil}} + \frac{1}{2}(\partial^\mu \sigma)^2 - \frac{1}{2}\left[m_0^2\left(\frac{G}{G_0}\right)^2 - c\right]\sigma^2 - \frac{1}{4}\left(\lambda_1 + \frac{\lambda_2}{2}\right)\sigma^4 + h\sigma. \quad (6)$$

Upon shifting the fields by their VEV's, $\sigma \rightarrow \sigma + \phi$ and $G \rightarrow G + G_0$, we obtain the masses of the states $\sigma = (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $G = gg$ [6],

$$M_\sigma^2 = m_0^2 - c + 3\left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi^2, \quad (7)$$

$$M_G^2 = m_0^2 \frac{\phi^2}{G_0^2} + m_G^2 \frac{G_0^2}{\Lambda^2} \left(1 + 3 \ln \left| \frac{G_0}{\Lambda} \right| \right).$$

Note that the pure glueball mass M_G depends also on the quark condensate ϕ , but correctly reduces to m_G in the limit $m_0^2 = 0$ (decoupling of quarkonia and glueball). In the presence of quarkonia, $m_0^2 \neq 0$, the VEV G_0 is given by the equation

$$-\frac{m_0^2 \phi^2 \Lambda^2}{m_G^2} = G_0^4 \ln \left| \frac{G_0}{\Lambda} \right|. \quad (8)$$

The shift of the fields by their VEV's introduces a bilinear mixing term $\sim \sigma G$ in the Lagrangian (6). The physical fields σ' and G' can be obtained through an $SO(2)$ rotation,

$$\begin{pmatrix} \sigma' \\ G' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sigma \\ G \end{pmatrix}, \quad (9)$$

with

$$M_{\sigma'}^2 = M_\sigma^2 \cos^2\theta + M_G^2 \sin^2\theta + 2m_0^2 \frac{\phi}{G_0} \sin(2\theta), \quad (10)$$

$$M_{G'}^2 = M_G^2 \cos^2\theta + M_\sigma^2 \sin^2\theta - 2m_0^2 \frac{\phi}{G_0} \sin(2\theta), \quad (11)$$

where the mixing angle θ reads

$$\theta = \frac{1}{2} \arctan \left[-4 \frac{\phi}{G_0} \frac{m_0^2}{M_G^2 - M_\sigma^2} \right]. \quad (12)$$

The quantity m_0^2 can be calculated from the masses of the pion, η_N , and the bare σ mass [see Ref. [6]]:

$$m_0^2 = \left(\frac{m_\pi}{Z}\right)^2 + \frac{1}{2} \left[\left(\frac{m_{\eta_N}}{Z}\right)^2 - M_\sigma^2 \right]. \quad (13)$$

If $m_0^2 - c < 0$, spontaneous breaking of chiral symmetry is realized.

III. RESULTS AND DISCUSSION

The Lagrangian (5) contains the following 12 free parameters: m_0 , λ_1 , λ_2 , m_1 , g_1 , c , h , h_1 , h_2 , h_3 , m_G , $\Lambda = \sqrt{11}C^2/(2m_G)$. The processes that we shall consider depend only on the combination $h_1 + h_2 + h_3$, thus reducing the number of parameters to ten. We replace the set of ten parameters by the following equivalent set: m_π , m_{η_N} , m_ρ , m_{a_1} , ϕ , Z , M_σ , m_G , m_1 , C . The masses $m_\pi (= 139.57 \text{ MeV})$ and $m_\rho (= 775.49 \text{ MeV})$ are fixed to their Particle Data Group (PDG) values [16].

As outlined in Refs. [6,12], the mass of the η_N meson can be calculated using the mixing of strange and non-strange contributions in the physical fields η and $\eta'(958)$:

$$\begin{aligned} \eta &= \eta_N \cos\varphi + \eta_S \sin\varphi, \\ \eta' &= -\eta_N \sin\varphi + \eta_S \cos\varphi, \end{aligned} \quad (14)$$

where η_S denotes a pure $\bar{s}s$ state and $\varphi \simeq -36^\circ$ [17]. In this way, we obtain the value $m_{\eta_N} = 716 \text{ MeV}$. (Given the well-known uncertainty of the value of the angle φ , one could also consider other values, e.g., $\varphi = -41.4^\circ$, as published by the KLOE Collaboration [18], which corresponds to $m_{\eta_N} = 755 \text{ MeV}$. Variations of the pseudoscalar mixing angle affect the results presented in this paper only slightly.)

The value of m_{a_1} is fixed to 1050 MeV according to the study of Ref. [13]. (We note that taking the present PDG estimate of 1230 MeV does not change our conclusions.) The chiral condensate is fixed as $\phi = Zf_\pi$ and the renormalization constant Z is determined by the study of the process $a_1 \rightarrow \pi\gamma$: $Z = 1.67 \pm 0.2$ [6].

A. Assigning σ' and G' to $f_0(1370)$ and $f_0(1500)$

The σ' field denotes an isoscalar $J^{PC} = 0^{++}$ state and its assignment to a physical state is a long-debated problem of low-energy QCD [2,3,5,19]. The two major candidates are the resonances $f_0(600)$ and the $f_0(1370)$ [16]. The study of Ref. [6] has shown that $f_0(1370)$ is favored to be a state which is predominantly $\bar{q}q$. As stated above, the resonance $f_0(1500)$ is a convincing glueball candidate. For these reasons, we first test the scenario in which $\{\sigma', G'\} = \{f_0(1370), f_0(1500)\}$, which turns out to be phenomenologically successful, see below.

We are left with the following four free parameters: C , $M_{\sigma'}$, m_G , and m_1 . They can be obtained by a fit to the five experimental quantities of Table I. The five experimental quantities are the masses of the resonances $f_0(1500)$ ($M_{G'} \equiv M_{f_0(1500)} = 1505$ MeV [16]) and $f_0(1370)$ (for which we use the mean value $M_{\sigma'}^{\text{ex}} = (1350 \pm 150)$ MeV taking into account the PDG mass range between 1200 MeV and 1500 MeV [16]) and the three well-known decay widths of the well-measured resonance $f_0(1500)$: $f_0(1500) \rightarrow \pi\pi$, $f_0(1500) \rightarrow \eta\eta$, and $f_0(1500) \rightarrow K\bar{K}$.

Using the Lagrangian (5), these observables can be expressed as functions of the parameters listed above. Note that, although our framework is based on $N_f = 2$, we can calculate the amplitudes for the decays into mesons containing strange quarks by making use of the flavor symmetry $SU(N_f = 3)$ [4]. It is then possible to calculate the following $f_0(1500)$ decay widths into pseudoscalar mesons containing s quarks: $f_0(1500) \rightarrow K\bar{K}$, $f_0(1500) \rightarrow \eta\eta$, and $f_0(1500) \rightarrow \eta\eta'$.

The χ^2 method yields $\chi^2/\text{d.o.f.} = 0.29$ (thus very small), $C = (699 \pm 40)$ MeV, $M_{\sigma'} = (1275 \pm 30)$ MeV, $m_G = (1369 \pm 26)$ MeV and $m_1 = (809 \pm 18)$ MeV. We have also examined the uniqueness of our fit. To this end, we have considered χ^2 fixing three of four parameters entering the fit at their best values and varying the

TABLE I. Fit in the scenario $\{\sigma', G'\} = \{f_0(1370), f_0(1500)\}$. Note that the $f_0(1370)$ mass ranges between 1200 MeV and 1500 MeV [16] and therefore, as an estimate, we are using the value $m_{\sigma'} = (1350 \pm 150)$ MeV in the fit.

Quantity	Our Value [MeV]	Experiment [MeV]
$M_{\sigma'}$	1191 ± 26	1200–1500
$M_{G'}$	1505 ± 6	1505 ± 6
$G' \rightarrow \pi\pi$	38 ± 5	38.04 ± 4.95
$G' \rightarrow \eta\eta$	5.3 ± 1.3	5.56 ± 1.34
$G' \rightarrow K\bar{K}$	9.3 ± 1.7	9.37 ± 1.69

remaining fourth parameter. In each of the four cases, we observe only one minimum of the χ^2 function; each minimum leads exactly to the parameter values stated in Table I. We also observe no changes of the results for the errors of the parameters. These findings give us confidence that the obtained minimum corresponds to the absolute minimum of the χ^2 function.

The consequences of this fit are the following:

- (i) The quarkonium-glueball mixing angle reads $\theta = (29.7 \pm 3.6)^\circ$. This, in turn, implies that the resonance $f_0(1500)$ consists to 76% of a glueball and to the remaining 24% of a quark-antiquark state. An inverted situation holds for $f_0(1370)$.
- (ii) Our fit allows us to determine the gluon condensate: $C = (699 \pm 40)$ MeV. This result implies that the upper value in Eq. (2) is favored by our analysis. It is remarkable that insights into this basic quantity of QCD can be obtained from the PDG data on mesons.
- (iii) Further results for the $f_0(1500)$ meson are reported in the first two entries of Table II. The decay into 4π is calculated as a product of an intermediate $\rho\rho$ decay. To this end the usual integration over the ρ spectral function is performed. Our result yields 30 MeV in the 4π decay channel and is about half of the experimental value $\Gamma_{f_0(1500) \rightarrow 4\pi} = (54.0 \pm 7.1)$ MeV. However, it should be noted that an intermediate state consisting of two $f_0(600)$ mesons (which is also expected to contribute in this decay channel) is not included in the present model. The decay into the $\eta\eta'$ channel is also evaluated; this channel is subtle because it is exactly on the threshold of the $f_0(1500)$ mass. Therefore, an integration over the spectral function of the decaying meson $f_0(1500)$ is necessary. The result is in a qualitative agreement with the experiment.
- (iv) The results for the $f_0(1370)$ meson are reported in the last four rows of Table II. They are in agreement with the experimental data regarding the full width: $\Gamma_{f_0(1370)} = (200\text{--}500)$ MeV [16]. Unfortunately, the experimental results in the different channels are not yet conclusive. Our theoretical results point towards a dominant direct $\pi\pi$ and a

TABLE II. Further results regarding the $\sigma' \equiv f_0(1370)$ and $G' \equiv f_0(1500)$ decays.

Quantity	Our Value [MeV]	Experiment [MeV]
$G' \rightarrow \rho\rho \rightarrow 4\pi$	30	54 ± 7.1
$G' \rightarrow \eta\eta'$	0.6	2.1 ± 1
$\sigma' \rightarrow \pi\pi$	284 ± 43	...
$\sigma' \rightarrow \eta\eta$	72 ± 6	...
$\sigma' \rightarrow K\bar{K}$	4.6 ± 2.1	...
$\sigma' \rightarrow \rho\rho \rightarrow 4\pi$	0.09	...

non-negligible $\eta\eta$ contribution; these results correspond well to the experimental analysis of Ref. [20] where $\Gamma_{f_0(1370) \rightarrow \pi\pi} = 325$ MeV and $\Gamma_{f_0(1370) \rightarrow \eta\eta} / \Gamma_{f_0(1370) \rightarrow \pi\pi} = 0.19 \pm 0.07$ are obtained. We find that the four-pion decay of $f_0(1370) \rightarrow \rho\rho \rightarrow 4\pi$ is strongly suppressed (as was also found in Ref. [6]). This is unlike Ref. [20], where a small but non-negligible value of about 50 MeV is found. However, it should be noted that due to interference effects, our result for this decay channel varies strongly when the parameters are even slightly modified.

- (v) The mass of the ρ meson can be expressed as $m_\rho^2 = m_1^2 + \phi^2(h_1 + h_2 + h_3)/2$. In order that the contribution of the chiral condensate is not negative, the condition $m_1 \leq m_\rho$ should hold. In the framework of our fit this condition is fulfilled at the two-sigma level. This result points towards a dominant m_1 contribution to the ρ mass. This property, in turn, means that the ρ mass is predominantly generated from the gluon condensate and not from the chiral condensate. It is therefore expected that the ρ mass in the medium scales as the gluon condensate rather than as the chiral condensate. In view of the fact that m_1 is slightly larger than m_ρ , we have also repeated the fit by fixing $m_1 = m_\rho$: the minimum has a $\chi^2/\text{d.o.f.} \simeq 1$ and the results are very similar to the previous case. The corresponding discussion about the phenomenology is unchanged.
- (vi) As already stressed in Refs. [6,21], the inclusion of (axial-)vector mesons plays a central role to obtain the present results. The artificial decoupling of (axial-)vector states would generate a, by far, too-wide $f_0(1370)$ state. For this reason the glueball-quarkonium mixing scenario above 1 GeV has been previously studied only in phenomenological models with flavor symmetry [2,4] but not in the context of chirally invariant models.

Given that the resonance $f_0(1370)$ has a large mass uncertainty, we have also examined the behavior of the fit at different points of the PDG mass interval. Considering the minimal value $m_{f_0(1370)}^{\min} = (1220 \pm 20)$ MeV, we obtain $\chi^2 = 0.2/\text{d.o.f.}$ The resulting value of the mixing angle $\theta = (30.3 \pm 3.4)^\circ$ is practically the same as the value $\theta = (29.7 \pm 3.6)^\circ$ obtained in the case where $m_{f_0(1370)} = (1350 \pm 150)$ MeV was considered. Other results are also qualitatively similar to the case of $m_{f_0(1370)} = (1350 \pm 150)$ MeV.

For the upper boundary of the $f_0(1370)$ mass, the error interval of ± 20 MeV turns out to be too restrictive as it leads to unacceptably large χ^2 values. Consequently, increasing the error interval decreases the χ^2 values—we observe that $m_{f_0(1370)}^{\max} = (1480 \pm 120)$ MeV leads to an acceptable χ^2 value of 1.14/d.o.f. We then obtain

$\theta = (30.0 \pm 3.5)^\circ$, practically unchanged in comparison with the value $\theta = (29.7 \pm 3.6)^\circ$ in the case where $m_{f_0(1370)} = (1350 \pm 150)$. Also other quantities remain basically the same as in the case of $m_{f_0(1370)} = (1350 \pm 150)$ MeV.

We have also considered the fit at several points between the lower and upper boundaries of the $m_{f_0(1370)}$ mass range. We have chosen points of 50 MeV difference starting at $m_{f_0(1370)} = 1250$ MeV (i.e., we have considered $m_{f_0(1370)} \in \{1250, 1300, 1350, 1400, 1450\}$ MeV) with errors chosen such that the $\chi^2/\text{d.o.f.}$ becomes minimal (error values are between ± 30 MeV for $m_{f_0(1370)} = 1250$ MeV and ± 100 MeV for $m_{f_0(1370)} = 1450$ MeV). We observe that the previous results presented in this section do not change significantly; most notably, the mixing angle θ attains values between 30.2° and 30.7° , with an average error value of $\pm 3.4^\circ$.

We therefore conclude that considering different values of $m_{f_0(1370)}$ within the (1200–1500) MeV interval does not change the results significantly. In particular, the quarkonium-glueball mixing angle θ changes only slightly (by approximately 1°) and thus we confirm our conclusion that $f_0(1370)$ is predominantly a quarkonium and $f_0(1500)$ is predominantly a glueball.

B. Assigning σ' and G' to $f_0(1370)$ and $f_0(1710)$

Although the resonance $f_0(1710)$ has also been regarded as a glueball candidate in a variety of works [22], its enhanced decay into kaons and its rather small decay width make it compatible with a dominant $\bar{s}s$ contribution in its wave function. Nonetheless, we have also tested the assumption that the pure quarkonium and glueball states mix to produce the resonances $f_0(1370)$ and $f_0(1710)$.

The resonance $f_0(1710)$ is experimentally well-known. Decays into $\pi\pi$, $\bar{K}K$, and $\eta\eta$ have been seen, while no decays into $\eta\eta'$ and into 4π have been detected. Using the total decay width $\Gamma_{f_0(1710)} = (135 \pm 8)$ MeV and the branching ratios reported in Ref. [16], it is possible to deduce the decay widths into $\pi\pi$, $\bar{K}K$, and $\eta\eta$, see Table III.

A fit analogous to the one in Table I yields too-large errors for the decay width $\sigma' \equiv f_0(1370) \rightarrow \pi\pi$. For this reason we repeat our fit by adding the following constraint: $\Gamma_{\sigma' \rightarrow \pi\pi} = (250 \pm 150)$ MeV. The large error assures that

TABLE III. Fit in the scenario $\{\sigma', G'\} = \{f_0(1370), f_0(1710)\}$.

Quantity	Our Value [MeV]	Experiment [MeV]
$M_{\sigma'}$	1386 ± 134	1350 ± 150
$M_{G'}$	1720 ± 6	1720 ± 6
$G' \rightarrow \pi\pi$	29.7 ± 6.5	29.3 ± 6.5
$G' \rightarrow \eta\eta$	6.9 ± 5.8	34.3 ± 17.6
$G' \rightarrow K\bar{K}$	16 ± 14	71.4 ± 29.1
$\sigma' \rightarrow \pi\pi$	379 ± 147	250 ± 150

TABLE IV. Further results from the fit with $\{\sigma', G'\} = \{f_0(1370), f_0(1710)\}$.

Decay	Width	Our Value [MeV]	Experimental value [MeV]
$G' \rightarrow 4\pi$		115	...
$G' \rightarrow \eta\eta'$		16	...
$\sigma' \rightarrow \eta\eta$		153 ± 79	...
$\sigma' \rightarrow K\bar{K}$		$2.1^{+13.6}_{-2.1}$	...

this value is in agreement with experimental data on this decay width. The results of the fit are reported in Table III.

The χ^2 fit yields $\chi^2/\text{d.o.f.} = 1.72$, $C = (764 \pm 256)$ MeV, $M_\sigma = (1516 \pm 80)$ MeV, $m_G = (1531 \pm 233)$ MeV, and $m_1 = (827 \pm 36)$ MeV. The mixing angle between the pure quarkonium σ and the pure glueball G calculated from Eq. (12) is $\theta = (37.2 \pm 21.4)^\circ$. The χ^2 is worse than in the previous case, but the overall agreement is acceptable. The mixing angle is large and could also overshoot the value of 45° , which would imply a somewhat unexpected and unnatural reversed ordering, in which $f_0(1370)$ is predominantly glueball and $f_0(1710)$ predominantly quarkonium.

In Table IV we report the decay widths $G' \rightarrow 4\pi$, $G' \rightarrow \eta\eta'$, $\sigma' \rightarrow \eta\eta$, and $\sigma' \rightarrow K\bar{K}$, which can be calculated as a consequence of the fit of Table III.

A clear problem of this scenario emerges: the decay width $G' \equiv f_0(1710) \rightarrow 4\pi$ is large, while experimentally it has not been seen. Therefore, we conclude that this scenario is not favored. Moreover, in this scenario the remaining resonance $f_0(1500)$ should then be interpreted as a predominantly $\bar{s}s$ state, contrary to what its experimentally dominant $\pi\pi$ decay pattern suggests. Consequently, $f_0(1710)$ is unlikely to be predominantly a glueball state; this is also in accordance with the results from the ZEUS Collaboration [23].

C. Scenarios with $\sigma' \equiv f_0(600)$

The scenarios $\{\sigma', G'\} = \{f_0(600), f_0(1500)\}$ and $\{\sigma', G'\} = \{f_0(600), f_0(1710)\}$ have also been tested. In both cases, the mixing angle turns out to be small

($\lesssim 15^\circ$), thus the state $f_0(600)$ is predominantly quarkonium. Then, in these cases, the analysis of Ref. [6] applies: a simultaneous description of the $\pi\pi$ scattering lengths and the $\sigma \rightarrow \pi\pi$ decay width cannot be achieved. For these reasons, the mixing scenarios with the resonance $f_0(600)$ as a quarkonium state are not favored.

IV. CONCLUSIONS AND OUTLOOK

We have presented a globally chirally invariant linear sigma model with (axial-)vector mesons and a dilaton/glueball degree of freedom. We have studied the phenomenology of the scalar states for the case $N_f = 2$: in the favored scenario, the resonance $f_0(1500)$ is predominantly a glueball with a subdominant $\bar{q}q$ component and, conversely, $f_0(1370)$ is predominantly a quark-antiquark ($\bar{u}u + \bar{d}d$)/ $\sqrt{2}$ state with a subdominant glueball contribution. It is interesting to observe that the success of the phenomenological description of these scalar resonances is due to the inclusion of the (axial-)vector mesons in the model. The gluon condensate is also an outcome of our study and turns out to be in agreement with lattice QCD results. Different scenarios in which $f_0(1710)$ is predominantly glueball and/or $f_0(600)$ is predominantly quarkonium do not seem to be in agreement with the present experimental data.

Natural extensions of the model are the case $N_f = 3$ [24] and the inclusion of a nonet of tetraquark states as additional low-lying scalar states. In this general scenario, a mixing of five scalar-isoscalar states takes place, which allows us to describe all relevant scalar-isoscalar resonances listed in the PDG below 1.8 GeV [16]. Applications of the model at nonzero temperature and density are also important because the presence of the dilaton field allows us to study the restoration of both the dilatation and the chiral symmetry of QCD.

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Strong and electromagnetic decays of the light scalar mesons interpreted as tetraquark states

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The study of two-pseudoscalar and two-photon decays for the scalar meson nonet below 1 GeV is performed within an effective approach in which the scalar resonances are described as (Jaffe's) tetraquark states. The dominant (fall apart decay) and the subdominant (one transverse gluon as intermediate state) decay amplitudes are systematically taken into account. The latter improves the agreement with the experimental data. Possible scenarios concerning the scalar-isoscalar mixing are discussed.

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I. INTRODUCTION

The interpretation of the mesonic scalar states below 2 GeV is not yet univocally established [1–3]. According to the most popular scenario, one interprets the isovector and isotriplet resonances $a_0(1400)$ and $K(1430)$ as the ground-state quark-antiquark bound states. The three isoscalar resonances $f_0(1300)$, $f_0(1500)$, and $f_0(1710)$ are a mixture of two isoscalar quarkonia and bare glueball configurations (we refer to [1,2,4–9] and references therein). As a consequence, the scalar states below 1 GeV [σ , k , $f_0(980)$, and $a_0(980)$] must be something else, like (loosely bound) mesonic molecular states [10,11] or Jaffe's tetraquark states [1,12–14]. In this work we explore by means of an effective approach the phenomenological implications of Jaffe's states, whose building blocks are a diquark (q^2) and an antidiquark ($\bar{q}^2$); calculations based on one-gluon exchange support a strong attraction among two quarks in a color antitriplet ($\bar{3}_C$), a flavor antitriplet ($\bar{3}_F$), and spinless configuration [1,12] (color and flavor triplets are realized for an antidiquark). Naively speaking, a diquark “behaves like an antiquark” from a flavor (and color) point of view [13]:

$$[u, d] \leftrightarrow \bar{s}, \quad [u, s] \leftrightarrow \bar{d}, \quad [d, s] \leftrightarrow \bar{u}, \quad (1)$$

therefore out of a diquark and an antidiquark one can build a full scalar nonet, whose most appealing property is the reversed mass order, thus explaining the (almost) degeneracy of the isoscalar state $f_0(980)$ [whose dominant contribution is the tetraquark structure “ $\bar{s}s(\bar{u}u + \bar{d}d)$ ”] and the isovector $a_0(980)$ [the neutral one interpreted as “ $\bar{s}s(\bar{u}u - \bar{d}d)$ ”]; the σ (dominantly “ $\bar{u}u\bar{d}d$ ”) is then the lightest, in between one expects the kaonic state $k(800)$ (k^+ interpreted as “ $\bar{d}d\bar{s}u$ ”), which is omitted in the compilation of PDG [15], but listed in many recent theoretical and experimental works ([16–23] and references therein).

Evidence for a broad σ state is also found, together with $a_0(980)$ and $f_0(980)$, in the theoretical work of [24,25] where a unitarized chiral perturbation theory is used; as shown in [26,27] a broad scalar state k exists as well. The fact that a full scalar nonet is generated within the same

approach points to a similar inner structure of the low-lying scalar states. Furthermore, the large- N_c behavior of the light scalar states indicates large non- $\bar{q}q$ amounts in their spectroscopic wave function [27,28]. Support for the Jaffe's states has also been found in Lattice calculations [29,30], where the diquark q^2 and the antidiquark $\bar{q}^2$ are connected by a flux tube.

In [31,32] the scalar states below 1 GeV have also a large tetraquark content; mixing among light tetraquark states and heavy quarkonia states, generating two scalar nonet of mixed states below and above 1 GeV is described.

In the present work we intend to analyze the two-pseudoscalar decays of the scalar states below 1 GeV when interpreted as Jaffe's tetraquark states; to this end we consider the dominant and the subdominant diagrams in the large- N_c expansion which describe the transition of a tetraquark scalar state into two pseudoscalars [12,14].

The dominant diagram, depicted in Fig. 1(a), occurs by switch of a quark belonging to the compact diquark with an antiquark of the antidiquark, thus generating two $\bar{q}q$ objects, which separate as pseudoscalar mesons: $q^2\bar{q}^2 \rightarrow (q\bar{q})(q\bar{q}) \rightarrow (q\bar{q}) + (q\bar{q})$. The two pseudoscalar mesons fall apart from the tetraquark configuration; this decay mechanism was denoted as Okuba-Zweig-Iizuka (OZI) superallowed in Ref. [12].

The subdominant diagram, depicted in Fig. 1(b), occurs via an annihilation of a quark and an antiquark (into one gluon), with subsequent $\bar{q}q$ creation and two-pseudoscalar decay: $q^2\bar{q}^2 \rightarrow qg\bar{q} \rightarrow (q\bar{q}) + (q\bar{q})$. Although suppressed by a factor N_c , the fact that the annihilation of the quark-antiquark pair can occur with only one gluon as an intermediate state (as already noted in Ref. [12]), i.e. to order α_s only, may indeed indicate that the corresponding amplitudes are not negligible.

In Ref. [12] the subdominant coupling has not been considered in the decay rates; in [14] it has been introduced as the last step of the analysis in order to improve the results of the superallowed decays. In the present work we intend to systematically write down the expressions for all scalar-to-pseudoscalar transition amplitudes as functions of the strengths of the dominant (“fall apart”) and subdo-

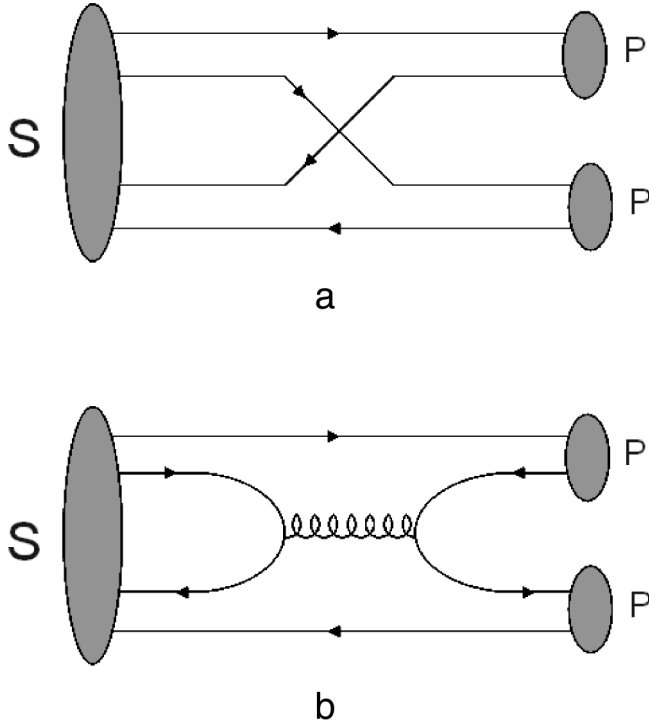


FIG. 1. Dominant (a) and subdominant (b) contributions to the transition amplitudes of a scalar-tetraquark state into two pseudoscalar mesons.

minant (“one intermediate gluon”) diagrams, denoted as c_1 and c_2 , respectively. Then, by a fitting procedure to experimental known branching ratios, we determine the quantity c_2/c_1 , which measures the intensity of the subdominant decay mechanism with respect to the dominant one. Two possible solutions with a nonzero value of the ratio c_2/c_1 are discussed, which in turn correspond to different isoscalar mixing configurations.

An interesting feature of the subdominant decay is the strong enhancement of the coupling $g_{f_0 \rightarrow \bar{K}K}^2$ with respect to $g_{a_0 \rightarrow \bar{K}K}^2$, thus possibly solving the problem of Jaffe’s model mentioned in [21] at the leading (OZI superallowed, Fig. 1(a)) order for which $g_{f_0 \rightarrow \bar{K}K}^2/g_{a_0 \rightarrow \bar{K}K}^2 = 1$ (if no isoscalar mixing occurs, even smaller if mixing is introduced), in clear contrast with the result $g_{f_0 \rightarrow \bar{K}K}^2/g_{a_0 \rightarrow \bar{K}K}^2 = 2.15 \pm 0.40$ reported in the analysis of Refs. [21,22]. The subdominant decay mechanism of Fig. 1(b) can explain the experimental value without introducing explicit $\bar{K}K$ clouds dressing the scalar resonances.

We then turn our attention to the decay of scalar states into two photons, which in line of the previous discussion is described by two decay mechanisms; they are analogous to the strong decay diagrams of Fig. 1, where one replaces the two pseudoscalar mesons in the final state with two photons. We present the theoretical ratios and the phenomenological discussion.

The paper is organized as follows: in Sec. II we write down the Lagrangian for the description of the scalar mesons below 1 GeV as tetraquark states; the mass term and the two-body strong decays are presented. In Sec. III we perform a phenomenological study with the available experimental data. In Sec. IV we describe the two-photon transitions and in Sec. V we drive our conclusions.

II. THE MODEL

A. The Lagrangian

The basic terms are the pseudoscalar fields, collected in the matrix $\mathcal{P} = \frac{1}{\sqrt{2}} \sum_{i=0}^8 P^i \lambda_i$ (the λ_i are Gell-Mann matrices), and low-lying scalar fields, collected in the matrix S defined as (see Appendix A)

$$S = \begin{pmatrix} \sigma_B & k^0 & k^+ \\ \bar{K}^0 & \sqrt{\frac{1}{2}}(f_B + a_0^0) & a_0^+ \\ k^- & a_0^- & \sqrt{\frac{1}{2}}(f_B - a_0^0) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}[u, d][\bar{u}, \bar{d}] & \frac{1}{2}[u, d][\bar{u}, \bar{s}] & \frac{1}{2}[u, d][\bar{d}, \bar{s}] \\ \frac{1}{2}[u, s][\bar{u}, \bar{d}] & \frac{1}{2}[u, s][\bar{u}, \bar{s}] & \frac{1}{2}[u, s][\bar{d}, \bar{s}] \\ \frac{1}{2}[d, s][\bar{u}, \bar{d}] & \frac{1}{2}[d, s][\bar{u}, \bar{s}] & \frac{1}{2}[d, s][\bar{d}, \bar{s}] \end{pmatrix}, \quad (2)$$

where in the second matrix the diquark-antidiquark decomposition has been made explicit. The states $\sigma_B = \frac{1}{2}[u, d] \times [\bar{u}, \bar{d}]$ and $f_B = \frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] + [d, s][\bar{d}, \bar{s}])$ refer to *bare* (unmixed) states. A mixing of these configurations, leading to the physical states σ and $f_0(980)$, is possible and considered below.

The Lagrangian for the scalar-pseudoscalar interaction reads

$$\mathcal{L} = \langle \frac{1}{2}(\partial_\mu \mathcal{P})^2 - \mathcal{P}^2 \chi_P \rangle + \mathcal{L}_{\text{mix}}^P + \langle \frac{1}{2}(\partial_\mu S)^2 - S^2 \chi_S \rangle + \mathcal{L}_{\text{mix}}^S + c_1 S_{ij} \langle A^i \mathcal{P}^j A^i \mathcal{P}^j \rangle - c_2 S_{ij} \langle A^i A^j \mathcal{P}^2 \rangle, \quad (3)$$

where $\langle \dots \rangle$ denotes trace over flavor. Some comments are in order:

- (i) In the first term the quantity $\chi_P = B \cdot \text{diag}\{m_u, m_d = m_u, m_s\}$ encodes flavor symmetry violation. It corresponds to the lowest order chiral perturbation theory result [33] (see also [34] and references therein), to which we refer for a careful description. The second term $\mathcal{L}_{\text{mix}}^P = -\frac{\chi_P}{2}(P^0)^2 - z_P P^0 P^8$ takes into account the enhanced flavor-singlet mass [$U_A(1)$ anomaly] and the octet-singlet mixing, leading to the physical states (we follow the notations of [9])

$$\begin{aligned} \eta &= P^8 \cos \theta_P - P^0 \sin \theta_P, \\ \eta' &= P^8 \sin \theta_P + P^0 \cos \theta_P, \end{aligned} \quad (4)$$

where θ_P is the pseudoscalar mixing angle. According to the standard procedure [35–37], we diagonalize the corresponding η^0 - η^8 mass ma-

trix to obtain the masses of η and η' . By using $M_\pi = 139.57$ MeV, $M_K = 493.677$ MeV (the physical charged pion and kaon masses), $M_\eta = 547.75$ MeV, and $M_{\eta'} = 957.78$ MeV, the mixing angle is determined as $\theta_P = -9.95^\circ$, which corresponds to the tree-level result (see details in Ref. [37]). Correspondingly, one finds $M_{P^0} = \sqrt{(M_\pi^2 + 2M_K^2)/3 + \gamma_P} = 948.10$ MeV and $z_P = -0.105$ GeV² [9].

- (ii) The third and the fourth terms of Eq. (3) refer to the quadratic part of the scalar-tetraquark nonet; $\chi_S = \text{diag}\{\alpha, \beta, \beta\}$ and $\alpha < \beta$ takes into account that the diquark $[u, d]$ is lighter than the partners (due to $m_u = m_d < m_s$). In this way the masses are given as

$$\begin{aligned} M_{a_0}^2 = M_{f_B}^2 = 2\beta, \quad M_k^2 = (\alpha + \beta), \\ M_{\sigma_B}^2 = 2\alpha, \end{aligned} \quad (5)$$

corresponding to the reversed mass ordering. The fourth term $\mathcal{L}_{\text{mix}}^S$ describes the mixing of σ_B and f_B ; details about it are presented in the next subsection.

It is interesting to note that the same (inverted) mass ordering for the light scalars can be obtained in the framework of the SU(3) linear σ model [38] with $U_A(1)$ anomaly.

It is important to stress that in this work we consider only tree-level expression for the decays (see Secs. II C and III) and no mass-shift via loop diagrams is calculated; the Lagrangian masses for the state $a_0(980)$ and k reported in Eq. (5) [derived from the Lagrangian (3)] are therefore the physical masses in our phenomenological approach. Similarly, M_{f_B} and M_{σ_B} of Eq. (5) would be physical masses if no isoscalar mixing among σ_B and f_B would occur. The physical masses for the states σ and $f_0(980)$ are determined in the next subsection when isoscalar mixing is considered.

- (iii) The last two terms of Eq. (3) correspond to the large- N_c dominant (proportional to c_1) and subdominant (proportional to c_2) two-pseudoscalar decays of the low-lying scalars when interpreted as tetraquark states. The matrices A^i entering in the expression (3) are the three antisymmetric real 3×3 matrices:

$$A^1 = i\lambda_2, \quad A^2 = i\lambda_5, \quad A^3 = i\lambda_7. \quad (6)$$

Both terms are SU(3)-flavor invariant; the details about the transformation properties are reported in Appendix A, where also the other flavor-invariant terms are listed [Eq. (A11)].

We do not consider in the present work decay terms which break flavor symmetry in virtue of $\alpha \neq \beta$

[Eq. (5)]. Generally flavor-breaking corrections to the decay amplitudes are not large and do not change the qualitative picture; furthermore the consideration of such terms would imply a too large number of parameters for the three-level decay amplitudes. For these reasons the consideration of such terms is beyond the goal of the present paper, but it represents a possible future development of our study.

- (iv) As described in Appendix A, the $S \rightarrow PP$ interaction term $c_3 \mathcal{S}_{ij} \langle A^i A^j \mathcal{P} \rangle \langle \mathcal{P} \rangle$ is also suppressed of a factor N_c with respect to the dominant OZI-superaligned one. It is coupled to the flavor-blind pseudoscalar configuration $\langle \mathcal{P} \rangle = \sqrt{3}P^0$, connected to the physical fields η and η' in Eq. (4). It can however occur with at least two intermediate transverse gluons attached to P^0 , i.e. to order α_s^2 . Its contribution is then believed to be smaller than the diagram of Fig. 1(b), which takes place at order α_s . A gluonic amount in the wave functions of the η and η' states would enhance this channel, but such an eventuality seems not to occur [39–41]. Furthermore, in the analysis of [42], where the two-pseudoscalar decays of the experimentally well-known tensor mesons are evaluated, a good description of data is obtained without an enhanced flavor-blind channel in the pseudoscalar mesonic sector. These arguments lead us not to consider the term $c_3 \mathcal{S}_{ij} \langle A^i A^j \mathcal{P} \rangle \langle \mathcal{P} \rangle$ in the present work. According to our view, the further systematical inclusion of this term, which affects the couplings involving the η and η' mesons only, would be then necessary when the experimental knowledge on the light scalar meson sector becomes more exhaustive.

B. Mixing

We discuss the term $\mathcal{L}_{\text{mix}}^S$ of Eq. (3) which generates mixing between σ_B and f_B . In line with the pseudoscalar sector, we consider the flavor-singlet and octet four-quark configurations $S_0 = \sqrt{2/3}f_B + \sqrt{1/3}\sigma_B$ and $S_8 = \sqrt{1/3}f_B - \sqrt{2/3}\sigma_B$ and define

$$\mathcal{L}_{\text{mix}}^S = -\frac{1}{2}\gamma_S S_0^2 - z_S S_0 S_8, \quad (7)$$

where a octet-singlet mixing and mass modification for the flavor-blind state are taken into account; we also refer to [14] for this point.

By using Eqs. (3) and (7), the quadratic part referring to the isoscalar states f_B and σ_B and their relative mixing reads

$$\mathcal{L}_{\text{isoscalar}}^S = -\frac{1}{2}(\sigma_B \ f_B)\Omega\begin{pmatrix} \sigma_B \\ f_B \end{pmatrix}, \quad (8)$$

where

$$\Omega = \begin{pmatrix} M_{\sigma_B}^2 & \varepsilon \\ \varepsilon & M_{f_B}^2 \end{pmatrix} = \begin{pmatrix} 2\alpha + \frac{1}{3}\gamma_S - \frac{2\sqrt{2}}{3}z_S & -\frac{1}{3}z_S + \frac{\sqrt{2}}{3}\gamma_S \\ -\frac{1}{3}z_S + \frac{\sqrt{2}}{3}\gamma_S & 2\beta + \frac{2}{3}\gamma_S + \frac{2\sqrt{2}}{3}z_S \end{pmatrix} \quad (9)$$

[note that $M_{\sigma_B}^2$ and $M_{f_B}^2$ receive extra contributions from (7) and therefore modified from Eq. (5)].

The physical states σ and $f_0 \equiv f_0(980)$ are then given by

$$\begin{pmatrix} \sigma \\ f_0(980) \end{pmatrix} = \begin{pmatrix} \cos(\theta_S) & \sin(\theta_S) \\ -\sin(\theta_S) & \cos(\theta_S) \end{pmatrix} \begin{pmatrix} \sigma_B \\ f_B \end{pmatrix} \quad (10)$$

where

$$B = \begin{pmatrix} \cos(\theta_S) & \sin(\theta_S) \\ -\sin(\theta_S) & \cos(\theta_S) \end{pmatrix}, \quad B\Omega B' = \text{diag}\{M_{\sigma'}^2, M_{f_0}^2\}. \quad (11)$$

The experimental fact that $M_{f_0} \approx M_{a_0}$ suggests a small scalar mixing angle θ_S . Unfortunately, to determine θ_S from the masses can be misleading because of the large widths of k and σ . Furthermore, small variation of the a_0 and f_0 masses and mixing can lead to different scenarios. We therefore prefer to determine the mixing angle from the decay amplitudes (see Sec. IV).

Nevertheless some interesting considerations can be done; exploiting the relation $\langle\Omega\rangle = M_{\sigma'}^2 + M_{f_0}^2$ [from Eq. (11); the matrix B is orthogonal], one finds $\gamma_S = M_{f_0}^2 + M_{\sigma'}^2 - 2 \cdot M_k^2$. The sign of γ_S strongly depends on the values of M_{σ} and M_k . One has then to tune the parameter z_S in order to generate the experimental value $M_{f_0} = M_{a_0} \approx 0.98$ GeV, then the mixing angle θ_S is also fixed.¹ We then distinguish among 2 possibilities:

- (a) $\gamma_S > 0 \rightarrow z_S < 0$ and $\theta_S < 0$; in the upper state f_0 the bare components σ_B and f_B are in phase, while in the lower state σ they are out of phase.
- (b) $\gamma_S < 0 \rightarrow z_S > 0$ and $\theta_S > 0$; the phases are reversed.

The sign of θ_S is important because of destructive or constructive interference phenomena in the decays of σ and f_0 . However, as stressed above, it cannot be determined from the knowledge of the masses because of the large uncertainties on their values. We consider two examples to elucidate this point. Fixing $M_k = 0.8$ GeV, we have $\gamma_S > 0$ (and $\theta_S < 0$) for $M_{\sigma} > 0.56$ GeV; vice versa for $M_{\sigma} < 0.56$ one has $\gamma_S < 0$ (and $\theta_S > 0$). Small changes of M_k generate very different result; fixing $M_k =$

¹The parameter z_S can be found from $\det[\Omega] = M_{\sigma'}^2 \cdot M_{f_0}^2$ [see Eq. (11)]. Indeed, this is a second order equation in z_S , so strictly speaking two solutions are possible. The solution corresponding to the smallest absolute value $|\theta_S|$ is the one that we take into account. The disregarded solution typically induces a very large mixing, in disagreement with the phenomenology (see Sec. IV).

0.72 (the pole of E791 [19]) one finds $\gamma_S > 0$ (and $\theta_S < 0$) for $M_{\sigma} > 0.277$ GeV (therefore, if $M_k \sim 0.7$ GeV the option $\theta_S < 0$ seems favored). We notice that the present results differ from those of [14].

In Sec. IV we will determine the values of the scalar mixing angle θ_S from the phenomenology and discuss the implications for the masses.

C. Two-pseudoscalar decay amplitudes

In the following we report the results for the two-pseudoscalar decay rates. We use the following notation: for a given decay mode $S \rightarrow P_1 P_2$ [where S refers to a scalar state and $P_1(P_2)$ to a pseudoscalar state] the decay width (for kinematically allowed decays) is written as

$$\Gamma_{S \rightarrow P_1 P_2} = \frac{p_{S \rightarrow P_1 P_2}}{8\pi M_S^2} g_{S \rightarrow P_1 P_2}^2, \quad (12)$$

where $p_{S \rightarrow P_1 P_2}$ is the three-momentum of (one of) the outgoing particle(s) (in the rest frame of S):

$$p_{S \rightarrow P_1 P_2} = \frac{1}{2M_S} \times \sqrt{M_S^4 + (M_{P_1}^2 - M_{P_2}^2)^2 - 2(M_{P_1}^2 + M_{P_2}^2)M_S^2}. \quad (13)$$

Notice that $g_{S \rightarrow P_1 P_2}^2$ already includes the charge multiplicities and the symmetry factors (see Appendix B).

In general, but especially when $M_S < M_{P_1} + M_{P_2}$, the expression (12) has to be modified taking into account the finite width of the resonance, thus integrating over the mass of the resonance by employing a suitable mass distribution (like a Breit-Wigner one). However, the expressions for the coupling constant are left invariant by these operations. We refer to Appendix B for a brief recall on the connection between Eq. (12) and the Lagrangian (3).

In Table I we list the coupling constants for the isovector $a_0(980)$ and the isodoublet(s) k to two pseudoscalar mesons as a function of the dominant and subdominant decay strengths c_1 and c_2 . The expressions in $\{...\}$ represent the invariant amplitudes, eventually multiplied by a factor 2 for decays into identical particles, while the coefficients in front of the parenthesis account for charge multiplicities and symmetry factors (see Appendix B).

In Table II we report the results for the bare states σ_B and f_B ; although not physical when $\theta_S \neq 0$, the expressions are easier to read and allow one to understand the role of the subleading decay mechanism of Fig. 1(b). It is important to note that the decay mode $f_B \rightarrow \bar{K}K$ is significantly enhanced with respect to $a_0 \rightarrow \bar{K}K$ when $c_2/c_1 > 0$. This fact shows that a non-negligible and positive ratio c_2/c_1 can explain why the $f_0 \rightarrow \bar{K}K$ coupling is larger than the $a_0 \rightarrow \bar{K}K$ one. This point will be discussed in the next section.

TABLE I. Decay coupling constants for a_0 and k .

$S \rightarrow P_1 P_2$	$g_{S \rightarrow P_1 P_2}$
$a_0 \rightarrow \bar{K}K$	$\sqrt{2}\{\sqrt{2}c_1 + \frac{1}{\sqrt{2}}c_2\}$
$a_0 \rightarrow \pi\eta$	$\sqrt{1}\{\frac{2}{\sqrt{3}}c_1[\sqrt{2}\cos(\theta_p) + \sin(\theta_p)] + \sqrt{\frac{2}{3}}c_2[\cos(\theta_p) - \sqrt{2}\sin(\theta_p)]\}$
$a_0 \rightarrow \pi\eta'$	$\sqrt{1}\{-\frac{2}{\sqrt{3}}c_1[\cos(\theta_p) - \sqrt{2}\sin(\theta_p)] + \sqrt{\frac{2}{3}}c_2[\sqrt{2}\cos(\theta_p) + \sin(\theta_p)]\}$
$k \rightarrow \pi K$	$\sqrt{3}\{\sqrt{2}c_1 + \frac{1}{\sqrt{2}}c_2\}$
$k \rightarrow K\eta$	$\sqrt{1}\{-\sqrt{\frac{2}{3}}c_1[\cos(\theta_p) - \sqrt{2}\sin(\theta_p)] - \sqrt{\frac{1}{6}}c_2[\cos(\theta_p) + 2\sqrt{2}\sin(\theta_p)]\}$
$k \rightarrow K\eta'$	$\sqrt{1}\{-\sqrt{\frac{2}{3}}c_1[\sqrt{2}\cos(\theta_p) + \sin(\theta_p)] + \sqrt{\frac{1}{6}}c_2[2\sqrt{2}\cos(\theta_p) - \sin(\theta_p)]\}$

TABLE II. Decay coupling constants of the bare states σ_B and f_B .

$S \rightarrow P_1 P_2$	$g_{S \rightarrow P_1 P_2}$
$\sigma_B \rightarrow \pi\pi$	$\sqrt{\frac{3}{2}}\{2c_1 + 2c_2\}$
$\sigma_B \rightarrow \bar{K}K$	$\sqrt{2}\{c_2\}$
$\sigma_B \rightarrow \eta\eta$	$\sqrt{\frac{1}{2}}\{\frac{2}{3}(-c_1 + c_2)(\cos(\theta_p) - \sqrt{2}\sin(\theta_p))^2\}$
$\sigma_B \rightarrow \eta'\eta'$	$\sqrt{\frac{1}{2}}\{\frac{2}{3}(-c_1 + c_2)(\sqrt{2}\cos(\theta_p) + \sin(\theta_p))^2\}$
$\sigma_B \rightarrow \eta\eta'$	$\sqrt{1}\{\frac{1}{3}(-c_1 + c_2)(2\sqrt{2}\cos(2\theta_p) - \sin(2\theta_p))\}$
$f_B \rightarrow \pi\pi$	$\sqrt{\frac{3}{2}}\{\sqrt{2}c_2\}$
$f_B \rightarrow \bar{K}K$	$\sqrt{2}\{(\sqrt{2}c_1 + \frac{3}{\sqrt{2}}c_2)\}$
$f_B \rightarrow \eta\eta$	$\sqrt{\frac{1}{2}}\{\frac{1}{6}(9\sqrt{2}c_2 + \sqrt{2}(8c_1 + c_2)\cos(2\theta_p) + 4(-c_1 + c_2)\sin(2\theta_p))\}$
$f_B \rightarrow \eta'\eta'$	$\sqrt{\frac{1}{2}}\{\frac{1}{6}(9\sqrt{2}c_2 - \sqrt{2}(8c_1 + c_2)\cos(2\theta_p) + 4(c_1 - c_2)\sin(2\theta_p))\}$
$f_B \rightarrow \eta\eta'$	$\sqrt{1}\{\frac{1}{6}(4(c_1 - c_2)\cos(2\theta_p) + \sqrt{2}(8c_1 + c_2)\sin(2\theta_p))\}$

TABLE III. Decay coupling constants of the physical states σ and f_0 .

$S \rightarrow P_1 P_2$	$g_{S \rightarrow P_1 P_2}$
$\sigma \rightarrow \pi\pi$	$\sqrt{\frac{3}{2}}\{2c_1\cos(\theta_s) + c_2(2\cos(\theta_s) + \sqrt{2}\sin(\theta_s))\}$
$\sigma \rightarrow \bar{K}K$	$\sqrt{2}\{\sqrt{2}c_1\sin(\theta_s) + c_2(\cos(\theta_s) + \frac{3}{\sqrt{2}}\sin(\theta_s))\}$
$f_0 \rightarrow \pi\pi$	$\sqrt{\frac{3}{2}}\{-2c_1\sin(\theta_s) + c_2(-2\sin(\theta_s) + \sqrt{2}\cos(\theta_s))\}$
$f_0 \rightarrow \bar{K}K$	$\sqrt{2}\{\sqrt{2}c_1\cos(\theta_s) + c_2(-\sin(\theta_s) + \frac{3}{\sqrt{2}}\cos(\theta_s))\}$

When considering the mixed physical states σ and $f_0(980)$ defined in Eq. (10), the coupling constants are modified as follows:

$$\begin{aligned}
 g_{\sigma \rightarrow P_1 P_2} &= \{g_{\sigma_B \rightarrow P_1 P_2} \cdot \cos(\theta_s) + g_{f_B \rightarrow P_1 P_2} \cdot \sin(\theta_s)\} \\
 g_{f_0 \rightarrow P_1 P_2} &= \{-g_{\sigma_B \rightarrow P_1 P_2} \cdot \sin(\theta_s) + g_{f_B \rightarrow P_1 P_2} \cdot \cos(\theta_s)\}.
 \end{aligned}
 \tag{14}$$

Because of their relevance, we report in Table III the explicit expressions for the decays of σ and $f_0(980)$ into $\pi\pi$ and $\bar{K}K$ as derived from Table II and Eqs. (14).

III. STRONG DECAYS: NUMERICAL RESULTS

The data about the decay widths reported in [15] are still not complete. The resonance σ is very broad ($M_\sigma =$

0.4–1.2 GeV, $\Gamma = 0.6$ –1 GeV), k is still not yet listed (in the recent result of [20] the pole is found at 0.76 GeV, the width is very large). The masses for the states $a_0(980)$ and $f_0(980)$ are well known, $M_{a_0} = 948.7 \pm 1.2$ MeV, $M_{f_0} = 980 \pm 10$ MeV, but the widths are not, between 40 and 100 MeV for both states. The presence of near-threshold $\bar{K}K$ decays complicates the experimental and theoretical analysis (see discussion in [25,43] and below).

Information about (some of) the coupling constants $g_{S \rightarrow P_1 P_2}^2$ has been extracted directly from experiment [20–22,43]. In the case of $a_0(980)$, for instance, the quantities $g_{a_0 \rightarrow \pi\eta}^2$, $g_{a_0 \rightarrow \bar{K}K}^2$, and M_{a_0} are free parameters of the meson-meson scattering amplitudes (generally the Flatté distribution is used [43,44]). They can be determined by fitting the theoretical amplitudes to the experimental ones. However, as explained in [43], relatively large differences in the absolute values of the coupling constants $g_{a_0 \rightarrow \pi\eta}^2$ and

$g_{a_0 \rightarrow \bar{K}K}^2$ are found in the literature. Fortunately, the ratio $g_{a_0 \rightarrow \pi\eta}^2 / g_{a_0 \rightarrow \bar{K}K}^2$ shows a stable behavior.

Furthermore, the presence of the strong coupling constant for the near-threshold decay mode $g_{a_0 \rightarrow \bar{K}K}$ affects also the channel $a_0 \rightarrow \pi\eta$ rendering the experimental width narrower than the theoretical result obtained using Eq. (12), see the discussion in [25].

In the following we will compare our results with the analysis of experimental results of Refs. [20–22], where the ratios of coupling constants for various decay channels are deduced; it should be stressed that the results of [20–22] are not yet conclusive and depend on the choice of the form factors and other assumptions, for which we refer to the above cited works for a careful description.

The ratios of coupling constants for the resonances $a_0(980)$ and $f_0(980)$ as reported in [20–22] are

$$\begin{aligned} \frac{g_{f_0 \rightarrow \bar{K}K}^2}{g_{f_0 \rightarrow \pi\pi}^2} &= 4.21 \pm 0.46, & \frac{g_{f_0 \rightarrow \bar{K}K}^2}{g_{a_0 \rightarrow \bar{K}K}^2} &= 2.15 \pm 0.40, \\ \frac{g_{a_0 \rightarrow \pi\eta}^2}{g_{a_0 \rightarrow \bar{K}K}^2} &= 0.75 \pm 0.11. \end{aligned} \quad (15)$$

These three results are indeed (at least qualitatively) common to (almost) all the analyses (see [43] and references therein, and the recent experimental analysis of [45,46]). The largest uncertainty is about the crossed ratio $g_{f_0 \rightarrow \bar{K}K}^2 / g_{a_0 \rightarrow \bar{K}K}^2$; however, the enhanced coupling of the $f_0 \rightarrow \bar{K}K$ mode with respect to $a_0 \rightarrow \bar{K}K$ is also a stable result. As we will see, all three ratios in (15) are all compatible with a sizable c_2/c_1 .

Before considering a nonzero value for c_2/c_1 , we analyze as a first step the case $c_2 = 0$, corresponding to the original work of [12] and described in the recent paper of [21]. We fit the only free parameter θ_S to the first and the second branching ratios of Eq. (15); the ratio $g_{a_0 \rightarrow \pi\eta}^2 / g_{a_0 \rightarrow \bar{K}K}^2 = 0.50$ does not depend on θ_S , hence not included in the fit (in [21] it is 0.4 because of a slightly different pseudoscalar mixing angle; here it is $\theta_P = -9.95^\circ$, while in [21] the value $\theta_P = -17.29^\circ$ is used).

We find the following value for θ_S (we refer to this case as solution A):

$$\begin{aligned} \text{solution A: } (c_2 = 0), \quad \theta_S &= \pm 21.6^\circ; \\ \left(\frac{\chi^2}{2} = 5.17 \right) \end{aligned} \quad (16)$$

corresponding to the values

$$\begin{aligned} \frac{g_{f_0 \rightarrow \bar{K}K}^2}{g_{f_0 \rightarrow \pi\pi}^2} &= 4.26, & \frac{g_{f_0 \rightarrow \bar{K}K}^2}{g_{a_0 \rightarrow \bar{K}K}^2} &= 0.86, \\ \frac{g_{a_0 \rightarrow \pi\eta}^2}{g_{a_0 \rightarrow \bar{K}K}^2} &= 0.50. \end{aligned} \quad (17)$$

The large and unsatisfactory χ^2 is generated by the mismatch between experiment and theory for the ratio $g_{f_0 \rightarrow \bar{K}K}^2 / g_{a_0 \rightarrow \bar{K}K}^2$. Two scalar mixing angles are reported in Eq. (16) because the fitted quantities are symmetric for $\theta_S \rightarrow -\theta_S$ when $c_2 = 0$. Our theoretical ratio for $g_{f_0 \rightarrow \bar{K}K}^2 / g_{f_0 \rightarrow \pi\pi}^2$ as function of θ_S is deduced from Table III and reads for $c_2 = 0$:

$$\frac{g_{f_0 \rightarrow \bar{K}K}^2}{g_{f_0 \rightarrow \pi\pi}^2} = \frac{2}{3} \cot^2(\theta_S). \quad (18)$$

We notice that in [21] the theoretical ratio is $g_{f_0 \rightarrow \bar{K}K}^2 / g_{f_0 \rightarrow \pi\pi}^2 = \frac{1}{3} \cot^2(\theta_S)$; for this reason the mixing angle found in [21] is $\theta_S = \pm 15.9^\circ$ and differs from our result. The extra factor 1/2 present in the result of [21] could not be verified when evaluating the traces of Eq. (3).

We now consider a nonzero c_2 : in principle one could determine the ratio c_2/c_1 from the quantity $g_{a_0 \rightarrow \pi\eta}^2 / g_{a_0 \rightarrow \bar{K}K}^2$ alone. In fact, the theoretical expression depends only on c_2/c_1 . When $c_2 = 0$ the result as derived from Table I is 0.5, then $g_{a_0 \rightarrow \pi\eta}^2 / g_{a_0 \rightarrow \bar{K}K}^2$ increases very slowly for increasing c_2/c_1 . The value $g_{a_0 \rightarrow \pi\eta}^2 / g_{a_0 \rightarrow \bar{K}K}^2 = 0.6$ is reached when $c_2/c_1 = 0.6$, but the value $g_{a_0 \rightarrow \pi\eta}^2 / g_{a_0 \rightarrow \bar{K}K}^2 = 0.75$ corresponds to $c_2/c_1 = 2.35$. Because of this strong sensibility it is not practicable to determine c_2/c_1 in this way. However, a sizable c_2 improves the agreement with the experiment.

The χ^2 -method is then used to find the free quantities c_2/c_1 and θ_S which correspond to the best description of (15) (solution B):

$$\begin{aligned} \text{solution B: } c_2/c_1 &= 0.62, \quad \theta_S = -12.8^\circ; \\ \left(\frac{\chi^2}{3} = 0.65 < 1 \right). \end{aligned} \quad (19)$$

The theoretical ratios evaluated with the parameters of solution B are

$$\begin{aligned} \frac{g_{f_0 \rightarrow \bar{K}K}^2}{g_{f_0 \rightarrow \pi\pi}^2} &= 4.21, & \frac{g_{f_0 \rightarrow \bar{K}K}^2}{g_{a_0 \rightarrow \bar{K}K}^2} &= 2.28, \\ \frac{g_{a_0 \rightarrow \pi\eta}^2}{g_{a_0 \rightarrow \bar{K}K}^2} &= 0.60, \end{aligned} \quad (20)$$

thus in good agreement with the results of (15). The inclusion of the subdominant decay diagram of Fig. 1(b) leads to a clear improvement of all three ratios without adding $\bar{K}K$ contributions to the wave functions of the resonances. Notice that within this solution a negative value for the scalar mixing angle is preferred.

In [21,22] a large ratio $g_{\sigma \rightarrow \bar{K}K}^2 / g_{\sigma \rightarrow \pi\pi}^2 = 0.6 \pm 0.1$ is deduced from analysis of $\phi \rightarrow \gamma \pi^0 \pi^0$ (such a large ratio can explain a problem in reproducing the overall normalizations; other explanations are however possible [47,48]). With the parameters of solution B (19), one finds a very

small ratio: $g_{\sigma \rightarrow \bar{K}K}^2 / g_{\sigma \rightarrow \pi\pi}^2 = 4.8 \times 10^{-7}$. Such a small value is caused by the destructive interference between the σ_B and f_B components (see Table III). If the value of [22] should be confirmed by future experimental analyses, our solution B should be rejected.

We make a second fit by adding to the data in (15) the value $g_{\sigma \rightarrow \bar{K}K}^2 / g_{\sigma \rightarrow \pi\pi}^2 = 0.6 \pm 0.1$. The minimum of χ^2 corresponds to solution C and reads

$$\text{solution C: } c_2/c_1 = 0.89, \quad \theta_S = 35.8^\circ; \quad \left(\frac{\chi^2}{4} = 2.04\right). \quad (21)$$

The results derived from the solutions A, B, and C are summarized in Table IV, where also other coupling ratios are presented. In solution C a large branching ratio $\bar{K}K/\pi\pi$ for the σ resonance is found (0.65), however $g_{f_0 \rightarrow \bar{K}K}^2 / g_{a_0 \rightarrow \bar{K}K}^2$ gets worse (1.12, generating a large χ^2 ; it is however still larger than 1).

Some comments are in order:

- (a) The first three ratios refer to $a_0(980)$ and to $f_0(980)$, which have been studied in detail in the literature for what concern the $\bar{K}K$, $\pi\eta$, and $\pi\pi$ channels. For these reasons our preferred solution is solution B (19), which is in good agreement with these three experimental ratios.

The other ratios refer to the broad σ and k states or to channels far from threshold for a_0 and f_0 , therefore are not free of ambiguities.

- (b) The ratio $g_{\sigma \rightarrow \bar{K}K}^2 / g_{\sigma \rightarrow \pi\pi}^2$ is the main difference between solutions B and C, which also generates different values for the scalar angle, negative in the first case and positive in the second. We notice also a large discrepancy of all three solutions for the branching ratio $g_{f_0 \rightarrow \eta\eta}^2 / g_{f_0 \rightarrow \pi\pi}^2$, whose experimental value reported in [21] is smaller than the theo-

retical results for Jaffe's tetraquark states. Future checks on these two quantities may help to disentangle the nature of the light scalar mesons.

- (c) In solutions B and C the ratio c_2/c_1 is not small. The inclusion of the decay mechanism of Fig. 1(b) (which can occur with one gluon as intermediate state) represents an improvement for the phenomenology of Jaffe's tetraquark states. Indeed, this result has been anticipated in [14], where the quantity c_2/c_1 varies between 0.7 and 1 (for comparison, the parameters a and b in [14] are $2c_1$ and $-c_2$ in the present work).

In particular, one can notice that the first three entries of Table IV are improved in solution B when compared to solution A (corresponding to $c_2 = 0$), thus a nonzero and sizable ratio c_2/c_1 is favored within the here presented phenomenological analysis.

In solution B, the ratio $c_2/c_1 = 0.62$ lies in between $1/N_c = 1/3$ and 1. Although larger than $1/3$, it is still clearly smaller than 1. Solution B is still in agreement with expectations from large N_c considerations. In solution C, one has $c_2/c_1 = 0.89$, which is of order 1. Such a value would imply a substantial violation from large N_c ; this fact constitutes a further hint in favor of solution B.

- (d) The coupling $g_{k \rightarrow \pi K}^2$ is smaller than $g_{\sigma \rightarrow \pi\pi}^2$ in solutions B and C; this is not in accord with the experimental result describing such a wide k resonance (curiously, solution A is better in this respect). Two experimental values are reported in Table IV (see [21]), the second one does not represent such a large mismatch with the theoretical values. It is noticeable that a similar problem exists when describing the scalar sector above 1 GeV ([9] and references therein): the experimental large width of the state $K_0(1430)$ cannot be explained theoretically.

TABLE IV. Comparison of coupling ratios with the analysis of experimental results of Refs. [21,22].^a

Ratios of g^2	Solution A (16)	Solution B (19)	Solution C (21)	Analysis of Refs. [21,22]
$g_{a_0 \rightarrow \pi\eta}^2 / g_{a_0 \rightarrow \bar{K}K}^2$	0.50	0.60	0.63	0.75 ± 0.11
$g_{f_0 \rightarrow \bar{K}K}^2 / g_{f_0 \rightarrow \pi\pi}^2$	4.26	4.21	4.35	4.21 ± 0.16
$g_{f_0 \rightarrow \bar{K}K}^2 / g_{a_0 \rightarrow \bar{K}K}^2$	0.86	2.28	1.11	2.15 ± 0.4
$g_{a_0 \rightarrow \pi\eta}^2 / g_{a_0 \rightarrow \pi\pi}^2$	1.01	0.16	0.05	...
$g_{f_0 \rightarrow \eta\eta}^2 / g_{f_0 \rightarrow \pi\pi}^2$	1.78 (1.57)	1.35	2.30	< 0.33
$g_{\sigma \rightarrow \bar{K}K}^2 / g_{\sigma \rightarrow \pi\pi}^2$	0.10	4.8×10^{-7}	0.65	0.6 ± 0.1
$g_{\sigma \rightarrow \eta\eta}^2 / g_{\sigma \rightarrow \pi\pi}^2$	0.30 (0.02)	0.05	0.10	0.20 ± 0.04
$g_{k \rightarrow \pi K}^2 / g_{\sigma \rightarrow \pi\pi}^2$	1.16	0.78	0.58	$(2.14 \pm 0.28) \text{ to } (1.35 \pm 0.10)$
$g_{k \rightarrow \eta K}^2 / g_{k \rightarrow \pi K}^2$	0.17	0.12	0.11	0.06 ± 0.02
$g_{k \rightarrow \eta K}^2 / g_{k \rightarrow \pi K}^2$	0.16	0.006	~ 0	0.29 ± 0.29

^aThe results for solution A are equal for the two mixing angles $\theta_S = \pm 21.6^\circ$, with the exception of two ratios: $g_{f_0 \rightarrow \eta\eta}^2 / g_{f_0 \rightarrow \pi\pi}^2$ and $g_{\sigma \rightarrow \eta\eta}^2 / g_{\sigma \rightarrow \pi\pi}^2$, for which the first result corresponds to $\theta_S = -21.6^\circ$, while the second (in parentheses) to $\theta_S = 21.6^\circ$.

- (e) The knowledge of the mixing angle does not allow us to deduce M_σ and M_k . However, we can consider reasonable values for M_σ and determine M_k for which the mixing angles of solutions B and C are realized: if we take $M_\sigma = 0.45$ GeV, then a scalar mixing angle $\theta_S = -12.8^\circ$ [solution B (19)] corresponds to $M_k = 0.69$ GeV, while a mixing angle $\theta_S = 35.8^\circ$ [solution C (21)] to $M_k = 0.944$ GeV. Similarly, if $M_\sigma = 0.55$ GeV the corresponding couple of values ($\theta_S = -12.8^\circ$, $M_k = 0.735$ GeV) and ($\theta_S = 31.8^\circ$, $M_k = 0.953$ GeV) are found. The present values for the k pole favor a light mass between 0.7 and 0.8 GeV [19,20], therefore solution B is in better agreement with the data. (However, the analysis of [49] points to a slightly heavier kaonic state: $M_k = 841 \pm 30_{-73}^{+81}$, therefore caution is still needed when driving conclusions.)
- (f) Keeping in mind the remarks at the beginning of this section, we nevertheless discuss some full widths as (naively) calculated from Eq. (12); in [43] various values for $g_{a_0 \rightarrow \pi\eta}^2$ are reported ($= \bar{g}_\eta 8\pi M_{a_0}^2$ in [43]). The results vary between 5 and 10 GeV². We take for simplicity a value in between: $g_{a_0 \rightarrow \pi\eta}^2 = 7.5$ GeV². For solution B such a value corresponds to $c_1 = 1.32$ GeV, resulting in (values in MeV) $\Gamma_{f_0 \rightarrow \pi\pi} = 136$, $\Gamma_{a_0 \rightarrow \pi\eta} = 98$, $\Gamma_{\sigma \rightarrow \pi\pi} = 795$, and $\Gamma_{k \rightarrow K\pi} = 251$. For solution C, we have $c_1 = 1.19$ GeV, implying $\Gamma_{f_0 \rightarrow \pi\pi} = 58$, $\Gamma_{a_0 \rightarrow \pi\eta} = 98$, $\Gamma_{\sigma \rightarrow \pi\pi} = 1013$, and $\Gamma_{k \rightarrow K\pi} = 241$. Clearly the widths change substantially when varying $g_{a_0 \rightarrow \pi\eta}^2$ in the mentioned range. The qualitative picture emerging is the presence in both cases of two broad states σ and k with $\Gamma_{\sigma \rightarrow \pi\pi} > \Gamma_{k \rightarrow K\pi}$ (but the latter not so broad as desired), and two narrower partial decay widths for f_0 and a_0 ; notice that the ordering $\Gamma_{f_0 \rightarrow \pi\pi} > \Gamma_{a_0 \rightarrow \pi\eta}$ in solution B is reversed in solution C. The inclusion of a mass distribution for the calculation of the decay widths, thus also evaluating the $\bar{K}K$ modes for a_0 and f_0 , is planned as a future step but beyond the goal of the present work.
- (g) Before moving to the two-photon decays, we wish to remind what has been considered and what omitted in the performed study on strong decays. The two interaction terms of the Lagrangian (3) are $SU_V(3)$ -invariant and correspond to the diagrams of Fig. 1; other flavor-symmetric terms, listed in Eq. (A11), and direct $SU_V(3)$ -breaking interactions have been omitted. They are supposed to represent corrections to the here presented scenario(s) (large N_c -suppressed terms and flavor symmetry breaking corrections) eventually useful for a more quantitative study. By performing a fit leading to solution B, we used as experimental quantities the values reported in [20–22]; as already noticed above, these three results, although qualitatively similar to other works on light scalars, depend on particular assump-

tion of form factors. Furthermore, the errors are only statistical and not systematic. Even more caution is needed with the other values reported in the right column of Table IV. Being aware of these limitations of both theoretical and experimental origin, we however intended to focus on a particular and interesting aspect of phenomenology of four-quark states encoded in the two diagrams of Fig. 1.

IV. TWO-PHOTON DECAYS

We now turn the attention to the $\gamma\gamma$ -decays of the Jaffe's tetraquark states. As for the strong decays we consider two analogous channels, where one has photons instead of mesons as final states in Fig. 1. The Lagrangian reads

$$\mathcal{L}_{\text{em}} = c_1^{\gamma\gamma} S_{ij} \langle A^i Q A^j Q \rangle F_{\mu\nu}^2 - c_2^{\gamma\gamma} S_{ij} \langle A^i A^j Q^2 \rangle F_{\mu\nu}^2, \quad (22)$$

where $Q = e \cdot \text{diag}\{2/3, -1/3, -1/3\}$ is the charge matrix ($e = \sqrt{4\pi\alpha}$ is the electron charge, $\alpha \simeq 1/137$), and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the electromagnetic field tensor. Note that the convention for the relative sign of the leading and subleading terms is the same as Eq. (3) (see also Appendix A).

The decay width into two photons reads

$$\Gamma_{S \rightarrow \gamma\gamma} = 16\pi\alpha^2 M_S^3 \cdot g_{S \rightarrow \gamma\gamma}^2, \quad (23)$$

where the nonzero contributions obviously correspond to $S = a_0^0, \sigma, f_0$. The coupling constants for a_0^0 and for the bare states σ_B and f_B are deduced from (22) and read

$$g_{a_0^0 \rightarrow \gamma\gamma} = \frac{2c_1^{\gamma\gamma} + c_2^{\gamma\gamma}}{3\sqrt{2}}, \quad g_{\sigma_B \rightarrow \gamma\gamma} = \frac{4c_1^{\gamma\gamma} + 5c_2^{\gamma\gamma}}{9},$$

$$g_{f_B \rightarrow \gamma\gamma} = \frac{2c_1^{\gamma\gamma} + 7c_2^{\gamma\gamma}}{9\sqrt{2}}. \quad (24)$$

When considering the mixed physical states σ and $f_0(980)$ defined in Eq. (10) the coupling constants are modified as [see also Eq. (14)]

$$g_{\sigma \rightarrow \gamma\gamma} = g_{\sigma_B \rightarrow \gamma\gamma} \cos(\theta_S) + g_{f_B \rightarrow \gamma\gamma} \sin(\theta_S), \quad (25)$$

$$g_{f_0 \rightarrow \gamma\gamma} = -g_{\sigma_B \rightarrow \gamma\gamma} \sin(\theta_S) + g_{f_B \rightarrow \gamma\gamma} \cos(\theta_S)$$

The experimental results for the decay width of a_0 and f_0 are given by [15]

$$\Gamma_{f_0 \rightarrow \gamma\gamma} = 0.39_{-0.13}^{+0.10} \text{ keV},$$

$$\Gamma_{a_0^0 \rightarrow \gamma\gamma} = 0.30 \pm 0.10 \text{ keV}. \quad (26)$$

The experimental value for $\Gamma_{a_0^0 \rightarrow 2\gamma}$ is not reported as an average in [15]; however, it is in accord with the quoted averages $\Gamma_{a_0^0 \gamma 2\gamma}(\Gamma_{\eta\pi}/\Gamma_{\text{tot}}) = 0.24_{-0.07}^{+0.08} \text{ keV}$ and $\Gamma_{\bar{K}K}/\Gamma_{\eta\pi} = 0.183 \pm 0.024$. Combining these two results, one finds $\Gamma_{a_0^0 \rightarrow \gamma\gamma} = 0.28 \pm 0.10 \text{ keV}$, well compatible with (26). In [50] it was shown that two-photon widths

about 0.3 keV are compatible with a tetraquark nature of the states.

Expressing the results of (26) in terms of squared coupling constants we find

$$\frac{\Gamma_{a_0^0 \rightarrow \gamma\gamma}}{\Gamma_{f_0 \rightarrow \gamma\gamma}} = \frac{g_{a_0^0 \rightarrow \gamma\gamma}^2}{g_{f_0 \rightarrow \gamma\gamma}^2} = 0.77 \pm 0.48 \quad (27)$$

(where $M_{a_0} = M_{f_0} = 0.98$ GeV and an average error of 0.115 keV for $\Gamma_{f_0 \rightarrow \gamma\gamma}$ have been used).

Large N_c results are still valid by invoking vector meson dominance, for which the decay into two photons occurs via two virtual vector mesons. Then, we first discuss the case $c_2^{\gamma\gamma} = 0$ (solution A), where only the dominant contribution is considered. By using the mixing angle $\theta_S = 21.6^\circ$ [see Eq. (16)], we find the totally wrong ratio $g_{a_0^0 \rightarrow \gamma\gamma}^2 / g_{f_0 \rightarrow \gamma\gamma}^2 = 724.7$, while the other option $\theta_S = -21.6^\circ$ implies $g_{a_0^0 \rightarrow \gamma\gamma}^2 / g_{f_0 \rightarrow \gamma\gamma}^2 = 2.3$, which is still unacceptable when compared to the experimental result of (27). Even by modifying the scalar angle θ_S the situation is not improved; one finds a minimum for the ratio $g_{a_0^0 \rightarrow \gamma\gamma}^2 / g_{f_0 \rightarrow \gamma\gamma}^2 = 1$ at $\theta_S = -70.5^\circ$. Such a value for the scalar mixing angle is however ruled out by the phenomenology of the strong decay analyzed in the previous section [a small mixing angle θ_S is common to all three analyzed scenarios and is in accord with the mass degeneracy of $a_0(980)$ and $f_0(980)$]. Therefore, also the two-photon decay shows that the dominant decay mechanism, corresponding to a switch of q and $\bar{q}$ analogous to Fig. 1(a), is not enough to describe the experimental data.

Let us then consider the case of a nonzero $c_2^{\gamma\gamma}$. We keep the scalar mixing angle θ_S as found in solutions B and C, respectively, and we determine the ratio $c_2^{\gamma\gamma} / c_1^{\gamma\gamma}$ in order to reproduce the experimental result of Eq. (27)²

$$\text{solution B [Eq. (19), } \theta_S = -12.8^\circ] \Rightarrow \frac{c_2^{\gamma\gamma}}{c_1^{\gamma\gamma}} = 0.73 \quad (28)$$

$$\text{solution C [Eq. (21), } \theta_S = 35.8^\circ] \Rightarrow \frac{c_2^{\gamma\gamma}}{c_1^{\gamma\gamma}} = -1.04. \quad (29)$$

In both cases a large ratio $c_2^{\gamma\gamma} / c_1^{\gamma\gamma}$ is found; while in solution B it is still safely smaller than unity, in solution C a large (and negative < -1) ratio is obtained.

Furthermore, in solution B we find a noticeable correspondence between the electromagnetic and strong transitions: $c_2^{\gamma\gamma} / c_1^{\gamma\gamma} = 0.73 \sim c_2 / c_1 = 0.62$. The two-photon decay, which in the framework of vector meson dominance is mediated by two virtual vector mesons, differs from the

two-pseudoscalar decay only in the final stage, therefore the contribution of the subleading diagram with respect to the leading one is naively expected to be of the same magnitude (and with the same sign). On the contrary in solution C, we have $c_2^{\gamma\gamma} / c_1^{\gamma\gamma} = -1.04 \ll c_2 / c_1 = 0.89$: this fact would mean very different contributions of the subleading diagram in the decay amplitude into two vector mesons (then converting into two photons) from the corresponding transition into two pseudoscalar mesons. The composite approach used in this work does not allow for a more microscopic insight to deal rigorously with this issue, therefore this discussion does not represent a proof in favor of solution B. We simply limit to notice that within solution B one has $c_2^{\gamma\gamma} / c_1^{\gamma\gamma} \sim c_2 / c_1$, which on the contrary does not take place in solution C.

The experimental situation concerning $\sigma \rightarrow 2\gamma$ is less clear; no average or fit is presented in [15], however two experiments listed in [15] find large $\gamma\gamma$ decay widths: 3.8 ± 1.5 keV and 5.4 ± 2.3 keV, respectively. In a footnote it is then stated that these values could be assigned to $f_0(1370)$ [actually, in an older version of Particle Data Group [51] these values were assigned to the resonance $f_0(1370)$]. It is not clear if the $\gamma\gamma$ signal comes from the high mass tail of the broad σ state or from $f_0(1370)$ (or even from both; in such case the experimental result would represent the sum, i.e. an upper limit for both resonances). Furthermore, the application of Eq. (23) is too naive for the broad σ state; in fact, due to the third power of the mass of the decaying resonance, the two-photon decay width is strongly influenced by a large width, especially from the right mass tail (the precise form of a mass distribution would be necessary for a more precise and quantitative analysis). Here we simply report the results for the ratio $g_{a_0^0 \rightarrow \gamma\gamma}^2 / g_{\sigma \rightarrow \gamma\gamma}^2$ in the two scenarios: $g_{a_0^0 \rightarrow \gamma\gamma}^2 / g_{\sigma \rightarrow \gamma\gamma}^2$ is 0.83 in solution B (28) and 0.42 in solution C (29). However, if a two-photon width between 3 and 5 keV should be entirely assigned to the σ -resonance as recently discussed in Ref. [52], the interpretation of the σ as a tetraquark state would be problematic, because in the four-quark scenario the $\gamma\gamma$ decay width of the σ would be of the same order of the $a_0^0(980)$ one (~ 0.3 keV), as the above reported coupling ratios $g_{a_0^0 \rightarrow \gamma\gamma}^2 / g_{\sigma \rightarrow \gamma\gamma}^2$ suggest. Future work on the $\sigma \rightarrow \gamma\gamma$ transition is needed both theoretically and experimentally in order to clarify this crucial issue about light scalar mesons.

V. SUMMARY AND CONCLUSIONS

The Jaffe four-quark states have appealing characteristics to be good candidates for the description of the scalar nonet below 1 GeV; this paper aimed to analyze this possibility by studying strong and the electromagnetic decays of scalar resonances.

We first considered the strong decays of the light scalar mesons below 1 GeV. Beyond the OZI-superalowed de-

²Once the ratio is determined, one can fix the strength of $c_1^{\gamma\gamma}$ to reproduce $\Gamma_{f_0 \rightarrow \gamma\gamma}$ [Eq. (26)]. The other width $\Gamma_{f_0 \rightarrow \gamma\gamma}$ is then also correctly described.

cay, in which the scalar-tetraquark state falls apart into two pseudoscalar mesons as depicted in Fig. 1(a), the next to leading order in the N_c expansion, which occurs with (only) one intermediate gluon as shown in Fig. 1(b), has been systematically taken into account for all decay coupling constants and presented in Tables I, II, and III. The two decay mechanisms are described by corresponding terms in an effective composite interaction Lagrangian and are parametrized by two strength parameters c_1 and c_2 , respectively.

As a first step we studied the case $c_2 = 0$, in which the diagram of Fig. 1(b) is switched off. The large ratio $g_{f_0 \rightarrow \bar{K}K}^2 / g_{a_0 \rightarrow \bar{K}K}^2 = 2.15 \pm 0.40 > 1$ reported in the experimental analysis of [22] cannot be described because the theoretical ratio is ≤ 1 for $c_2 = 0$. We referred to the case $c_2 = 0$ as “solution A.”

We then allowed for a nonzero ratio c_2/c_1 : by fitting c_2/c_1 and the scalar-isoscalar mixing angle θ_S to the three values reported in Eq. (15) deduced in the analysis of [22] and involving the $\bar{K}K$, $\pi\eta$, and $\pi\pi$ channels for the $a_0(980)$ and $f_0(980)$ states, a sizable ratio $c_2/c_1 = 0.62$ is obtained. The subdominant transition of Fig. 1(b) is non-negligible in the present analysis and improves the theoretical description of the mentioned decay modes of $a_0(980)$ and $f_0(980)$. The mixing angle $\theta_S = -12.8^\circ$ is small and negative: the bare components $\sigma_B \equiv \bar{u}u\bar{d}d$ and $f_B \equiv \bar{s}s(\bar{u}u + \bar{d}d)$ are out of phase in the physical state σ and in phase in the resonance $f_0(980)$. We called this scenario “solution B,” which represents our preferred solution, as discussed in Secs. III and IV.

Within solution B, one finds a very small ratio $g_{\sigma \rightarrow \bar{K}K}^2 / g_{\sigma \rightarrow \pi\pi}^2 \sim 10^{-7}$, while the result reported in the analysis of Ref. [21] reads 0.6 ± 0.1 . Solution C has been then built by including this ratio when minimizing the χ -squared. As a result, agreement with [22] is obtained for the branching ratio $\bar{K}K/\pi\pi$, but the ratio $g_{f_0 \rightarrow \bar{K}K}^2 / g_{a_0 \rightarrow \bar{K}K}^2$ is found to be 1.11 within solution C, which is, although larger than 1, clearly worsened with respect to solution B. Future experimental check on $g_{\sigma \rightarrow \bar{K}K}^2 / g_{\sigma \rightarrow \pi\pi}^2$ may help to clarify this point. The mixing angle in solution C is positive: $\theta_S = 31.8^\circ$, i.e. the bare components are in phase for σ and out of phase for $f_0(980)$. The sign difference of θ_S is responsible of such different results concerning the quantity $g_{\sigma \rightarrow \bar{K}K}^2 / g_{\sigma \rightarrow \pi\pi}^2$.

The results of solutions A, B, and C are summarized in Table IV, where also other ratios of squared coupling constants are listed.

We then considered the two-photon decay of the scalar resonances. The diagrams for such transitions are similar to those of Fig. 1(a) and 1(b) but with photons instead of mesons as final states. The corresponding intensities are parametrized by $c_1^{\gamma\gamma}$ and $c_2^{\gamma\gamma}$, respectively.

The discussions follow the same line as before: the case $c_2^{\gamma\gamma} = 0$ (solution A) is analyzed first. The experimental

result $\Gamma_{a_0 \rightarrow 2\gamma} / \Gamma_{f_0 \rightarrow 2\gamma} = g_{a_0 \rightarrow \gamma\gamma}^2 / g_{f_0 \rightarrow \gamma\gamma}^2 = 0.77 \pm 0.48$ cannot be obtained if $c_2^{\gamma\gamma} = 0$, thus again pointing to a nonzero contribution of the subleading diagram with one intermediate gluon.

We then analyzed solutions B and C: by taking the scalar mixing angle θ_S from solution B or C the ratio $c_2^{\gamma\gamma} / c_1^{\gamma\gamma}$ has been determined in order to reproduce the experimental value $g_{a_0 \rightarrow \gamma\gamma}^2 / g_{f_0 \rightarrow \gamma\gamma}^2 = 0.77 \pm 0.48$. In case B one has the noticeable correspondence $c_2^{\gamma\gamma} / c_1^{\gamma\gamma} \sim c_2 / c_1$, that is the contribution of the subleading decay is similar (and with the same sign) in strong and electromagnetic transitions. This fact does not take place in solution C where the two ratios are large but with opposite sign.

The interpretation of the light scalar states as tetraquark objects is a viable and interesting possibility. The inclusion of the subleading decay mechanism of Fig. 1(b) improves the description of experimental data for both strong and electromagnetic transitions within our solution B.

Future work is however needed: on an experimental side a clear experimental establishment of the resonance k and a better understanding of the parameters describing the broad states σ and k would constitute a decisive improvement.

We would like to mention possible future theoretical studies: the decays of heavier states, such as tensor mesons or scalar mesons above 1 GeV, into two light scalar mesons as $\sigma\sigma, kk, \dots$ should be evaluated by explicitly taking into account the four-quark nature of the light scalars. Such an analysis would therefore extend and complete the ones in [9,42]. In particular, the fact that the tensor meson nonet is well established and well known experimentally would offer a suitable environment for such a study. Furthermore, the two-body decay of excited pseudoscalar states (eventually mixed with a pseudoscalar glueball) would also constitute an interesting development. In the recent work of [53], attention is focused on electromagnetic decays of the type $S \rightarrow V\gamma$, which may also be helpful in disentangling the nature of the light scalar states below 1 GeV.

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APPENDIX A: TETRAQUARK SCALAR NONET

The wave function for a “good” diquark is [13]

$$|qq\rangle = |\text{space: } L = 0\rangle |\text{spin: } S = 0\rangle |\text{color: } \bar{3}_C\rangle |\text{flavor: } \bar{3}_F\rangle. \quad (\text{A1})$$

The state $|qq\rangle$ is antisymmetric by exchange of the two particles in accord with the Pauli principle; the parity quantum number is $P = (-1)^L = +1$. In the following we consider the flavor decomposition for a diquark (and for an antidiquark). To this end we define the vector q such that $q^i = (u, d, s)$; the antisymmetric $\bar{3}_F$ decomposition of a diquark is described by the antisymmetric matrix D :

$$D = \sqrt{\frac{1}{2}}(q_1 \cdot q_2' - q_2 \cdot q_1') = \varphi_i A^i, \quad (\text{A2})$$

where the matrices A^i are the three antisymmetric 3×3 real matrices of Eq. (6) and the quantities φ_i are given by

$$\varphi_1 = \sqrt{\frac{1}{2}}(u_1 d_2 - u_2 d_1) \equiv \sqrt{\frac{1}{2}}[u, d], \quad (\text{A3})$$

$$\varphi_2 = \sqrt{\frac{1}{2}}(u_1 s_2 - u_2 s_1) \equiv \sqrt{\frac{1}{2}}[u, s], \quad (\text{A4})$$

$$\varphi_3 = \sqrt{\frac{1}{2}}(d_1 s_2 - d_2 s_1) \equiv \sqrt{\frac{1}{2}}[d, s]. \quad (\text{A5})$$

The antidiquark is then described by the matrix

$$D^\dagger = \bar{\varphi}_i (A^i)^\dagger = -\bar{\varphi}_i A^i \quad (\text{A6})$$

with $\bar{\varphi}_1 = \sqrt{1/2}[\bar{u}, \bar{d}]$, $\bar{\varphi}_2 = \sqrt{1/2}[\bar{u}, \bar{s}]$, and $\bar{\varphi}_3 = \sqrt{1/2}[\bar{d}, \bar{s}]$.

The matrix D has the following transformation properties:

$$\text{SU}(3)\text{-flavor: } D \rightarrow U D U^\dagger, \quad U \in \text{SU}(3), \quad (\text{A7})$$

$$P\text{-parity: } D \rightarrow D, \quad (\text{A8})$$

$$C\text{-charge conjugation: } D \rightarrow D^\dagger. \quad (\text{A9})$$

The flavor transformation (A7), corresponding to $q \rightarrow Uq$, follows directly from the definition (A2).

The matrix S defined in Eq. (2) and entering in Eq. (3) is given by $S_{ij} = \varphi_i \bar{\varphi}_j$.

In a generic Lagrangian term involving linear tetraquark scalar states (i.e. one matrix D and one matrix $D^\dagger$), one has a schematic form like

$$(\dots D \dots D^\dagger \dots) = S_{ij}(\dots A^i \dots (A^j)^\dagger \dots), \quad (\text{A10})$$

where dots indicate operation with matrices, such as multiplication, traces, etc. The form on the left-hand side is useful to check the SU(3)-flavor transformations, while the form in the right-hand side allows the calculations of the decay amplitudes (see Tables I, II, and III and Appendix B). When describing the scalar $\rightarrow$ two-pseudoscalar decay, we then have the following possible flavor-invariant terms:

$$\begin{aligned} \mathcal{L}_{S \rightarrow PP} = & -c_1 \langle D P^\dagger D^\dagger P \rangle + c_2 \langle D D^\dagger P^2 \rangle + c_3 \langle D D^\dagger P \rangle \\ & \times \langle P \rangle + c_4 \langle D D^\dagger \rangle \langle P^2 \rangle + c_5 \langle D D^\dagger \rangle \langle P \rangle^2, \end{aligned} \quad (\text{A11})$$

where $P = \frac{1}{\sqrt{2}} \sum_{i=0}^8 P^i \lambda_i$. The first two terms correspond to the two diagrams of Fig. 1, i.e. to the last two terms of Eq. (3) [by applying the operation (A10) the equality is easily seen] described throughout all the work. The third term is also suppressed by a factor N_c only with respect to the OZI-superallowed decay, but the transition can occur with at least two transverse gluons as intermediate states attached to the pseudoscalar flavor-singlet state. The fourth and the fifth terms are further suppressed.

The Lagrangian $\mathcal{L}_{S \rightarrow PP}$ is clearly charge and parity invariant (we recall that $P \rightarrow U P U^\dagger$, $P \rightarrow -P$, and $P \rightarrow P^t$ under flavor, parity, and charge conjugation transformations).

Formally the two-photon Lagrangian (22) can be obtained from Eq. (A11) by replacing $P \rightarrow Q F_{\mu\nu}$, $c_i \rightarrow c_i^{\gamma\gamma}$ (and then contracting the Lorentz indices). In virtue of $\langle Q \rangle = 0$, the first, the second, and the fourth terms “survive” the replacing; then, neglecting the latter we get the Lagrangian (22) (by invoking vector meson dominance, large N_c arguments can be applied also for the two-photon decays).

When considering the quadratic term for the scalar-tetraquark states care is needed; let us introduce an extra index $a = 1, 2$ with $D^a = \varphi_i^a A^i$. The elements S_{ij} [Eq. (2)] are formed by the two corresponding diquark φ_i^a and antidiquark $\bar{\varphi}_j^a$: $S_{ij} = \varphi_i^a \bar{\varphi}_j^a$. Let us consider the flavor-invariant term $\langle D^\dagger (D^2)^\dagger \rangle \langle D^2 (D^1)^\dagger \rangle$; we aim to show that this is nothing else but $4\langle S^2 \rangle$ [proportional to the mass term of Eq. (3) in the flavor-symmetric limit]:

$$\begin{aligned} \langle D^\dagger (D^2)^\dagger \rangle \langle D^2 (D^1)^\dagger \rangle &= \varphi_i^1 \bar{\varphi}_j^2 \varphi_k^1 \bar{\varphi}_l^2 \langle A^i A^j \rangle \langle A^k A^l \rangle \\ &= 4(\varphi_i^1 \bar{\varphi}_l^1)(\varphi_k^2 \bar{\varphi}_j^2) \delta_{ij} \delta_{kl} \\ &= 4S_{ii} S_{ll} = 4\langle S^2 \rangle. \end{aligned} \quad (\text{A12})$$

Then, in Eq. (3) modification from the flavor-symmetric limit are parametrized by the matrix $\chi_S = \text{diag}\{\alpha, \beta, \beta\}$ and $\alpha < \beta$.

In Sec. II B [Eq. (7)], we considered extra terms in the scalar singlet-octet and singlet-singlet channels. The singlet-singlet channel, proportional to S_0^2 , is flavor symmetric and corresponds to $\langle D^\dagger (D^1)^\dagger \rangle \langle D^2 (D^2)^\dagger \rangle$. The singlet-octet mixing introduced in Eq. (7) breaks flavor invariance. Here we notice that a flavor-symmetric term, also including singlet-octet mixing (but affecting also the isovector and isodoublet states) can be constructed for the quadratic scalar sector:

$$\langle D^\dagger (D^1)^\dagger D^2 (D^2) \rangle = S_{ii} S_{kk} \langle A^i A^j A^k A^l \rangle. \quad (\text{A13})$$

Further study on the mass sector for tetraquark scalar states including this term could be interesting.

APPENDIX B: STRONG COUPLING CONSTANTS

We briefly recall the connection between the Lagrangian (3) and the coupling constants $g_{S \rightarrow P_1 P_2}^2$ entering in Eq. (12).

After evaluating the traces, the Lagrangian (3) can be rewritten as a sum over all the decay channels $S \rightarrow P_1 P_2$, where $S = \{a_0^\pm, a_0^0, k^\pm, k_0, \bar{k}_0, \sigma, f_0\}$ and $P = \{\pi^\pm, \pi^0, K^\pm, K_0, \bar{K}_0, \eta, \eta'\}$:

$$\begin{aligned} \mathcal{L}_{S \rightarrow PP} &= c_1 \mathcal{S}_{ij} \langle A^i \mathcal{P}^j A^j \mathcal{P} \rangle - c_2 \mathcal{S}_{ij} \langle A^i A^j \mathcal{P}^2 \rangle \\ &= \left(\sqrt{2} c_1 + \frac{c_2}{\sqrt{2}} \right) (a_0^0 K^- K^+ - K^0 \bar{K}^0) \\ &\quad + (c_1 + c_2) (2\sigma_B \pi^+ \pi^- + \sigma_B (\pi^0)^2) + \dots \\ &= \sum_{S, P_1, P_2} \lambda_{S \rightarrow P_1 P_2} S P_1 P_2, \end{aligned} \quad (\text{B1})$$

where in the second line we wrote the a_0^0 to $\bar{K}K$ and the σ_B to $\pi\pi$ couplings explicitly; they serve as illustrative examples of the adopted conventions. The coupling constants at this stage (without sum/average over isomultiplets) are simply given by $g_{S \rightarrow P_1 P_2} = s \cdot \lambda_{S \rightarrow P_1 P_2}$, where $s = 2/\sqrt{2} = \sqrt{2}$ when $P_1 = P_2$, 1 otherwise.

We have, for instance, $g_{a_0^0 \rightarrow K^+ K^-} = (\sqrt{2} c_1 + \sqrt{1/2} c_2)$, $g_{a_0^0 \rightarrow \bar{K}^0 K^0} = -(\sqrt{2} c_1 + \sqrt{1/2} c_2)$, $g_{\sigma_B \rightarrow \pi^+ \pi^-} = 2(c_1 + c_2)$, $g_{\sigma_B \rightarrow \pi^0 \pi^0} = \sqrt{2}(c_1 + c_2)$. Notice that by plugging these coupling constants into (12) we get the partial decay width for the corresponding channel, such as $\Gamma_{a_0^0 \rightarrow K^+ K^-}$.

Let us then group the final states in corresponding isomultiplets: $P = \{\pi, K, \eta, \eta'\}$. One has to perform the sum over the members of the isomultiplets, for instance:

$$\begin{aligned} g_{a_0^0 \rightarrow \bar{K}K}^2 &= g_{a_0^0 \rightarrow K^+ K^-}^2 + g_{a_0^0 \rightarrow \bar{K}^0 K^0}^2 \rightarrow g_{a_0^0 \rightarrow \bar{K}K}^2 \\ &= \sqrt{2} g_{a_0^0 \rightarrow K^+ K^-} \end{aligned} \quad (\text{B2})$$

$$\begin{aligned} g_{\sigma_B \rightarrow \pi\pi}^2 &= g_{\sigma_B \rightarrow \pi^+ \pi^-}^2 + g_{\sigma_B \rightarrow \pi^0 \pi^0}^2 \rightarrow g_{\sigma_B \rightarrow \pi\pi}^2 \\ &= \sqrt{\frac{3}{2}} g_{\sigma_B \rightarrow \pi^+ \pi^-} \end{aligned} \quad (\text{B3})$$

The sign of the resulting coupling constant can be chosen arbitrarily for the isotriplet and isodoublets a_0 and k states. In the a_0^0 to $\bar{K}K$ case, we chose the sign of the charged decay channel $K^+ K^-$ (which is opposite to the $\bar{K}^0 K^0$ one); Table I is compiled following this convention. This however does not affect the decay rates expressed in (12). Care is needed in the scalar-isoscalar sector because of mixing occurring in Eq. (14), such as $g_{\sigma_B \rightarrow \pi\pi}$: the sign of $g_{\sigma_B \rightarrow \pi\pi}$ is taken to be the same as $g_{\sigma_B \rightarrow \pi^+ \pi^-}$ and $g_{\sigma_B \rightarrow \pi^0 \pi^0}$. In the scalar-isoscalar sector the various contributions to the sum over final isomultiplets have the same overall sign, thus making the definition unambiguous. Tables II and III follow this convention.

As a last step an average over the initial isospin multiplets is performed, that is we consider $S = \{a_0, k, \sigma, f_0\}$. For instance, $g_{a_0 \rightarrow \bar{K}K}^2$ is given by

$$g_{a_0 \rightarrow \bar{K}K}^2 = \frac{1}{3} (g_{a_0^0 \rightarrow \bar{K}K}^2 + g_{a_0^+ \rightarrow \bar{K}K}^2 + g_{a_0^- \rightarrow \bar{K}K}^2) = g_{a_0^0 \rightarrow \bar{K}K}^2. \quad (\text{B4})$$

The terms of the average are equal (this is true for each decay of a_0 and k ; for σ and f_0 no average is needed).

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Mixing of scalar tetraquark and quarkonia states in a chiral approach

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A chiral invariant Lagrangian describing the tetraquark-quarkonia interaction is considered at the leading and subleading order in the large- N_c expansion. Spontaneous chiral symmetry breaking generates mixing of scalar tetraquark and quarkonia states and nonvanishing tetraquark condensates. In particular, the mixing strength is related to the decay strengths of tetraquark states into pseudoscalar mesons. The results show that scalar states below 1 GeV are mainly four-quark states and the scalars between 1 and 2 GeV quark-antiquark states, probably mixed with the scalar glueball in the isoscalar sector.

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I. INTRODUCTION

The spectroscopic interpretation of the scalar states below 1 GeV represents an important issue of modern hadronic physics. It is not yet clear if the dominant contribution to their wave function constitutes of quarkonia, mesonic molecules or Jaffe's tetraquark states. In turn, this subject is strongly connected to the nature of the scalar states above 1 GeV (we refer to the review papers [1–3]).

Various interpretations have been proposed in the literature about the scalar resonances below and above 1 GeV. According to the most popular scenario, one interprets the isovector and isotriplet resonances $a_0(1450)$ and $K(1430)$ as the ground-state quark-antiquark bound states. The three isoscalar resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ are an admixture of two isoscalar quarkonia and bare glueball configurations (we refer to [1,2,4–11] and references therein; recently the inclusion of hybrids in the mixing scheme has been performed in Ref. [12]). As a consequence, the scalar states below 1 GeV [$f_0(600)$, $k(800)$, $f_0(980)$, and $a_0(980)$] must be something else, like (loosely bound) mesonic molecular states [13,14], dynamical generated resonances [15], or Jaffe's tetraquark states [1,3,16–18]. It is indeed possible that an interplay of these three possibilities takes place.

The tetraquark states, whose building blocks are a diquark (q^2) and an antidiquark ($\bar{q}^2$), play a central role in this paper. Calculations based on one-gluon exchange [1,19], instantons [20,21], Nambu Jona-Lasinio model (NJL) [22], and Dyson-Schwinger equation (DSE) [23] support a strong attraction among two quarks in a color antitriplet ($\bar{3}_C$), a flavor antitriplet ($\bar{3}_F$), and spinless configuration [1,16] (color and flavor triplets are realized for an antidiquark). Naively speaking, such a scalar diquark “behaves like an antiquark” from a flavor (and color) point of view, thus a nonet of light scalar tetraquark states naturally emerges in this context. Support for the existence of Jaffe's states below 1 GeV is in agreement with the Lattice studies of Refs. [24–26].

In the recent work of Ref. [18], the present author analyzed the strong and the electromagnetic decays of the light scalar states $\{f_0(600), k(800), f_0(980)$ and $a_0(980)\}$ ¹ interpreted as Jaffe's tetraquark states, which naturally account for the mass degeneracy of $f_0(980)$ and $a_0(980)$ and their large $\bar{K}K$ decay strength. The dominant [Fig. 1(a)] and the subdominant [Fig. 1(b)] decay mechanisms in the large- N_c expansion, respectively, proportional to the decay strengths c_1 and c_2 , have been systematically taken into account in an effective $SU_V(3)$ -invariant interaction Lagrangian.

In the present work we extend the model of Ref. [18], which was built under the requirement of flavor symmetry $SU_V(3)$, by considering invariance under the chiral group $SU_R(3) \times SU_L(3)$. The explicit inclusion of a scalar-quarkonia nonet, lying between 1 and 2 GeV (see discussion in Sec. II A) as the chiral partner of the pseudoscalar nonet, and the inclusion of the pseudoscalar diquark, as the chiral partner of the scalar diquark, are required. As in [18], we keep the leading and the subleading terms in the large- N_c expansion.

As a consequence of chiral symmetry breaking, mixing among tetraquark and quarkonia states takes place. The most important theoretical result of the present work is the possibility to relate the mixing strength between the scalar tetraquark and quarkonia nonets to the tetraquark decay strengths c_1 and c_2 of Fig. 1 and to the pion and kaon decay constants. Furthermore, the tetraquark-quarkonia mixing in the scalar sector is responsible for the emergence of nonvanishing tetraquark condensates.

The connection of the decay strengths c_1 and c_2 to the mixing allows us to evaluate its strength. As a result, we find that the tetraquark assignment for the light scalar states is consistent: by analyzing the isovector channel the resonance $a_0(980)$ has a dominant tetraquark content; the

¹The resonance $k(800)$ is now listed in the compilation of the Particle Data Group [27] but it still needs confirmation, and is omitted from the summary table. The resonance is also found in many recent theoretical and experimental works ([15,28–32] and references therein).

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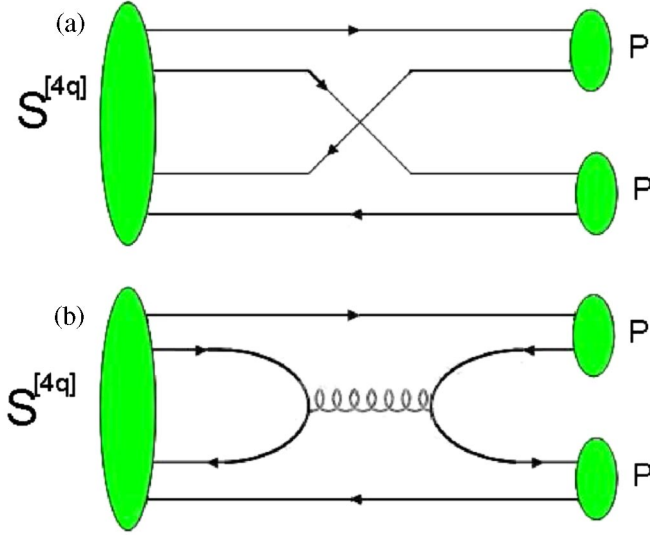


FIG. 1 (color online). Dominant (a) and subdominant (b) contributions to the transition amplitudes of a scalar tetraquark state into two pseudoscalar mesons. They correspond to the Lagrangian in Eqs. (15) and (16).

quarkonium amount in its spectroscopic wave function turns out to be relatively small ($\lesssim 10\%$). An analogous result is obtained in the kaonic sector.

The use of chiral Lagrangian for the analysis of tetraquark-quarkonia mixing has been studied in Refs. [33–36], where sizable admixtures in the scalar physical resonances below and above 1 GeV are found. In the present work a different chiral Lagrangian is utilized and only the scalar (and not the pseudoscalar) diquarks are considered as a basic constituent for low-energy mesonic resonances. Our results point to a smaller mixing strength and thus to a substantial separation of four-quark states below 1 GeV and quarkonia states above 1 GeV.

The paper is organized as follows. In Sec. II, the model is constructed: we recall the basics of the chiral treatment of the scalar and pseudoscalar nonets, we introduce the scalar diquark and briefly review Ref. [18], we describe the pseudoscalar diquark and write down the chiral invariant tetraquark-quarkonia interaction Lagrangian. In Sec. III the phenomenological implications are studied: the scalar

tetraquark-quarkonia mixing and the magnitude of the tetraquark condensates. In Sec. IV we present the summary and the conclusions.

II. SETUP OF THE MODEL

A. Quarkonia nonets

We briefly recall the basic elements for the setup of the pseudoscalar and the scalar-quarkonia nonets. At a microscopic level, one has the quark field $q_i(x)$ with $i = u, d, s$. The right and left spinors are given by

$$q_{i,R} = P_R q_i, \quad q_{i,R}^\dagger = q_i P_R, \quad \bar{q}_{i,R} = \bar{q}_i P_L \quad (1)$$

$$q_{i,L} = P_L q_i, \quad q_{i,L}^\dagger = q_i P_L, \quad \bar{q}_{i,L} = \bar{q}_i P_R, \quad (2)$$

where $P_R = \frac{1}{2}(1 + \gamma_5)$ and $P_L = \frac{1}{2}(1 - \gamma_5)$.

The $SU_R(3) \times SU_L(3)$ transformation on the quark fields is defined as

$$q_i = q_{i,R} + q_{i,L} \rightarrow R_{ij} q_{j,R} + L_{ij} q_{j,L} \quad \text{with } R \in SU_R(3), \\ L \in SU_L(3). \quad (3)$$

Out of quark fields, one can build up operators (currents) with the correct quantum numbers of the physical resonances. In fact, at a composite level one deals with mesons, which have the same transformation properties of the underlying quark currents. In Table I we summarize the properties of the pseudoscalar and scalar-quarkonia Hermitian matrices $\mathcal{P}$ and $\mathcal{S}$ and of the matrix $\Sigma = \mathcal{S} + i\mathcal{P}$: the corresponding matrix elements and the components in the Gell-Mann basis (denoted as “currents” in Table I), the transformations under parity (P), charge conjugation (C), $SU_V(3)$ [occurring for $R = L = U$ in Eq. (3)], chiral $SU_R(3) \times SU_L(3)$ and $U_A(1)$ (occurring for $q_i \rightarrow e^{i\nu\gamma_5} q_i$, i.e. $q_{i,R} \rightarrow e^{i\nu} q_{i,R}$ and $q_{i,L} \rightarrow e^{-i\nu} q_{i,L}$) are reported.

Following [30] and references therein, which we refer to for a careful treatment, we introduce the Lagrangian $\mathcal{L}_\Sigma$

$$\mathcal{L}_\Sigma = \frac{1}{4} \text{Tr}[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger] - V_0(\Sigma, \Sigma^\dagger) - V_{\text{SB}}(\Sigma, \Sigma^\dagger) \quad (4)$$

(Tr denotes trace over flavor) which describes the dynamics of the pseudoscalar and scalar-quarkonia mesons. As

TABLE I. Summary of the properties of $\mathcal{P}$, $\mathcal{S}$, and Σ .

	$\mathcal{P} = \frac{1}{\sqrt{2}} \sum_{i=0}^8 P^i \lambda_i$	$\mathcal{S} = \frac{1}{\sqrt{2}} \sum_{i=0}^8 S^i \lambda_i$	$\Sigma = \mathcal{S} + i\mathcal{P}$
Matrix elements	$\mathcal{P}_{ij} \equiv \bar{q}_j i \gamma^5 q_i$	$\mathcal{S}_{ij} \equiv \bar{q}_j q_i$	$\Sigma_{ij} \equiv 2\bar{q}_{j,R} q_{i,L}$
Currents	$P^i \equiv \bar{q} i \gamma^5 \frac{\lambda_i}{\sqrt{2}} q$	$S^i \equiv \bar{q} \frac{\lambda_i}{\sqrt{2}} q$	$\Sigma^i \equiv 2\bar{q}_R \frac{\lambda_i}{\sqrt{2}} q_L$
P	$-\mathcal{P}(x^0, -\mathbf{x})$	$\mathcal{S}(x^0, -\mathbf{x})$	$\Sigma^\dagger(x^0, -\mathbf{x})$
C	$\mathcal{P}^t$	$\mathcal{S}^t$	Σ^t
$SU_V(3)$	$U\mathcal{P}U^\dagger$	$U\mathcal{S}U^\dagger$	$U\Sigma U^\dagger$
$SU_R(3) \times SU_L(3)$	$\frac{1}{2i}(\mathcal{L}\Sigma R^\dagger - R\Sigma^\dagger L^\dagger)$	$\frac{1}{2}(\mathcal{L}\Sigma R^\dagger + R\Sigma^\dagger L^\dagger)$	$\mathcal{L}\Sigma R^\dagger$
$U_A(1)$ ($q_i \rightarrow e^{i\nu\gamma_5} q_i$)	$\frac{1}{2i}(e^{-2i\nu}\Sigma - e^{2i\nu}\Sigma^\dagger)$	$\frac{1}{2}(e^{-2i\nu}\Sigma + e^{2i\nu}\Sigma^\dagger)$	$e^{-2i\nu}\Sigma$

usual, V_0 represents the chiral invariant potential while V_{SB} encodes symmetry breaking due to the nonzero current quark masses.² In the present work we are not concerned with the detailed description of the properties of the potentials V_0 and V_{SB} . What is important for us is spontaneous chiral symmetry breaking (χ SB); that is, the minimum of the potential $V_0 + V_{SB}$ is realized for nonzero vacuum expectation values (vev's):

$$\langle S_{ij} \rangle = \alpha_i \delta_{ij} \quad (\text{with } \alpha_u = \alpha_d: \text{isospin symmetry}). \quad (5)$$

The expectation values α_i are related to the pion and the kaon decay constants in a model-independent way [30,37]:

$$\begin{aligned} \langle S_{11} \rangle + \langle S_{22} \rangle &= 2\alpha_u = \sqrt{2}F_\pi, \\ \langle S_{11} \rangle + \langle S_{33} \rangle &= \alpha_u + \alpha_s = \sqrt{2}F_K. \end{aligned} \quad (6)$$

We use $F_\pi = 0.0924$ GeV and $F_K = 1.22F_\pi$. This leads us to shift the matrix Σ as

$$\begin{aligned} \Sigma &= S + i\mathcal{P} \rightarrow \tilde{\Sigma} = \Sigma_0 + S + i\mathcal{P} \\ \text{where } \Sigma_0 &= \text{diag}\{\alpha_u, \alpha_u, \alpha_s\}. \end{aligned} \quad (7)$$

The pseudoscalar nonet is well established: $\mathcal{P} = \{\pi, K, \eta, \eta'\}$. The identification of the scalar states is controversial. Some models [22,38–40] identify the resonance $f_0(600)$ as the chiral partner of the pion, hence a quarkonium $\bar{n}n = \sqrt{1/2}(\bar{u}u + \bar{d}d)$. This assignment encounters a series of well-known problems: (i) in this scheme the resonances $a_0^0(980)$ and $f_0(980)$ would be respectively $\sqrt{1/2}(\bar{u}u - \bar{d}d)$ and $\bar{s}s$. Their mass degeneracy is then hard to be explained from their quark content (see also the different point of view in Refs. [39,40]). (ii) The strong coupling of $a_0(980)$ to $\bar{K}K$ cannot be explained within this assignment [the points (i)–(ii) are naturally explained when interpreting the light scalar resonance as mainly Jaffe's four-quark states, see Refs. [3,16–18] and next subsection]. (iii) The scalar-quarkonia states are p -wave ($L = S = 1$), therefore expected to have a mass comparable to the p -wave nonets of tensor and axial-vector mesons which lie well above 1 GeV. (iv) The lattice results of Refs. [26,41] predict a mass for the quarkonium state $u\bar{d}$ about 1.4–1.5 GeV, thus well above 1 GeV (see also the

different result of Ref. [42]). (v) As shown in Ref. [43] [and recently confirmed in Ref. [44] at two-loop order in unitarized chiral perturbation theory for the resonance $f_0(600)$], the large- N_c behavior of the masses of the scalar states below 1 GeV is *not* compatible with a dominant quarkonium content, thus further pointing to a heavier bare mass of the latter.

Thus, we expect that the bare quarkonia masses lie above 1 GeV. We will then analyze the mixing of the quarkonia states with the (lighter) four-quark states in Sec. III.

B. Scalar diquark and corresponding tetraquark states

We turn our attention to the scalar diquark current. To this end we consider the following scalar flavor-antisymmetric ($\bar{3}_F$) diquark-matrix D :

$$D_{ij} \equiv \sqrt{\frac{1}{2}}(q_j^t C \gamma^5 q_i - q_i^t C \gamma^5 q_j) = \sum_{i=1}^3 \varphi_i A^i \quad (8)$$

$$(A^i)_{jk} = \varepsilon_{ijk}, \quad \varphi_i = \sqrt{\frac{1}{2}}\varepsilon_{ijk} q_j^t C \gamma^5 q_k, \quad (9)$$

where the superscript t refers to transposition in the Dirac space. Color indices, formally identical to the flavor ones ($\bar{3}_C$), are understood. We refer to the quantities φ_i , arising from the decomposition of D in the basis of the antisymmetric matrices A^i , as the scalar-diquark currents and to the Hermitian conjugate $\varphi_i^\dagger$ as the scalar antidiquark currents.

In terms of flavor the currents φ_i read

$$\begin{aligned} \varphi_1 &= \sqrt{\frac{1}{2}}[d, s] \leftrightarrow \bar{u}, & \varphi_2 &= -\sqrt{\frac{1}{2}}[u, s] \leftrightarrow \bar{d}, \\ \varphi_3 &= \sqrt{\frac{1}{2}}[u, d] \leftrightarrow \bar{s}, \end{aligned} \quad (10)$$

where the correspondence $\leftrightarrow$ refers to the fact that a diquark in the flavor (and color) antisymmetric decomposition behaves like an antiquark, as already anticipated in the Introduction.

The spinor structure of the kind $q^t C \gamma^5 q$ corresponds to a diquark with parity $+1$ (ergo to $L = S = 0$). Schematically:

$$|qq\rangle_{L=S=0} = |\text{space: } L=0\rangle |\text{spin: } S=0\rangle |\text{color: } \bar{3}_C\rangle |\text{flavor: } \bar{3}_F\rangle; \quad J^P = 0^+. \quad (11)$$

As discussed in the Introduction, the scalar diquark $|qq\rangle_{L=S=0}$ forms a compact and stable object, as one-gluon exchange, instanton-based calculations, NJL and DSE ap-

²Notice that we are considering nonets of states and not only octets (the sum in Table I runs from $i = 0, \dots, 8$). Thus, in V_{SB} also $U_A(1)$ breaking, mixing, and large- N_c suppressed terms are (implicitly) included.

proaches show, rendering it a good constituent for light meson (and baryon) spectroscopy [3].

In Table II we recall the microscopic decomposition of the elements of the diquark matrix D and the corresponding currents φ_i of Eq. (8) and the properties under $SU_V(3)$, parity and charge-conjugation transformations.

As one can notice, the $SU_V(3)$ -transformation of the diquark currents $\varphi_i \rightarrow \varphi_k U_{ki}^\dagger$ is exactly analogous to the

TABLE II. Properties of the scalar-diquark matrix D and components φ_i .

	$D = \sum_{i=1}^3 \varphi_i A^i$	φ_i
Elements/Currents	$D_{ij} = \sqrt{\frac{1}{2}}(q_j^t C \gamma^5 q_i - q_i^t C \gamma^5 q_j)$	$\varphi_i = \sqrt{\frac{1}{2}} \varepsilon_{ijk} q_j^t C \gamma^5 q_k$
P	D	φ_i
C	$D^\dagger$	$\varphi_i^\dagger$
$SU_V(3)$	UDU^t	$\varphi_k U_{ki}^\dagger$

$SU_V(3)$ -transformation of an antiquark: $\bar{q}_i \rightarrow \bar{q}_k U_{ki}^\dagger$. This is the formal way to express the correspondences in Eq. (10).

The scalar tetraquark nonet is given by the composition of a scalar diquark and a scalar antidiquark, resulting in the following diquark-current:

$$\mathcal{S}_{ij}^{[4q]} = \varphi_i^\dagger \varphi_j, \quad (12)$$

$$\mathcal{S}^{[4q]} = \frac{1}{2} \begin{pmatrix} [\bar{d}, \bar{s}][d, s] & -[\bar{d}, \bar{s}][u, s] & [\bar{d}, \bar{s}][u, d] \\ -[\bar{u}, \bar{s}][d, s] & [\bar{u}, \bar{s}][u, s] & -[\bar{u}, \bar{s}][u, d] \\ [\bar{u}, \bar{d}][d, s] & -[\bar{u}, \bar{d}][u, s] & [\bar{u}, \bar{d}][u, d] \end{pmatrix} \quad (13)$$

$$= \begin{pmatrix} \sqrt{\frac{1}{2}}(f_B[4q] - a_0^0[4q]) & -a_0^+[4q] & k^+[4q] \\ -a_0^-[4q] & \sqrt{\frac{1}{2}}(f_B[4q] + a_0^0[4q]) & -k^0[4q] \\ k^-[4q] & -\bar{k}^0[4q] & \sigma_B[4q] \end{pmatrix}, \quad (14)$$

where in Eq. (14) we explicitly introduced the tetraquark fields. In particular, the states $\sigma_B[4q] = \frac{1}{2}[u, d][\bar{u}, \bar{d}]$ and $f_B[4q] = \frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] + [d, s][\bar{d}, \bar{s}])$ refer to *bare* (unmixed) tetraquark scalar-isoscalar states.

In Ref. [18] the $SU_V(3)$, C and P invariant interaction Lagrangian describing the decay of a tetraquark meson into two pseudoscalar quarkonia mesons has been introduced as

$$\mathcal{L}_{\mathcal{S}^{[4q]}PP} = -c_1 \text{Tr}[D \mathcal{P}' D^\dagger \mathcal{P}] + c_2 \text{Tr}[D D^\dagger \mathcal{P}^2] \quad (15)$$

$$= c_1 \mathcal{S}_{ij}^{[4q]} \text{Tr}[A^j \mathcal{P}' A^i \mathcal{P}] - c_2 \mathcal{S}_{ij}^{[4q]} \text{Tr}[A^j A^i \mathcal{P}^2] \quad (16)$$

where the dominant and the subdominant terms in the large- N_c expansion are considered and correspond to the decay diagrams expressed in Figs. 1(a) and 1(b), which are proportional to c_1 and c_2 , respectively. In Eq. (15) the interaction Lagrangian is expressed in terms of the diquark matrices D and $D^\dagger$: in this way invariance under $SU_V(3)$, C , and P transformation is easily verified by using the transformation properties listed in Tables I and II. In the form (16) the tetraquark states are made explicit by using Eq. (12): the decay amplitudes for the tetraquark states into pseudoscalar mesons can be easily evaluated from Eq. (16), see Ref. [18].

where the superscript $[4q]$ refers to four-quark states and avoid confusion with the scalar-quarkonia nonet introduced previously.

In flavor components $\mathcal{S}^{[4q]}$ explicitly reads [from Eqs. (8) and (9)]

Identifying the light scalar mesons as tetraquark states means the following assignment [18]:

$$\mathcal{S}^{[4q]} \equiv \begin{pmatrix} \sqrt{\frac{1}{2}}(f_B - a_0^0(980)) & -a_0^+(980) & k^+ \\ -a_0^-(980) & \sqrt{\frac{1}{2}}(f_B + a_0^0(980)) & -k^0 \\ k^- & -\bar{k}^0 & \sigma_B \end{pmatrix}, \quad (17)$$

where $a_0[4q]$ and $k[4q]$ of Eqs. (13) and (14) are identified with the physical resonances $a_0(980)$ and $k(800)$. Then, a mixing of the isoscalar tetraquark states $\sigma_B[4q] \equiv \sigma_B$ and $f_B[4q] \equiv f_B$, leading to the physical states $f_0(600)$ and $f_0(980)$, occurs [18]. The nonet $\mathcal{S}^{[4q]}$ transforms as a usual scalar nonet under flavor, parity, and charge transformations: $\mathcal{S}^{[4q]} \rightarrow U \mathcal{S}^{[4q]} U^\dagger$ ($U \subset SU_V(3)$), $\mathcal{S}^{[4q]}$, and $(\mathcal{S}^{[4q]})^t$ respectively.

The assignment of Eq. (17), i.e. the interpretation of the light scalar states as tetraquark resonances, has some characteristics able to explain some enigmatic properties of the light scalar mesons: the almost mass degeneracy of the state $a_0(980)$ and $f_0(980)$ and the strong decay rates into $\bar{K}K$ are an immediate consequence of the quark content of such states in this scenario. Then, in the analysis of [18] the strengths of diagrams of Figs. 1(a) and 1(b) are analyzed

quantitatively: it has been found that the sizable contribution of the subdominant decay mechanism [Fig. 1(b)], resulting in the ratio $c_2/c_1 = 0.62$, improves the theoretical prediction of the important branching ratio $g_{f_0 \rightarrow \bar{K}K}^2/g_{a_0 \rightarrow \bar{K}K}^2$ [31]. In fact, at the leading order [Okubo-Zweig-Iizuka–superallowed, Fig. 1(a), $c_2 = 0$], one has $|g_{f_0 \rightarrow \bar{K}K}^2/g_{a_0 \rightarrow \bar{K}K}^2| \leq 1$, in clear contrast with the result $g_{f_0 \rightarrow \bar{K}K}^2/g_{a_0 \rightarrow \bar{K}K}^2 = 2.15 \pm 0.40$ reported in the analysis of Refs. [31,32].

In the Lagrangian (15) only flavor symmetry, and not chiral symmetry, is present. The basic question which we address in the present work is what happens when extending the symmetry group. As we shall see, we obtain mixing of the tetraquark and quarkonia scalar states. That is, the strict equivalence of Eq. (17) is not anymore valid: the physical scalar resonances below and above 1 GeV will be an admixture of four-quark and $\bar{q}q$ configurations. One crucial question is if the tetraquark content for the light scalar states below 1 GeV (and correspondingly quarkonia above 1 GeV) is the dominant one or not.

In order to see these phenomena at work, we first consider the chiral partner of the scalar diquark of Eq. (8), a necessary step in order to write down a chiral invariant interaction Lagrangian.

C. Pseudoscalar diquark

The pseudoscalar diquark is the chiral partner of the scalar diquark and is described by the diquark-matrix $\tilde{D}$ and by the currents $\tilde{\varphi}_i$:

$$\begin{aligned}\tilde{D}_{ij} &\equiv \sqrt{\frac{1}{2}}(q_j^t C q_i - q_i^t C q_j) = \sum_{i=1}^3 \tilde{\varphi}_i A^i; \\ \tilde{\varphi}_i &= \sqrt{\frac{1}{2}} \varepsilon_{ijk} q_j^t C q_k.\end{aligned}\quad (18)$$

The pseudoscalar diquark has the same flavor (and color) substructure ($\bar{3}_F, \bar{3}_C$) as the scalar diquark but negative parity. It corresponds to

$$|qq\rangle_{L=S=1} = |\text{space: } L=1\rangle |\text{spin: } S=1\rangle |\text{color: } \bar{3}_C\rangle |\text{flavor: } \bar{3}_F\rangle; \quad J^P = 0^-.\quad (19)$$

The matrix $\tilde{D}$ and the pseudoscalar diquarks $\tilde{\varphi}_i$ transform exactly as in Table II but with opposite parity.

In the chiral limit the scalar and the pseudoscalar diquarks have the same mass. However, chiral symmetry is spontaneously broken by the QCD vacuum. Calculations based on instantons show that a strong attraction is generated in the scalar channel and a strong repulsion in the pseudoscalar one [20,21]. Support for this picture is found in the recent Lattice calculation of Ref. [45], in the chiral model for diquarks of Ref. [46], in which the pseudoscalar diquark $\tilde{D}$ is about 600 MeV heavier than the scalar partner, and in the framework of Dyson-Schwinger equation [23], where the mass difference is of the same order of magnitude.

The common result of the above cited works is that the pseudoscalar diquark is loosely bound and heavier when compared to the scalar partner. Indeed, it is not clear if the pseudoscalar diquark can play the role of a constituent for hadronic states. As emphasized in Ref. [47], in the large- N_c limit only quarkonia states survive in the mesonic sector, a fact which also explains why nonquarkonia states are rare in the mesonic spectrum. The scalar diquark, being the most compact diquark state, can represent an exception and play a role in the physical world at $N_c = 3$. For all these reasons we will consider only the scalar diquark, and not the pseudoscalar diquark, as a basic and compact constituent of low-energy physical resonances. The inclusion of the pseudoscalar diquark is however a necessary intermediate step in order to write down a chiral invariant Lagrangian, see below.

D. Chiral invariant interaction Lagrangian

Out of the scalar and pseudoscalar matrices D and $\tilde{D}$ of Eqs. (8) and (18) we define the matrices D_R and D_L :

$$D_R = \sqrt{\frac{1}{2}}(\tilde{D} + D) = \sum_{i=1}^3 \varphi_i^R A^i; \quad \varphi_i^R = \sqrt{\frac{1}{2}}(\tilde{\varphi}_i + \varphi_i),\quad (20)$$

$$D_L = \sqrt{\frac{1}{2}}(\tilde{D} - D) = \sum_{i=1}^3 \varphi_i^L A^i; \quad \varphi_i^L = \sqrt{\frac{1}{2}}(\tilde{\varphi}_i - \varphi_i).\quad (21)$$

The transformation properties of the matrices D_R and D_L are summarized in Table III.

Under chiral transformations the diquark components φ_i^R transform as a right-handed antiquark, while the components φ_i^L as a left-handed antiquark:

TABLE III. Properties of the diquark matrices D_R and D_L .

	$D_R = \sum_{i=1}^3 \varphi_i^R A^i$	$D_L = \sum_{i=1}^3 \varphi_i^L A^i$
Currents	$\varphi_i^R = \varepsilon_{ijk} q_j^t C P_R q$	$\varphi_i^L = \varepsilon_{ijk} q_j^t C P_L q_k$
P	$-D_L$	$-D_R$
C	$D_R^\dagger$	$D_L^\dagger$
$SU_V(3)$	$UD_R U^t$	$UD_L U^t$
$SU_R(3) \times SU_L(3)$	$RD_R R^t$	$LD_L L^t$
$U_A(1) (q_i \rightarrow e^{i\nu\gamma_5} q_i)$	$e^{2i\nu} D_R$	$e^{-2i\nu} D_L$

$$\varphi_i^R \rightarrow \varphi_k^R R_{ki}^\dagger, \quad \varphi_i^L \rightarrow \varphi_k^L L_{ki}^\dagger \text{ under } SU_R(3) \times SU_L(3). \quad (22)$$

We are now in the position to write a chiral invariant interaction Lagrangian in terms of the diquark matrices D_R and D_L and the quarkonia nonet matrix Σ . By taking into account the transformation properties in Tables I and III the chiral invariant interaction Lagrangian at leading and sub-leading order in the large- N_c expansion reads

$$\begin{aligned} \mathcal{L}_{\text{c.i.}} = & -c_1 \text{Tr}(D_R \Sigma' D_L^\dagger \Sigma + D_L \Sigma^* D_R^\dagger \Sigma^\dagger) \\ & + c_2 \text{Tr}(D_R D_R^\dagger \Sigma^\dagger \Sigma + D_L D_L^\dagger \Sigma \Sigma^\dagger). \end{aligned} \quad (23)$$

A diquark matrix and an antidiquark matrix are coupled to two Σ 's: in both cases two quarks and two antiquarks are present. The Lagrangian (23) is also invariant under parity, charge conjugation, and $U_A(1)$ axial transformations.

The constants c_1 and c_2 are exactly those of Eq. (15). In fact, the flavor invariant Lagrangian (15) has to emerge out of the chiral invariant Lagrangian. We discuss the precise relation between Eqs. (15) and (23) in the next section.

The presence of two different diquark types leads to 4 tetraquark nonets: two scalars given by $\varphi_i^\dagger \varphi_j$ ($= \mathcal{S}^{[4q]}$) and $\tilde{\varphi}_i^\dagger \tilde{\varphi}_j$ and two pseudoscalars given by $\varphi_i^\dagger \tilde{\varphi}_j$ and $\tilde{\varphi}_i^\dagger \varphi_j$ (admixture of these nonets with definite properties under chiral transformations are found, see Appendix A).

As discussed in Sec. II C we do not consider the pseudoscalar diquark of Eq. (19) as a suitable constituent for mesonic states. For this reason, we consider only the scalar diquark as relevant constituent for low-energy spectroscopy, thus only the tetraquark nonet $\mathcal{S}^{[4q]} = \varphi_i^\dagger \varphi_j$ is taken into account. The other three nonets may eventually exist, but be heavier, and/or too broad to be measured. In the QCD spectrum below 2 GeV, one notices the presence of supernumerary scalar states, which can accommodate a nonquarkonia nonet like $\mathcal{S}^{[4q]}$ (and probably a scalar glueball), but the presence of a second nonquarkonia scalar nonet, such as the composition of two pseudoscalar diquarks $\tilde{\varphi}_i^\dagger \tilde{\varphi}_j$, seems to be excluded by present data [27].

The pseudoscalar sector is less clear: beyond the well established low-energy pseudoscalar nonet $\{\pi, K, \eta, \eta'\}$, a second nonet shows up at around 1.3 GeV: the state $\pi(1300)$ is usually interpreted as the radial excitation of the pion [1]. A kaonic state $K(1460)$ is also reported in [27]. The two isoscalar states $\eta(1295)$ and $\eta(1475)$ are usually interpreted as the excited η and η' mesons. The resonance $\eta(1405)$ is ambiguous, and various interpretations have been proposed, such as a pseudoscalar glueball, but some authors do not accept its existence [48]. Other massive pseudoscalar states such as $\pi(1800)$, $K(1830)$, $\eta(1760)$ are identified and interpreted as the second radial excitation [1] (but this assignment is not yet conclusive).

The fact that we take into account only scalar diquarks and the corresponding scalar nonet $\mathcal{S}^{[4q]} = \varphi_i^\dagger \varphi_j$ is the basic difference with Refs. [30,34–36], where a scalar

and a pseudoscalar nonet are considered (see also Appendix A). For instance, the resonance $\pi(1300)$ is mainly a four-quark state in Ref. [34]. Furthermore, the Lagrangian interaction of Refs. [30,35] breaks $U_A(1)$ invariance, while Eq. (23) does not. Here we do not evaluate the masses of quarkonia (we did not specify the potential $V_0 + V_{\text{SB}}$ in Sec. II A) and of tetraquark states, but we concentrate on their interaction. Theoretical evaluation of masses of bare states is, on the contrary, an important part of Refs. [30,34–36].

III. LIGHT TETRAQUARK STATES: MIXING WITH SCALAR QUARKONIA AND CONDENSATES

A. The “remnant” interaction Lagrangian

By isolating in the interaction Lagrangian (23) only those terms involving the scalar-diquark matrix D (and not the pseudoscalar matrix $\tilde{D}$) we obtain

$$\begin{aligned} \mathcal{L}_{\mathcal{S}^{[4q]}\Sigma\Sigma} = & \frac{c_1}{2} \text{Tr}[D \Sigma' D^\dagger \Sigma + D \Sigma^* D^\dagger \Sigma^\dagger] \\ & + \frac{c_2}{2} \text{Tr}[D D^\dagger \Sigma^\dagger \Sigma + D D^\dagger \Sigma \Sigma^\dagger] \end{aligned} \quad (24)$$

$$\begin{aligned} = & -\frac{c_1}{2} \mathcal{S}_{ij}^{[4q]} \text{Tr}[A^j \Sigma' A^i \Sigma + D \Sigma^* D^\dagger \Sigma^\dagger] \\ & -\frac{c_2}{2} \mathcal{S}_{ij}^{[4q]} \text{Tr}[A^j A^i \Sigma^\dagger \Sigma + A^j A^i \Sigma \Sigma^\dagger], \end{aligned} \quad (25)$$

where in the last line the expression is explicitly presented in terms of the tetraquark scalar nonet $\mathcal{S}^{[4q]}$ defined in Eq. (12).

For completeness we report the total Lagrangian under consideration. It is the sum of the Lagrangian $\mathcal{L}_\Sigma$ in Eq. (4), which involves the pseudoscalar and the scalar-quarkonia nonets, of a quadratic Lagrangian involving the kinematic and the mass terms of the scalar tetraquark nonet $\mathcal{S}^{[4q]}$ and of the quarkonia-tetraquark interaction $\mathcal{L}_{\mathcal{S}^{[4q]}\Sigma\Sigma}$ of Eq. (24):

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_\Sigma + \mathcal{L}_{\mathcal{S}^{[4q]}\text{-quadratic}} + \mathcal{L}_{\mathcal{S}^{[4q]}\Sigma\Sigma}. \quad (26)$$

The term $\mathcal{L}_{\mathcal{S}^{[4q]}\text{-quadratic}}$ is described in Ref. [18], where the nonet mass splitting and the isoscalar mixing are taken into account. In the present work we do not need to specify it. Our attention is focused on the quarkonia-tetraquark interaction term $\mathcal{L}_{\mathcal{S}^{[4q]}\Sigma\Sigma}$.

The phenomenon of chiral symmetry breaking, encoded in the nonvanishing vev for the field Σ in Eq. (5), introduces further terms beyond the tetraquark-quarkonia decay diagrams of Fig. 1: a mixing term among the two scalar nonets $\mathcal{S}^{[4q]}$ and $\mathcal{S}$ and a linear term in $\mathcal{S}^{[4q]}$, corresponding to nonvanishing tetraquark condensates, are generated. In fact, when substituting $\Sigma = \Sigma_0 + \mathcal{S} + i\mathcal{P}$ into (24), we can decompose it into four different terms:

$$\mathcal{L}_{S^{[4q]}\Sigma\Sigma} = \mathcal{L}_{S^{[4q]}PP} + \mathcal{L}_{S^{[4q]}SS} + \mathcal{L}_{\text{mix}} + \mathcal{L}_{4q\text{-cond}}. \quad (27)$$

The Lagrangian $\mathcal{L}_{S^{[4q]}PP}$ of Eq. (15) is reobtained (with the same coupling strengths c_1 and c_2). The Lagrangian $\mathcal{L}_{S^{[4q]}SS}$ is analogous to $\mathcal{L}_{S^{[4q]}PP}$, where one has two scalar-quarkonia mesons instead of two pseudoscalar ones. We will not study the phenomenological implications of this term because in the present work the bare tetraquark states are lighter than the quarkonia states, thus such a decay is not kinematically allowed.

The term $\mathcal{L}_{\text{mix}}$ is linear in Σ_0 and describes the mixing of $S^{[4q]}$ and S , see the next subsection. The term $\mathcal{L}_{4q\text{-cond}}$ is quadratic in Σ_0 and linear in $S^{[4q]}$ and is responsible for the nonzero vacuum expectation value of the isoscalar tetraquark fields (see Sec. III C).

In Eq. (27) we obtained a decomposition of the tetraquark interaction terms by expanding Σ around Σ_0 , which is a minimum for the potential $V_0 + V_{\text{SB}}$ as discussed in Sec. II A. Care is however needed: when including the interaction term $\mathcal{L}_{S^{[4q]}\Sigma\Sigma}$ of Eq. (24) in the total Lagrangian $\mathcal{L}_{\text{tot}}$ of Eq. (26), the potential involving the Σ -matrix has been extended to $V_0 + V_{\text{SB}} - \mathcal{L}_{S^{[4q]}\Sigma\Sigma}$: the matrix Σ_0 is not anymore the minimum. Strictly speaking, one should not expand around Σ_0 as in Eq. (27) but around the new minimum of $V_0 + V_{\text{SB}} - \mathcal{L}_{S^{[4q]}\Sigma\Sigma}$. We will discuss the issue in detail in Sec. III C, where we show that in our case Σ_0 still represents a good approximation for the minimum and that the expansion of Eq. (27) is justified. We also illustrate the point by means of a simple toy model.

B. Scalar tetraquark-quarkonia mixing in the isovector sector

1. The mixing Lagrangian

The tetraquark-quarkonia mixing Lagrangian $\mathcal{L}_{\text{mix}}$ is derived from Eq. (24) by using $\Sigma = \Sigma_0 + S + i\mathcal{P}$ with $\Sigma_0 = \text{diag}\{\alpha_u, \alpha_u, \alpha_s\}$ and keeping terms linear in Σ_0 :

$$\begin{aligned} \mathcal{L}_{\text{mix}} &= c_1 \text{Tr}[D\Sigma_0 D^\dagger S + DS' D^\dagger \Sigma_0] \\ &\quad + c_2 \text{Tr}[DD^\dagger \Sigma_0 S + DD^\dagger S \Sigma_0] \\ &= -c_1 S_{ij}^{[4q]} \text{Tr}[A^j \Sigma_0 A^i S + A^j S' A^i \Sigma_0] \\ &\quad - c_2 S_{ij}^{[4q]} \text{Tr}[A^j A^i (\Sigma_0 S + S \Sigma_0)]. \end{aligned} \quad (28)$$

We depict the process corresponding to $\mathcal{L}_{\text{mix}}$ in Fig. 2, where the two diagrams resemble Figs. 1(a) and 1(b), but at one vertex the vacuum expectation matrix Σ_0 enters in the game. If Σ_0 vanishes, such terms vanish as well. It is noticeable that the decay-strength parameters c_1 and c_2 also regulate the intensity of the mixing. In Eq. (17) and in Refs. [16–18] the scalar states below 1 GeV are interpreted as pure tetraquark states. The present analysis shows that such an assignment cannot be strictly valid because mixing occurs. We aim now to evaluate the intensity of this mixing in the isovector channel.

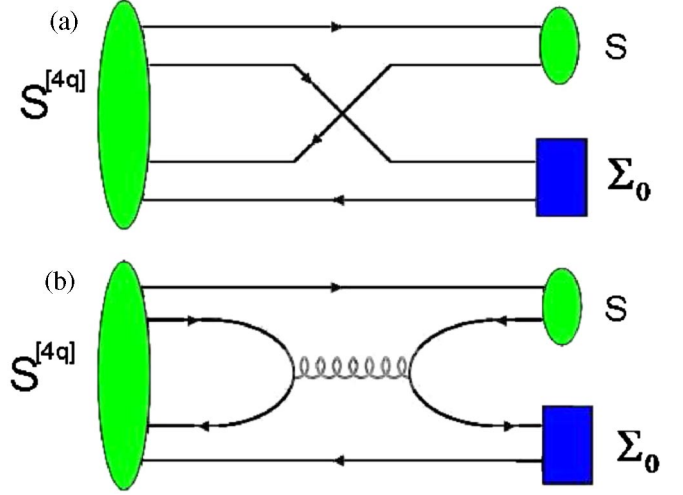


FIG. 2 (color online). Dominant (a) and subdominant (b) contributions to the tetraquark-quarkonia mixing, corresponding to Eq. (28). At one vertex the vev's for the scalar-isoscalar quarkonia fields, encoded in the matrix Σ_0 of Eq. (7), appears. As a consequence a tetraquark-quarkonia mixing is generated, whose strength is related to Σ_0 , i.e. to the pion and kaon decay constants.

An important point is the following: in Sec. II A we discussed various arguments in favor of bare quarkonia masses well above 1 GeV. At the same time in the Introduction and in Sec. II D we recalled that the scalar diquark emerges a compact light object within different approaches (one-gluon exchange [1,19], instantons [20,21], NJL model, and DSE [22,23]). The s -wave tetraquark states arising by composition of a diquark and anti-

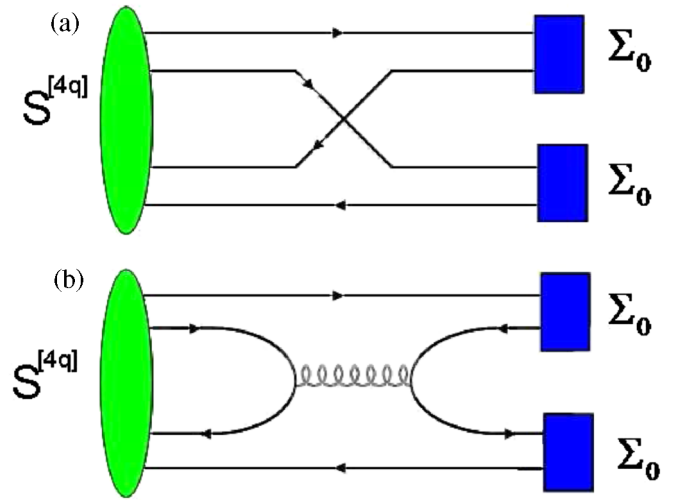


FIG. 3 (color online). Dominant (a) and subdominant (b) contributions to the linear terms in the scalar-isoscalar tetraquark fields, which generate a nonzero vacuum expectation value for the latter, see Eq. (44). At both vertices the vev's of the scalar-isoscalar quarkonia fields, encoded in the matrix Σ_0 of Eq. (7), appear.

diquark [as expressed in Eq. (12)–(14)] is expected to have a mass below (or about) 1 GeV, as discussed in Secs. II A and II B by means of phenomenological arguments and as suggested by the lattice works of Refs. [24–26]. These facts lead us to consider the bare level ordering $M_{4q} < M_{\bar{q}q}$.

2. Mixing in the isovector channel

We analyze the mixing of the two neutral a_0^0 states, denoted as $a_0^0[4q]$ [from the tetraquark nonet $\mathcal{S}^{[4q]}$ of Eq. (13) and (14)] and as $a_0^0[\bar{q}q]$ (from the quarkonia nonet $\mathcal{S}$ of Table I). The isovector channel is free from isoscalar-mixing (and glueball) complications, and is experimentally better known than the kaonic sector.

We isolate in $\mathcal{L}_{\text{mix}}$ of Eq. (28) the part concerning the neutral a_0 states:

$$\mathcal{L}_{\text{mix}-a_0^0} = 2(c_1\alpha_s + c_2\alpha_u)(a_0^0[4q] \cdot a_0^0[\bar{q}q]). \quad (29)$$

When including the kinematic and (bare) mass contributions, one has to diagonalize the following Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a_0^0[4q])^2 + \frac{1}{2}(\partial_\mu a_0^0[\bar{q}q])^2 + \frac{1}{2}\nu^\dagger \Omega \nu, \quad (30)$$

where

$$\begin{aligned} \nu &= \begin{pmatrix} a_0^0[4q] \\ a_0^0[\bar{q}q] \end{pmatrix}, \\ \Omega &= \begin{pmatrix} -M_{a_0^0[4q]}^2 & 2(c_1\alpha_s + c_2\alpha_u) \\ 2(c_1\alpha_s + c_2\alpha_u) & -M_{a_0^0[\bar{q}q]}^2 \end{pmatrix}. \end{aligned} \quad (31)$$

The orthogonal transformation matrix B , given by

$$B\Omega B^t = -\text{diag}\{M_{a_0(980)}^2, M_{a_0(1470)}^2\}, \quad (32)$$

connects the bare tetraquark and quarkonia states to the physical ones:

$$\begin{aligned} \begin{pmatrix} a_0^0(980) \\ a_0^0(1470) \end{pmatrix} &= B \cdot \begin{pmatrix} a_0^0[4q] \\ a_0^0[\bar{q}q] \end{pmatrix}, \\ B &= \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \end{aligned} \quad (33)$$

The physical masses read [27]

$$\begin{aligned} M_{a_0(980)} &= 984.7 \pm 1.2 \text{ MeV} \quad \text{and} \\ M_{a_0(1450)} &= 1474 \pm 19 \text{ MeV}. \end{aligned} \quad (34)$$

The decay rates for the decay channel $a_0(980) \rightarrow \eta\pi$ and $a_0(1450) \rightarrow \eta\pi$ are given by

$$\begin{aligned} \Gamma_{a_0(980) \rightarrow \eta\pi} &= \frac{p_{a_0(980) \rightarrow \eta\pi}}{8\pi M_{a_0(980)}^2} g_{a_0(980) \rightarrow \eta\pi}^2, \\ \Gamma_{a_0(1450) \rightarrow \eta\pi} &= \frac{p_{a_0(1450) \rightarrow \eta\pi}}{8\pi M_{a_0(1450)}^2} g_{a_0(1450) \rightarrow \eta\pi}^2, \end{aligned} \quad (35)$$

where $p_{a_0(980) \rightarrow \eta\pi}$ and $p_{a_0(1450) \rightarrow \eta\pi}$ represent the phase-

space factors and the decay amplitudes $g_{a_0(980) \rightarrow \eta\pi}$ and $g_{a_0(1450) \rightarrow \eta\pi}$ are a superposition of the tetraquark and quarkonia contributions:

$$\begin{aligned} g_{a_0(980) \rightarrow \eta\pi}^2 &= [g_{a_0[4q] \rightarrow \eta\pi} \cos(\theta) + g_{a_0[\bar{q}q] \rightarrow \eta\pi} \sin(\theta)]^2, \\ g_{a_0(1450) \rightarrow \eta\pi}^2 &= [-g_{a_0[4q] \rightarrow \eta\pi} \sin(\theta) + g_{a_0[\bar{q}q] \rightarrow \eta\pi} \cos(\theta)]^2. \end{aligned} \quad (36)$$

The amplitude $g_{a_0[4q] \rightarrow \eta\pi}$ is calculated from $\mathcal{L}_{\mathcal{S}^{[4q]}pp}$ of Eq. (15) and reads [18]

$$\begin{aligned} g_{a_0[4q] \rightarrow \eta\pi} &= \frac{2c_1}{\sqrt{3}} [\sqrt{2} \cos(\theta_P) + \sin(\theta_P)] + \sqrt{\frac{2}{3}} c_2 [\cos(\theta_P) \\ &\quad - \sqrt{2} \sin(\theta_P)], \end{aligned} \quad (37)$$

where $\theta_P = -9.95^\circ$ at tree level [10,49].

The quantity $g_{a_0[\bar{q}q] \rightarrow \eta\pi}$ depends on the Lagrangian describing the decay of scalar quarkonia into pseudoscalar mesons, which we did not specify in this work. In the following we will treat it as a free parameter, see below.

We now turn the attention to the experimental information about the coupling constants in Eq. (37). The coupling constant $g_{a_0(980) \rightarrow \eta\pi}^2$ as extracted from experimental analyses varies between 5 and 10 GeV² [50]. We then consider the following three values in the above range:³

$$g_{a_0(980) \rightarrow \eta\pi}^2 = 5, 7.5, 10 \text{ GeV}^2 \quad (38)$$

which corresponds to $\Gamma_{a_0(980) \rightarrow \eta\pi} = 65.8, 98.7, 131.6$ MeV, respectively. These values are compatible with the data reported by Particle Data Group (PDG) [27], which are however not yet precise. By using [27]

$$\begin{aligned} \Gamma_{a_0(980)} &= \Gamma_{a_0(980) \rightarrow \bar{K}K} + \Gamma_{a_0(980) \rightarrow \eta\pi} \quad \text{and} \\ \frac{\Gamma_{a_0(980) \rightarrow \bar{K}K}}{\Gamma_{a_0(980) \rightarrow \eta\pi}} &= 0.183 \pm 0.024, \end{aligned}$$

we obtain (ignoring the error on the last ratio) $\Gamma_{a_0(980) \rightarrow \eta\pi} = \Gamma_{a_0}/1.183$. In PDG [27], the value $\Gamma_{a_0(980)} = 50\text{--}100$ MeV is reported, thus implying $\Gamma_{a_0(980) \rightarrow \eta\pi}$ equal to 42.3 and 84.6 MeV, respectively. The largest value $g_{a_0(980) \rightarrow \eta\pi}^2 = 10 \text{ GeV}^2$ seems disfavored and we regard it as an upper limit.

We now turn the attention to $g_{a_0(1450) \rightarrow \eta\pi}^2$. In Ref. [27] the averages for the following branching ratios are reported:

³Because of the large uncertainties in the experimental analyses, we do not report in Eq. (38) a single value with corresponding errors, but three possible values in agreement with present experimental information.

$$\frac{\Gamma_{a_0(1450) \rightarrow \eta' \pi}}{\Gamma_{a_0(1450) \rightarrow \eta \pi}} = 0.35 \pm 0.16,$$

$$\frac{\Gamma_{a_0(1450) \rightarrow \bar{K} K}}{\Gamma_{a_0(1450) \rightarrow \eta \pi}} = 0.88 \pm 0.23.$$

The full width amounts to $\Gamma_{a_0(1450)} = 265 \pm 13$ MeV. The contribution of the two-pseudoscalar decays to the full width is unknown. By assuming it to be dominant, and thus that the $\omega\rho$ mode is suppressed, we obtain $\Gamma_{a_0(1450) \rightarrow \eta \pi} \simeq 119$ MeV, corresponding to $g_{a_0(1450) \rightarrow \eta \pi}^2 \simeq 10.4$ GeV². We do not include errors because we ignore the contribution of the $\omega\rho$ decay to the full width. Furthermore, the experimental result $\Gamma_{a_0(1450) \rightarrow \omega\rho} / \Gamma_{a_0(1450) \rightarrow \eta \pi} = 10.7 \pm 2.3$ reported in Ref. [51] would indicate a dominant $\omega\rho$ mode. This value is however not listed as an average or fit in [27]. We will consider the value

$$g_{a_0(1450) \rightarrow \eta \pi}^2 = 10.4 \text{ GeV}^2, \quad (39)$$

being aware that it could be smaller.

We now turn to the evaluation of the mixing angle. We consider the theoretical coupling $g_{a_0[\bar{q}q] \rightarrow \eta \pi}$ of Eq. (36) as a free parameter, thus we are left with five parameters: $\{c_1, c_2, M_{a_0^0[4q]}, M_{a_0^0[\bar{q}q]}, g_{a_0[\bar{q}q] \rightarrow \eta \pi}\}$. We fix the ratio $c_2/c_1 = 0.62$ as obtained in [18], where the light scalars are interpreted as tetraquark states. Although this choice cannot be *a priori* justified, we will then vary the ratio c_2/c_1 checking the dependence of the results on it.

We fix the remaining four parameters to the physical masses of Eq. (34), to the intermediate value $g_{a_0(980) \rightarrow \eta \pi}^2 = 7.5$ GeV² of Eq. (38), and to Eq. (39).

By using the bare level ordering $M_{a_0^0[4q]} < M_{a_0^0[\bar{q}q]}$, the parameters are determined as (values in GeV, ratio $c_2/c_1 = 0.62$ fixed)

$$\begin{aligned} |c_1| &= 0.96, & M_{a_0^0[4q]} &= 1.01, \\ M_{a_0^0[\bar{q}q]} &= 1.45, & g_{a_0[\bar{q}q] \rightarrow \eta \pi} &= 3.75. \end{aligned} \quad (40)$$

corresponding to a quarkonium amount in the resonance $a_0(980)$:

$$(\sin\theta)^2 = 4.93\%. \quad (41)$$

According to our result, the resonance $a_0(980)$ has a by far dominant tetraquark substructure and only a small quarkonium amount. Similarly, the resonance $a_0(1450)$ has a dominant quarkonium substructure with a small 4.93% tetraquark content. The mixing between the tetraquark and quarkonia states turns out to be small.

To have used $c_2/c_1 = 0.62$ from [18] is then justified *a posteriori*. Anyway, when varying the ratio c_2/c_1 and the couplings of Eqs. (38) and (39), the results show a stable behavior: the mixing turns out to be small for all reasonable parameter choices, see Appendix B.

Notice that we cannot determine the sign of the mixing angle θ and c_1 . In fact, we have no information about the sign of $g_{a_0(1450) \rightarrow \eta \pi}$ and $g_{a_0(980) \rightarrow \eta \pi}$ from experiment. For this reason, the modulus $|c_1|$ is reported in Eq. (40). If $c_1 > 0$, then $\theta > 0$ and vice versa. The two possibilities are however indistinguishable here.

3. Further discussion

Some comments are in order:

- (a) In the kaonic sector the situation is similar. For instance, the part of the Lagrangian $\mathcal{L}_{\text{mix}}$ (28) describing the $k^-[4q]-K_0^-[\bar{q}q]$ mixing reads

$$\begin{aligned} \mathcal{L}_{\text{mix-}k} &= -(2c_1\alpha_u + c_2(\alpha_u + \alpha_s)) \\ &\quad \times (k^-[4q] \cdot K_0^+[\bar{q}q]). \end{aligned} \quad (42)$$

By using the solution reported in Eq. (40) and the masses $M_{k(800)} \simeq 800$ MeV and $M_{K_0(1430)} = 1414 \pm 6$ MeV, we obtain a quarkonium amount in $k(800)$ of the order of 3%, i.e. very small. As a consequence, the state $K_0(1430)$ has a dominant quarkonium content. The corresponding bare masses of the tetraquark and quarkonia states are $M_{k[4q]} = 823$ MeV and $M_{K[\bar{q}q]} = 1400$ MeV, thus only slightly shifted from the physical masses.

Let us turn to the problematic πK -decay of the two resonances: the smallness of the mixing allows us to consider the approximate relations $g_{k(800) \rightarrow \pi K} \simeq g_{k[4q] \rightarrow \pi K} [\sqrt{3}(\sqrt{2}c_1 + c_2/\sqrt{2})]$ as evaluated in [18] and $g_{K(1430) \rightarrow \pi K} \simeq g_{K[\bar{q}q] \rightarrow \pi K}$. Using the parameters of (40), one finds $\Gamma_{k(800) \rightarrow \pi K} \simeq 130$ MeV. The uncertainty on the decay strengths c_1 and c_2 allows for a width between 100 and 220 MeV. The values in this range are smaller than the present (not yet conclusive) experimental value of about 600 MeV [27]. We refer to the discussions about the experimental caveats in Ref. [31], where it is also pointed out that quarkonium, molecular, or tetraquark interpretations all fail in reproducing the large width of $k(800)$. Meson-meson interaction governed by chiral symmetry as presented in Ref. [15] can play an important role to explain the large width of $k(800)$. It is important to stress that also the decay $K(1430) \rightarrow \pi K$, evaluated in Ref. [10], turns out to be smaller than the present data of a factor 4. This problem has been analyzed in detail in Ref. [10] and is rather model independent. Further work both on theoretical and experimental sides is clearly needed to fit all the properties of the kaonic states $k(800)$ and $K_0(1430)$ in a unified picture.

- (b) We report the mixing Lagrangian in the isoscalar sector in terms of the bare states $\sigma_B[4q] = \frac{1}{2} \times [u, d][\bar{u}, \bar{d}]$, $f_B[4q] = \frac{1}{2\sqrt{2}} ([u, s][\bar{u}, \bar{s}] + [d, s][\bar{d}, \bar{s}])$

and $N = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$, $S = \bar{s}s$:

$$\begin{aligned}\mathcal{L}_{\text{mix-iso}} = & 2(c_1\alpha_s + c_2\alpha_u)Nf_B[4q] \\ & + 2\sqrt{2}(c_1\alpha_u + c_2\alpha_s)Sf_B[4q] \\ & + 2\sqrt{2}(c_1 + c_2)\alpha_u N\sigma_B[4q].\end{aligned}\quad (43)$$

The intensity of the mixing is of the same order of magnitude of the isovector and isodoublet channel, that is small. The system is then complicated by internal mixing terms like $\sigma_B f_B$ and NS and glueball mixing, which lead to the resonances $f_0(600)$ and $f_0(980)$ below 1 GeV, and to $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ above 1 GeV. Here we do not analyze this system quantitatively (see Ref. [36] for such a study). However, the tetraquark-quarkonia mixing is small as verified in the isovector and isodoublet channels, thus it is still valid to deal with two separated tetraquark and quarkonia nonets with the scalar glueball intruding in the scalar-isoscalar quarkonia sector between 1 and 2 GeV. We therefore expect smaller mixing than in Ref. [36].

- (c) We did not take into account the momentum dependence of the theoretical amplitudes $g_{a_0[\bar{q}q] \rightarrow \eta\pi}$ and $g_{a_0[4q] \rightarrow \eta\pi}$. For instance, within a chiral perturbation theory framework the quantity $g_{a_0[\bar{q}q] \rightarrow \eta\pi}$ has a (dominant) momentum-squared p^2 dependence of the form $g_{a_0[\bar{q}q] \rightarrow \eta\pi} \propto (p^2 - M_\pi^2 - M_\eta^2)$ [10]. When including such a form in the calculation the results are similar. In general (reasonable) momentum dependence does not change the picture, see Appendix B.
- (d) One of the basic starting points of the evaluation of the mixing has been the bare level ordering $M_{a_0^0[4q]} < M_{a_0^0[\bar{q}q]}$. The reasons for this choice have been listed in Secs. II A and III B 1. Here we notice that solutions are possible also for the reversed bare level ordering $M_{a_0^0[4q]} > M_{a_0^0[\bar{q}q]}$, for which the quarkonium content is dominant in $a_0(980)$. Although the case $M_{a_0^0[4q]} > M_{a_0^0[\bar{q}q]}$ seems unlikely for the above-mentioned discussions, it cannot be ruled out.
- (e) The interaction Lagrangian $\mathcal{L}_{\text{c.i.}}$ of Eq. (23) contains the dominant and subdominant terms in large- N_c expansion. Further large- N_c suppressed terms and flavor-symmetry breaking terms were not included in the present analysis. Although they can quantitatively influence the results, they are not believed to change the qualitative picture emerging from this work.

C. Tetraquark condensates

1. Vacuum expectation values and estimation of tetraquark condensates

The term $\mathcal{L}_{4q\text{-cond}}$ of Eq. (27) is linear in $\mathcal{S}^{[4q]}$ and quadratic in Σ_0 (see Fig. 3). It explicitly reads

$$\begin{aligned}\mathcal{L}_{4q\text{-cond}} = & c_1 \text{Tr}[D\Sigma_0 D^\dagger \Sigma_0] + c_2 \text{Tr}[DD^\dagger \Sigma_0^2] \\ = & 2(c_1 + c_2)\alpha_u^2 \sigma_B[4q] \\ & + \sqrt{2}(2c_1\alpha_u\alpha_s + c_2(\alpha_u^2 + \alpha_s^2))f_B[4q],\end{aligned}\quad (44)$$

where in the second line the flavor trace has been performed and a linear dependence in the isoscalar fields $\sigma_B[4q]$ and $f_B[4q]$ is found. This implies nonzero vacuum expectation values for these two (bare) fields:

$$\begin{aligned}\langle \sigma_B[4q] \rangle = & \frac{2(c_1 + c_2)\alpha_u^2}{M_{\sigma_B[4q]}^2}, \\ \langle f_B[4q] \rangle = & \sqrt{2} \frac{2c_1\alpha_u\alpha_s + c_2(\alpha_u^2 + \alpha_s^2)}{M_{f_B[4q]}^2},\end{aligned}\quad (45)$$

where $M_{\sigma_B[4q]}$ and $M_{f_B[4q]}$ refer to the (bare) tetraquark masses of the states $\sigma_B[4q]$ and $f_B[4q]$.

The nonzero vacuum expectation value (vev) of the nonet $\mathcal{S}^{[4q]}$ (13) and (14) reads

$$\begin{aligned}\langle \mathcal{S}_{ij}^{[4q]} \rangle = & \beta_i \delta_{ij}, \quad \beta_1 = \beta_2 = \frac{\langle f_B[4q] \rangle}{\sqrt{2}}, \\ \beta_3 = & \langle \sigma_B[4q] \rangle.\end{aligned}\quad (46)$$

We can estimate the corresponding tetraquark condensate following the discussion of [34]:

$$|\langle \frac{1}{2}(\epsilon_{ikl}\bar{q}_k^t C\gamma^5 \bar{q}_l)(\epsilon_{jrm}q_r^t C\gamma^5 q_m) \rangle| \sim \Lambda_{\text{QCD}}^5 \beta_i \delta_{ij}. \quad (47)$$

The scale factor Λ_{QCD}^5 enters on dimensional ground. In virtue of the flavor content of the fields $\sigma_B[4q] = \frac{1}{2} \times [u, d][\bar{u}, \bar{d}]$ and $f_B[4q] = \frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] + [d, s][\bar{d}, \bar{s}])$ and using the parameter set of Eq. (40) together with $\Lambda_{\text{QCD}} \sim 0.25$ GeV we obtain

$$|\langle \frac{1}{2}[u, d][\bar{u}, \bar{d}] \rangle| \sim \Lambda_{\text{QCD}}^5 \beta_3 \simeq 3.1 \times 10^{-5} \text{ GeV}^6 \quad (48)$$

$$\begin{aligned}|\langle \frac{1}{2}[u, s][\bar{u}, \bar{s}] \rangle| = & |\langle \frac{1}{2}[d, s][\bar{d}, \bar{s}] \rangle| \sim \Lambda_{\text{QCD}}^5 \beta_1 \\ \simeq & 1.9 \times 10^{-5} \text{ GeV}^6,\end{aligned}\quad (49)$$

where the typical bare tetraquark masses $M_{\sigma_B[4q]} \sim 0.65$ GeV and $M_{f_B[4q]}^2 \sim 1$ GeV have been employed [18]. The precise value of the bare tetraquark masses is not relevant for our estimation. It is interesting to notice that the magnitude of the condensates is similar to [34].

2. Self-consistency problem

The tetraquark nonet $\mathcal{S}^{[4q]}$ acquires nonzero vev's $\langle \mathcal{S}_{ij}^{[4q]} \rangle = \beta_i \delta_{ij}$ [Eqs. (45) and (46)]. Then, one has to shift the nonet as $\mathcal{S}_{ij}^{[4q]} \rightarrow \mathcal{S}_{ij}^{[4q]} + \beta_i \delta_{ij}$ and substitute it back into the Lagrangian (24). In particular, when considering the mixing term of Eq. (43) the shift generates linear terms in the quarkonium scalar-isoscalar fields $N = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ and $S = \bar{s}s$:

$$\begin{aligned} \mathcal{L}_{\text{mix-iso}} \rightarrow & 2\sqrt{2}(c_1\alpha_s + c_2\alpha_u)N\beta_1 \\ & + 4\sqrt{2}(c_1\alpha_u + c_2\alpha_s)S\beta_1 \\ & + 2\sqrt{2}(c_1 + c_2)\alpha_u N\beta_3 + \dots \end{aligned} \quad (50)$$

Then, these linear terms modify the vacuum expectation values for the scalar quarkonium nonet of Eq. (5) as

$$\begin{aligned} \langle \mathcal{S}_{11} \rangle &= \langle \mathcal{S}_{22} \rangle \\ &= \alpha_u \rightarrow \alpha_u + 2 \frac{(c_1\alpha_s + c_2\alpha_u)\beta_1 + (c_1 + c_2)\alpha_u\beta_3}{M_N^2} \end{aligned} \quad (51)$$

$$\langle \mathcal{S}_{33} \rangle = \alpha_s \rightarrow \alpha_s + \frac{4\sqrt{2}(c_1\alpha_u + c_2\alpha_s)}{M_S^2} \beta_1, \quad (52)$$

where M_N and M_S refer to the bare quarkonia masses; we employ the typical values $M_N \sim 1.3$ GeV and $M_S \sim 1.6$ GeV [10].

The modification of the vacuum expectation values of the scalar-quarkonia fields acknowledges the problem mentioned in Sec. III A: the minimum Σ_0 of the potential $V_0 + V_{\text{SB}}$ is not anymore a minimum of our entire potential $V_0 + V_{\text{SB}} - \mathcal{L}_{\mathcal{S}^{[4q]}\Sigma\Sigma}$.

We have to take it into account when evaluating the values of α_u and α_s from Eq. (6). When using the modified expressions (51) and (52) in the Eq. (5), here rewritten as

$$\langle \mathcal{S}_{11} \rangle + \langle \mathcal{S}_{22} \rangle = \sqrt{2}F_\pi, \quad \langle \mathcal{S}_{11} \rangle + \langle \mathcal{S}_{33} \rangle = \sqrt{2}F_K, \quad (53)$$

the constants α_u and α_s change as follows [the parameter set of Eq. (40) and the above listed scalar masses are employed]:

$$\alpha_u = \frac{F_\pi}{\sqrt{2}} \rightarrow \frac{F_\pi}{\sqrt{2}}(1 - 0.11), \quad (54)$$

$$\alpha_s = -\frac{F_\pi}{\sqrt{2}} + \sqrt{2}F_K \rightarrow \left(-\frac{F_\pi}{\sqrt{2}} + \sqrt{2}F_K\right)(1 - 0.06). \quad (55)$$

The corrections to the vacuum expectation values are 11% and 6%, respectively, which is safely small. The results of the mixing evaluation in Sec. III B and in Appendix B are therefore confirmed. Indeed, the imprecise

knowledge of the experimental coupling constants of Eqs. (38) and (39) generates a larger uncertainty than the neglect of the vev's corrections. The smallness of the latter originates from factors of the kind $(\sqrt{2}\alpha_u/M_{\text{scalar}})^2 \sim (F_\pi/M_{\text{scalar}})^2$ in Eqs. (51) and (52), where $M_{\text{scalar}} \sim 1$ GeV refers to the typical order of magnitude for the bare scalar tetraquark and quarkonia fields.

One should then proceed iteratively, by shifting again the vev of the scalar-quarkonia fields according to Eqs. (51) and (52) and subsequently finding in the Lagrangian the new linear terms in the scalar tetraquark fields (which arise from the scalar-tetraquark mixing terms); in turn, this procedure allows one to determine the next-to-leading order correction to the tetraquark vev of Eq. (46). For instance, by using the new “alfa-values” of Eqs. (54) and (55) and calculating the next-order correction to the vev β_3 , a slight increase of 4% is found, a small fraction which does not change the results of the previous subsection about the tetraquark condensates. This result is expected because the n th correction involves factors like $(F_\pi/M_{\text{scalar}})^{2n}$, thus decreasing very fast. Naively, the minimum of the extended potential $V_0 + V_{\text{SB}} - \mathcal{L}_{\mathcal{S}^{[4q]}\Sigma\Sigma}$ is close to the minimum Σ_0 of $V_0 + V_{\text{SB}}$. In the next subsection, an explicit study of this issue by means of a simple toy model is performed avoiding complicated algebraic expressions. Notice that the iterative process is the unique way to proceed because the exact form of the potential $V_0 + V_{\text{SB}}$ is not specified.

The shift of the tetraquark nonet also induces contributions to the pseudoscalar and scalar-quarkonia masses [terms $\mathcal{L}_{\mathcal{S}^{[4q]}PP}$ and $\mathcal{L}_{\mathcal{S}^{[4q]}SS}$ in Eq. (27)]. This fact has no influence in this work because we do not evaluate the bare quarkonia and tetraquark masses.

3. Toy potential

Let us consider only the light quarks u and d : as well known, chiral symmetry invariance under $SU_R(2) \times SU_L(2)$ is fulfilled by considering $\Sigma = N\tau^0 + i\pi^i\tau^i$, where $N \equiv \bar{n}n = \sqrt{1/2}(\bar{u}u + \bar{d}d)$ is the isoscalar-quarkonium field, π^i the pseudoscalar pionic fields, and τ^i the 3 Pauli matrices (τ^0 is the 2×2 identity matrix).

In the $SU(2)$ limit, only one scalar-diquark field survives: $\varphi = \sqrt{\frac{1}{2}}\epsilon_{jk}q_j^t C\gamma^5 q_k$. The scalar-diquark matrix is given by $D = \varphi A$ where $A = i\tau^2$. Thus, we are left with only one tetraquark field: $T = \sigma_B[4q] = \frac{1}{2}[u, d][\bar{u}, \bar{d}]$.

The Lagrangian $\mathcal{L}_{\mathcal{S}^{[4q]}\Sigma\Sigma}$ in Eq. (24) reduces to the very simple form:

$$\mathcal{L}_{\mathcal{S}^{[4q]}\Sigma\Sigma} \rightarrow gT(N^2 + \vec{\pi}^2), \quad g = c_1 + c_2. \quad (56)$$

[In the $SU(2)$ case, the expressions for the dominant and subdominant terms in large- N_c expansion coincide.] For illustrative purposes, we use the usual Mexican-hat potential $V_0 = \frac{A}{4}(N^2 + \vec{\pi}^2 - F^2)^2$ and neglect V_{SB} . Thus, the toy potential of the reduced $SU(2)$ -problem reads

$$V_{\text{toy}} = \frac{\lambda}{4}(N^2 + \vec{\pi}^2 - F^2)^2 + \frac{1}{2}M_T^2 T^2 - gT(N^2 + \vec{\pi}^2), \quad (57)$$

where a mass term for the tetraquark field has been included.

The minimum of V_0 is at $\{N_0 = F, \vec{\pi} = \vec{0}\}$. By expanding around this point, i.e. shifting $N \rightarrow F + N$, the quantity $\mathcal{L}_{S^{[4q]}\Sigma\Sigma}$ in the $SU(2)$ limit generates analogous terms to those discussed throughout this section:

$$\mathcal{L}_{S^{[4q]}\Sigma\Sigma} \rightarrow gT(N^2 + \vec{\pi}^2) = gT(N^2 + \vec{\pi}^2 + 2FN + F^2). \quad (58)$$

In fact, we recognize the tetraquark-mesons decay terms, the mixing term (whose strength amounts to $2gF$), and the linear term in the field T .

But the minimum of V_0 is not the minimum of V_{toy} . The corresponding minimum point of V_{toy} , denoted as $P_{\min} = \{N_0, T_0, \vec{\pi}^2 = \vec{0}\}$, can be analytically calculated:

$$N_0 = \frac{F}{\sqrt{1 - \frac{2g^2}{\lambda M_T^2}}} = F \left(1 + \frac{g^2}{\lambda M_T^2} + \dots \right), \quad (59)$$

$$T_0 = \frac{g}{M_T^2} N_0^2 = \frac{g}{M_T^2} \frac{F^2}{1 - \frac{2g^2}{\lambda M_T^2}} = \frac{gF^2}{M_T^2} \left(1 + \frac{2g^2}{\lambda M_T^2} + \dots \right). \quad (60)$$

When $g = 0$ we reobtain the minimum at $N_0 = F$ and $T_0 = 0$. If the term $g^2/\lambda M_T^2$ entering in the expansions is small, one simply has small corrections to the value $N_0 = F$. The toy model clarifies what kind of correction terms one evaluates in the iterative process sketched in the previous subsection leading to Eqs. (54) and (55).

The bare mass of the field N and the mixing strength can be also exactly evaluated by expanding around the minimum $P_{\min}$:

$$M_N^2 = \left(\frac{\partial^2 V_{\text{toy}}}{\partial N^2} \right)_{P=P_{\min}} = \frac{2\lambda F^2}{1 - \frac{2g^2}{\lambda M_T^2}}, \quad (61)$$

$$\left(\frac{\partial^2 V_{\text{toy}}}{\partial N \partial T} \right)_{P=P_{\min}} = -2gN_0.$$

Let us estimate the corrections. The parameter λ at first order is $\lambda \simeq M_N^2/2F^2$. This value is accurate if $(2g^2)/(\lambda M_T^2) \simeq (4g^2 F^2)/(M_T^2 M_N^2) \ll 1$. The condition is satisfied in our case. In fact, using the typical values $g \sim 1.5$ GeV [as in Eq. (40)] $F \sim F_\pi$, $M_T \sim 0.65$ GeV and $M_N \sim 1.3$ GeV, one has $(4g^2 F_\pi^2)/(M_T^2 M_N^2) \simeq 0.1$; we also see the appearance of a factor like $(F_\pi/M_{\text{scalar}})^2$ in the Taylor expansion, as already discussed in Sec. III B.

IV. SUMMARY AND CONCLUSIONS

This work aimed to study the implications of Jaffe's tetraquark states as a necessary component to correctly interpret the scalar low-energy QCD sector. We summarize the relevant points.

- (a) The scalar and the pseudoscalar quarkonia nonets are introduced in the usual fashion. We did not specify the potential for these fields, but we solely assumed chiral symmetry breaking to occur, thus nonvanishing vev's for the isoscalar quarkonia fields, in turn related to the pion and kaon decay constants F_π and F_K , are generated. The bare scalar-quarkonia masses are set above 1 GeV (in accord with the Lattice study of Refs. [26,41,45]), where the other p -wave nonets of axial-vector and tensor mesons lie.
- (b) The scalar diquark in the flavor ($\bar{3}_F$) and color ($\bar{3}_C$) antitriplet configurations is a compact and stable object, thus a good candidate for the basic building block of the light scalar mesons, which naturally emerge as a tetraquark scalar nonet. This assignment is in agreement with the mass degeneracy of $a_0(980)$ and $f_0(980)$, their large $\bar{K}K$ decay strengths, and their nonquarkonia behavior for large- N_c analysis. These facts, together with point (a), support the bare level ordering $M_{4q} < M_{\bar{q}q}$.
- (c) In a chiral framework the ($\bar{3}_F, \bar{3}_C$) pseudoscalar diquark is introduced as the chiral partner of the scalar diquark. Chiral symmetry breaking driven by instantons predicts a strong attraction in the scalar channel and a repulsion in the pseudoscalar one. This fact makes the pseudoscalar diquark heavier and loosely bound, thus we do not consider it as a relevant constituent for the light meson spectroscopy. For instance, an extra nonquarkonia nonet built out two-pseudoscalar diquark is not seen in the spectrum below 2 GeV.
- (d) A tetraquark-quarkonia interaction Lagrangian invariant under $SU_R(3) \times SU_L(3) \times U_A(1)$ is written down at the leading and subleading order in the large- N_c expansion. Both scalar and pseudoscalar diquark constituents enter in its expression. Then, in virtue of point (c) only the scalar diquark and the corresponding tetraquark nonet are taken into account.
- (e) The χ SB of point (a) generates a mixing term among the scalar quarkonia and tetraquark nonet. The corresponding mixing strengths are a linear combination of F_π and F_K and the tetraquark decay strengths c_1 and c_2 , which parametrize the processes of Figs. 1(a) and 1(b). The mixing is then evaluated in the isovector channel: $a_0(980)$ is mainly a Jaffe's tetraquark state, with a small quarkonium amount ($\lesssim 10\%$), and $a_0(1450)$ has a dominant quarkonium content. The results are similar in the kaonic sector

and are stable under changes of the employed parameters, as long as the bare level ordering $M_{4q} < M_{\bar{q}q}$ holds.

- (f) The χ SB at a quarkonia level induces also linear terms in the isoscalar tetraquark fields, thus non-vanishing vev's for the latter emerge. They are also related to the magnitude of corresponding four-quark condensate(s), whose values have been estimated about $2-3 \times 10^{-5} \text{ GeV}^6$. As a last step, a self-consistency check about the minimum of scalar-isoscalar fields has been done and a simple toy model for the reduced $SU(2)$ problem discussed.

We found a substantial separation of the tetraquark states (below 1 GeV) and quarkonia states (between 1–2 GeV, where the scalar glueball intrudes in the isoscalar sector). The confirmation of the falsification of this scenario is an important issue of low-energy hadronic QCD. Furthermore, decays of heavy states in the charmonia region involve the scalar mesons below 2 GeV. Thus, the correct interpretation of the latter is a crucial step for the analysis of the decays of charmonia and heavy-glueball states, which according to lattice QCD are believed to show up in the mass region between 3–5 GeV [52], in turn related to the planned experimental search of PANDA at FAIR [53].

As an interesting development, the analysis of electromagnetic decay of (and into) a vector meson such as $V \rightarrow S^{[4q]}\gamma$ [54] and $S^{[4q]} \rightarrow V\gamma$ [55] within a phenomenological composite Lagrangian can constitute a useful step in disentangling the nature of the light scalar states below 1 GeV and is planned as a future work. Along the same line, possible interactions involving the experimentally well-known tensor mesons within a composite approach as in [56] can also be performed.

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APPENDIX A: NONETS AND THEIR TRANSFORMATION

Out of the introduced diquarks we can (formally) identify four nonets, with definite properties under chiral transformations. We first consider the matrix of tetraquark states T (analogous to $\hat{\Phi}$ in [35] and M' in [34]):

$$T_{ij} = \sqrt{2}\varphi_i^{L\dagger}\varphi_j^R; \quad T = T^S + iT^P, \quad (\text{A1})$$

which constitutes a scalar and a pseudoscalar nonet of tetraquark states given by the Hermitian matrices:

$$T_{ij}^S = \frac{1}{\sqrt{2}}(\tilde{\varphi}_i^\dagger \tilde{\varphi}_j - \varphi_i^\dagger \varphi_j), \quad T_{ij}^P = \frac{i}{\sqrt{2}}(\varphi_i^\dagger \tilde{\varphi}_j - \tilde{\varphi}_i^\dagger \varphi_j). \quad (\text{A2})$$

The matrices T , T^S , and T^P transform as Σ , S , and P in Table I, except for the $U_A(1)$ transformation, which now reads $T \rightarrow e^{-4i\nu}T$. In particular, for chiral transformations: $T \rightarrow LTR^\dagger$. Notice that the scalar nonet $S^{[4q]}$ of Eqs. (12) and (17) is now a part of T^S . In the chiral context the scalar nonet T^S is an admixture of both diquarks. In [30,35] the tetraquark-quarkonia mixing occurs via the chirally invariant [but not $U_A(1)$ invariant] term

$$\mathcal{L}_{\text{mix}} = e \cdot \text{Tr}[\Sigma T^\dagger + T \Sigma^\dagger], \quad (\text{A3})$$

where e is a free parameter.

Other two tetraquark meson nonets can be formed:

$$T_{ij}^R = \sqrt{2}\varphi_i^{R\dagger}\varphi_j^R, \quad T_{ij}^L = \sqrt{2}\varphi_i^{L\dagger}\varphi_j^L \quad (\text{A4})$$

which under chiral transformations transform as $T^R \rightarrow RT^R R^\dagger$ and $T^L \rightarrow LT^L L^\dagger$, i.e. such as quark's left and right currents, connected to vector and axial-vector mesons. In the present context we still deal with scalar and pseudoscalar tetraquark states, which we denote as Π^S and Π^P :

$$\Pi^S = \frac{1}{2}(T^R + T^L); \quad \Pi_{ij}^S = \frac{1}{\sqrt{2}}(\tilde{\varphi}_i^\dagger \tilde{\varphi}_j + \varphi_i^\dagger \varphi_j) \quad (\text{A5})$$

$$\Pi^P = \frac{1}{2}(T^R - T^L); \quad \Pi_{ij}^P = \frac{1}{\sqrt{2}}(\varphi_i^\dagger \tilde{\varphi}_j + \tilde{\varphi}_i^\dagger \varphi_j). \quad (\text{A6})$$

The scalar and pseudoscalar nonets Π^S and Π^P transform as vector and axial-vector under chiral transformation. Out of a quark and an antiquark such scalar and pseudoscalar objects do not exist because they vanish identically (direct product $P_R \cdot P_L = 0$ in the expression for the currents). They are however possible for tetraquark states: four nonets can be then formed.

After chiral symmetry breaking at a diquark level occurs, there is no reason that the physical nonets are those listed in the present Appendix. The scalar nonets T^S and Π^S can mix and split. This fact resembles the flavor wave functions of the vector mesons ω and ϕ , where the quark mass splitting generates a separation of (u, d) and s quark dynamics. We assumed that the splitting is large enough to generate two separated nonets of scalar and pseudoscalar diquark constituents:

$$T^S, \Pi^S \rightarrow S^{[4q]} = \varphi_i^\dagger \varphi_j, \tilde{\varphi}_i^\dagger \tilde{\varphi}_j. \quad (\text{A7})$$

The scalar nonet $\tilde{\varphi}_i^\dagger \tilde{\varphi}_j$ does not show up in the spectrum below 2 GeV. It could be heavier, too broad, or simply not realized in nature. Here we simply concentrated on $S^{[4q]}$. A more quantitative analysis of the splitting of scalar and pseudoscalar diquarks would represent an interesting subject on its own.

APPENDIX B: RESULTS FOR PARAMETER VARIATION

We evaluate the quarkonium amount in the resonance $a_0(980)$, represented by the quantity $\sin^2\theta$, for different choices of the parameters. We consider all three values for $g_{a_0(980)\rightarrow\eta\pi}^2 = 5, 7.5, 10 \text{ GeV}^2$ listed in Eq. (38). We first employ $g_{a_0(1450)\rightarrow\eta\pi}^2 = 10.4 \text{ GeV}^2$ (Table IV) and we consider different values for the ratio c_2/c_1 . (The value $c_2/c_1 = 1$ would imply large- N_c violation. Here it is used to show the stability of the results under changes of this ratio.)

Notice that the dependence on the ratio c_2/c_1 is extremely weak.

As stressed in Sec. III B 2, the value for $g_{a_0(1450)\rightarrow\eta\pi}^2$ can be smaller than 10.4 GeV^2 . We evaluate $\sin^2\theta$ for $g_{a_0(1450)\rightarrow\eta\pi}^2 = 5.2 \text{ GeV}^2$ (Table V). This value corresponds to a width 4 times smaller: $\Gamma_{a_0(1450)\rightarrow\eta\pi} \simeq 119/4 = 29.75 \text{ MeV}$, probably too small. It can be regarded as a lower limit.

The previous results are confirmed.

As a last step we include a possible momentum dependence for the quarkonium coupling constant: we use the dominant term of Ref. [10] obtained in the framework of a chiral Lagrangian: $g_{a_0[\bar{q}q]\rightarrow\eta\pi} = \gamma(p^2 - M_\pi^2 - M_\eta^2)$, where γ is a constant involving the pseudoscalar angle θ_P and a nonet decay strength. Strictly speaking, one should consistently also include a running coupling constant for the four-quark amplitude $g_{a_0[4q]\rightarrow\eta\pi}$. This operation goes beyond the goal of this work. The present aim is to show the stability of the results even in the presence of an explicit momentum dependence of the amplitude $g_{a_0[\bar{q}q]\rightarrow\eta\pi}$ (Table VI).

The results point to a slightly larger quarkonium content, which is however still smaller than 11%. Furthermore, this

TABLE IV. $\sin^2\theta$ when varying $g_{a_0(980)\rightarrow\eta\pi}^2$ and $\frac{c_2}{c_1}$ ($g_{a_0(1450)\rightarrow\eta\pi}^2 = 10.4 \text{ GeV}^2$).

$g_{a_0(980)\rightarrow\eta\pi}^2 (\text{GeV}^2)$	$(\frac{c_2}{c_1} = \frac{1}{3})$ $\sin^2\theta$	$(\frac{c_2}{c_1} = 0.62)$ $\sin^2\theta$	$(\frac{c_2}{c_1} = 1)$ $\sin^2\theta$
5	3.23%	3.22%	3.22%
7.5	4.82%	4.81%	4.80%
10	6.40%	6.39%	6.38%

TABLE V. $\sin^2\theta$ when varying $g_{a_0(980)\rightarrow\eta\pi}^2$ and $\frac{c_2}{c_1}$ ($g_{a_0(1450)\rightarrow\eta\pi}^2 = 5.2 \text{ GeV}^2$).

$g_{a_0(980)\rightarrow\eta\pi}^2 (\text{GeV}^2)$	$(\frac{c_2}{c_1} = \frac{1}{3})$ $\sin^2\theta$	$(\frac{c_2}{c_1} = 0.62)$ $\sin^2\theta$	$(\frac{c_2}{c_1} = 1)$ $\sin^2\theta$
5	3.79%	3.78%	3.77%
7.5	5.66%	5.65%	5.63%
10	7.51%	7.50%	7.48%

TABLE VI. $\sin^2\theta$ when varying $g_{a_0(980)\rightarrow\eta\pi}^2$ and $\frac{c_2}{c_1}$ (running $g_{a_0[\bar{q}q]\rightarrow\eta\pi}$).

$g_{a_0(980)\rightarrow\eta\pi}^2 (\text{GeV}^2)$	$(\frac{c_2}{c_1} = \frac{1}{3})$ $\sin^2\theta$	$(\frac{c_2}{c_1} = 0.62)$ $\sin^2\theta$	$(\frac{c_2}{c_1} = 1)$ $\sin^2\theta$
5	4.97%	4.96%	4.94%
7.5	7.68%	7.66%	7.64%
10	10.56%	10.53%	10.51%

value is realized for $g_{a_0(980)\rightarrow\eta\pi}^2 = 10 \text{ GeV}^2$, which, as discussed in Sec. III B 2, can be considered as an upper limit.

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Decay of light scalar mesons into vector-photon and into pseudoscalar mesons

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Abstract

The decays of a light scalar meson into a vector meson and a photon ($S \rightarrow V\gamma$) are evaluated in the tetraquark and quarkonium assignments of the scalar states. A link with the radiative decays $\phi \rightarrow S\gamma$ is established: experimental results for $S \rightarrow V\gamma$ will make it possible to decide if the direct, quark-loop contribution to $\phi \rightarrow S\gamma$ or the kaon-loop contribution is dominant. Also strong decays $S \rightarrow PP$ – where P denotes a pseudoscalar meson – are investigated: within our scheme, the tetraquark assignment works better than the quarkonium one.

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1. Introduction

The nature of the light scalar states $S = \{\sigma \equiv f_0(600), k \equiv K_0^*(800), f_0 \equiv f_0(980), a_0 \equiv a_0(980)\}$ is a long-standing puzzle of low-energy QCD. Studies of strong decays $S \rightarrow PP$, where P represents a pseudoscalar meson $\{\pi, K, \eta\}$, have been performed by many authors (see Refs. [1–3] and references therein). Due to the large width of σ and k and some enigmatic fea-

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tures of f_0 and a_0 mesons – mainly due to the nearby kaon–antikaon threshold – a consensus about the dominant substructure of the light scalars was not achieved: tetraquark, quarkonia and molecules are the prominent scenarios. The radiative decay $\phi \rightarrow S\gamma$, where $S = f_0, a_0$, analyzed experimentally by the SND and KLOE Collaborations [4–7] and theoretically by many authors [8–11], has also initiated a debate about the nature of the scalar states; also in this case, however, a clear signature in favour of one of the mentioned interpretations of the scalar states could not be found.

Recently, theoretical studies were focused on the new class of the reactions $S \rightarrow V\gamma$, where $S = f_0 \equiv f_0(980)$, $a_0 \equiv a_0(980)$ and $V = \rho, \omega$ [9,12–16]: in Ref. [12] a general description of the phenomenology of these decays has been presented, in Ref. [9] a general approach to study $S \rightarrow V\gamma$ valid for both tetraquark/quarkonia interpretations of scalar mesons has been described in the framework of $SU(3)$ flavor symmetry, in Refs. [13–15] the importance of meson loops has been studied in different hadronic approaches and in Ref. [16] the processes are evaluated in the framework of a quark-level Nambu–Jona-Lasinio model. On the experimental side effort is on-going at Wasa@Cosy to determine the corresponding decay widths [17]. The goal is to find new and clear signatures which can help to identify the nature of the scalar resonances.

The aim of this work is to (i) review strong decays $S \rightarrow PP$ and (ii) calculate $S \rightarrow V\gamma$ in the tetraquark ($\bar{q}^2 q^2$) and quarkonium ($\bar{q}q$) pictures of the light scalar mesons. The different nature of tetraquark and quark–antiquark states corresponds to distinct large- N_c dominant interaction Lagrangians, and thus to different Clebsch–Gordan coefficients in the decay amplitudes of both strong and radiative processes. Moreover, we aim to establish a link between the already performed and the future experimental work at KLOE on the reaction $\phi \rightarrow S\gamma$ [6,7] and the on-going analysis at Cosy, where complementary processes involving the f_0 and a_0 mesons are studied.

For what concerns strong decays $S \rightarrow PP$, we test the $\bar{q}^2 q^2$ and $\bar{q}q$ pictures by using the decay amplitudes of f_0 and a_0 measured in Refs. [18,19]. We then evaluate the decay widths of the other nonet members σ and k . Quite remarkably, the tetraquark assignment works better than the quarkonium one: in fact, large decay widths for σ and k are found, while in the quarkonium assignment small – and thus unrealistic – values for the widths are obtained. Moreover, the scalar mixing angle in the $\bar{q}^2 q^2$ assignment is compatible with zero, in agreement with the degeneracy of f_0 and a_0 being $\frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] + [d, s][\bar{d}, \bar{s}])$ and $\frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] - [d, s][\bar{d}, \bar{s}])$, respectively. (A large value of the mixing angle would inevitably spoil the mass degeneracy, which is one of the key features in favour of a tetraquark interpretation of scalars.) On the contrary, in the $\bar{q}q$ assignment the mixing angle turns out to have a sign which is opposite to the one expected from basic considerations of the axial anomaly.

For what concerns the radiative decays $S \rightarrow V\gamma$ two distinct ways have been followed in this paper:

In way (1) we use previous results [11] obtained by studying the processes, precisely measured by the SND and KLOE Collaborations, $\phi \rightarrow f_0(980)\gamma \rightarrow \pi^0\pi^0\gamma$ and $\phi \rightarrow a_0(980)\gamma \rightarrow \pi^0\eta\gamma$. The analysis of Ref. [11] was based on the assumption that the direct (i.e. quark-loop driven) decay mechanism of Fig. 1(a) (via derivative coupling of scalar-to-pseudoscalar mesons) dominates; see the next section for details. The results for $S \rightarrow V\gamma$ turn out to be large as compared to other works [12,9,13–16] and seemingly unlikely (especially in the $\bar{q}q$ assignment). In particular, the decay $\rho \rightarrow \sigma\gamma$ turns out to be sizably larger than the corresponding experimental value [20,21]. Also, using (Sakurai’s version of) Vector Meson Dominance (VMD), the $\gamma\gamma$ decays turn out to be 2 (3) orders of magnitudes larger than the experimental results, in the tetraquark (quarkonium) scenario. For all these reasons, the scenario in which the mechanism in Fig. 1(a)

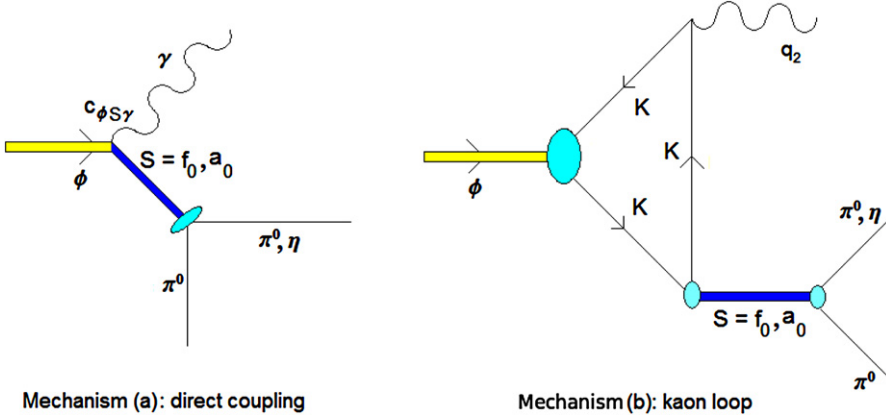


Fig. 1. Mechanism (a): direct (quark-loop) contribution. Mechanism (b): kaon-loop contribution.

dominates the radiative ϕ decay is disfavored. However, its final rejection (or, eventually, resurrection) will be possible as soon as the experimental results for $S \rightarrow V\gamma$ will be known.

In way (2) we start from $\gamma\gamma$ data of f_0 and a_0 reported in Ref. [20]. Using VMD, the decays $S \rightarrow V\gamma$ are evaluated: smaller results are obtained, in line with other studies and with the experimental value for $\rho \rightarrow \sigma\gamma$, both in the tetraquark and the quarkonium scenarios. These results for $S \rightarrow V\gamma$, if confirmed, show that the mechanism in Fig. 1(a) is negligible when studying ϕ decays, in turn meaning that the kaon-loop diagram of Fig. 1(b) dominates.

The paper is organized as follows: after a brief review of radiative ϕ decays (Section 2), we turn the attention to the results in the tetraquark (Section 3) and quarkonium scenarios (Section 4), respectively. Then, in Section 5 further discussions and conclusions are presented.

2. Recall of $\phi \rightarrow S\gamma$

2.1. General considerations

As a first step we briefly review the mechanisms of the radiative ϕ decays: the reaction $\phi \rightarrow S\gamma$, where $S = f_0, a_0$ is followed by a subsequent decay of S into two pseudoscalar mesons. In the case in which the mesons f_0 and a_0 are tetraquark or quarkonium states (i.e. correspond to preformed and preexisting, non-dynamically generated states; see the discussion in Ref. [3]) the chain $\phi \rightarrow S\gamma \rightarrow PP\gamma$ where $P = \pi, \eta$ occurs following the two mechanisms described in Fig. 1: Fig. 1(a) corresponds to a direct coupling of the ϕ meson to a photon and S , while in Fig. 1(b) ϕ couples to γ and S via a kaon-loop. If f_0, a_0 are loosely bound $\bar{K}K$ molecular states, then the diagram in Fig. 1(b) is naturally expected to be the dominant contribution.

In the case of the f_0 meson, the general interaction Lagrangian which describes both mechanisms is given by:

$$\mathcal{L}_{\text{int}, f_0} = c_{f_0\gamma} \phi_{\mu\nu} f_0 F^{\mu\nu} + c_{f_0\pi\pi} f_0 (\partial_\mu \vec{\pi})^2 + d_{f_0\pi\pi} f_0 \vec{\pi}^2 + c_{f_0KK} f_0 (\partial_\mu K^+) (\partial^\mu K^-) + d_{f_0KK} f_0 (K^+ K^-) + \dots \quad (1)$$

where dots refer to the analogous terms with the neutral kaon states. One has:

(a) The f_0 meson decays into $\pi^0\pi^0$ via the parameters $c_{f_0\pi\pi}$ and $d_{f_0\pi\pi}$. They parameterize the derivative interaction $f_0(\partial_\mu \vec{\pi})^2$ and the non-derivative interaction $f_0\vec{\pi}^2$, respectively. The

presence of these two types of interactions is the outcome of resonance chiral theory [22]¹ and also of linear models in which (axial-)vector mesons are included [23]. (An argument based on dilatation invariance which explains why couplings with higher derivatives are not present is given in Refs. [3,24].) The contributions to the line shapes of radiative ϕ -decays is different if derivative or non-derivative couplings are used, as studied in details in Refs. [11,10]; see also the following subsection.

(b) The direct coupling $c_{\phi f_0 \gamma} \phi_{\mu\nu} f_0 F^{\mu\nu}$ (where $\phi_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$ with ϕ_μ the field of the ϕ meson, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic strength tensor) induces the ϕ decay, Fig. 1(a). The present form is gauge invariant and has been derived in Ref. [9] in the context of vector meson dominance. The same form has also been used in Refs. [7,10,25].

(c) The ϕ couples to f_0 via a kaon-loop; see Fig. 1(b). Then, the f_0 subsequently decays. This mechanism, although large- N_c suppressed, has been regarded as dominant by many authors [8].

Similarly in the a_0 case: a coupling $c_{\phi a_0 \gamma} \phi_{\mu\nu} a_0 F^{\mu\nu}$ describes the point-like (quark-loop driven) coupling of the ϕ mesons to a_0 and γ , and analogous coupling constants $c_{a_0 \pi \eta}$, $d_{a_0 \pi \eta}$, $c_{a_0 K K}$, $d_{a_0 K K}$ are introduced. It is important to stress that in the considered tetraquark/quarkonium interpretation of a_0 and f_0 both mechanisms necessarily take place. It is however still unsettled which one, if any, is dominant.

On the contrary, in the molecular assignment only the parameters $d_{f_0 K K}$ and $d_{a_0 K K}$ can be taken into account as a first approximation since they represent the leading contribution (all other constants can be set to zero, in particular $c_{\phi f_0 \gamma}$). The coupling to ϕ and also the coupling to $\pi\pi$ are driven by kaon loops via a subsequent $\bar{K} K \pi \pi$ and $\bar{K} K \pi \eta$ couplings [15].

2.2. Fit to line shapes via Fig. 1(a)

In Ref. [11], by using the formalism developed in Ref. [26], only the diagram of Fig. 1(a) has been taken into account in the limit $d_{f_0 \pi \pi} = d_{f_0 K K} = d_{a_0 \pi \eta} = d_{a_0 K K} = 0$, which corresponds to the limit in which *only* derivative couplings of scalar to pseudoscalar mesons are retained [9,27]. As originally discussed in Ref. [10], derivative interactions are potentially interesting because, due to the extra dependence on the phase space, allow to describe peaked line shapes $\phi \rightarrow f_0(980)\gamma \rightarrow \pi^0 \pi^0 \gamma$ and $\phi \rightarrow a_0(980)\gamma \rightarrow \pi^0 \eta \gamma$; see details in [11].²

By fitting the theoretical curves to the experimental data one can determine – among others – the coupling constants $c_{\phi f_0 \gamma}$ and $c_{\phi a_0 \gamma}$, which parametrize the point-like coupling. A precise determination was not possible because the fits depend only marginally on the couplings to kaons $c_{f_0 K K}$ and $c_{a_0 K K}$; for this reasons fits with different values of the latter have been performed. Nevertheless, a clear and parameter-independent outcome has been obtained:

¹ In Ref. [22] the nonet of scalar states is coupled to pseudoscalar mesons following a quarkonium interpretation of the scalars; see Section 4.1 for details. However, the important point in this discussion is the use in Eq. (1) of both derivative and non-derivative interaction vertices, independently on the particular assignment of the scalar states.

² Due to spontaneous and explicit breaking of chiral symmetry one obtains that $d_{f_0 \pi \pi} \propto m_\pi^2$, $d_{f_0 K K} \propto m_K^2$, $d_{a_0 \pi \eta} \propto \frac{m_\pi^2 + m_\eta^2}{2}$ [22]. Thus, the assumption of neglecting the non-derivative couplings is solid in the pionic case but only approximate in the other channels due to the higher masses of the kaon and eta mesons. Nevertheless, we shall restrict to this approximation in this work. As commented later on in this subsection, non-derivative interactions have also been tested in the study of radiative ϕ decays and similar order of magnitude for the constants $c_{\phi f_0 \gamma}$ and $c_{\phi a_0 \gamma}$ have been obtained. What has not yet been done, is a fit of ϕ line shapes with both non-derivative and derivative interactions. However, the order of magnitude of the constants $c_{\phi f_0 \gamma}$ and $c_{\phi a_0 \gamma}$ is not expected to change abruptly. We thus believe that the arguments of the present work would also be valid in this generalized context.

$$c_{\phi f_0 \gamma} \gtrsim 0.25 \text{ GeV}^{-1}, \quad c_{\phi a_0 \gamma} \gtrsim 0.25 \text{ GeV}^{-1}. \quad (2)$$

Moreover, the strong decay amplitudes have been obtained in the experimental works of Refs. [18,19] (see also Ref. [28]): In Ref. [18] the decay amplitudes of $f_0 \equiv f_0(980)$ have been obtained from the BES data of j/ψ decays, while in Ref. [19] the decay amplitudes of $a_0 \equiv a_0(980)$ have been obtained from an analysis of the Crystall-Barrel data of $\bar{p}p$ collisions:

$$\begin{aligned} |A_{f_0 \pi \pi}^{\text{exp-BES}}| &= 2.88 \pm 0.22, & |A_{f_0 K K}^{\text{exp-BES}}| &= 5.91 \pm 0.77, \\ |A_{a_0 \pi \eta}^{\text{exp-CB}}| &= 3.33 \pm 0.15, & |A_{a_0 K K}^{\text{exp-CB}}| &= 3.59 \pm 0.44. \end{aligned} \quad (3)$$

(We adopt the following convention: if kinematically allowed, the decay width in a given channel is given by $\Gamma_{S \rightarrow P_1 P_2} = \frac{k}{8\pi M_S^2} |A_{S P_1 P_2}|^2$ where k is the three-momentum of an outgoing particle. All the symmetry and isospin factors have been absorbed in the amplitudes; see also Section 3.2.)

When using the experimental amplitudes of Eq. (3) as a further constraint on the fit on radiative ϕ decays, the following coupling constants have been found (tiny errors omitted):

$$c_{\phi f_0 \gamma} = 0.801 \text{ GeV}^{-1}, \quad c_{\phi a_0 \gamma} = 0.385 \text{ GeV}^{-1}. \quad (4)$$

Other choices for the amplitudes are possible by taking the values listed in the compilation of Refs. [29,27], where a variety of results is summarized. They would lead to slightly different values for the point-like couplings but would not change qualitatively the conclusions.

If, instead of derivative couplings of scalars to pseudoscalars, non-derivative ones are used, the coupling constants $c_{\phi f_0 \gamma}$ and $c_{\phi a_0 \gamma}$ correspond to the lower limit of Eq. (2): this study has been performed in Ref. [25]. However, as we shall describe later on, this case seems disfavored when the resulting strong decays of all scalar states are evaluated.

3. Results in the tetraquark assignment

3.1. The Lagrangian

The original discussion of tetraquark states as candidates for the light scalar mesons was presented in Ref. [30] and revisited in Ref. [31,27]. Here we introduce the tetraquark scalar nonet $S^{[4q]}$ by using the formalism of Refs. [32,33]:

$$\begin{aligned} S^{[4q]} &= \frac{1}{2} \begin{pmatrix} [\bar{d}, \bar{s}][d, s] & -[\bar{d}, \bar{s}][u, s] & [\bar{d}, \bar{s}][u, d] \\ -[\bar{u}, \bar{s}][d, s] & [\bar{u}, \bar{s}][u, s] & -[\bar{u}, \bar{s}][u, d] \\ [\bar{u}, \bar{d}][d, s] & -[\bar{u}, \bar{d}][u, s] & [\bar{u}, \bar{d}][u, d] \end{pmatrix} \\ &= \begin{pmatrix} \sqrt{\frac{1}{2}}(f_B - a_0^0) & -a_0^+ & k^+ \\ -a_0^- & \sqrt{\frac{1}{2}}(f_B + a_0^0) & -k^0 \\ k^- & -\bar{k}^0 & \sigma_B \end{pmatrix}, \end{aligned} \quad (5)$$

where the tetraquark content, in terms of diquark–antidiquark composition, has been made explicit. Note, the commutator $[\cdot, \cdot]$ indicates that the all diquarks are in the antisymmetric antitriplet $\bar{3}_F$ representation. In particular, the states $\sigma_B[4q] = \frac{1}{2}[u, d][\bar{u}, \bar{d}]$ and $f_B[4q] = \frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] + [d, s][\bar{d}, \bar{s}])$ refer to bare (unmixed) tetraquark scalar–isoscalar states. The physical states σ and $f_0 \equiv f_0(980)$ are then given by

$$\begin{pmatrix} \sigma \\ f_0(980) \end{pmatrix} = \begin{pmatrix} \cos(\varphi_S) & \sin(\varphi_S) \\ -\sin(\varphi_S) & \cos(\varphi_S) \end{pmatrix} \begin{pmatrix} \sigma_B \\ f_B \end{pmatrix}, \quad (6)$$

where φ_S is the scalar mixing angle.

The pseudoscalar nonet is described, as usual, by the matrix

$$\mathcal{P} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{2}}\eta_N & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{2}}\eta_N & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}, \quad (7)$$

where $\eta_N \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ and $\eta_S \equiv \bar{s}s$. The physical fields η and η' listed in Ref. [20] are given by:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos(\varphi_P) & \sin(\varphi_P) \\ -\sin(\varphi_P) & \cos(\varphi_P) \end{pmatrix} \begin{pmatrix} \eta_N \\ \eta_S \end{pmatrix}. \quad (8)$$

The value $\varphi_P = -36.0^\circ$ [34] is used in the following for definiteness; it lies roughly in the middle of the phenomenological range from -32° to -44° found in various studies (variation within this range does not imply any qualitative change in the following).

We now turn to the neutral vector mesons. The strength-energy tensor is defined as $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, where the diagonal matrix V_μ reads: $V_\mu = \text{diag}\{\frac{1}{\sqrt{2}}(\rho^0 + \omega_N), \frac{1}{\sqrt{2}}(-\rho^0 + \omega_N), \phi_S\}$; ρ^0 is the charge-neutral ρ meson, while the fields ω_N and ϕ_S refer to the bare, unmixed quark–antiquark configurations $\omega_N = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ and $\phi_S = \bar{s}s$. Due to (a small) mixing between the non-strange and strange sectors, the physical states ω and ϕ listed in Ref. [20] are given by:

$$\begin{pmatrix} \omega \\ \phi \end{pmatrix} = \begin{pmatrix} \cos(\varphi_V) & \sin(\varphi_V) \\ -\sin(\varphi_V) & \cos(\varphi_V) \end{pmatrix} \begin{pmatrix} \omega_N \\ \phi_S \end{pmatrix}. \quad (9)$$

The numerical value for the vector mixing angle $\varphi_V = -3.74^\circ$, which was determined in Ref. [35], is used in this work. The interaction Lagrangian involves the (derivative) scalar–pseudoscalar $SP\bar{P}$ couplings (parametrized by c_1 and c_2) and γVS couplings (parametrized by b_1 and b_2):

$$\begin{aligned} \mathcal{L}_{\bar{q}^2 q^2} = & b_1 \mathcal{S}_{ij}^{[4q]} \text{Tr}[Y^j V_{\mu\nu} Y^i Q] F^{\mu\nu} - b_2 \mathcal{S}_{ij}^{[4q]} \text{Tr}[Y^j Y^i V_{\mu\nu} Q] F^{\mu\nu} \\ & + c_1 \mathcal{S}_{ij}^{[4q]} \text{Tr}[Y^j (\partial_\mu \mathcal{P}^t) Y^i (\partial^\mu \mathcal{P})] - c_2 \mathcal{S}_{ij}^{[4q]} \text{Tr}[Y^j Y^i (\partial_\mu \mathcal{P}) (\partial^\mu \mathcal{P})], \end{aligned} \quad (10)$$

where $(Y^i)_{jk} = \varepsilon_{ijk}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic strength tensor and $Q = \text{diag}\{2/3, -1/3, -1/3\}$ the charge matrix. Note also that the Lagrangian of Eq. (1) with $d_{f_0\pi\pi} = d_{f_0KK} = 0$ is part of Eq. (10).

The trace structure is such that the terms b_1, b_2 (c_1, c_2) describe the $V\gamma$ (PP) decays according to Figs. 2(a) and 2(b), respectively. In Fig. 2(a) a rearrangement and a subsequent decay of the tetraquark state take place, while in Fig. 2(b) one has first a quark–antiquark annihilation into one (or more) gluon(s) and then the decay. Although the mechanism of Fig. 2(b) is suppressed by a factor N_c w.r.t. Fig. 2(a), it has an important role in phenomenology as discussed in Ref. [32].

3.2. Strong decays

The strong decays are parametrized by c_1 (Fig. 2(a), fall apart decay) and c_2 (Fig. 2(b)). We recall that, in virtue of the derivative interactions, the (tree-level) decay widths read:

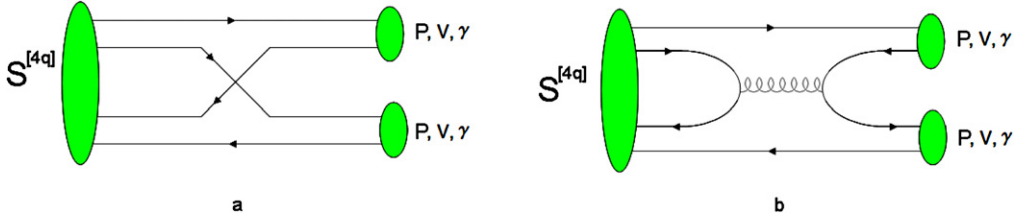


Fig. 2. Decay of a tetraquark into two pseudoscalars (PP), vector and photon ($V\gamma$), or two photons ($\gamma\gamma$) via quark rearrangement (a) and annihilation in (at least) one gluon (b).

Table 1
Relevant decay coupling constants.

$S \rightarrow P_1 P_2$	$c_{S \rightarrow P_1 P_2}$
$f_B \rightarrow \bar{K} K$	$\sqrt{2}(\sqrt{2}c_1 + \frac{3}{\sqrt{2}}c_2)$
$f_B \rightarrow \pi\pi$	$\sqrt{\frac{3}{2}} \cdot (\sqrt{2}c_2)$
$a_0 \rightarrow \bar{K} K$	$\sqrt{2} \cdot (\sqrt{2}c_1 + \frac{1}{\sqrt{2}}c_2)$
$a_0 \rightarrow \pi\eta$	$-2c_1 \sin(\varphi_P) + \sqrt{2}c_2 \cos(\varphi_P)$
$k \rightarrow \pi K$	$\sqrt{3} \cdot (\sqrt{2}c_1 + \frac{1}{\sqrt{2}}c_2)$
$\sigma_B \rightarrow \pi\pi$	$\sqrt{\frac{3}{2}} \cdot (2c_1 + 2c_2)$

$$\Gamma_{S \rightarrow P_1 P_2} = \frac{k}{8\pi M_S^2} |A_{SP_1 P_2}|^2, \quad \text{with amplitude} \quad A_{SP_1 P_2} = c_{SP_1 P_2} \frac{M_S^2 - M_{P_1}^2 - M_{P_2}^2}{2}, \quad (11)$$

where k is the three-momentum of an outgoing particle. The coefficients $c_{SP_1 P_2}$ are obtained by evaluating the traces of Eq. (10). Some relevant ones are reported in Table 1. Most notably, the term proportional to c_2 (Fig. 2(b)) has two effects: it sizably increases $f_B \rightarrow \bar{K} K$ (thus allowing for a correct description of the enhanced $f_B \rightarrow \bar{K} K$ amplitude in comparison to $a_0 \rightarrow \bar{K} K$) and it is responsible for a non-zero $f_B \rightarrow \pi\pi$ amplitude. (Note, in Ref. [36] an instanton-induced term is responsible for a non-zero $f_B \rightarrow \pi\pi$ amplitude; here such an instanton term is not needed because the mechanism of Fig. 2(b) is capable of a proper description of phenomenology. Future studies are required to understand which possibility is realized.)

We perform a fit with 3 parameters c_1 , c_2 and φ_S to the four experimental values of Eqs. (3). The minimum is found for

$$c_1 = 5.19 \pm 1.78 [\text{GeV}^{-1}], \quad c_2 = 3.84 \pm 1.80 [\text{GeV}^{-1}], \\ \varphi_S = 1.2^\circ \pm 8.0^\circ; \quad \chi^2 = 1.17. \quad (12)$$

The small value $\chi^2 = 1.17$ implies that all the amplitudes can be well reproduced. The scalar mixing angle φ_S is small and compatible with zero. This is an important fact: a large scalar mixing angle would spoil the experimentally well measured degeneracy of f_0 and a_0 .

As a result of Eq. (12), one has $\Gamma_{\sigma \rightarrow \pi\pi} = 379 \pm 52 \text{ MeV}$, $\Gamma_{k \rightarrow K\pi} = 330 \pm 82 \text{ MeV}$ for masses $M_\sigma = 0.6 \text{ GeV}$ and $M_k = 0.9 \text{ GeV}$. While the errors refer to the parameters c_1 , c_2 and φ_S only (and are thus underestimated), it is quite remarkable that large decays of σ and k can be obtained from parameters which were fitted in the f_0 and a_0 sector *only*. One obtains a qualitative

acceptable description of the full scalar nonet below 1 GeV. This is not the case in the quark–antiquark assignment; see next sections.

Two comments are still in order: (i) Derivative (used here) and non-derivative couplings lead to different results for the ratios of on-shell decays of the scalar nonet. For instance, the amplitudes based on the derivative interactions of $\sigma \rightarrow \pi\pi$ and $f_0 \rightarrow \pi\pi$ show the contributions $\frac{1}{2}(M_\sigma^2 - 2M_\pi^2)$ and $\frac{1}{2}(M_{f_0}^2 - 2M_\pi^2)$, respectively (see Eq. (11)), which are absent in the case of non-derivative couplings. In virtue of the strong dependence on the mass of the scalars, we also notice that too small decay widths would be obtained if we reduce M_σ below 0.5 GeV and M_k below 0.75 GeV.³ (ii) We limited the study to the flavor symmetric contributions depicted in Fig. 2. The role played by terms which explicitly break flavor symmetry is surely interesting and should be investigated in more refined studies. Unfortunately, the present experimental knowledge does not allow such a precision study. It is anyway interesting to notice that in the case of the experimentally well-known tensor mesons, flavor-breaking terms are subleading [37].

We conclude this subsection by noticing that the use of non-derivative couplings of scalar to pseudoscalar mesons would lead to a somewhat worse χ^2 ($\simeq 3$) and to smaller decay widths $\sigma \rightarrow \pi\pi$ and $k \rightarrow K\pi$: 157 and 137 MeV, for $M_\sigma = 0.6$ GeV and $M_k = 0.9$ GeV, respectively. Both are too small when compared to the experimental data. This result represents a further hint that couplings of derivative type play an important role in the phenomenology of scalar mesons.

3.3. Radiative decays

We first recall that the tree-level decay rates read

$$\Gamma_{S \rightarrow V\gamma} = \frac{(M_S^2 - M_V^2)^3}{8\pi M_S^3} g_{SV\gamma}^2, \quad \Gamma_{V \rightarrow S\gamma} = \frac{(M_V^2 - M_S^2)^3}{24\pi M_V^3} g_{SV\gamma}^2. \quad (13)$$

The couplings $g_{SV\gamma}$ are functions of the two coefficients b_1 and b_2 entering in Eq. (10). They are reported in Table 2 where, for transparency, the limiting case $\varphi_V = 0$ and $\varphi_S = 0$ is considered, which corresponds to the amplitudes of the bare fields ω_N , ϕ_S and σ_B , f_B . Being the mixing angles φ_V and φ_S small, these amplitudes are already close to their real values. A simple use of the relations of Eqs. (9)–(6) would allow to obtain the amplitude for the physical states. For completeness we report also the ϕ decay channels, but we stress that, also when assuming that Fig. 1(a) is dominant, the tree-level decay rates cannot be used due to the closeness to threshold and a full study of the line shapes, as for instance the one in Ref. [11], should be performed.

Independently from the precise value of the parameters, the tetraquark scenario makes the following predictions: $\Gamma_{a_0 \rightarrow \omega\gamma} \simeq \Gamma_{f_0 \rightarrow \rho\gamma}$ and $\Gamma_{a_0 \rightarrow \rho\gamma} \simeq \Gamma_{f_0 \rightarrow \omega\gamma}$. Also, the mechanism of Fig. 2(a) strongly enhances the (quark-loop driven) decay mode $\phi \rightarrow f_0\gamma$.

We now calculate the radiative decays in the two ways mentioned in the Introduction: first, by using the results of Ref. [11], which assume a dominance of the diagram of Fig. 1(a) in radiative

³ The value of the σ mass is strongly model dependent. While the PDG quotes a mass between 400–1200 MeV, recent results lie in the low-side of this broad range [38]. In particular, some recent investigations of the pole position in the complex energy plane of the σ meson provide the result $\simeq 0.45 - i250$ GeV [39]. A value of $M_\sigma = 0.45$ would generate a too narrow σ meson in our study. However, one should not forget that the value 0.45 GeV, which refers to the real part of the pole, does not necessary correspond to the peak width (or Breit–Wigner mass) of the corresponding mass distribution, which is the value of the sigma mass tacitly used in this paper when evaluating the on-shell decays. For broad states the deviations can be large, and a precise study of rescattering and unitarization effects would be necessary to this end [40]. While surely interesting, we leave this study for the future. Similar considerations hold for the k case.

Table 2

SV γ decay coupling constants in the $\bar{q}^2 q^2$ case.

ϕ_S	$g_{SV\gamma}$	ω_N	$g_{SV\gamma}$	ρ	$g_{SV\gamma}$
$\phi_S \rightarrow a_0 \gamma$	$-\frac{b_1}{\sqrt{2}}$	$a_0 \rightarrow \omega_N \gamma$	$\frac{b_2}{2}$	$a_0 \rightarrow \rho \gamma$	$\frac{2b_1+b_2}{6}$
$\phi_S \rightarrow f_B \gamma$	$-\frac{b_1+2b_2}{3\sqrt{2}}$	$f_B \rightarrow \omega_N \gamma$	$\frac{2b_1+b_2}{6}$	$f_B \rightarrow \rho \gamma$	$\frac{b_2}{2}$
$\phi_S \rightarrow \sigma_B \gamma$	0	$\omega_N \rightarrow \sigma_B \gamma$	$\frac{-b_1+b_2}{3\sqrt{2}}$	$\rho \rightarrow \sigma_B \gamma$	$\frac{b_1+b_2}{\sqrt{2}}$

Table 3

 $S \rightarrow V\gamma$ in the $\bar{q}^2 q^2$ case using Ref. [11].

Mode	Decay [keV]	Mode	Decay [keV]
$a_0 \rightarrow \rho \gamma$	535	$a_0 \rightarrow \omega \gamma$	1406
$f_0 \rightarrow \rho \gamma$	1005	$f_0 \rightarrow \omega \gamma$	463
$\rho \rightarrow \sigma \gamma$	876	$\omega \rightarrow \sigma \gamma$	20

Moreover: $\Gamma_{\phi \rightarrow \sigma \gamma} = 0.06$ keV (small).

ϕ decays, and then starting from $\gamma\gamma$ data of f_0 and a_0 . As we shall see, different results are obtained.

Way 1: We fix $\varphi_S = 1.2^\circ$ as determined by the fit to strong decays. Using Eq. (4) we determine $b_1 = 0.61 \text{ GeV}^{-1}$ and $b_2 = 1.46 \text{ GeV}^{-1}$. Note, the ratio $b_2/b_1 \sim 2$ is already problematic because it implies a dominance of the large- N_c suppressed contribution of Fig. 2(b). The results for $S \rightarrow V\gamma$ decays are summarized in Table 3.

The values in Table 3 are large when compared to the results obtained via meson-loop contributions, which are typically smaller than 20 keV [12,13,15]. The main point is that large decay widths such as those in Table 3 are possible only if the direct mechanism of Fig. 1(a) dominates.

Both decay modes $\omega \rightarrow \sigma \gamma$ and $\rho \rightarrow \sigma \gamma$ turn out to be larger than the present experimental knowledge ($\Gamma_{\rho \rightarrow \sigma \gamma} \simeq 6$ keV and $\Gamma_{\omega \rightarrow \sigma \gamma} \leq 6$ keV; see Ref. [20,21]). While this represents an argument against this scenario, one should not forget that the theoretical expressions for these decays strongly depend on the σ mass and on finite-width corrections.

Vector meson dominance can be introduced in the model by applying the shift $V_{\mu\nu} \rightarrow V_{\mu\nu} + \frac{\sqrt{2}\alpha}{g_\rho} Q F_{\mu\nu}$, where $g_\rho = 6.1$ and $\alpha \simeq 1/137$ is the fine structure constant. Although VMD cannot be used for precise calculations of on-shell decays of photons (see discussion in Ref. [41]), it is a valuable phenomenological tool to estimate the order of magnitude. The results are:

$$\Gamma_{f_0 \rightarrow \gamma \gamma} \simeq 72 \text{ keV}, \quad \Gamma_{a_0 \rightarrow \gamma \gamma} \simeq 40 \text{ keV}, \quad \Gamma_{\sigma \rightarrow \gamma \gamma} \simeq 26 \text{ keV}. \quad (14)$$

These results are sizably larger than the experimental results [20]:

$$\Gamma_{f_0 \rightarrow \gamma \gamma}^{\text{exp}} = 0.29_{-0.9}^{+0.7} \text{ keV}, \quad \Gamma_{a_0 \rightarrow \gamma \gamma}^{\text{exp}} = 0.3 \pm 0.1 \text{ keV}, \quad \Gamma_{\sigma \rightarrow \gamma \gamma}^{\text{exp}} = 0.5\text{--}4 \text{ keV}. \quad (15)$$

The present experimental evidence is against these solutions and thus the hypothesis that mechanism of Fig. 1(a) plays a dominant role in the ϕ decay.

In principle, we could also have used the results of the non-structure model of Ref. [25], in which Fig. 1(a) is also regarded as dominant, but non-derivative interactions of scalar to pseudoscalar mesons are used. The coupling constants $c_{\phi f_0 \gamma}$ and $c_{\phi a_0 \gamma}$ correspond indeed to the lower limit of Eq. (2). Smaller results (for instance, $\Gamma_{a_0 \rightarrow \omega \gamma} \simeq 77$ keV) follow, but the strong amplitudes $A_{SP_1 P_2}$ (fixed by fitting to KLOE line shapes) determined in Ref. [25] turn out to be

Table 4
 $S \rightarrow V\gamma$ in the $\bar{q}^2 q^2$ case using VMD.

Mode	Decay [keV]	Mode	Decay [keV]
$a_0 \rightarrow \rho\gamma$	4.0	$a_0 \rightarrow \omega\gamma$	4.9
$f_0 \rightarrow \rho\gamma$	3.1	$f_0 \rightarrow \omega\gamma$	3.4
$\rho \rightarrow \sigma\gamma$	5.1	$\omega \rightarrow \sigma\gamma$	0.003

Moreover: $\Gamma_{\phi \rightarrow \sigma\gamma} = 0.004$ keV (very small).

significantly smaller than Eq. (3). As a consequence, in this scheme the σ and k mesons would have a width of ~ 100 MeV (or even smaller), which is in disagreement with the data.

Way 2: We first obtain $b_1 = 0.075 \text{ GeV}^{-1}$ and $b_2 = 0.083 \text{ GeV}^{-1}$ from the experimental values for $\gamma\gamma$ decays: $\Gamma_{f_0 \rightarrow \gamma\gamma}^{\text{exp}} = 0.29_{-0.9}^{+0.7} \text{ keV}$, $\Gamma_{a_0 \rightarrow \gamma\gamma}^{\text{exp}} = 0.30 \pm 0.1 \text{ keV}$ [20] (we work under the assumptions that they are dominated by the direct, quark-loop in the tetraquark assignment). Note, in this case $b_2/b_1 \simeq 1$ in line with the ratio c_2/c_1 in the strong sector. As a first consequence a small $\Gamma_{\sigma \rightarrow \gamma\gamma} = 0.14 \text{ keV}$ (which corresponds to the direct, quark-loop decays and neglects pion loops) is found. Then, via VMD, we evaluate the $S \rightarrow V\gamma$ decay rates, which are summarized in Table 4.

In this case the predicted $\rho \rightarrow \sigma\gamma$ and $\omega \rightarrow \sigma\gamma$ are in agreement with the experiment [21]. As a further important consequence, the couplings $c_{\phi f_0\gamma} = 0.050 \text{ GeV}^{-1}$ and $c_{\phi a_0\gamma} = 0.054 \text{ GeV}^{-1}$ are determined. They are a factor of 10 *smaller* than the values of Eq. (4). It is evident that the contribution of Fig. 1(a) is reduced by a factor of 100 and is negligible in this scenario. The only possibility is that the kaon loop of Fig. 1(b) dominates the radiative decay of the ϕ mesons.

At this stage one may ask if mesons loops are not expected to sizably contribute also to $\gamma\gamma$ and to $S \rightarrow V\gamma$ processes. For a general description of this issue see Ref. [12], where the properties of kaon loops are investigated and the following scale of regimes has been outlined: for $S \rightarrow \gamma\gamma$, the quark loop (if present, i.e. if the scalar states are not loosely bound molecular objects) should be regarded as dominant. This justifies the here performed parameter by fitting $\gamma\gamma$ data to quark-driven contributions. For processes of the type $S \rightarrow V\gamma$ one rather expects a similar contribution of quark and meson loops diagrams. This implies that the results of Table 4 are roughly half of the total rate. While this may change some of the ratios, it does not change the here obtained orders of magnitude. Finally, it is interesting to notice that, based on the same arguments, in Ref. [12] it is predicted that $\phi \rightarrow \gamma S$ is dominated by kaon loops, which is in line with the results of this subsection.

4. Results in the quarkonium assignment

4.1. The Lagrangian

A quark–antiquark interpretation of the light scalars seems disfavored by the mass pattern, large N_c and various phenomenological arguments (see Refs. [2,42] and references therein; see, however, also Ref. [43]). Being not yet ruled out, it is instructive to perform the study within this assignment. The quark–antiquark nonet is encoded in the matrix

$$S^{[\bar{q}q]} = \begin{pmatrix} \bar{u}u & \bar{d}u & \bar{s}u \\ \bar{u}d & \bar{d}d & \bar{s}d \\ \bar{u}s & \bar{d}s & \bar{s}s \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{2}}\sigma_B & a_0^+ & k^+ \\ a_0^- & -\frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{2}}\sigma_B & k^0 \\ k^- & \bar{k}^0 & f_B \end{pmatrix}, \quad (16)$$

where the quark content has been made explicit. The scalar–isoscalar states read $\sigma_B = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ and $f_B = \bar{s}s$. The physical states σ and $f_0 \equiv f_0(980)$ arise via mixing:

$$\begin{pmatrix} \sigma \\ f_0(980) \end{pmatrix} = \begin{pmatrix} \cos(\varphi_S) & \sin(\varphi_S) \\ -\sin(\varphi_S) & \cos(\varphi_S) \end{pmatrix} \begin{pmatrix} \sigma_B \\ f_B \end{pmatrix}. \quad (17)$$

The Lagrangian for the full-nonet of quark–antiquark, including the non-derivative flavor symmetry breaking term, reads [22,44]:

$$\begin{aligned} \mathcal{L}_{\bar{q}q} = & c_d \text{Tr}[\mathcal{S}^{[\bar{q}q]} \partial_\mu \mathcal{P} \partial^\mu \mathcal{P}] - c_m \frac{B}{2} \text{Tr}[2\mathcal{S}^{[\bar{q}q]} \mathcal{P} M \mathcal{P} + \mathcal{S}^{[\bar{q}q]} \mathcal{P} \mathcal{P} M + \mathcal{S}^{[\bar{q}q]} M \mathcal{P} \mathcal{P}] \\ & + b \text{Tr}[\mathcal{S}^{[\bar{q}q]} V_{\mu\nu} Q] F^{\mu\nu} \end{aligned} \quad (18)$$

where $M = \text{diag}\{m_u, m_d, m_s\}$ is the diagonal matrix of current quark masses and the parameter B is related to the pion and kaon masses by $M_\pi^2 = 2Bm_u$ and $M_K^2 = B(m_u + m_s)$.

4.2. Strong decays

We proceed as in the tetraquark case by performing a fit of the three constants c_d , c_m and φ_S to the four experimental amplitudes of Eq. (3). The theoretical expressions for all the decays can be found in Ref. [44]. The global minimum of the χ^2 is found for:

$$\begin{aligned} c_d &= 8.72 \pm 0.44 \text{ GeV}^{-1}, & c_m &= 6.01 \pm 1.56 \text{ GeV}^{-1}, \\ \varphi_S &= -23.7^\circ \pm 2.3^\circ; & \chi^2 &= 0.02. \end{aligned} \quad (19)$$

Although the χ^2 is very small, and thus the amplitudes of Eq. (3) can be correctly reproduced for the above values of the parameters, one obtains as a result that $\Gamma_{\sigma \rightarrow \pi\pi} = 170 \pm 17 \text{ MeV}$, $\Gamma_{k \rightarrow K\pi} = 218 \pm 25 \text{ MeV}$. ($M_\sigma = 0.6 \text{ GeV}$ and $M_k = 0.9 \text{ GeV}$ have been used. Smaller masses would provide even smaller and unrealistic decay widths.) These decay widths are too small when compared to experiments. This is a clear drawback of the quark–antiquark assignment: the parameters obtained from f_0 and a_0 resonances do *not* allow for a description of the broad k and σ states. Note that as a result of the fit $c_m/c_d \sim 0.7$, thus sizable. A fit based only on the chiral symmetric c_d term would provide a large χ^2 .

A second more subtle but interesting drawback is the following: a negative mixing angle φ_S is a clear outcome of the fit. In the generalized Nambu–Jona-Lasinio model, which provides still one of the main reasons in favour of a $\bar{q}q$ interpretation of light scalars, the mixing in the scalar sector is driven by the 't Hooft term which solves the $U_A(1)$ anomaly problem in the pseudoscalar sector. In fact, the 't Hooft term induces the mixing of isoscalar states both in the pseudoscalar and the scalar sectors and it turns out that the mixing angles φ_P and φ_S should have opposite sign [45]. Being φ_P negative, a positive φ_S is expected for a quark–antiquark nonet (see also the discussion in Ref. [44] and references therein). Note, the same conclusion has been obtained in Ref. [46] where a generalized σ model with an anomaly term is studied. Thus, the fact that our fit provides a negative φ_S represents a further argument against a quarkonium interpretation of light scalar mesons. Indeed, a positive φ_S is the outcome of Refs. [44], where the scalar quarkonium nonet is placed above 1 GeV.

Due to the problematic results of the performed fit it is important to search for other local minima of the χ^2 . In fact, it is a priori possible that other solutions to the minimization with other values of the parameters and a larger but still acceptable value of χ^2 exist. By a careful numerical study only one additional local minimum has been found for

Table 5
SV γ decay coupling constants in the $\bar{q}q$ case.

ϕ_S	$g_{SV\gamma}$	ω_N	$g_{SV\gamma}$	ρ	$g_{SV\gamma}$
$\phi_S \rightarrow a_0\gamma$	0	$a_0 \rightarrow \omega_N\gamma$	$\frac{b}{2}$	$a_0 \rightarrow \rho\gamma$	$\frac{b}{6}$
$\phi_S \rightarrow f_0\gamma$	$-\frac{b \cos(\varphi_S)}{3}$	$f_0 \rightarrow \omega_N\gamma$	$-\frac{b \sin(\varphi_S)}{6}$	$f_0 \rightarrow \rho\gamma$	$-\frac{b \sin(\varphi_S)}{2}$
$\phi_S \rightarrow \sigma\gamma$	$-\frac{b \sin(\varphi_S)}{3}$	$\omega_N \rightarrow \sigma\gamma$	$\frac{b \cos(\varphi_S)}{6}$	$\rho \rightarrow \sigma\gamma$	$\frac{b \cos(\varphi_S)}{2}$

$$c_d = 8.65 \pm 0.44 \text{ GeV}^{-1}, \quad c_m = 9.66 \pm 2.02 \text{ GeV}^{-1},$$

$$\varphi_S = 22.4^\circ \pm 2.0^\circ; \quad \chi^2 = 8.37. \quad (20)$$

Although the scalar mixing angle is now positive, the corresponding χ^2 is worsened. (The reduced χ^2 reads $\chi^2_{\text{reduced}} = \chi^2/4 \simeq 2$.) Moreover, the strong decay widths $\Gamma_{\sigma \rightarrow \pi\pi} \simeq 145 \text{ MeV}$, $\Gamma_{k \rightarrow K\pi} \simeq 150 \text{ MeV}$ are even worsened w.r.t. the previous solution.

As a further and more stringent test, we perform a numerical study of the χ^2 over the whole parameter space by using a 3D grid with c_d and c_m varying from -15 to 15 with 300 steps (units GeV^{-1}) and φ_S from $-\pi/2$ to $\pi/2$ with 100 steps. We calculate for each case the values of χ^2 , $\Gamma_{\sigma \rightarrow \pi\pi}$ and $\Gamma_{k \rightarrow K\pi}$. It turns out that there are no values of the parameters for which $\chi^2 \leq 16$ (i.e., $\chi^2/4 \leq 4$) and $\Gamma_{\sigma \rightarrow \pi\pi} \geq 250 \text{ MeV}$, $\Gamma_{k \rightarrow K\pi} \geq 250 \text{ MeV}$. We have performed also two other tests: we have included in the fit two more experimental points corresponding to $\Gamma_{\sigma \rightarrow \pi\pi} = 450 \text{ MeV}$ and $\Gamma_{k \rightarrow K\pi} = 500 \text{ MeV}$ with errors of 100 MeV. Results: $\chi^2 = 14.7$, $\Gamma_{\sigma \rightarrow \pi\pi} = 185 \text{ MeV}$, $\Gamma_{k \rightarrow K\pi} = 244 \text{ MeV}$. Another strategy is to fix $\Gamma_{k \rightarrow K\pi} = 400 \text{ MeV}$ for example and obtain c_m as a function of c_d . Then we perform the fit but with the parameters c_d and φ_S . Results: $\chi^2 = 12.9$ and $\Gamma_{\sigma \rightarrow \pi\pi} = 235 \text{ MeV}$. Too small widths are clearly obtained and the χ^2 is worsened in both tests. Therefore, the conclusion that a tree-level study of strong decays of quark–antiquark states is in disagreement with experimental data seems to be upheld also when performing other numerical tests.

4.3. Radiative decays

The theoretical amplitudes for $S \rightarrow V\gamma$ decays in the $\bar{q}q$ case are reported in Table 5, where φ_S is kept free but $\varphi_V = 0$. Independently from the way (1) or (2) described later on, the following ratios are obtained: $\Gamma_{a_0 \rightarrow \omega\gamma} / \Gamma_{a_0 \rightarrow \rho\gamma} \simeq \Gamma_{f_0 \rightarrow \rho\gamma} / \Gamma_{f_0 \rightarrow \omega\gamma} \simeq \Gamma_{\rho \rightarrow \sigma\gamma} / \Gamma_{\omega \rightarrow \sigma\gamma} \simeq 9$. (Deviations from these simple ratios may occur when the physical value of the vector mixing angle $\varphi_V = -3.74^\circ$ together with particular values of the scalar mixing angle φ_S are considered; see later on for practical examples.)

Way I: When assuming the mechanism of Fig. 1(a) as dominant, a problem arises: the ratio $c_{\phi f_0\gamma} / c_{\phi a_0\gamma}$ reads

$$\frac{c_{\phi f_0\gamma}}{c_{\phi a_0\gamma}} = \frac{2 \cos(\varphi_V) \cos(\varphi_S) - \sin(\varphi_V) \sin(\varphi_S)}{3 \sin(\varphi_V)}. \quad (21)$$

Using Eq. (4) one obtains the ratio $c_{\phi f_0\gamma} / c_{\phi a_0\gamma} = 2.1$. This value cannot be reproduced for a small value of the scalar mixing angle φ_S . Indeed, a value of φ_S close to $\pm 90^\circ$ is required to explain this ratio: this is in disagreement with the result of Eq. (19) and implies an unnatural, dominant $\bar{s}s$ content for $f_0(600)$. More in details, one can determine the parameters b and φ_S by using Eq. (4): $b = -11.8 \text{ GeV}^{-1}$ and $\varphi_S = -99.9^\circ$. As a consequence of these values of

Table 6

 $S \rightarrow V\gamma$ in the $\bar{q}q$ case using Ref. [11].

Mode	Decay [MeV]	Mode	Decay [MeV]
$a_0 \rightarrow \rho\gamma$	10.4	$a_0 \rightarrow \omega\gamma$	85.6
$f_0 \rightarrow \rho\gamma$	69.88	$f_0 \rightarrow \omega\gamma$	6.73
$\rho \rightarrow \sigma\gamma$	0.41	$\omega \rightarrow \sigma\gamma$	0.15

the parameters, we determine the radiative decays (summarized in Table 6). They turn out to be too large (note the MeV scale!). For instance, the value $\Gamma_{a_0 \rightarrow \omega\gamma} \sim 85$ MeV is unrealistic and $\Gamma_{\rho \rightarrow \sigma\gamma} \sim 410$ keV is incompatible with Ref. [21].

Note that the expected ratios $\Gamma_{a_0 \rightarrow \omega\gamma} / \Gamma_{a_0 \rightarrow \rho\gamma} \simeq \Gamma_{f_0 \rightarrow \rho\gamma} / \Gamma_{f_0 \rightarrow \omega\gamma} \simeq 9$ are well verified by the results in Table 6. However, the ratio $\Gamma_{\rho \rightarrow \sigma\gamma} / \Gamma_{\omega \rightarrow \sigma\gamma}$ turns out to be 2.7, which is smaller than the expected value of 9. This is due to the particular – and unusual – value of the employed scalar mixing angle $\varphi_S = -99.9^\circ$. The theoretical ratio reads

$$\frac{\Gamma_{\rho \rightarrow \sigma\gamma}}{\Gamma_{\omega \rightarrow \sigma\gamma}} = k \left(3 \frac{\cos \varphi_S}{\cos \varphi_S \cos \varphi_V - 2 \sin \varphi_S \sin \varphi_V} \right)^2 \quad (22)$$

where k refers to the phase space correction [$k = (m_\omega/m_\rho)^3(m_\rho^2 - m_\sigma^2)^3/(m_\omega^2 - m_\sigma^2)^3$, see Eq. (13)] and is of order 1 because $m_\rho \simeq m_\omega$. When φ_V is strictly zero, the ratio reads $9k$ and holds for each value of φ_S , just as expected. However, a small but negative value of φ_V together with a large value of $\varphi_S = -99.9^\circ$ enhances the $\omega \rightarrow \sigma\gamma$ amplitude and explains the deviation from the expected value of $9k$.

As a last point we turn to the $\gamma\gamma$ decays, which are evaluated by using VMD. The results $\Gamma_{f_0 \rightarrow \gamma\gamma} \simeq 772$ keV, $\Gamma_{a_0 \rightarrow \gamma\gamma} \simeq 1767$ keV, $\Gamma_{\sigma \rightarrow \gamma\gamma} \simeq 92$ keV, which are 3 order of magnitudes larger than the values of Eq. (15), are obtained. Varying φ_S does not improve the overall situation. There is no need to discuss this scenario any further: a quarkonium scenario together with a dominant Fig. 1(a) is strongly disfavored by these results.

Way 2: Using the experimental values $\Gamma_{a_0 \rightarrow \gamma\gamma}^{\text{exp}} = 0.30$ keV and $\Gamma_{f_0 \rightarrow \gamma\gamma}^{\text{exp}} = 0.29$ keV [20], we obtain $b_1 = 0.23$ GeV $^{-1}$ and $\varphi_S = -20^\circ$. In this case the mixing angle is in agreement with the strong fit of Eq. (19) (but also in this case in disagreement with the sign expected by the axial anomaly arguments discussed previously). The corresponding quark–antiquark contribution of the σ decay reads $\Gamma_{\sigma \rightarrow \gamma\gamma} = 0.13$ keV, thus small.⁴ The $S \rightarrow V\gamma$ results are summarized in Table 7. (A deviation from the ideal ratio of 9 is now found for $f_0 \rightarrow \rho\gamma/f_0 \rightarrow \omega\gamma$, where the value of 5.4 is obtained. This, however, implies a ratio of the amplitudes equal to $\sqrt{5.4} \sim 2.3$, which is slightly shifted from the ideal value of 3 in virtue of the mixing angles $\varphi_V = -3.74^\circ$ and $\varphi_S = -20^\circ$.)

The decay widths $\rho \rightarrow \sigma\gamma$ and $\omega \rightarrow \sigma\gamma$ are in agreement with the experiment [21]. The order of magnitude is similar to the one of the values in Table 4. The coupling constants $c_{\phi f_0\gamma} = 0.072$ GeV $^{-1}$ and $c_{\phi a_0\gamma} = -0.0075$ GeV $^{-1}$ are determined, thus implying that Fig. 1(a) is negligible (compare with Eq. (4)). There is, however, a drawback: $\Gamma_{\phi \rightarrow \sigma\gamma} = 3.28$ keV, which is much larger than the experimental value $\Gamma_{\phi \rightarrow \sigma\gamma}^{\text{exp}} \lesssim 0.6$ keV obtained in Ref. [6], and thus

⁴ A small quark-loop contribution of $\sigma \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ into $\gamma\gamma$ is also the outcome of Ref. [47]. Namely, the finite dimension, which is a necessary property of a bound state such as the quarkonium one, is responsible for a smaller $\gamma\gamma$ result than what a local calculation, which neglects the finite extension of the σ field – would deliver.

Table 7
 $S \rightarrow V\gamma$ in the $\bar{q}q$ case using VMD.

Mode	Decay [keV]	Mode	Decay [keV]
$a_0 \rightarrow \rho\gamma$	4.0	$a_0 \rightarrow \omega\gamma$	33
$f_0 \rightarrow \rho\gamma$	3.3	$f_0 \rightarrow \omega\gamma$	0.61
$\rho \rightarrow \sigma\gamma$	4.7	$\omega \rightarrow \sigma\gamma$	0.53

seems to be excluded. Such a large $\Gamma_{\phi \rightarrow \sigma\gamma} = 3.28$ keV would probably produce a much more pronounced σ peak, which is, however, not present in the KLOE line shapes [7].

5. Discussions and conclusions

In this work we studied the strong decays $S \rightarrow PP$ and the radiative decays $S \rightarrow \gamma\gamma$, $S \rightarrow V\gamma$ in the tetraquark and quark–antiquark pictures of the nonet of scalar states $\{\sigma \equiv f_0(600), k \equiv K_0^*(800), f_0 \equiv f_0(980), a_0 \equiv a_0(980)\}$. The guiding principle in writing down the corresponding interaction Lagrangians has been the large- N_c expansion. In the following we briefly summarize and discuss the results.

5.1. Strong decays

Both in the tetraquark and quarkonium scenarios we made a fit of the three free parameters by using the experimental amplitudes for the $f_0(980)$ and $a_0(980)$ mesons obtained by the analyses of BES and Crystal-Barrel data in Refs. [18,19] which are reported in Eq. (3) and in the following:

$$\begin{aligned} |A_{f_0\pi\pi}^{\text{exp-BES}}| &= 2.88 \pm 0.22, & |A_{f_0KK}^{\text{exp-BES}}| &= 5.91 \pm 0.77, \\ |A_{a_0\pi\eta}^{\text{exp-CB}}| &= 3.33 \pm 0.15, & |A_{a_0KK}^{\text{exp-CB}}| &= 3.59 \pm 0.44. \end{aligned} \quad (23)$$

We notice that these amplitudes imply that the tree-level decays of the f_0 and a_0 mesons are large:

$$\Gamma_{f_0 \rightarrow \pi\pi}^{\text{tl}} = 161 \pm 25 \text{ MeV}, \quad \Gamma_{a_0 \rightarrow \pi\eta}^{\text{tl}} = 146 \pm 13 \text{ MeV}. \quad (24)$$

Without the inclusion of the $\bar{K}K$ channel, these values are already larger than the 50–100 MeV *full* widths reported by PDG [20]. However, the PDG widths refer to the peak widths, which are strongly distorted due to the nearby $\bar{K}K$ threshold. We also refer to the detailed discussion in Ref. [48] about this subject. As indeed stated also in the PDG compilation, the reported decay widths “are very model dependent”. Peak widths are about 50 MeV, but the decay widths “can be much larger”. In Ref. [29] this issue has also been carefully discussed and results of different works on the resonance f_0 and a_0 have been reviewed: in this context it is interesting to notice that most studies point to a large decay width ($\gtrsim 100$ MeV) for both resonances. For all these reasons we decided not to use the PDG values when performing the fit, but we decided to use the amplitudes reported in Refs. [18,19], which were obtained by a direct fit to experimental results.

After having clarified the reasons behind the choice of the employed experimental values, we turn to the results. The fit to the amplitudes of Eq. (23) is successful for both tetraquark and quarkonia cases. Once the parameters have been fixed, the following consequences emerge: within the tetraquark assignment a – albeit qualitative – consistent description of a full tetraquark

nonet $\{\sigma, k, f_0, a_0\}$ below 1 GeV is achieved, as we presented in Section 3.2. The scalar mixing angle is small, in agreement with the degeneracy of f_0 and a_0 being $\frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] + [d, s][\bar{d}, \bar{s}])$ and $\frac{1}{2\sqrt{2}}([u, s][\bar{u}, \bar{s}] - [d, s][\bar{d}, \bar{s}])$. The decay widths of σ and k are, as expected, large. We believe that it is quite remarkable that large decays of σ and k can be obtained from parameters which were fitted in the f_0 and a_0 sector *only*. On the contrary, such a consistent description is *not* achieved in the $\bar{q}q$ case: too small σ and k widths are found and the scalar mixing angle is in disagreement with general arguments based on the $U_A(1)$ anomaly. Baring in mind that our evaluation of strong decays is based on a tree-level study (rescattering and loop effects have been neglected) and that mixing of tetraquark/quarkonium components has not been considered, we find remarkable that the tetraquark scenario works rather successfully on a qualitative level, while the quarkonium scenario is disfavored.

As a last subject of the study of strong decays we argue that also the molecular scenario seems not to be favoured when the amplitudes of Eq. (23), and thus the large tree-level decay widths of Eq. (24), are considered. In fact, in the framework of a loosely bound kaon molecular state, small decay widths are predicted: $\Gamma_{a_0 \rightarrow \pi\eta} = 10\text{--}60$ MeV and $\Gamma_{f_0 \rightarrow \pi\pi} \simeq 10\text{--}50$ MeV [15,49–51]. Indeed, a small width ($\lesssim 50$ MeV) is a typical characteristic of a loosely bound kaonic state, as emphasized in Ref. [51]. These predictions are, however, in disagreement with Eq. (24). (Note, in the recent work Ref. [50] a small width $\Gamma_{a_0 \rightarrow \pi\eta}$ of at most 30 MeV is found.)

5.2. Radiative decays

A general outcome of $S \rightarrow V\gamma$ ratios (Tables 2 and 5) is the prediction of the following properties. Tetraquark scenario: $\Gamma_{a_0 \rightarrow \omega\gamma} \simeq \Gamma_{f_0 \rightarrow \rho\gamma}$ and $\Gamma_{a_0 \rightarrow \rho\gamma} \simeq \Gamma_{f_0 \rightarrow \omega\gamma}$. Quarkonium scenario: $\Gamma_{a_0 \rightarrow \omega\gamma} / \Gamma_{a_0 \rightarrow \rho\gamma} \simeq \Gamma_{f_0 \rightarrow \rho\gamma} / \Gamma_{f_0 \rightarrow \omega\gamma} \simeq \Gamma_{\rho \rightarrow \sigma\gamma} / \Gamma_{\omega \rightarrow \sigma\gamma} \simeq 9$. (Deviations from these ratios might emerge if the scalar mixing angle is large.)

In order to obtain numerical values, two ways have been followed. When assuming Fig. 1(a) as the dominant process in the radiative ϕ decays (denoted as way 1, which necessarily implies a preformed tetraquark or quarkonium substructure of the f_0/a_0 mesons), the results for $S \rightarrow V\gamma$ are summarized in Tables 3 and 6 in the $\bar{q}^2 q^2$ and $\bar{q}q$ cases, respectively. The $\bar{q}q$ case seems to be excluded because of the unrealistically large values of the $S \rightarrow V\gamma$ decays (see Table 6). The $\bar{q}^2 q^2$ case is also disfavored in view of too large rates for $\rho, \omega \rightarrow \sigma\gamma$ and the (VMD deduced) $f_0, a_0, \sigma \rightarrow \gamma\gamma$, but still not completely ruled out. Experimental results are needed: if, however, large decays will be found (such as, for instance, $\Gamma_{a_0 \rightarrow \omega\gamma} \gtrsim 50$ keV) one could infer that a compact structure is compulsory. Being the quarkonium interpretation problematic – as we have shown in the context of the assumptions described in this work – one would be left with the tetraquark interpretation as the best candidate.

When starting from $\gamma\gamma$ data of f_0 and a_0 (way 2), the results, summarized in Tables 4 and 7 for the $\bar{q}^2 q^2$ and $\bar{q}q$ cases, turn out to be of the same order of the kaon-loop based calculations [12,13,15]. The outcome is in agreement with the $\rho, \omega \rightarrow \sigma\gamma$ data. However, in this case one would need to consider both direct and meson-loop driven (not considered here) contributions to $S \rightarrow \gamma\gamma$ and $S \rightarrow V\gamma$ decays for a full and consistent analysis (the $\bar{q}q$ case is anyway disfavored because of a too large $\phi \rightarrow \sigma\gamma$ branching ratio). As described in Ref. [12] (see also the end of Section 3.3), they are expected to be small for $S \rightarrow \gamma\gamma$ but of the same order of the quark-loop contribution for $S \rightarrow V\gamma$. While meson loops may change some of the obtained ratios, they do not modify the orders of magnitude. Moreover, a model-independent conclusion can be achieved: small $S \rightarrow V\gamma$ radiative decays – if confirmed experimentally – would imply that the

ϕ decay is dominated by the kaon loop of Fig. 1(b), independently from the nature of the scalar states (in agreement with the discussion presented in Ref. [52]). This, in turn, may allow for a precise determination of the amplitudes in future updates of the KLOE experiment. It is indeed interesting to notice that the present strong amplitudes as determined by fits to the line shapes of Ref. [7] assuming the dominance of Fig. 1(b) read (in GeV):

$$\begin{aligned} |A_{f_0\pi\pi}^{\text{kl-kloe}}| &= 1.71 \pm 0.7, & |A_{f_0KK}^{\text{kl-kloe}}| &= 5.4 \pm 1.6, \\ |A_{a_0\pi\eta}^{\text{kl-kloe}}| &= 2.8 \pm 0.1, & |A_{a_0KK}^{\text{kl-kloe}}| &= 3.06 \pm 0.06, \end{aligned} \quad (25)$$

and are in rough agreement with the experimentally independent results of Eq. (3), what indeed constitutes a remarkable fact. Note, when using the KLOE amplitudes to calculate the tree-level decay widths one obtains that (i) the $f_0 \rightarrow \pi\pi$ is about 60 MeV, but the large error on the amplitude does not allow for a conclusive result, while $a_0 \rightarrow \pi\eta$ is 103 ± 10 MeV, which is also sizable and in qualitative agreement with the value reported in Eq. (24).

5.3. Concluding remarks

At first sight one may wonder if there is a conflict in the presented results: on the one hand we stated that the tetraquark scenario is favoured by strong decays once that the amplitudes in Eq. (23) are considered, on the other hand we stated that the ϕ decay is likely to occur via the kaon-loop mechanism. Is not the latter fact an hint that a molecular nature is favoured? In our view this is not the case. Within the tetraquark approach the couplings of both f_0 and a_0 mesons to kaons are large. As a consequence, a dressing via kaon-loops takes place and it can happen that some reactions are dominated by the kaon-loops rather than the corresponding quark-loop diagrams. In the radiative ϕ decays this is also due to the strong coupling of the ϕ meson itself to kaons. However, while dressing of a bare tetraquark state via kaon loops induces a kaon–antikaon amount in the wave function (e.g. in the explicit calculation in Ref. [26]), the very existence of these resonances rely on a preformed and compact tetraquark structure. This is different from what a molecular state is expected to be: a loosely bound kaon–antikaon object due to the exchange of some intermediate meson (such as the ρ meson exchange). When the attraction is smaller than a critical value, no bound state occurs. This is the molecular picture described in Refs. [49–51], in which the strong decay amplitudes are predicted to be small and are in disagreement with Eq. (3). We also refer to the discussion of Ref. [3], where the distinction of the tetraquark and molecular state has been discussed in detail.

Many effects have not been included in this work. Further studies including unitarization effects, explicit symmetry breaking and both derivative and non-derivative interactions in the tetraquark case, possible tetraquark/quarkonium mixing configurations, should be performed for a more complete treatment. Nevertheless, hints toward a dominance of a tetraquark component for the light scalar mesons have been presented and discussed, which might be helpful for future theoretical and experimental work on the subject.

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Vacuum phenomenology of the chiral partner of the nucleon in a linear sigma model with vector mesons

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We investigate a linear sigma model with global chiral $U(2)_R \times U(2)_L$ symmetry. The mesonic degrees of freedom are the standard scalar and pseudoscalar mesons and the vector and axial-vector mesons. The baryonic degrees of freedom are the nucleon, N , and its chiral partner, N^* , which is usually identified with $N(1535)$. The chiral partner is incorporated in the so-called mirror assignment, where the nucleon mass is not solely generated by the chiral condensate but also by a chirally invariant mass term, m_0 . The presence of (axial-) vector fields modifies the expressions for the axial-coupling constants of the nucleon, g_A^N , and its partner, $g_A^{N^*}$. Using experimental data for the decays $N^* \rightarrow N\pi$ and $a_1 \rightarrow \pi\gamma$, as well as lattice results for $g_A^{N^*}$ we infer that in our model $m_0 \sim 500$ MeV, i.e., an appreciable amount of the nucleon mass originates from sources other than the chiral condensate. We test our model by evaluating the decay $N^* \rightarrow N\eta$ and the s -wave nucleon-pion scattering lengths $a_0^{(\pm)}$.

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I. INTRODUCTION

The theory of the strong interaction, quantum chromodynamics (QCD), has a global chiral $U(N_f)_R \times U(N_f)_L$ symmetry, for N_f flavors of massless quarks. This symmetry is spontaneously broken in the vacuum, which has important consequences for hadron phenomenology. Because of confinement of color charges, all low-energy hadronic properties, such as masses, decay widths, scattering lengths, etc. cannot be inferred from perturbative QCD calculations. Therefore, effective chiral models are widely used in order to study the vacuum properties of hadrons. Viable candidates should obey a well-defined set of low-energy theorems [1–3], but they may still differ in some interesting aspects such as the generation of the nucleon mass and the behavior at nonzero temperature T and chemical potential μ .

A nucleon mass term $\sim m_N \bar{\Psi}\Psi$ explicitly breaks the chiral $U(N_f)_R \times U(N_f)_L$ symmetry and thus should not occur in a chiral linear sigma model. Therefore, in the standard linear sigma model of Refs. [2,4], the nucleon mass is (mostly) generated by the chiral condensate, $\langle \bar{q}q \rangle$. (A small contribution also arises from the explicit breaking of chiral symmetry due to the nonzero current quark masses.) Similarly, in the framework of QCD sum rules Ioffe [5] formulated a connection between the quark condensate and the nucleon mass, now called Ioffe formula: $m_N \sim -4\pi^2 \Lambda_B^{-2} \langle \bar{q}q \rangle$, where $\Lambda_B \simeq 1$ GeV.

However, also other condensates exist, e.g. a gluon condensate, and it is not yet known to what extent they

contribute to the nucleon mass [6]. This problem can be studied in a chiral model via the so-called mirror assignment for the chiral partner of the nucleon, which was first discussed in Ref. [4] and extensively analyzed in Refs. [7,8]. In this assignment, there exists a chirally invariant mass term $\sim m_0$ which does *not* originate from the quark condensate. The mirror assignment has been subsequently used in Ref. [9] to study the properties of cold and dense nuclear matter.

In this work we consider a linear sigma model with global chiral $U(2)_R \times U(2)_L$ symmetry which includes scalar and pseudoscalar mesons as well as vector and axial-vector mesons [10]. We extend this model by including the nucleon and its chiral partner in the mirror assignment. The most natural candidate for the chiral partner of the nucleon is the resonance $N(1535)$ which is the lightest state with the correct quantum numbers ($J^P = \frac{1}{2}^-$) listed in the PDG [11]. We also investigate two other possibilities: the well-identified resonance $N(1650)$ and a speculative, very broad, and not yet discovered resonance with mass about 1.2 GeV, which has been proposed in Ref. [9].

We first study their axial charges which have been the focus of interest in recent studies of hadron phenomenology [see Ref. [12] and refs. therein]. We show that, in the present model, including (axial-) vector mesons drastically changes the relations of the original model [7]. Without (axial-) vector mesons, N and N^* have opposite axial charge, $g_A^N = -g_A^{N^*} \leq 1$. (We remind the reader that, in the so-called “naive assignment,” where the nucleon partner transforms just as the nucleon, one has $g_A^N = g_A^{N^*} = 1$

[8]). With (axial-) vector mesons, this is no longer true and we are free to adjust the two axial charges independently, employing experimental knowledge about g_A^N and recent lattice QCD data for $g_A^{N^*}$ [13].

Using the decays $N^* \rightarrow N\pi$ and $a_1 \rightarrow \pi\gamma$ to determine the other parameters of the model, the mass parameter turns out to be $m_0 \sim 500$ MeV. This value is between the one derived in Ref. [7] and the one from Ref. [9].

We then test our model studying the decay $N^* \rightarrow N\eta$ and pion-nucleon scattering. For $N(1535)$ as chiral partner of the nucleon, the decay width $N^* \rightarrow N\eta$ comes out too small, while for $N(1650)$, it agrees well with experimental data. Pion-nucleon scattering has been studied in a large variety of approaches [see Refs. [14–17] and refs. therein]. Here, we evaluate the scattering lengths in the framework of the mirror assignment. We find that the isospin-odd s -wave scattering length $a_0^{(-)}$ is in good agreement with experimental data, while the isospin-even scattering length $a_0^{(+)}$ strongly depends on the value for the mass of the sigma meson.

Finally, we discuss two possible extensions of our work. The first is an enlarged mixing scenario. A second pair of chiral partners is added, e.g. $N(1440)$ and $N(1650)$, which also mix with $N(939)$ and $N(1535)$. The second is the generalization of the chirally invariant mass term $\sim m_0$ to a dilatation-invariant mass term. In this case, we argue that m_0 is a sum of two contributions, arising from the tetraquark and the gluon condensates, respectively. The dilatation-invariant mass term also couples a tetraquark state to the nucleon. We discuss possible implications for nuclear physics and the behavior of the nucleon mass at nonzero temperature.

This paper is organized as follows. In Sec. II we present the Lagrangian of our model and the expressions for the axial charges, the decay widths $N^* \rightarrow N\pi$ and $N^* \rightarrow N\eta$, and the s -wave scattering lengths. Section III contains our results. In Sec. IV, we present a short summary of our work and discuss the two possible extensions mentioned above, i.e., the enlarged mixing scenario and the dilatation-invariant mass term. Details of our calculations are relegated to the appendices.

Our units are $\hbar = c = 1$, the metric tensor is $g^{\mu\nu} = \text{diag}(+, -, -, -)$.

II. THE MODEL AND ITS IMPLICATIONS

A. The Lagrangian

In this section we present the chirally symmetric linear sigma model considered in this work. It contains scalar,

pseudoscalar, vector, and axial-vector fields, as well as nucleons and their chiral partners including all globally symmetric terms up to fourth order, see Refs. [10,18]. While higher-order terms are in principle possible, we do not consider them here. In fact, one can argue that they should be absent in dilation-invariant theories, cf. the discussion in Sec. IV.

The scalar and pseudoscalar fields are included in the matrix

$$\Phi = \sum_{a=0}^3 \phi_a t_a = (\sigma + i\eta_N)t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}, \quad (1)$$

where $\vec{t} = \vec{\tau}/2$, with the vector of Pauli matrices $\vec{\tau}$, and $t^0 = \mathbf{1}_2/2$. Under the global $U(2)_R \times U(2)_L$ chiral symmetry, Φ transforms as $\Phi \rightarrow U_L \Phi U_R^\dagger$. The vector and axial-vector fields are represented by the matrices

$$V^\mu = \sum_{a=0}^3 V_a^\mu t_a = \omega^\mu t^0 + \vec{\rho}^\mu \cdot \vec{t}, \quad (2a)$$

$$A^\mu = \sum_{a=0}^3 A_a^\mu t_a = f_1^\mu t^0 + \vec{a}_1^\mu \cdot \vec{t}. \quad (2b)$$

From these fields, we define right- and left-handed vector fields $R^\mu \equiv V^\mu - A^\mu$, $L^\mu \equiv V^\mu + A^\mu$. Under global $U(2)_R \times U(2)_L$ transformations, these fields behave as $R^\mu \rightarrow U_R R^\mu U_R^\dagger$, $L^\mu \rightarrow U_L L^\mu U_L^\dagger$.

The identification of mesons with particles listed in Ref. [11] is straightforward in the pseudoscalar and (axial-) vector sectors, as already indicated in Eqs. (1) and (2): the fields $\vec{\pi}$ and η_N correspond to the pion and the $SU(2)$ counterpart of the η meson, $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$, with a mass of about 700 MeV. This value can be obtained by “unmixing” the physical η and η' mesons, which also contain $\bar{s}s$ contributions. The fields ω^μ and $\vec{\rho}^\mu$ represent the $\omega(782)$ and $\rho(770)$ vector mesons, respectively, and the fields f_1^μ and $\vec{a}_1^\mu$ represent the $f_1(1285)$ and $a_1(1260)$ axial-vector mesons, respectively. (In principle, the physical ω and f_1 states also contain $\bar{s}s$ contributions, however their admixture is negligible small.)

Unfortunately, the identification of the σ and $\vec{a}_0$ fields is controversial, the possibilities being the pairs $\{f_0(600), a_0(980)\}$ and $\{f_0(1370), a_0(1450)\}$. In Sec. IV B a more detailed discussion of this problem is presented. In the present work, the scalar assignment affects only the isospin-even πN scattering length and we study its dependence on the sigma mass. The Lagrangian describing the meson fields reads

$$\begin{aligned} \mathcal{L}_{\text{mes}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda_2 (\Phi^\dagger \Phi)^2] - \lambda_1 (\text{Tr}[\Phi^\dagger \Phi])^2 + c(\det \Phi^\dagger + \det \Phi) + h_0 \text{Tr}[(\Phi^\dagger + \Phi)] \\ & - \frac{1}{4} \text{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \frac{m_1^2}{2} \text{Tr}[(L^\mu)^2 + (R^\mu)^2] + \frac{h_1}{2} \text{Tr}[\Phi^\dagger \Phi] \text{Tr}[(L^\mu)^2 + (R^\mu)^2] \\ & + h_2 \text{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi] + 2h_3 \text{Tr}[\Phi R_\mu \Phi^\dagger L^\mu] + \mathcal{L}_3 + \mathcal{L}_4, \end{aligned} \quad (3)$$

where $D^\mu \Phi = \partial^\mu \Phi + ig_1(\Phi R^\mu - L^\mu \Phi)$, and $R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$, $L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu$ are the field-strength tensors of the vector fields. The terms $\mathcal{L}_3$ and $\mathcal{L}_4$ describe three- and four-particle interactions of the (axial-) vector fields [10], which are not important for this work. We list them in Appendix A. For $c = h_0 = 0$, the Lagrangian $\mathcal{L}_{\text{mes}}$ is invariant under global $U(2)_R \times U(2)_L$ transformations. For $c \neq 0$, the $U(1)_A$ symmetry, where $A = L - R$, is explicitly broken, thus parametrizing the $U(1)_A$ anomaly of QCD. For $h_0 \neq 0$, the $U(2)_R \times U(2)_L$ symmetry is explicitly broken to the vectorial subgroup $U(2)_V$, where $V = L + R$.

The chiral condensate $\varphi = \langle 0 | \sigma | 0 \rangle = Z f_\pi$ emerges upon spontaneous chiral symmetry breaking in the mesonic sector. The parameter $f_\pi = 92.4$ MeV is the pion decay constant and Z is the wave function renormalization constant of the pseudoscalar fields [10,19], also related to π - a_1 mixing, see Appendix B for more details.

We now turn to the baryon sector which involves the baryon doublets Ψ_1 and Ψ_2 , where Ψ_1 has positive parity and Ψ_2 negative parity. In the mirror assignment they transform as follows:

$$\begin{aligned} \Psi_{1R} &\rightarrow U_R \Psi_{1R}, & \Psi_{1L} &\rightarrow U_L \Psi_{1L}, \\ \Psi_{2R} &\rightarrow U_L \Psi_{2R}, & \Psi_{2L} &\rightarrow U_R \Psi_{2L}, \end{aligned} \quad (4)$$

i.e., Ψ_2 transforms in a “mirror way” under chiral transformations [4,7]. These field transformations allow to write down a baryonic Lagrangian with a chirally invariant mass term for the fermions, parametrized by m_0 :

$$\begin{aligned} \mathcal{L}_{\text{bar}} = & \bar{\Psi}_{1L} i \gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i \gamma_\mu D_{1R}^\mu \Psi_{1R} \\ & + \bar{\Psi}_{2L} i \gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i \gamma_\mu D_{2L}^\mu \Psi_{2R} \\ & - \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi^\dagger \Psi_{1L}) \\ & - \hat{g}_2 (\bar{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \bar{\Psi}_{2R} \Phi \Psi_{2L}) \\ & - m_0 (\bar{\Psi}_{1L} \Psi_{2R} - \bar{\Psi}_{1R} \Psi_{2L} - \bar{\Psi}_{2L} \Psi_{1R} + \bar{\Psi}_{2R} \Psi_{1L}), \end{aligned} \quad (5)$$

where $D_{1R}^\mu = \partial^\mu - ic_1 R^\mu$, $D_{1L}^\mu = \partial^\mu - ic_1 L^\mu$, and $D_{2R}^\mu = \partial^\mu - ic_2 R^\mu$, $D_{2L}^\mu = \partial^\mu - ic_2 L^\mu$ are the covariant derivatives for the nucleonic fields, with the coupling constants c_1 and c_2 . (Note that in the case of local chiral symmetry one has $c_1 = c_2 = g_1$). The interaction of the baryonic fields with the scalar and pseudoscalar mesons is parametrized by $\hat{g}_1$ and $\hat{g}_2$.

The term proportional to m_0 generates a mixing between the fields Ψ_1 and Ψ_2 . The physical fields N and N^* , referring to the nucleon and its chiral partner, arise by diagonalizing the corresponding mass matrix in the Lagrangian (5):

$$\begin{pmatrix} N \\ N^* \end{pmatrix} = \frac{1}{\sqrt{2} \cosh \delta} \begin{pmatrix} e^{\delta/2} & \gamma_5 e^{-\delta/2} \\ \gamma_5 e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}. \quad (6)$$

The masses of the nucleon and its partner are obtained as

$$m_{N,N^*} = \sqrt{m_0^2 + [\frac{1}{4}(\hat{g}_1 + \hat{g}_2)\varphi]^2} \pm \frac{1}{4}(\hat{g}_1 - \hat{g}_2)\varphi. \quad (7)$$

The coupling constants $\hat{g}_{1,2}$ are uniquely determined by the values of m_N , m_{N^*} , and the parameter m_0 ,

$$\hat{g}_{1,2} = \frac{1}{\varphi} [\pm(m_N - m_{N^*}) + \sqrt{(m_N + m_{N^*})^2 - 4m_0^2}]. \quad (8)$$

From Eq. (7) one observes that, in the chirally restored phase where $\varphi \rightarrow 0$, the masses of the nucleon and its partner become degenerate, $m_N = m_{N^*} = m_0$. The mass splitting is generated by breaking chiral symmetry, $\varphi \neq 0$.

Note that the nucleon mass *cannot* be expressed as $m_N = m_0 + \lambda\varphi$, thus m_0 should not be interpreted as a linear contribution to the nucleon mass. Such a linearization is only possible in the case when m_0 dominates or the chiral condensate dominates. As we shall see, this does not happen and both quantities are sizable.

The parameter δ in Eq. (6) is related to the masses and the parameter m_0 by the expression

$$\cosh \delta = \frac{m_N + m_{N^*}}{2m_0}. \quad (9)$$

When $\delta \rightarrow \infty$, corresponding to $m_0 \rightarrow 0$, there is no mixing and $N = \Psi_1$, $N^* = -\Psi_2$. In this case, $m_N = \hat{g}_1 \varphi/2$ and $m_{N^*} = \hat{g}_2 \varphi/2$, thus the nucleon mass is solely generated by the chiral condensate as in the standard linear sigma model of Refs. [2,4] with the naive assignment for the baryons.

B. Axial coupling constants

The expressions for the axial coupling constants of the nucleon and the partner are derived in Appendix C. The result is

$$\begin{aligned} g_A^N &= \frac{1}{2 \cosh \delta} (g_A^{(1)} e^\delta + g_A^{(2)} e^{-\delta}), \\ g_A^{N^*} &= \frac{1}{2 \cosh \delta} (g_A^{(1)} e^{-\delta} + g_A^{(2)} e^\delta), \end{aligned} \quad (10)$$

where

$$g_A^{(1)} = 1 - \frac{c_1}{g_1} \left(1 - \frac{1}{Z^2}\right), \quad g_A^{(2)} = -1 + \frac{c_2}{g_1} \left(1 - \frac{1}{Z^2}\right) \quad (11)$$

are the axial coupling constants of the bare fields Ψ_1 and Ψ_2 . At this point, it should be emphasized that the interaction with the (axial-) vector mesons generates additional contributions to g_A^N and $g_A^{N^*}$, proportional to c_1 and c_2 . We now discuss several limiting cases, using the fact that Z is required to be larger than 1, cf. Eq. (B7):

- (i) *Local chiral symmetry*: In this case, the coupling constants $c_1 = c_2 = g_1$. This implies $g_A^N = -g_A^{N^*} = Z^{-2} \tanh \delta < 1$, which is at odds with the experimental value $g_A^N = 1.267 \pm 0.004$ [11].
- (ii) *Decoupling of vector mesons*: Here, $Z = 1$ and $c_1 = c_2 = 0$, and we obtain the results of Ref. [7]: $g_A^N = -g_A^{N^*} = \tanh \delta$. In the limit $\delta \rightarrow \infty$, this reduces to $g_A^N = 1$ and $g_A^{N^*} = -1$. Also in this case the experimental value for g_A^N cannot be obtained for any choice of the parameters. Moreover, a positive value of $g_A^{N^*}$, as found in the lattice simulation of Ref. [13], is also impossible.
- (iii) *Decoupling of the chiral partner*: This is achieved in the limit $\delta \rightarrow \infty$, where $N = \Psi_1$ and $N^* = -\Psi_2$. One has $g_A^N = g_A^{(1)}$ and $g_A^{N^*} = g_A^{(2)}$. Since $Z > 1$, it is evident that the ratio c_1/g_1 must be negative in order to obtain the experimental value $g_A^N = 1.267 \pm 0.004$ [11].

Note that, in the case of local chiral symmetry, the axial charge of the nucleon can be also correctly reproduced when introducing dimension-6 terms in the Lagrangian $\mathcal{L}_{\text{bar}}$, cf. Refs. [1,2,17,20,21], because the coefficients of these so-called Weinberg-Tomozawa (WT) terms [22,23] can be adjusted accordingly. However, such WT terms naturally arise when integrating out the axial-vector mesons from our Lagrangian, just as in chiral perturbation theory [15]. In this sense, it would be double-counting to simultaneously consider axial-vector mesons and WT

terms. Our generalization to a *global* chiral symmetry allows a description of the axial charge without explicitly introducing WT terms.

C. Decay widths

We now turn to the decays $N^* \rightarrow N\pi$ and $N^* \rightarrow N\eta$. The calculation of the tree-level decay width for $N^* \rightarrow N\pi$ from the Lagrangian (5) is straightforward. However, the decay $N^* \rightarrow N\eta$ cannot be directly evaluated because of the absence of the s quark. In order to proceed, we have to take into account that

$$\eta = \eta_N \cos \phi_P + \eta_S \sin \phi_P, \quad (12)$$

where $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$, $\eta_S \equiv \bar{s}s$ and ϕ_P lies between -32° and -45° [24]. Then, the decay amplitude $\mathcal{A}_{N^* \rightarrow N\eta}$ can be expressed as

$$\mathcal{A}_{N^* \rightarrow N\eta} = \mathcal{A}_{N^* \rightarrow N\eta_N} \cos \phi_P + \mathcal{A}_{N^* \rightarrow N\eta_S} \sin \phi_P. \quad (13)$$

In the following, we assume that the OZI-suppressed amplitude $\mathcal{A}_{N^* \rightarrow N\eta_S}$ is small, so that to good approximation the decay width $\Gamma_{N^* \rightarrow N\eta} \simeq \cos^2 \phi_P \Gamma_{N^* \rightarrow N\eta_N}$. Note that the *physical* η meson mass, $m_\eta = 547$ MeV, enters $\Gamma_{N^* \rightarrow N\eta}$. Therefore, also the decay width $\Gamma_{N^* \rightarrow N\eta_N}$ has to be evaluated for the physical mass m_η , not for m_{η_N} .

The expression for the decay width $N^* \rightarrow NP$, where $P = \pi, \eta$, is (for details, see Appendix C)

$$\begin{aligned} \Gamma_{N^* \rightarrow NP} = \lambda_P \frac{k_P}{2\pi} \frac{m_N}{m_{N^*}} \frac{Z^2}{32 \cosh^2 \delta} \Big\{ & w^2 (c_1 + c_2)^2 \left[(m_{N^*}^2 - m_N^2 - m_P^2) \frac{E_P}{m_N} + m_P^2 \left(1 - \frac{E_N}{m_N} \right) \right] + (\hat{g}_1 - \hat{g}_2)^2 \left(\frac{E_N}{m_N} + 1 \right) \right. \\ & \left. + 2w(\hat{g}_1 - \hat{g}_2)(c_1 + c_2) \left(\frac{m_{N^*}^2 - m_N^2 - m_P^2}{2m_N} + E_P \right) \right\}. \end{aligned} \quad (14)$$

The coefficients $\lambda_\pi = 3$, $\lambda_\eta = \cos^2 \phi_P$, $w \equiv g_1 \varphi / m_{a_1}^2$, and the momentum of the pseudoscalar particle is given by

$$k_P = \frac{1}{2m_{N^*}} \sqrt{(m_{N^*}^2 - m_N^2 - m_P^2)^2 - 4m_N^2 m_P^2}. \quad (15)$$

The energies are $E_P = \sqrt{k_P^2 + m_P^2}$ and $E_N = \sqrt{k_P^2 + m_N^2}$, because the momenta of the nucleon and the pseudoscalar particles are equal in the rest frame of N^* .

It is important to stress that, in the mirror assignment, the only way to obtain a nonzero $N^* N \pi$ coupling is a nonzero value of the parameter m_0 . In fact, the coupling is proportional to $\cosh^{-1} \delta \propto m_0$, i.e., when m_0 increases, also the decay width increases.

In the naive assignment, in which the field Ψ_2 transforms just like the field Ψ_1 , a term proportional to m_0 is not possible, because it would break chiral symmetry. In this case, a mixing term of the form $\propto \bar{\Psi}_2 \gamma^5 \Phi \Psi_1 + \text{h.c.}$ is allowed. This leads to a term $\propto \bar{\Psi}_2 \gamma^5 (\sigma + i \gamma^5 \vec{\pi} \cdot \vec{\tau}) \Psi_1$,

where the pion is coupled to Ψ_1 and Ψ_2 in a chirally symmetric way. However, the very same term also generates a mixing of Ψ_2 and Ψ_1 due to the nonzero vacuum expectation value of the field $\sigma = \sigma_0$. When performing the diagonalization one obtains two physical fields N and N^* , to be identified with the nucleon and a negative-parity state such as $N^*(1535)$. In terms of the physical fields N^* and N the coupling $\bar{N}^* i \vec{\pi} \cdot \vec{\tau} N$ vanishes; for the explicit calculation see Ref. [8]. Thus, in the naive assignment and in the minimal framework with only one multiplet of scalar and pseudoscalar fields the decay $N^* \rightarrow N\pi$ vanishes. One could go beyond this minimal setup: a possibility is to include the (axial-) vector mesons into the Lagrangian of the naive assignment. In this way a nonzero derivative coupling $\propto \bar{N}^* \gamma^\mu \partial_\mu \vec{\pi} \cdot \vec{\tau} N$ survives. A complete study of this scenario, involving also the scattering lengths, is in preparation.

Alternatively, the inclusion of a second (or more) multiplet(s) of (pseudo-)scalar mesons, see Refs. [25,26],

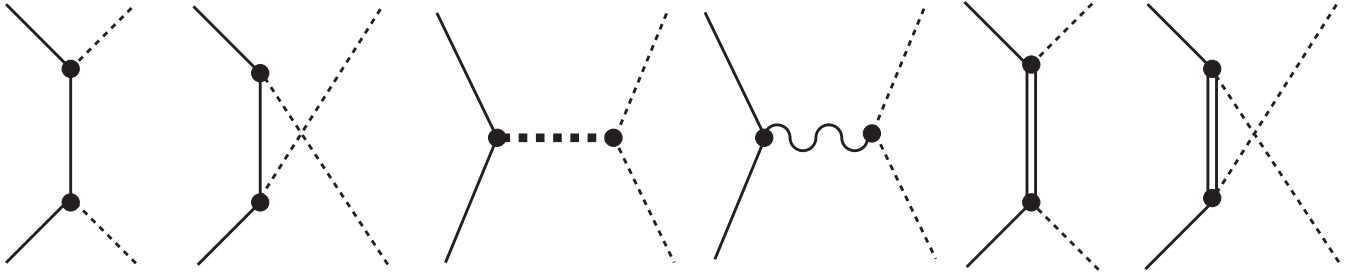


FIG. 1. Tree-level diagrams contributing to πN scattering. Dashed lines represent the pion, the bold dashed line the σ meson, the wavy line the ρ meson, full lines the nucleon, and double full lines the N^* , respectively.

coupled to the baryon fields also leads to a nonvanishing coupling between N^* , the nucleon, and the pion.

D. πN scattering lengths

The general form of the πN scattering amplitude is [16]

$$T_{ab} = [A^{(+)} + \frac{1}{2}(q_1^\mu + q_2^\mu)\gamma_\mu B^{(+)}]\delta_{ab} + [A^{(-)} + \frac{1}{2}(q_1^\mu + q_2^\mu)\gamma_\mu B^{(-)}]i\epsilon_{bac}\tau_c, \quad (16)$$

where the subscripts a and b refer to the isospin of the initial and final states and the superscripts $(+)$ and $(-)$ denote the isospin-even and isospin-odd amplitudes, respectively. The πN scattering amplitudes, $A^{(\pm)}$ and $B^{(\pm)}$, evaluated from the Lagrangian (5) at tree level, involve

exchange of σ and ρ mesons in the t -channel and intermediate N and N^* states in the s - and u -channels, cf. Fig. 1.

The s -wave scattering lengths, $a_0^{(\pm)}$, are given by

$$a_0^{(\pm)} = \frac{1}{4\pi(1 + m_\pi/m_N)}(A_0^{(\pm)} + m_\pi B_0^{(\pm)}), \quad (17)$$

where the subscript 0 at the amplitudes $A^{(\pm)}$, $B^{(\pm)}$ indicates that they are taken at threshold, i.e., for the following values of the Mandelstam variables s , t , u : $s = (m_N + m_\pi)^2$, $t = 0$, $u = (m_N - m_\pi)^2$.

The explicit expression for the isospin-even scattering length can be obtained from Eq. (17) by applying the Feynman rules resulting from the Lagrangians (3) and (5) to the diagrams shown in Fig. 1. The result is

$$\begin{aligned} a_0^{(+)} = & \frac{1}{4\pi(1 + \frac{m_\pi}{m_N})} \left(\frac{Z}{2 \cosh \delta} \right)^2 \left(-\frac{1}{2} \left[\hat{g}_1 - \hat{g}_2 + \frac{Z f_\pi}{2} w(c_1 + c_2)(\hat{g}_2 - \hat{g}_1) \right]^2 \frac{(m_N + m_{N^*})(m_N^2 + m_\pi^2 - m_{N^*}^2)}{(m_N^2 + m_\pi^2 - m_{N^*}^2)^2 - 4m_N^2 m_\pi^2} \right. \\ & - w(c_1 + c_2)(\hat{g}_1 - \hat{g}_2) + \frac{Z f_\pi}{4} (\hat{g}_1 - \hat{g}_2) w^2 (c_1 + c_2)^2 - w(c_1 e^\delta - c_2 e^{-\delta})(\hat{g}_1 e^\delta + \hat{g}_2 e^{-\delta}) \\ & + w^2 m_N (c_1 e^\delta - c_2 e^{-\delta})^2 + (\hat{g}_1 e^\delta - \hat{g}_2 e^{-\delta}) \frac{\cosh \delta}{Z f_\pi} \left\{ 1 + \frac{m_\pi^2}{m_\sigma^2} \frac{1}{Z^4} \left[Z^2 - 2 + 2(Z^2 - 1) \left(1 - \frac{Z^2 m_1^2}{m_{a_1}^2} \right) \right] \right\} \\ & + m_\pi \left[\left[\hat{g}_1 - \hat{g}_2 + \frac{Z f_\pi}{2} w(c_1 + c_2)(\hat{g}_2 - \hat{g}_1) \right]^2 \frac{m_N m_\pi}{(m_N^2 + m_\pi^2 - m_{N^*}^2)^2 - 4m_N^2 m_\pi^2} \right. \\ & \left. \left. + [\hat{g}_1 e^\delta + \hat{g}_2 e^{-\delta} - 2m_N w(c_1 e^\delta - c_2 e^{-\delta})]^2 \frac{m_N}{m_\pi} \frac{1}{m_\pi^2 - 4m_N^2} \right] \right). \end{aligned} \quad (18)$$

Similarly, the expression for the isospin-odd scattering length is given by

$$\begin{aligned} a_0^{(-)} = & \frac{1}{4\pi(1 + \frac{m_\pi}{m_N})} \left(\frac{Z}{2 \cosh \delta} \right)^2 \left(\left[\hat{g}_1 - \hat{g}_2 + \frac{Z f_\pi}{2} w(c_1 + c_2)(\hat{g}_2 - \hat{g}_1) \right]^2 \frac{(m_N + m_{N^*})m_N m_\pi}{(m_N^2 + m_\pi^2 - m_{N^*}^2)^2 - 4m_N^2 m_\pi^2} \right. \\ & + \frac{m_\pi}{2} \left[\left[\hat{g}_1 - \hat{g}_2 + \frac{Z f_\pi}{2} w(c_1 + c_2)(\hat{g}_2 - \hat{g}_1) \right]^2 \frac{m_N^2 + m_\pi^2 - m_{N^*}^2}{(m_N^2 + m_\pi^2 - m_{N^*}^2)^2 - 4m_N^2 m_\pi^2} \right. \\ & - [\hat{g}_1 e^\delta + \hat{g}_2 e^{-\delta} - 2m_N w(c_1 e^\delta - c_2 e^{-\delta})]^2 \frac{1}{m_\pi^2 - 4m_N^2} - w^2 [(c_1 + c_2)^2 - (c_1 e^\delta - c_2 e^{-\delta})^2] \\ & \left. \left. + \frac{g_1}{m_\rho^2} \frac{4 \cosh \delta}{Z^2} (c_1 e^\delta - c_2 e^{-\delta}) \right] \right). \end{aligned} \quad (19)$$

Although it is not obvious from these expressions, one can show that the s -wave scattering lengths $a_0^{(\pm)}$ vanish in the chiral limit, as required by low-energy theorems for theories with spontaneously broken chiral symmetry.

III. RESULTS AND DISCUSSION

In this section we present our results. We first discuss the case where the resonance $N(1535)$ is interpreted as the chiral partner of the nucleon. This is the most natural assignment because this resonance is the lightest with the correct quantum numbers. We then consider some important limiting cases. Finally, we also discuss two different assignments: the resonance $N(1650)$, which is the next heavier state with the correct quantum numbers listed in Ref. [11], and a *speculative* candidate $N(1200)$ with a mass $M_{N^*} \sim 1200$ MeV and a very large width $\Gamma_{N^* \rightarrow N\pi} \gtrsim 800$ MeV, such as to have avoided experimental detection up to now [9].

A. $N(1535)$ as partner

The resonance $N(1535)$ has a mass $m_{N^*} = (1535 \pm 10)$ MeV [11]. The theoretical expressions for g_A^N , $g_A^{N^*}$, $\Gamma_{N^* \rightarrow N\pi}$, $\Gamma_{a_1 \rightarrow \pi\gamma}$ depend on the four parameters c_1 , c_2 , Z , and m_0 . Here, Z is the only parameter entering from the meson sector, see Appendix B.

We determine the parameters c_1 , c_2 , Z , and m_0 by using the experimental results [11] for the decay width $\Gamma_{N^* \rightarrow N\pi}^{\text{exp}} = (67.5 \pm 23.6)$ MeV, the radiative decay of the $a_1(1260)$ meson, $\Gamma_{a_1 \rightarrow \pi\gamma}^{\text{exp}} = (0.640 \pm 0.246)$ MeV, and the axial coupling constant $g_A^{N,\text{exp}} = 1.267 \pm 0.004$, as well as the lattice result $g_A^{N^*,\text{latt}} = 0.2 \pm 0.3$ [13]. With the help of a standard χ^2 procedure it is also possible to determine the errors for the obtained parameters:

$$\begin{aligned} c_1 &= -3.0 \pm 0.6, & c_2 &= 11.6 \pm 3.6, \\ Z &= 1.67 \pm 0.2, \end{aligned} \quad (20)$$

and

$$m_0 = (460 \pm 136) \text{ MeV}. \quad (21)$$

The coupling constants $\hat{g}_1$ and $\hat{g}_2$ can be deduced from Eq. (8),

$$\hat{g}_1 = 11.0 \pm 1.5, \quad \hat{g}_2 = 18.8 \pm 2.4. \quad (22)$$

The value obtained for m_0 is larger than the one originally found in Ref. [7] and points to a sizable contribution of other condensates to the nucleon mass. However, because of the nonlinear relation (7) between the nucleon mass, m_0 , and the chiral condensate, when switching off m_0 the nucleon mass is not simply by an amount m_0 smaller than the physical value, rather $m_N = \hat{g}_1 \varphi / 2 \simeq 850$ MeV, and thus only slightly smaller than 939 MeV. The Ioffe formula is thus still approximately justified also in this context. On the other hand, when varying φ from 0 to the

physical value Zf_π , the nucleon mass goes from $m_0 = 460$ MeV to 939 MeV. Interestingly, the coupling constant c_2 which parametrizes the interaction of the nucleon's partner with the (axial-) vector mesons is larger than the constant c_1 which parametrizes the interaction of the nucleon with the (axial-) vector mesons. Nevertheless, when compared with the coupling $g_1 \sim 6$ (similar in all models with vector mesons and pions) the constants c_1 and c_2 are $|c_1| \sim g_1/2$, $c_2 \sim 2g_1$ i.e., they are related to g_1 by some numerical factor of order one. The direct comparison of c_1 and c_2 leads to $|c_1| \sim c_2/4$.

We now test the validity of our model by considering the πN scattering lengths (some preliminary results were already presented in Ref. [27]). The quantity $a_0^{(-)}$ depends on c_1 , c_2 , Z , and m_0 , and in addition on m_ρ and g_1 . The latter is a function of Z and m_{a_1} , cf. Eq. (B7). The values of m_ρ and m_{a_1} are known to reasonably good precision [11], and thus our uncertainty in determining $a_0^{(-)}$ is small. (This will be different for $a_0^{(+)}$ which also depends on the poorly known value of the σ meson mass, m_σ .) We obtain

$$a_0^{(-)} = (6.04 \pm 0.63) \cdot 10^{-4} \text{ MeV}^{-1}, \quad (23)$$

in agreement with the experimental value measured by the ETH Zürich-Neuchatel-PSI collaboration in pionic hydrogen and deuterium X-ray experiments [28]:

$$a_{0,\text{exp}}^{(-)} = (6.4 \pm 0.1) \cdot 10^{-4} \text{ MeV}^{-1}. \quad (24)$$

An even better agreement is expected when including the Δ resonance [17].

The scattering length $a_0^{(+)}$ depends also on c_1 , c_2 , Z , and m_0 , but in addition on m_1 and m_σ . The former parametrizes the contribution to the ρ mass which does not originate from the chiral condensate: $m_\rho^2 = m_1^2 + \frac{\phi^2}{2}(h_1 + h_2 + h_3)$. Notice that in the present theoretical framework with global chiral symmetry the KSFR relation [29] is obtained for $m_1 = 0$, $h_1 + h_2 + h_3 = g_1^2/Z^2$. A physically reasonable range of values for m_1 is between 0 and m_ρ . For the lower boundary, the mass of the ρ meson is exclusively generated by chiral symmetry breaking, thus it becomes massless when $\varphi \rightarrow 0$. This is similar to Georgi's vector limit [30] or Brown-Rho scaling [31]. In principle, the mass of the σ meson varies over a wide range of values; we could choose $m_\sigma \sim 0.4$ GeV or 1.37 GeV, according to the assignment $f_0(600)$ and $f_0(1370)$.

Since the allowed range of values for m_1 and m_σ is large, we choose to plot the scattering length $a_0^{(+)}$ as function of m_1 for different choices of m_σ ; the result is shown in Fig. 2. The experimental result [28]

$$a_{0,\text{exp}}^{(+)} = (-8.8 \pm 7.2) \cdot 10^{-6} \text{ MeV}^{-1} \quad (25)$$

is shown as grey (yellow) band. One observes that for small values of m_σ one requires a large value for m_1 in order to reproduce experimental data. For increasing m_σ , the re-

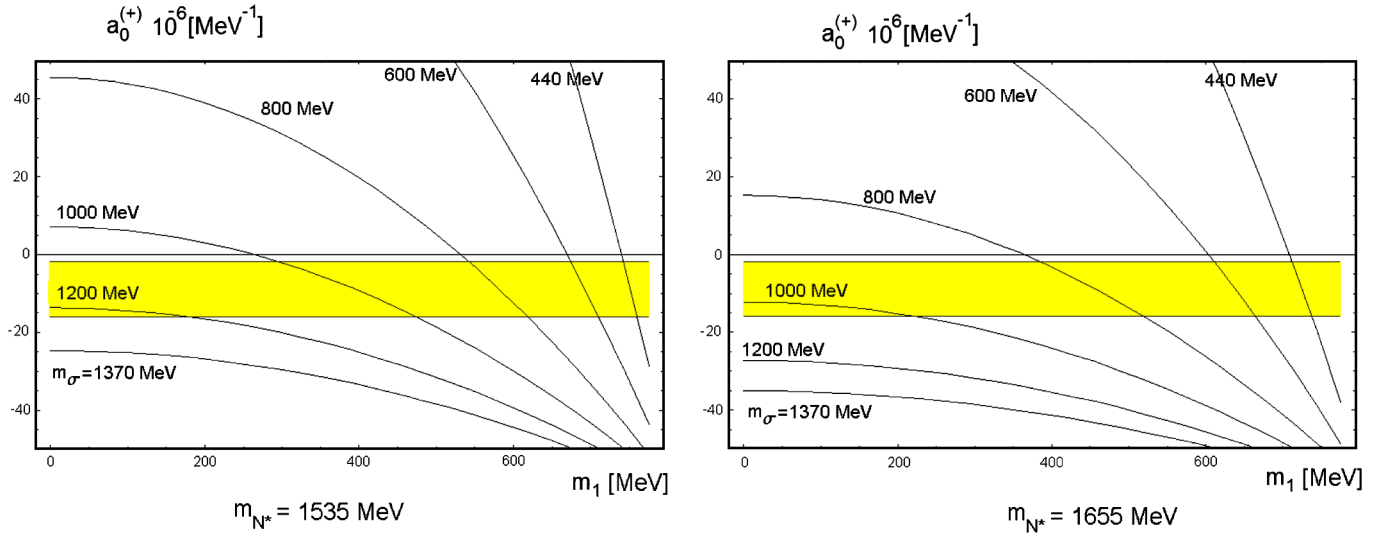


FIG. 2 (color online). The isospin-even scattering length $a_0^{(+)}$ as a function of m_1 for fixed values of m_σ , for the assignment $N^* = N(1535)$ (left panel) and $N^* = N(1650)$ (right panel). The experimental range is shown by the grey (yellow) band.

quired values for m_1 decrease. For $m_\sigma \geq 1.37$ GeV, $a_{0,\text{exp}}^{(+)}$ cannot be reproduced for any value of m_1 . This, however, does not exclude a heavy σ meson, rather, it indicates that an additional light scalar-isoscalar resonance needs to be included as discussed in Sec. IV B.

For the decay $N^* \rightarrow N\eta$, we obtain with Eq. (14) the result

$$\Gamma_{N^* \rightarrow N\eta} = (10.9 \pm 3.8) \text{ MeV}, \quad (26)$$

where the error also takes into account the uncertainty in the pseudoscalar mixing angle $\phi_P = -38.7^\circ \pm 6^\circ$. We observe that $\Gamma_{N^* \rightarrow N\eta}$ is about a factor 7 smaller than $\Gamma_{N^* \rightarrow N\pi}$, which is in reasonable agreement with the naive expectation based on the relation $\lambda_\eta/\lambda_\pi = \cos^2 \phi_P/3 \approx 0.097$. However, it is clearly smaller than the experimental value $\Gamma_{N^* \rightarrow N\eta}^{\text{exp}} = (78.7 \pm 24.3) \text{ MeV}$ [11]. The agreement could be improved if one generalizes our discussion to the $SU(3)$ case and includes a large OZI-violating contribution, or if one considers an enlarged mixing scenario as discussed in Sec. IV A.

B. Limiting cases

We now consider three important limiting cases. In all of these $N(1535)$ is taken as chiral partner of the nucleon.

- (i) *Local chiral symmetry*: This case is obtained by setting $g_1 = c_1 = c_2$ and $h_1 = h_2 = h_3 = 0$. As a consequence, $m_\rho = m_1$, $m_{a_1}^2 = m_\rho^2 + (g_1 \varphi)^2$, $Z = m_{a_1}/m_\rho$. Using the experimental values for $\Gamma_{N^* \rightarrow N\pi}$ and $\Gamma_{a_1 \rightarrow \gamma\pi}$ one obtains

$$m_0 = (730 \pm 229) \text{ MeV}. \quad (27)$$

As a consequence, $g_A^N = -g_A^{N^*} \equiv Z^{-2} \tanh \delta = 0.33 \pm 0.02$, both at odds with experimental and

lattice data. The scattering length $a_0^{(-)}$ is in the range of the experimental data, $a_0^{(-)} = (4.9 \pm 1.7) \times 10^{-4} \text{ MeV}^{-1}$. Since $m_1 = m_\rho$ is fixed, the isospin-even scattering length only depends on m_σ . Thus, for a given value of m_σ , we obtain a single value with theoretical errors: $a_0^{(+)} = (7.06 \pm 3.12) \times 10^{-6} \text{ MeV}^{-1}$ for $m_\sigma = 1.37$ GeV and $a_0^{(+)} = (4.46 \pm 0.11) \times 10^{-5} \text{ MeV}^{-1}$ for $m_\sigma = 0.44$ GeV, which is outside the range of the experimental error band. As already argued in Refs. [10,21] we conclude that the case of local chiral symmetry (in the present model without higher-order terms) is not capable of properly reproducing low-energy phenomenology.

- (ii) *Decoupling of vector mesons*: This corresponds to $g_1 = c_1 = c_2 = h_1 = h_2 = h_3 = 0$, and thus $Z = 1$ and $w = 0$. Using the decay width $\Gamma_{N^* \rightarrow N\pi} = (67.5 \pm 23.6) \text{ MeV}$ one obtains

$$m_0 = (262 \pm 46) \text{ MeV}, \quad (28)$$

in agreement with Ref. [7]. As a result $g_A^N = -g_A^{N^*} = 0.97 \pm 0.01$, in disagreement with both experimental and lattice data. The description of the scattering lengths also becomes worse; the isospin-odd scattering length $a_0^{(-)} = (5.7 \pm 0.47) \times 10^{-4} \text{ MeV}^{-1}$, which is just outside the experimental error band. Also in this case, the isospin-even scattering length assumes a single value (with theoretical errors) for given m_σ : $a_0^{(+)} = (1.08 \pm 0.05) \times 10^{-4} \text{ MeV}^{-1}$ for $m_\sigma = 1.37$ GeV and $a_0^{(+)} = (-7.55 \pm 0.19) \times 10^{-4} \text{ MeV}^{-1}$ for $m_\sigma = 0.44$ GeV, i.e., 2 orders of magnitude away from the experimental value. We thus conclude that vec-

tor mesons cannot be omitted for a correct description of pion-nucleon scattering lengths.

- (iii) *Decoupling of the chiral partner*: This is obtained by sending $m_0 \rightarrow 0$ or $\delta \rightarrow \infty$. The partner decouples and we are left with a linear σ model with vector and axial-vector mesons. The decay $\Gamma_{N^* \rightarrow N\pi}$ vanishes in this case and is obviously at odds with the experiment. Using the experimental values for g_A^N and $\Gamma_{a_1 \rightarrow \gamma\pi}$ to fix the parameters c_1 and Z (c_2 and $\hat{g}_2$ play no role here because of the decoupling of the partner), we obtain

$$c_1 = -2.59 \pm 0.51, \quad Z = 1.66 \pm 0.2. \quad (29)$$

The scattering lengths are

$$a_0^{(-)} = (5.99 \pm 0.66) \times 10^{-4} \text{ MeV}^{-1}, \quad (30)$$

and $a_0^{(+)}$ shows a similar behavior as shown in Fig. 2. We conclude that the role of the partner is marginal in improving the scattering lengths. It could be omitted, unless one wants to consider, in the framework of the mirror assignment, its decay into nucleon and pseudoscalar particles.

C. Other candidates

In this subsection, we discuss two more exotic possibilities for the chiral partner of the nucleon.

- (i) *$N(1650)$ as partner*. The resonance $N(1650)$ has a mass $m_{N^*} = (1655 \pm 15) \text{ MeV}$ and a decay width $\Gamma_{N^* \rightarrow N\pi}^{\text{exp}} = (128 \pm 44) \text{ MeV}$ [11]. The axial coupling constant measured in the lattice simulation of Ref. [13] reads $g_A^{N^*, \text{latt}} = 0.55 \pm 0.2$. By following the previous steps we obtain

$$c_1 = -3.3 \pm 0.7, \quad c_2 = 14.8 \pm 3.4, \quad (31)$$

$$Z = 1.67 \pm 0.2,$$

and

$$m_0 = (709 \pm 157) \text{ MeV}. \quad (32)$$

This leads to the coupling constants

$$\hat{g}_1 = 9.45 \pm 1.81, \quad \hat{g}_2 = 18.68 \pm 2.68. \quad (33)$$

In this case m_0 is even larger than before. However, as in the case of $N(1535)$, the quantity $\hat{g}_1 \varphi / 2 \simeq 730 \text{ MeV}$ is still sizable and similar to m_0 . The result for the isospin-odd scattering length

$$a_0^{(-)} = (5.90 \pm 0.46) \times 10^{-4} \text{ MeV}^{-1} \quad (34)$$

is similar to the case of $N(1535)$. The isospin-even scattering lengths behave similarly as before, cf. Fig. 2, however, slightly smaller values for m_σ are required in order to reproduce the experimental data.

The decay width into $N\eta$ is $\Gamma_{N^* \rightarrow N\eta} = (18.3 \pm 8.5) \text{ MeV}$, which should be compared to $\Gamma_{N^* \rightarrow N\eta}^{\text{exp}} = (10.7 \pm 6.7) \text{ MeV}$. Thus, in this case the decay width is in agreement with the experimental value. However, we then face the problem of how to describe the $N(1535)$ resonance, cf. the discussion in Sec. IV A.

- (ii) *Speculative candidate $N(1200)$ as partner*. We consider a *speculative* candidate $N(1200)$ with a mass $m_{N^*} \sim 1200 \text{ MeV}$ and a very large width $\Gamma_{N^* \rightarrow N\pi} \gtrsim 800 \text{ MeV}$, such as to have avoided experimental detection up to now. The reason for its introduction was motivated by properties of nuclear matter [9] and further on investigated in Ref. [32] in the context of asymmetric nuclear matter present in a neutron star. Regardless of the precise value of the axial coupling constant of the partner (which is unknown for this hypothetical resonance) one obtains $m_0 > 1 \text{ GeV}$. This, in turn, implies a large interaction of N and N^* . As a consequence, both scattering lengths turn out to be off by two order of magnitudes: $a_0^{(-)} \sim 10^{-2} \text{ MeV}^{-1}$ and $a_0^{(+)} \sim 10^{-4} \text{ MeV}^{-1}$. Thus, we are led to *discard* the possibility that a hypothetical, not yet discovered $N(1200)$ exists.

IV. SUMMARY AND OUTLOOK

In this paper, we investigated a linear sigma model with global chiral $U(2)_R \times U(2)_L$ symmetry, where the mesonic degrees of freedom are the standard scalar and pseudoscalar mesons and the vector and axial-vector mesons. In addition to the mesons, we included baryonic degrees of freedom, namely, the nucleon and its chiral partner, which is incorporated in the model in the so-called mirror assignment.

We used this model to study the origin of the mass of the nucleon, the assignment and decay properties of its chiral partner and the pion-nucleon scattering lengths. The mass of the nucleon results as an interplay of the chiral condensate and a chirally invariant baryonic mass term, proportional to the parameter m_0 . When the chiral partner of the nucleon is identified with the resonance $N^* \equiv N(1535)$, the parameter $m_0 \simeq 500 \text{ MeV}$ is obtained as a result of a fitting procedure which involves the three experimentally measured quantities $N^* \rightarrow N\pi$, $a_1 \rightarrow \pi\gamma$, g_A^N , and the quantity $g_A^{N^*}$ evaluated on the lattice. The isospin-odd scattering length $a_0^{(-)}$ is then fixed and found to be in good agreement with experimental data. The isospin-even scattering length depends, in addition, strongly on the mass of the σ meson, see Fig. 2 and the discussion in Sec. IV B. The decay width $N^* \rightarrow N\eta$ turns out to be a factor of 8 smaller than the experimental value.

The obtained value $m_0 \simeq 500 \text{ MeV}$ implies that a sizable amount of the nucleon mass does not originate from the chiral condensate. As this result is subject to the

assumptions and the validity of the employed chiral model, most notably due to the identification of the chiral partner with $N(1535)$ and to the mathematical properties of the mirror assignment, future studies of other scenarios, incorporating new results both from the experiment and the lattice, are necessary to further clarify this important issue of hadron physics.

It should also be noted that the results presented in this work are based on a tree-level calculation. The inclusion of loops represents a task for the future. Nevertheless, we expect that the results will not change qualitatively: On the one hand, while the dimensionless couplings of the model $g_1, c_1, c_2, \hat{g}_1, \hat{g}_2$ are large, the contribution of loops is suppressed according to large- N_c arguments [33]. On the other hand, in our model we have included from the very beginning the relevant resonances which contribute as virtual states to processes, thus reducing the effects of loops in the model. To clarify the latter point, consider the ρ meson exchange in πN scattering. In an approach in which the ρ meson is not directly included, its contribution could only be obtained after a corresponding loop resummation, while in our approach it is taken directly into account by a tree-level exchange diagram.

We studied three important limiting cases: (i) In the framework of local chiral symmetry it is not possible to correctly reproduce low-energy phenomenology. (ii) It is not admissible to neglect (axial-) vector mesons. They are crucial in order to obtain a correct description of the axial coupling constants and πN scattering lengths. (iii) The role of the partner N^* has only a marginal influence on the scattering lengths.

We have also tested other assignments for the partner of the nucleon: a broad, not-yet discovered partner with a mass of about 1.2 GeV must be excluded on the basis of scattering data. The well-established resonance $N(1650)$ provides qualitatively similar results as $N(1535)$ and, in this case, the theoretical value of the decay width $N(1650) \rightarrow N\eta$ is in agreement with the experimental one. However, in this scenario it is not clear how $N(1535)$ fits into the baryonic resonance spectrum. This issue is discussed in Sec. IV A presented below. In Sec. IV B we discuss the origin of m_0 in terms of tetraquark and gluon condensates and the implications for future studies.

A. Outlook 1: Enlarged mixing scenario

In this section we briefly describe open problems of the previous results and present a possible outlook to improve the theoretical description.

A simultaneous description of both resonances $N(1525)$ and $N(1650)$ requires an extension of the model. In the framework of the mirror assignment, instead of only two bare nucleon fields Ψ_1 and Ψ_2 one should include two additional bare fields Ψ_3 and Ψ_4 with positive and negative parity, respectively. The latter two are assumed to trans-

form like Ψ_1 and Ψ_2 in Eq. (4). The interesting part of the enlarged Lagrangian are the bilinear chirally invariant mass terms:

$$\begin{aligned} \mathcal{L}_{\text{mass}} = & m_0^{(1,2)}(\bar{\Psi}_2\gamma^5\Psi_1 - \bar{\Psi}_1\gamma^5\Psi_2) \\ & + m_0^{(3,4)}(\bar{\Psi}_4\gamma^5\Psi_3 - \bar{\Psi}_3\gamma^5\Psi_4) \\ & + m_0^{(1,4)}(\bar{\Psi}_4\gamma^5\Psi_1 - \bar{\Psi}_1\gamma^5\Psi_4) \\ & + m_0^{(2,3)}(\bar{\Psi}_2\gamma^5\Psi_3 - \bar{\Psi}_3\gamma^5\Psi_2). \end{aligned} \quad (35)$$

In the limit $m_0^{(1,4)} = m_0^{(2,3)} = 0$ the bare fields Ψ_1 and Ψ_2 do not mix with the fields Ψ_3 and Ψ_4 . The fields Ψ_1 and Ψ_2 generate the states $N(939)$ and $N(1535)$, just as described in this paper with $m_0^{(1,2)} = m_0$, while the fields Ψ_3 and Ψ_4 generate the states $N(1440)$ and $N(1650)$, which are regarded as chiral partners. The term proportional to $m_0^{(3,4)}$ induces a decay of the form $N(1650) \rightarrow N(1440)\pi$ (or η), but still $N(1650)$ and $N(1440)$ do not decay into $N\pi(\eta)$.

When in addition the coefficients $m_0^{(1,4)}$ and $m_0^{(2,3)}$ are nonzero, a more complicated mixing scenario involving four bare fields arises. As a consequence, it is possible to account for the decay of both resonances $N(1550)$ and $N(1650)$ into $N\pi(\eta)$. Moreover, it is well conceivable that the anomalously small value of the decay width $N(1550) \rightarrow N\eta$ arises because of destructive interference. Interestingly, a mixing of bare configurations generating $N(1535)$ and $N(1650)$ is necessary also at the level of the quark model [34]. Note that in the framework of the generalized mixing scenario, the fields $N(1535)$ and $N(1650)$ are chiral partners of $N(939)$ and $N(1440)$. Because of mixing phenomena, it is not possible to isolate the chiral partner of the nucleon, which is present in both resonances $N(1535)$ and $N(1650)$. However, also in this case the nonzero decay widths of both fields $N(1535)$ and $N(1650)$ are obtained as a result of nonvanishing m_0 -like parameters.

The mixing scenario outlined above may look at first sight not very useful, because it involves too many new parameters. However, a quick counting shows that this is not the case. In addition to the four mass parameters $m_0^{(i,j)}$, we have the already discussed parameters $c_1, c_2, \hat{g}_1$, and $\hat{g}_2$, plus similar parameters $c_3, c_4, \hat{g}_3$, and $\hat{g}_4$ which describe the interactions of $\Psi_{3,4}$ with mesons. These 12 parameters can be used to describe the following 14 quantities: the masses of the states $N \equiv N(939), N(1535), N(1440), N(1650)$, the decay widths $N(1535) \rightarrow N\pi, N(1535) \rightarrow N\eta, N(1650) \rightarrow N\pi, N(1650) \rightarrow N\eta, N(1440) \rightarrow N\pi, N(1440) \rightarrow N\eta$ (the latter by taking into account the nonzero width of the $N(1440)$ resonance), and the four axial coupling constants $g_A^N, g_A^{N(1535)}, g_A^{N(1440)}$, and $g_A^{N(1650)}$. A detailed study of this enlarged scenario, in which the four lightest $J^P = \frac{1}{2}^\pm$ baryonic resonances are simultaneously included, will be performed in the future.

B. Outlook 2: Origin of m_0

The scattering length $a_0^{(+)}$ shows a strong dependence on the mass of the σ meson. A similar situation occurs for $\pi\pi$ scattering at low energies [10]. While a light σ is favored by the scattering data, many other studies show that the σ meson—as the chiral partner of the pion in the linear sigma model—should be placed above 1 GeV and identified with the resonance $f_0(1370)$ rather than the light $f_0(600)$ [see Refs. [35,36] and refs. therein]. Indeed, also in the framework of the linear sigma model used in this paper, the decay width $f_0(600) \rightarrow \pi\pi$ turns out to be too small when the latter is identified with the chiral partner of the pion [10].

When identifying σ with $f_0(1370)$, two possibilities are left for $f_0(600)$: (i) It is a dynamically generated state arising from the pion-pion interaction. The remaining scalar states below 1 GeV, $f_0(980)$, $a_0(980)$, and $K_0^*(800)$ can be interpreted similarly. (ii) The state $f_0(600)$ is predominantly composed of a diquark $[u, d]$ (in the flavor and color antitriplet representation) and an antidiquark $[\bar{u}, \bar{d}]$, i.e., $f_0(600) \simeq [\bar{u}, \bar{d}][u, d]$. In this case the light scalar states $f_0(600)$, $f_0(980)$, $a_0(980)$, and $K_0^*(800)$ form an additional tetraquark nonet [37–41]. Note that in both cases the resonance $f_0(600)$ —which is needed to explain $\pi\pi$ and πN scattering experiments and also to understand the nucleon-nucleon interaction potential—is *not* the chiral partner of the pion. In the following we concentrate on the implications of scenario (ii) at a qualitative level, leaving a more detailed study for the future. First, a short digression on the dilaton field is necessary.

Dilatation invariance of the QCD Lagrangian in the chiral limit is broken by quantum effects. This situation can be taken into account in the framework of a chiral model by introducing the dilaton field G [42]. The corresponding dilaton potential reflects the trace anomaly of QCD as underlying theory and has the form $V(G) \propto G^4(\log \frac{G}{\Lambda_G} - \frac{1}{4})$, where $\Lambda_G \sim \Lambda_{\text{QCD}}$ is the only dimensional quantity which appears in the full effective Lagrangian in the chiral limit. Because of the nonzero expectation value of G , a shift $G \rightarrow G_0 + G$ is necessary: the fluctuations around the minimum correspond to the scalar glueball, whose mass is placed at $M_G \sim 1.7$ GeV by lattice QCD calculations [43] and by various phenomenological studies [44]. [Beyond the chiral limit, also the parameter h_0 in Eq. (3), which describes explicit symmetry breaking due to the nonzero valence quark masses, appears as an additional dimensional quantity.]

We assume that, in the chiral limit, the full interaction potential $V(\Phi, L_\mu, R_\mu, \Psi_1, \Psi_2, G, \chi)$ is dilatation invariant up to the term $\propto \log \frac{G}{\Lambda_G}$ and that it is finite for any finite value of the fields, i.e., only terms of the kind $G^2 \text{Tr}[\Phi^\dagger \Phi]$, $\text{Tr}[\Phi^\dagger \Phi]^2$, ... are retained. By performing the shift $G \rightarrow G_0 + G$, the term $G^2 \text{Tr}[\Phi^\dagger \Phi]$ becomes $G_0^2 \text{Tr}[\Phi^\dagger \Phi] + \dots$, where the dots refer to glueball-meson interactions.

Identifying $\mu^2 \sim G_0^2$, a term $G_0^2 \text{Tr}[\Phi^\dagger \Phi]$ is already present in our Lagrangian (3), but the glueball-hadron interactions are neglected. Note that a term of the kind $G^{-4} \text{Tr}[\partial_\mu \Phi^\dagger \partial^\mu \Phi]^2$ is not allowed because of our assumption that the potential is finite. Following this line of arguments, our Lagrangian (3) cannot contain operators of order higher than four [36], because such operators must be generated from terms with inverse powers of G . E.g., upon shifting G , the above mentioned term would generate an order-eight operator of the kind $G_0^{-4} \text{Tr}[\partial_\mu \Phi^\dagger \partial^\mu \Phi]^2$.

Let us now turn to the mass term $\sim m_0$ in Eq. (5),

$$m_0(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}). \quad (36)$$

The parameter m_0 has the dimension of mass and is the only term in the baryon sector, which is not dilatation invariant. In order to render it dilatation invariant while simultaneously preserving chiral symmetry, we can couple it to the chirally invariant dilaton field G . Moreover, in the framework of $U(2)_R \times U(2)_L$ chiral symmetry also the above mentioned tetraquark field, denoted as $\chi \equiv \frac{1}{2} \times [\bar{u}, \bar{d}][u, d]$, is invariant under chiral transformations. We then write the following dilatation-invariant interaction term:

$$(a\chi + bG)(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L}), \quad (37)$$

where a and b are dimensionless coupling constants.

When shifting both fields around their vacuum expectation values $\chi \rightarrow \chi_0 + \chi$ and $G \rightarrow G_0 + G$ we recover the term (36) of our Lagrangian by identifying

$$m_0 = a\chi_0 + bG_0, \quad (38)$$

where χ_0 and G_0 are the tetraquark and gluon condensates, respectively.

Note that the present discussion holds true also in the highly excited part of the baryon sector: as described in Refs. [12,25], the heavier the baryons, the less important becomes the quark condensate φ : For two heavy chiral partners B and B^* , one expects a mass degeneracy of the form $m_B \simeq m_{B^*} \simeq m_0$. We expect the gluon condensate G_0 to be the dominant term in this sector, $m_0 \simeq bG_0$. In fact, the tetraquark condensate is also related to the chiral condensate in the vacuum [40,45] and—while potentially important for low-lying states like the nucleon and its partner—its role should also diminish when considering very heavy baryons.

We now return to the nucleon and its partner and concentrate on their interaction with the tetraquark field χ . From the point of low-energy phenomenology, the tetraquark field χ is very interesting because the corresponding excitation is expected to be lighter than the gluonium and the scalar quarkonium states, for instance $m_\chi \sim M_{f_0(600)} \sim 0.6$ GeV. A nucleon-tetraquark interaction of the kind $a\chi(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} + \bar{\Psi}_{2R}\Psi_{1L})$ arising

from Eq. (37) would then contribute to pion-pion and nucleon-pion scattering and possibly improve the agreement with experimental data.

Moreover, there is also another interesting consequence: in virtue of Eq. (37) the state χ appears as intermediate state in nucleon-nucleon interactions and, due to its small mass, is likely to play an important role in the one-meson exchange picture for the nucleon-nucleon potential. This raises the interesting question whether a *tetraquark* is the scalar state which mediates the middle-range attraction among nucleons, in contrast to the standard picture where this task is performed by a quark-antiquark state. Let us further elucidate this picture by a simple and intuitive example. Let us consider the nucleon as a quark-diquark bound state. The standard picture of one-boson exchange in the nucleon-nucleon interaction consists of exchanging the two quarks between the nucleons. However, one could well imagine that instead of the quarks one exchanges the two diquarks between the nucleons. Note that these diquarks are in the correct color and flavor antitriplet representations in order to form a tetraquark of the type suggested by Jaffe [37], such as the meson χ discussed here. A full analysis must include a detailed study of mixing between all scalar states.

As a last subject we discuss how the nucleon mass might evolve at nonzero temperature and density. In particular, in the high-density region of the so-called “quarkyonic phase” [46] hadrons are confined but chiral symmetry is (almost) restored, i.e., the chiral condensate (approximately) vanishes. What are the properties of the nucleon in this phase? In the framework of the Lagrangian (5), when $\varphi \rightarrow 0$, the masses of both the nucleon and its partner approach a constant value m_0 . Then, the first naive answer is that we expect a nucleon mass of about 500 MeV in this phase. The situation is, however, more complicated than this. In fact, as discussed in this section the term m_0 is not simply a constant but is related to other condensates. The behavior of these condensates at nonzero T and μ is then crucial for the determination of the nucleon mass. Interestingly, in Ref. [45] it is shown that the tetraquark condensate does *not* vanish but rather increases for increasing T . A future study at nonzero T and μ must include both the tetraquark and the gluon condensate in the same framework.

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APPENDIX A: VECTOR-MESON SELF-INTERACTIONS

In this appendix, we present the terms $\mathcal{L}_3$ and $\mathcal{L}_4$ of Eq. (3):

$$\begin{aligned} \mathcal{L}_3 = & -2ig_2(\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & - 2g_3(\text{Tr}[(\partial_\mu L_\nu + \partial_\nu L_\mu)\{L^\mu, L^\nu\}] \\ & + \text{Tr}[(\partial_\mu R_\nu + \partial_\nu R_\mu)\{R^\mu, R^\nu\}]), \end{aligned} \quad (\text{A1})$$

and

$$\begin{aligned} \mathcal{L}_4 = & g_4\{\text{Tr}[L^\mu L^\nu L_\mu L_\nu] + \text{Tr}[R^\mu R^\nu R_\mu R_\nu]\} \\ & + g_5\{\text{Tr}[L^\mu L_\mu L^\nu L_\nu] + \text{Tr}[R^\mu R_\mu R^\nu R_\nu]\} \\ & + g_6\text{Tr}[R^\mu R_\mu]\text{Tr}[L^\nu L_\nu] + g_7\{\text{Tr}[L^\mu L_\mu]\text{Tr}[L^\nu L_\nu] \\ & + \text{Tr}[R^\mu R_\mu]\text{Tr}[R^\nu R_\nu]\}. \end{aligned} \quad (\text{A2})$$

The coupling constants g_k with $k = 2, \dots, 7$ are not relevant for the present work.

APPENDIX B: MESON SECTOR

In the mesonic Lagrangian (3), there are ten parameters: λ_1 , λ_2 , c , h_0 , h_1 , h_2 , h_3 , μ^2 , g_1 , and m_1 . In the following, we describe how to relate them to the physical meson masses and the pion decay constant.

If chiral symmetry is spontaneously broken, the scalar-isoscalar field σ develops a nonvanishing vacuum expectation value (v.e.v.), $\langle\sigma\rangle \equiv \varphi$, the so-called chiral condensate. In order to proceed, we have to shift σ by its v.e.v., $\sigma \rightarrow \varphi + \sigma$. The chiral condensate is identified with the minimum of the potential energy density $V(\varphi)$, cf. Eq. (3):

$$V(\varphi) = \frac{1}{2}(\mu^2 - c)\varphi^2 + \frac{1}{4}\left(\lambda_1 + \frac{\lambda_2}{2}\right)\varphi^4 - h_0\varphi, \quad (\text{B1})$$

$$0 = \frac{dV}{d\varphi} = \left[\mu^2 - c + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\varphi^2\right]\varphi - h_0. \quad (\text{B2})$$

After the shift $\sigma \rightarrow \varphi + \sigma$ a mixing term between axial-vector and pseudoscalar mesons arises; for instance between a_1 -meson and pion it is of the form $-g_1\vec{a}_1^\mu \cdot \partial_\mu \vec{\pi}$. The standard way to treat this term is to eliminate it by a shift of the axial-vector fields. Then, in order to recover the canonical normalization of the pseudoscalar fields, one has to introduce a corresponding wave function renormalization factor. For a_1 -meson and pion this operation has the form

$$\begin{aligned} \vec{a}_1^\mu & \rightarrow \vec{a}_1^\mu + Zw\partial^\mu \vec{\pi}, \quad \vec{\pi} \rightarrow Z\vec{\pi}, \quad \text{where } w = \frac{g_1\varphi}{m_{a_1}^2}, \\ Z^2 & = \frac{m_{a_1}^2}{m_{a_1}^2 - (g_1\varphi)^2}. \end{aligned} \quad (\text{B3})$$

The meson masses are then given by

$$m_\sigma^2 = \mu^2 - c + 3\left(\lambda_1 + \frac{\lambda_2}{2}\right)\varphi^2, \quad (\text{B4})$$

$$m_{a_0}^2 = \mu^2 + c + \left(\lambda_1 + 3\frac{\lambda_2}{2}\right)\varphi^2,$$

$$m_{\eta_N}^2 = Z^2\left[\mu^2 + c + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\varphi^2\right], \quad (\text{B5})$$

$$m_\pi^2 = Z^2\left[\mu^2 - c + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\varphi^2\right] = \frac{Z^2 h_0}{\varphi},$$

$$m_\omega^2 = m_\rho^2 = m_1^2 + \frac{\varphi^2}{2}(h_1 + h_2 + h_3), \quad (\text{B6})$$

$$m_{f_1}^2 = m_{a_1}^2 = m_1^2 + (g_1\varphi)^2 + \frac{\varphi^2}{2}(h_1 + h_2 - h_3).$$

Note that only the linear combination $h_1 + h_2$ enters these equations, so only nine out of the original ten parameters are determined by the meson masses. However, in the following considerations, only the sum $h_1 + h_2$ will enter, so we do not need to determine h_1 and h_2 independently. We therefore have six physical meson masses in order to determine nine parameters. A seventh physical quantity is the pion decay constant, f_π , which we determine from the axial current, $J_{A\mu}^a = \frac{\varphi}{Z}\partial_\mu\pi^a + \dots \equiv f_\pi\partial_\mu\pi^a + \dots$, i.e., $\varphi = Zf_\pi$.

This leaves us with two independent parameters, which turn out to be g_1 and m_1 . The latter only enters the isospin-even pion-nucleon scattering length. We shall leave it as a free parameter to study the dependence of $a_0^{(+)}$ on m_1 .

For the sake of convenience, we shall replace the coupling constant g_1 by the pseudoscalar wave function renormalization factor Z . This is achieved with the help of the relation (B3),

$$g_1(Z) = \frac{m_{a_1}}{Zf_\pi}\sqrt{1 - \frac{1}{Z^2}}. \quad (\text{B7})$$

In this work we fix $m_{a_1} = 1.23$ GeV which is the central value quoted in Ref. [11]. In fact, it is technically easier to use Z than g_1 as independent parameter: while g_1 is a unique function of Z , the function $Z(g_1)$ would be multivalued.

It remains to determine Z . For this purpose we use the decay width $a_1 \rightarrow \pi\gamma$. The experimental value quoted by the PDG is $\Gamma_{a_1 \rightarrow \pi\gamma}^{\text{exp}} = (640 \pm 246)$ keV [11]. The theoretical expression is obtained by minimal coupling of the photon in the meson sector and only depends on Z :

$$\Gamma_{a_1 \rightarrow \pi\gamma} = \frac{\alpha}{24} m_{a_1} (Z^2 - 1) \left(1 - \frac{m_\pi^2}{m_{a_1}^2}\right)^3, \quad (\text{B8})$$

where $\alpha = 1/137$. Using the experimental value quoted above we derive $Z = 1.67 \pm 0.19$. The quantities m_1 and Z are the only independent parameters from the mesonic sector, which enter the determination of the axial coupling

constants of the nucleon and its chiral partner, the decay widths for $N^* \rightarrow NP$, and the $N\pi$ scattering lengths. Because of the large uncertainty, we shall employ Eq. (B8) together with the constraints from the baryon sector, cf. Sec. III, to perform a simultaneous fit of all relevant parameters in the baryonic sector, i.e., c_1 , c_2 , Z , and m_0 .

It should be noted that the inclusion of the axial-vector degrees of freedom is the ultimate reason which allows for a correct determination of the axial-coupling constants g_A^N and $g_A^{N^*}$. We can easily convince ourselves of this fact by assuming the contrary, i.e., studying the case where the axial-vector mesons are absent. This can be achieved either by setting g_1 to zero, or by sending the a_1 mass to infinity. In both cases, $Z = [1 - (g_1\varphi/m_{a_1})^2]^{-1/2} \rightarrow 1 + O[(g_1\varphi/m_{a_1})^2]$. Then, from Eq. (11), we obtain $g_A^{(1)} = -g_A^{(2)} = 1$, and the physical axial coupling constants are $g_A^{(N)} = -g_A^{(N^*)} = \tanh\delta \leq 1$, in contradiction to the experimental values.

The next question is, whether the experimental value of the a_1 mass is not too large compared to the “natural scale” of the problem, so that the correct description of the axial-coupling constants is impossible. The natural scale is given by the scale of chiral symmetry breaking, i.e., by the value of φ , possibly multiplied by a constant of order one. If we take the natural scale to be $g_1\varphi \sim g_1 f_\pi \approx 600$ MeV, then indeed $g_1\varphi/m_{a_1} \sim 1$, i.e., the a_1 mass is not too large compared to the natural scale of the problem. This can also be seen from the fact that the $a_1 \rightarrow \pi\gamma$ decay requires $Z \approx 1.67 > 1$, i.e., $g_1\varphi$ must be of order m_{a_1} . If m_{a_1} were large, a fit of $g_A^{(N^*)}$ to the lattice data would lead to an unnaturally large c_2 . But this problem does not emerge because m_{a_1} is not large when compared to the natural scale of the model.

APPENDIX C: DETAILS OF THE CALCULATIONS

1. Axial coupling constants

From the baryonic Lagrangian (5) we select the terms which are relevant for the derivation of the baryonic axial coupling constants:

$$\begin{aligned} \mathcal{L}^{ax} = & i\bar{\Psi}_1\gamma^\mu\partial_\mu\Psi_1 + i\bar{\Psi}_2\gamma^\mu\partial_\mu\Psi_2 - c_1\bar{\Psi}_1\gamma^\mu\gamma^5\vec{t} \\ & \cdot \vec{a}_{1\mu}\Psi_1 + c_2\bar{\Psi}_2\gamma^\mu\gamma^5\vec{t} \cdot \vec{a}_{1\mu}\Psi_2, \end{aligned}$$

in which the interactions of Ψ_1 and Ψ_2 with the a_1 -meson are retained. After performing the shift of the axial field $\vec{a}_1^\mu \rightarrow \vec{a}_1^\mu + Zw\partial^\mu\vec{\pi}$ we obtain:

$$\begin{aligned} \mathcal{L}^{ax} = & i\bar{\Psi}_1\gamma^\mu\partial_\mu\Psi_1 + i\bar{\Psi}_2\gamma^\mu\partial_\mu\Psi_2 - Zw c_1\bar{\Psi}_1\gamma^\mu\gamma^5\vec{t} \\ & \cdot \partial_\mu\vec{\pi}\Psi_1 + Zw c_2\bar{\Psi}_2\gamma^\mu\gamma^5\vec{t} \cdot \partial_\mu\vec{\pi}\Psi_2 + \dots \quad (\text{C1}) \end{aligned}$$

The axial current is calculated as

$$\begin{aligned}\mathcal{J}_{A\mu}^i &= \frac{\partial \mathcal{L}}{\partial(\partial^\mu \Psi_1)} (\delta \Psi_1)^i + \frac{\partial \mathcal{L}}{\partial(\partial^\mu \Psi_2)} (\delta \Psi_2)^i \\ &+ \frac{\partial \mathcal{L}}{\partial(\partial^\mu \pi^j)} (\delta \pi^j)^i,\end{aligned}\quad (C2)$$

where $(\delta \Psi_1)^i = i\gamma^5 t^i \Psi_1$, $(\delta \Psi_2)^i = -i\gamma^5 t^i \Psi_2$, $(\delta \pi^j)^i = \delta^{ij}(\sigma + \varphi)/Z$.

We obtain:

$$\mathcal{J}_{A\mu}^i = g_A^{(1)} \bar{\Psi}_1 \gamma_\mu \gamma^5 t^i \Psi_1 + g_A^{(2)} \bar{\Psi}_2 \gamma_\mu \gamma^5 t^i \Psi_2 + \dots, \quad (C3)$$

where, taking into account that $w = (1 - Z^{-2})/(g_1 \varphi)$:

$$\begin{aligned}g_A^{(1)} &= 1 - \varphi w c_1 = 1 - \frac{c_1}{g_1} \left(1 - \frac{1}{Z^2}\right), \\ g_A^{(2)} &= -1 + \frac{c_2}{g_1} \left(1 - \frac{1}{Z^2}\right),\end{aligned}\quad (C4)$$

which are Eqs. (11).

In order to obtain the axial coupling constants of the physical fields, we make use of Eq. (6):

$$\mathcal{J}_{A\mu}^i = g_A^N \bar{N} \gamma_\mu \gamma^5 t^i N e^\delta + g_A^{N^*} \bar{N}^* \gamma_\mu \gamma^5 t^i N^* e^{-\delta} + \dots \quad (C5)$$

where

$$\begin{aligned}g_A^N &= \frac{1}{2 \cosh \delta} (e^\delta g_A^{(1)} + e^{-\delta} g_A^{(2)}), \\ g_A^{N^*} &= \frac{1}{2 \cosh \delta} (e^{-\delta} g_A^{(1)} + e^\delta g_A^{(2)}),\end{aligned}\quad (C6)$$

which are Eqs. (10).

2. Decay widths

After the field transformation $\sigma \rightarrow \varphi + \sigma$ and (B3) discussed in Appendix B have been performed, we isolate the terms relevant for the decay $N^* \rightarrow NP$. In the following, we only discuss $P = \pi^0$, η_N , the other isospin components can be obtained similarly:

$$\begin{aligned}\mathcal{L}_{NN^*P} &= iA \bar{N}^* N P + B \bar{N}^* \gamma^\mu N \partial_\mu P - iA \bar{N} N^* P \\ &+ B \bar{N} \gamma^\mu N^* \partial_\mu P.\end{aligned}\quad (C7)$$

where

$$A = -\frac{Z(\hat{g}_1 - \hat{g}_2)}{4 \cosh \delta}, \quad B = -\frac{Zw(c_1 + c_2)}{4 \cosh \delta}. \quad (C8)$$

The decay amplitude for the process $N^* \rightarrow NP$ reads

$$-iM_{\alpha\beta} = i\bar{u}_\beta^N(\vec{k}_1) C u_\alpha^{N^*}(\vec{k} = 0), \quad (C9)$$

where $C = -iA + iB\gamma_\rho k_2^\rho$. Averaging over initial states and summing over final states, we obtain the following squared amplitude:

$$\overline{|-iM_{N^* \rightarrow NP}|^2} = \frac{1}{2} \sum_{\alpha, \beta} |-iM_{\alpha\beta}|^2 = \frac{1}{2} \sum_{\alpha, \beta} [\bar{u}_\beta^N(\vec{k}_1) C u_\alpha^{N^*}(\vec{k} = 0)] [\bar{u}_\alpha^{N^*}(\vec{k} = 0) C' u_\beta^N(\vec{k}_1)], \quad (C10)$$

with $C' = iA - iB\gamma_\rho k_2^\rho = -C$. Using the well-known properties of the traces of γ matrices leads to the result

$$\begin{aligned}\overline{|-iM|^2} &= \frac{1}{2} \sum_{\alpha, \beta} |-iM_{\alpha\beta}|^2 = \frac{1}{2} \text{Tr} \left[C \frac{\gamma^\mu k_\mu + m_{N^*}}{2m_{N^*}} C' \frac{\gamma^\nu k_{1,\nu} + m_N}{2m_N} \right] \\ &= \frac{A^2}{2} \text{Tr} \left[\frac{\gamma^\mu k_\mu + m_{N^*}}{2m_{N^*}} \frac{\gamma^\mu k_{1,\mu} + m_N}{2m_N} \right] + \frac{B^2}{2} \text{Tr} \left[\gamma_\rho k_2^\rho \frac{\gamma^\mu k_\mu + m_{N^*}}{2m_{N^*}} \gamma_\rho k_2^\rho \frac{\gamma^\mu k_{1,\mu} + m_N}{2m_N} \right] \\ &\quad - AB \text{Tr} \left[\frac{\gamma^\mu k_\mu + m_{N^*}}{2m_{N^*}} \gamma_\rho k_2^\rho \frac{\gamma^\mu k_{1,\mu} + m_N}{2m_N} \right] \\ &= \frac{A^2}{2} \left(\frac{E_N}{m_N} + 1 \right) + \frac{B^2}{2} \left[(m_{N^*}^2 - m_N^2 - m_P^2) \frac{E_P}{m_N} + m_P^2 \left(1 - \frac{E_N}{m_N} \right) \right] - AB \left(\frac{m_{N^*}^2 - m_N^2 - m_P^2}{2m_N} + E_P \right).\end{aligned}\quad (C11)$$

The full decay width is obtained including all isospin states for the pion and by replacing the unphysical state η_N with the physical η meson. The result is

$$\Gamma_{N^* \rightarrow NP} = \lambda_P \frac{k_P}{2\pi} \frac{m_N}{m_{N^*}} \overline{|-iM_{N^* \rightarrow NP}|^2}, \quad (C12)$$

which leads to Eq. (14).

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Nuclear matter within a dilatation-invariant parity doublet model: The role of the tetraquark at nonzero density

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Abstract

We investigate the role of a scalar tetraquark state for the description of nuclear matter within the parity doublet model in the mirror assignment. In the dilatation-invariant version of the model a nucleon–nucleon interaction term mediated by the lightest scalar tetraquark field naturally emerges. At nonzero density one has, beyond the usual chiral condensate, also a tetraquark condensate. The behavior of both condensates and the restoration of chiral symmetry at high density are studied. It is shown that this additional scalar degree of freedom affects non-negligibly the properties of the medium.

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1. Introduction

The properties of strongly interacting matter at finite baryon density have been widely investigated in the past by use of chiral models [1,2]. This paper aims to study the effect of a light tetraquark field on such system. This subject is interesting for two reasons: (i) many works on light mesons spectroscopy show that the light scalar resonances below 1 GeV listed in the PDG [3] can be successfully explained as a nonet of tetraquark states, see Refs. [4–9] and

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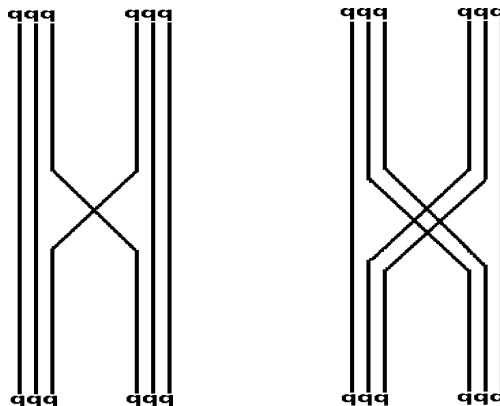


Fig. 1. The left picture represents the exchange of quarks between nucleons leading the nucleon–nucleon interactions mediated by quark–antiquark mesons. The right picture represents the exchange of diquarks leading to an additional contribution to the nucleon–nucleon potential mediated by tetraquark mesonic states.

references therein. (ii) The diquark correlation plays an important role in the nucleon [10]. It is then conceivable that two nucleons, in addition to the usual quark exchange leading to a quark–antiquark meson as intermediate state, interact via an exchange of a diquark, which leads to a tetraquark as mediator, see Fig. 1 for a pictorial representation.

In order to perform this study we use the linear sigma model for $N_f = 2$ described in Ref. [11], in which the nucleon N and its chiral partner N^* form a baryon doublet, (N, N^*) , where N^* is usually identified with the resonance $N(1535)$ [3]. The doublet is introduced in the so-called mirror assignment, first discussed in Ref. [12] and extensively analyzed in Refs. [11,13,14]. The particularity of the mirror assignment is the possibility to introduce a chirally invariant mass term $\sim m_0$, which does *not* originate from the quark condensate. In the framework of dilatation invariant interactions, this term can only originate from the condensation of two scalar and chirally invariant fields: the dilaton/glueball field and the tetraquark field. In the present work we neglect the effect of the glueball since, due to its relatively high mass of about 1.5 GeV [15], its exchange between nucleons is negligible in a first approximation. On the contrary, a light tetraquark state with a mass of about 600 MeV and identified predominantly with the resonance $f_0(600)$ is potentially very interesting for the properties of nuclear matter. In addition, the usual (pseudo)scalar and (axial-)vector quark–antiquark mesons are present in the model.

Applications of the parity doublet model to nuclear matter and neutron stars were studied in Refs. [16–19] where it was shown that, at variance with the normal linear sigma model, it is able to describe the saturation of nuclear matter and it predicts a maximum mass for neutron stars compatible with observations. Some problems however arise from these studies: the nuclear matter compressibility turns out to be larger than the measured one and the value of m_0 needed to describe saturation is large, ~ 800 MeV, and thus in disagreement with the results of the analysis of Ref. [11] in which a fit of vacuum properties resulted in $m_0 \sim 500$ MeV. Moreover the mass of the scalar–isoscalar meson responsible for the nucleon–nucleon attraction turns out to be very small, $m_\sigma \sim 350$ MeV, a value that does not correspond to any particle in the PDG.

In the present study, we remove these inconsistencies of the parity doublet model between vacuum physics and finite density physics by regarding the resonance $f_0(600)$ as a predominantly tetraquark state and the resonance $f_0(1370)$ as the chiral partner of the pion, hence a predominantly quark–antiquark state. We show that nuclear matter saturation can be correctly reproduced

with values for the compressibility compatible with data. The value of m_0 needed for fitting high density properties is the same of the one indicated by Ref. [11] in vacuum decays studies, i.e. $m_0 \sim 500$ MeV. Quite remarkably, in addition to the parity doublet model, hints for the need of having two scalar–isoscalar mesons come from the most recent Bonn parameterization of the nucleon–nucleon potential [20]: two scalar mesons are introduced having masses of ~ 500 and ~ 1200 MeV, thus very close to our assignment. One should however notice that good fits to the experimental data are obtained also by introducing only one scalar meson as done in the previous version of the Bonn potential [21] or in the Nijmegen potential [22].

The paper is organized as follows: in Section 2 we introduce the Lagrangian of the model, in Section 3 we obtain the corresponding thermodynamic potential in the mean field approximation, in Section 4 we presents our results for nuclear matter and, finally, in Section 5 we draw our conclusions.

Our units are $\hbar = c = 1$, the metric tensor is $g^{\mu\nu} = \text{diag}(+, -, -, -)$.

2. The parity doublet model in the baryon sector

We present here the chirally symmetric linear sigma model with scalar, pseudoscalar, vector, axial-vector mesons, the nucleon and its chiral partner [11]. The scalar and pseudoscalar fields are included in the matrix

$$\Phi = \sum_{a=0}^3 \phi_a t_a = (\sigma + i\eta_N)t_0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}, \quad (1)$$

where $\vec{t} = \vec{\tau}/2$, with the vector of Pauli matrices $\vec{\tau}$, and $t^0 = \mathbf{1}_2/2$. Under the global $U(2)_R \times U(2)_L$ chiral symmetry, Φ transforms as $\Phi \rightarrow U_L \Phi U_R^\dagger$, where U_L and U_R are 2×2 unitary matrices. The vector and axial-vector fields are represented by the matrices

$$V^\mu = \sum_{a=0}^3 V_a^\mu t_a = \omega^\mu t^0 + \vec{\rho}^\mu \cdot \vec{t}, \quad A^\mu = \sum_{a=0}^3 A_a^\mu t_a = f_1^\mu t_0 + \vec{a}_1^\mu \cdot \vec{t}. \quad (2)$$

From these fields we define right- and left-handed vector fields $R^\mu \equiv V^\mu - A^\mu$, $L^\mu \equiv V^\mu + A^\mu$. Under global $U(2)_R \times U(2)_L$ transformations, these fields transform as $R^\mu \rightarrow U_R R^\mu U_R^\dagger$, $L^\mu \rightarrow U_L L^\mu U_L^\dagger$.

The identification of mesons with particles listed in the PDG [3] is as follows: the fields $\vec{\pi}$ and η_N correspond to the pion and the $SU(2)$ counterpart of the η meson, $\eta_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$, with a mass of about 700 MeV. This value can be obtained by “unmixing” the physical η and η' mesons, which also contain $\bar{s}s$ contributions. The vector fields ω^μ and $\vec{\rho}^\mu$ represent the resonances $\omega(782)$ and $\rho(770)$ and the axial-vector fields f_1^μ and $\vec{a}_1^\mu$ represent the resonances $f_1(1285)$ and $a_1(1260)$. Two possibilities for the identification of the σ and $\vec{a}_0$ fields exist: $\{f_0(600), a_0(980)\}$ and $\{f_0(1370), a_0(1450)\}$. The first assignment is however unfavoured [23, 24] (for a general discussion of the issue of scalar mesons see also Ref. [25] and references therein), while the second is in agreement with the phenomenology.

The Lagrangian describing the meson fields is presented in detail in Refs. [11,23]. For our purposes we notice that: (i) The chiral condensate $\sigma_0 = \langle 0|\sigma|0 \rangle = Z f_\pi$ emerges upon spontaneous chiral symmetry breaking in the mesonic sector. The parameter $f_\pi = 92.4$ MeV is the pion decay constant and Z is the wave-function renormalization constant of the pseudoscalar fields [23,26] and takes the value $Z = 1.67 \pm 0.2$, which can be fixed from the process $a_1 \rightarrow \pi\gamma$.

(ii) In the case $N_f = 2$ only one tetraquark state χ exists. It can be coupled to the model following Refs. [9,27]. (iii) The dilaton/glueball field G can also be easily added to the meson sector according to the requirement of dilatation invariance and the corresponding trace anomaly [24]. (For a general discussion of the glueball see Refs. [25,28] and references therein.) (iv) The chiral symmetry of the Lagrangian is ensured by the fact that the glueball field G and the tetraquark χ are both invariant under chiral transformation $SU(2)_R \times SU(2)_L$ [29]. It is important to notice that no other relevant scalar and chirally invariant fields besides G and χ exist. According to lattice calculations [15] the mass of the glueball lies above 1.5 GeV, therefore it is natural – in accordance with the original idea of Ref. [4] – to identify the tetraquark field χ as the lightest scalar field with a mass of about 0.6 GeV. (v) Quark–antiquark mesons, glueballs and tetraquark states differ among each other when the limit of a large number of colors N_c is considered [30]. Quark–antiquark and glueball fields have in the large- N_c limit a constant mass and a width decreasing as $1/N_c$ and $1/N_c^2$, respectively. Tetraquark fields do not exist in the large- N_c limit, although a generalization to a state made of an antisymmetric $(N_c - 1)$ -quark configuration and $(N_c - 1)$ -antiquark configuration, which has a well-defined behavior in the large- N_c limit, is possible [31]. In the work of Ref. [32] it has indeed been shown that the lightest resonance $f_0(600)$ has a large- N_c behavior which is not compatible with a (predominantly) quark–antiquark state and is in agreement with a tetraquark configuration.

We now turn to the baryon sector, in which we have the baryon doublets Ψ_1 and Ψ_2 , where Ψ_1 has positive parity and Ψ_2 negative parity. In the mirror assignment they transform as follows:

$$\Psi_{1R} \longrightarrow U_R \Psi_{1R}, \quad \Psi_{1L} \longrightarrow U_L \Psi_{1L}, \quad \Psi_{2R} \longrightarrow U_L \Psi_{2R}, \quad \Psi_{2L} \longrightarrow U_R \Psi_{2L}, \quad (3)$$

i.e., Ψ_2 transforms in a “mirror way” under chiral transformations [12,13]. These field transformations allow to write down a chiral baryonic Lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{bar}} = & \bar{\Psi}_{1L} i \gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i \gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i \gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i \gamma_\mu D_{2L}^\mu \Psi_{2R} \\ & - \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi^\dagger \Psi_{1L}) - \hat{g}_2 (\bar{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \bar{\Psi}_{2R} \Phi \Psi_{2L}) \\ & - (a\chi + bG)(\bar{\Psi}_{1L} \Psi_{2R} - \bar{\Psi}_{1R} \Psi_{2L} - \bar{\Psi}_{2L} \Psi_{1R} + \bar{\Psi}_{2R} \Psi_{1L}), \end{aligned} \quad (4)$$

where $D_{1R}^\mu = \partial^\mu - i c_1 R^\mu$, $D_{1L}^\mu = \partial^\mu - i c_1 L^\mu$, and $D_{2R}^\mu = \partial^\mu - i c_2 R^\mu$, $D_{2L}^\mu = \partial^\mu - i c_2 L^\mu$ are the covariant derivatives for the nucleonic fields, with the coupling constants c_1 and c_2 . The interactions of the baryonic fields with the scalar and pseudoscalar mesons are parametrized by $\hat{g}_1$ and $\hat{g}_2$. The last term in Eq. (4) generates a mass term when the tetraquark field χ and the glueball field G condense:

$$m_0 = a\chi_{\text{vac}} + bG_{\text{vac}}. \quad (5)$$

In Ref. [11] the quantity m_0 has been obtained through a fit procedure to known experimental and lattice quantities, obtaining:

$$m_0 = 460 \pm 130 \text{ MeV}. \quad (6)$$

In this paper we work under the simplified assumption $b = 0$, i.e. the parameter m_0 is saturated by the tetraquark condensate and the corresponding nucleon–nucleon interaction is depicted in Fig. 1, right side. In this context the coupling constant a is fixed as soon as the tetraquark condensate is specified, see next section. The physical fields N and N^* are related to the spinors Ψ_1 and Ψ_2 through:

$$\Psi_1 = \frac{1}{\sqrt{2 \cosh \delta}} (N e^{\delta/2} + \gamma_5 N^* e^{-\delta/2}), \quad \Psi_2 = \frac{1}{\sqrt{2 \cosh \delta}} (\gamma_5 N e^{-\delta/2} - N^* e^{\delta/2}). \quad (7)$$

The masses of the nucleons are obtained by diagonalizing the corresponding mass matrix in the Lagrangian. As function of the two condensates σ_{vac} and χ_{vac} they read:

$$m_{N,N^*} = \sqrt{\left(\frac{\widehat{g}_1 + \widehat{g}_2}{4}\right)^2 \sigma_{vac}^2 + (a\chi_{vac})^2} \pm \frac{\widehat{g}_1 - \widehat{g}_2}{4} \sigma_{vac}. \quad (8)$$

In Ref. [11] this model has been used to study the properties of the vacuum, such as πN scattering lengths and decay widths. In this respect it is important to recall that if the axial-vector mesons are neglected, the nucleon and its partner have the same axial charge in modulus but opposite sign. However, by correctly including (axial-)vector mesons, this is no longer true and we are free to adjust the two axial charges independently, employing experimental knowledge about the axial charge and the recent lattice QCD data reported in Ref. [33]. In this work, axial-vector mesons play indeed an indirect but important role through the parameter Z .

3. Mean field approximation

We now turn to the effective model in the mean field approximation suitable for describing nuclear matter. We thus retain only those fields which are relevant for such a study: the scalar fields σ and χ and the vector field ω . The effective Lagrangian is given by:

$$\begin{aligned} \mathcal{L}_{eff} = & \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{4} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu)^2 \\ & + \frac{1}{2} m^2 \sigma^2 + \frac{1}{2} m_1^2 \omega_\mu \omega^\mu - \frac{1}{2} m_\chi^2 \chi^2 - \frac{\lambda}{4} \sigma^4 + g \chi \sigma^2 + \varepsilon \sigma \\ & + \bar{\Psi}_1 i \gamma_\mu \partial^\mu \Psi_1 + \bar{\Psi}_2 i \gamma_\mu \partial^\mu \Psi_2 - \frac{\widehat{g}_1}{2} \bar{\Psi}_1 \sigma \Psi_1 - \frac{\widehat{g}_2}{2} \bar{\Psi}_2 \sigma \Psi_2 \\ & - g_\omega^{(1)} \bar{\Psi}_1 i \gamma_\mu \omega^\mu \Psi_1 - g_\omega^{(2)} \bar{\Psi}_2 i \gamma_\mu \omega^\mu \Psi_2 + a \chi (\bar{\Psi}_2 \gamma_5 \Psi_1 - \bar{\Psi}_1 \gamma_5 \Psi_2). \end{aligned} \quad (9)$$

If we take their vacuum expectation values, $\sigma \rightarrow \sigma + \sigma_{vac}$ and $\chi \rightarrow \chi + \chi_{vac}$, the potential for the fields σ and χ reads:

$$V(\sigma, \chi) = \frac{1}{2} \begin{pmatrix} \chi & \sigma \end{pmatrix} \begin{pmatrix} m_\chi^2 & -2g\sigma_{vac} \\ -2g\sigma_{vac} & m_\sigma^2 \end{pmatrix} \begin{pmatrix} \chi \\ \sigma \end{pmatrix}.$$

A non-diagonal term, which is proportional to the parameter g and mixes the quarkonium field σ and the tetraquark field χ , has emerged [34]. The physical fields, denoted as h and s , are obtained through a standard diagonalization:

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos \theta_0 & \sin \theta_0 \\ -\sin \theta_0 & \cos \theta_0 \end{pmatrix} \begin{pmatrix} \chi \\ \sigma \end{pmatrix}, \quad \theta_0 = \frac{1}{2} \arctan \frac{4g\sigma_{vac}}{m_\sigma^2 - m_\chi^2}.$$

Then the physical masses of the scalar states are:

$$\begin{aligned} m_h^2 &= m_\chi^2 \cos^2 \theta_0 + m_\sigma^2 \sin^2 \theta_0 - 2g\sigma_{vac} \sin(2\theta_0), \\ m_s^2 &= m_\sigma^2 \cos^2 \theta_0 + m_\chi^2 \sin^2 \theta_0 + 2g\sigma_{vac} \sin(2\theta_0). \end{aligned} \quad (10)$$

We identify here the predominately tetraquark state h with $f_0(600)$ (the vacuum mass $m_h = 600$ MeV is used) and the predominately quarkonium state s with $f_0(1370)$ (the vacuum mass $m_s = 1300$ MeV is used). We now turn to the thermodynamic potential in the mean field approximation:

$$\frac{\Omega}{V} = -\mathcal{L}_M + \sum_i \frac{\gamma_i}{(2\pi)^3} \int_0^{p_{F_i}} d^3 p (E_i^*(p) - \mu_i^*),$$

where the mesons term reads

$$\mathcal{L}_M = \frac{1}{2} m_1^2 \omega_0^2 + \frac{1}{2} m^2 \sigma^2 - \frac{\lambda}{4} \sigma^4 + \epsilon \sigma + g \chi \sigma^2 - \frac{1}{2} m_\chi^2 \chi^2.$$

The index $i = N, N^*$ denotes the nucleon type (positive and negative parity nucleons), $\gamma_i = 2 \times 2$ is the fermionic degeneracy (spin and isospin), p_{F_i} are the Fermi momenta, $E_i^*(p) = \sqrt{p_i^2 + m_i^2}$, and $\mu_i^* = \mu_i - g_\omega \omega_0 = \sqrt{p_{F_i}^2 + m_i^2}$. The single particle energy of each parity partner i is given by $E_i(p) = E_i^*(p) + g_\omega \omega_0$. The mean mesons fields are obtained by minimizing the thermodynamic potential, i.e. by solving the following system of non-linear equations:

$$\begin{aligned} \frac{\partial(\Omega/V)}{\partial \sigma} &= \lambda \sigma^3 - m^2 \sigma - \epsilon - 2g \chi \sigma + \rho_N^*(\sigma, \omega_0, \chi) \frac{\partial m_N}{\partial \sigma} + \rho_{N^*}^*(\sigma, \omega_0, \chi) \frac{\partial m_N^*}{\partial \sigma} = 0, \\ \frac{\partial(\Omega/V)}{\partial \omega} &= m_\omega^2 \omega_0 - g_{\omega, N} \rho_N(\sigma, \omega_0, \chi) - g_{\omega, N^*} \rho_{N^*}(\sigma, \omega_0, \chi) = 0, \\ \frac{\partial(\Omega/V)}{\partial \chi} &= -g \sigma^2 + m_\chi^2 \chi + \rho_N^*(\sigma, \omega_0, \chi) \frac{\partial m_N}{\partial \chi} + \rho_{N^*}^*(\sigma, \omega_0, \chi) \frac{\partial m_N^*}{\partial \chi} = 0. \end{aligned} \quad (11)$$

The scalar densities ρ_i^* and the baryon densities ρ_i are given by the expressions:

$$\rho_i^* = \gamma_i \int_0^{p_{F_i}} \frac{d^3 \vec{p}}{(2\pi)^3} \frac{m_i}{\sqrt{p_i^2 + m_i^2}} = \frac{m_i}{\pi^2} \left[p_{F_i} E_{F_i}^* - m_i^2 \ln \left(\frac{p_{F_i} + E_{F_i}^*}{m_i} \right) \right], \quad (12)$$

$$\rho_i = \gamma_i \int_0^{p_{F_i}} \frac{d^3 \vec{p}}{(2\pi)^3} = 2 \frac{p_{F_i}^3}{3\pi^2}. \quad (13)$$

The parameters of the model are given by [23,27]:

$$\begin{aligned} \lambda &= \frac{1}{2(Zf_\pi)^2} \left(m_\sigma^2 - \frac{m_\pi^2}{Z^2} \right) + \frac{2g^2}{m_\chi^2}, \quad m^2 = \frac{1}{2} \left(m_\sigma^2 - 3 \frac{m_\pi^2}{Z^2} \right), \\ \epsilon &= \frac{f_\pi m_\pi^2}{Z}, \quad a = \frac{m_0 m_\chi^2}{g(Zf_\pi)^2}. \end{aligned} \quad (14)$$

Numerically one has $m_\pi = 139$ MeV and $f_\pi = 92.4$ MeV. The parameter Z is the wave-function renormalization of the pseudoscalar fields [23,26]. It is the only ‘remnant’ of the (axial-)vector mesons of the model. However, it should be stressed that the value of $Z = 1.67$ strongly affects the properties of nuclear matter, for instance reducing the value of the chiral symmetry explicit breaking term, as we will show in the following. As already discussed in Ref. [23], the presence of (axial-)vector degrees of freedom in a chiral framework has non-negligible consequences on the (pseudo)scalar sector as well. The coupling constants of the nucleons and the scalar σ meson are uniquely determined by the vacuum properties, $m_0 = 460 \pm 130$ MeV, $m_{N_{vac}} = 938$ MeV and $m_{N_{vac}^*} = 1535$ MeV: $\hat{g}_1 = 11.0 \pm 1.5$, $\hat{g}_2 = 18.8 \pm 2.4$ [11]. Finally, g parametrizes the interactions of the tetraquark field with pions and the parameters $g_\omega^{(1)}$, $g_\omega^{(2)}$ describe the interactions of

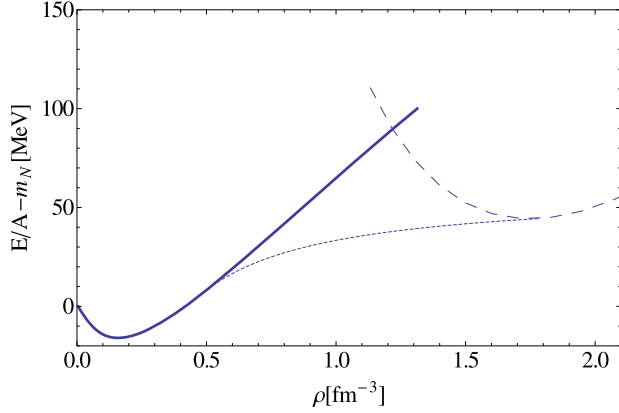


Fig. 2. Energy per baryon as a function of the density for the case $m_0 = 500$ MeV for the chiral symmetry broken solution (solid line) and the chiral symmetry restored solution (dashed line). The dotted line represents the Maxwell construction which matches the two solutions. The model can describe the saturation of nuclear matter.

the ω meson with the nucleons. For sake of simplicity we assume that $g_\omega^{(1)} = g_\omega^{(2)} = g_\omega$ and thus, $g_{\omega,N} = g_{\omega,N^*}$. The quantities g and g_ω are obtained from the following constraints of nuclear matter at saturation:

$$\left. \frac{\partial}{\partial p_{F_N}} \left(\frac{E}{A} - m_{N_{vac}} \right) \right|_{p_{F_N}=p_{F_0}} = 0 \quad \text{and} \quad \left. \left(\frac{E}{A} - m_{N_{vac}} \right) \right|_{p_{F_N}=p_{F_0}} = -16 \text{ MeV}, \quad (15)$$

where E/A is the energy per nucleon and p_{F_N} is the Fermi momentum of N . At the saturation point the latter equals the value $p_{F_N} = p_{F_0} = 258$ MeV.

4. Results

Let us fix m_0 to the intermediate value $m_0 = 500$ MeV. From Eqs. (10) and (15) we can determine the four free parameters of the model which turn out to be: $g_\omega = 4.87$, $g = 450$ MeV, $m_\sigma = 1294$ MeV, $m_\chi = 612$ MeV (the latter two quantities generate the previously mentioned masses $m_h = 600$ MeV and $m_s = 1300$ MeV). Quite remarkably the values of the masses of the two scalar mesons of our model are very close the values obtained within the “CD-Bonn parametrization” of the nucleon–nucleon potential extracted by scattering data [20]. In Fig. 2, we show the energy per baryon E/A as a function of the baryon density ρ : one can notice the typical feature of the saturation of nuclear matter as a minimum of E/A located at $\rho = \rho_0 = 0.16 \text{ fm}^{-3}$ and with a nucleon binding energy of 16 MeV.

The value of the compressibility, computed through the derivative of the pressure P with respect to the density:

$$K = 9 \left. \frac{\partial P}{\partial \rho} \right|_{\rho=\rho_0} = 194 \text{ MeV}, \quad (16)$$

a value which is very close to the standard range indicated by phenomenology $K = 200\text{--}300$ MeV [35,36]. In the figure we also display the energy per baryon for the chiral symmetry restored solution of the mean field equations (dashed line) and the interpolating Maxwell construction (dotted line) which we will explain in the following.

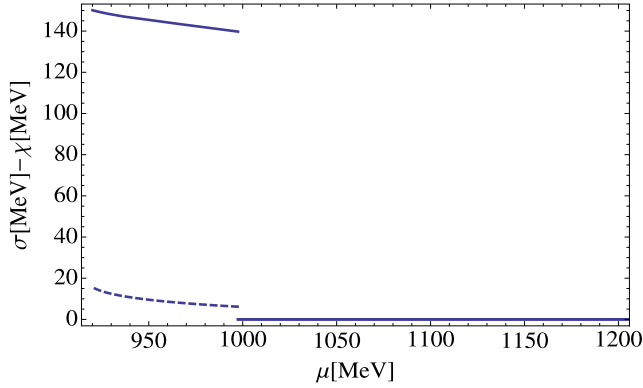


Fig. 3. Mean σ (solid line) and χ (dashed line) fields. A strong first order phase transition is present at $\mu \sim 1$ GeV. The transition point is the same for the two fields. Note, the minimal value for the chemical potential corresponds to nuclear matter.

At high densities, our model predicts a strong first order chiral phase transition occurring at a chemical potential $\mu \sim 1$ GeV as displayed in Fig. 3 where we show the mean scalar mesons fields (σ with solid line and χ with dashed line). The big jump in both the σ and χ fields is computed by using a Maxwell construction: at a fixed value of the chemical potential the mean mesons fields are the solutions of Eqs. (11) which minimize the thermodynamic potential. In the energy per baryon-density diagram, the Maxwell construction corresponds to a hyperbolic branch (see Fig. 2, dotted line) given by the thermodynamic equation relating the pressure and the energy density: $e/\rho = \mu_{crit} - p_{crit}/\rho$ where μ_{crit} and p_{crit} are the critical chemical potential and pressure. Notice that in absence of the tetraquark field and without the corrections related to Z [17], the phase transition, while being still a first order, is much weaker. The parameter $Z = 1.67$ indeed suppresses the chiral symmetry explicit breaking term and enhances the vacuum expectation value of σ , thus making the results for the chiral phase transition similar to the ones obtained in the chiral limit. In our model the chiral phase transition is accompanied by the appearance of the chiral partners of the nucleon. While at low density only the nucleons are present, after the phase transitions both, the nucleons and their chiral partners, contribute the same amount. As one can notice in Fig. 4, in which we show the masses of the nucleons and their chiral partners as a function of the chemical potential, the nucleons become massless after the phase transition. This result is different from the one of Ref. [17] in which the nucleon mass, at high density, saturates to the fixed value of m_0 . On the other hand, in our model, m_0 itself is a function of the density and at the chiral phase transition it vanishes since the tetraquark condensate vanishes (see Eq. (5)).

As a further step in our study we investigate the effect of varying the parameter m_0 . For each value of m_0 we compute the values of the four parameters described before: while we can describe saturation for the whole range of Eq. (6), the value of K turns out to be very small for small values of m_0 . We show the corresponding result in Fig. 5 together with the range of K indicated by phenomenology (dashed lines). By combining the information of Eq. (6) and the constraint obtained from the value of K we can further restrict the range of the allowed values for the “bare mass” of the nucleon $500 \lesssim m_0 \lesssim 600$ MeV.

Finally, in Fig. 6, we show the values of the critical densities ρ_1 and ρ_2 which correspond to the onset and the end of the chiral phase transition. One can notice that the strength of the phase

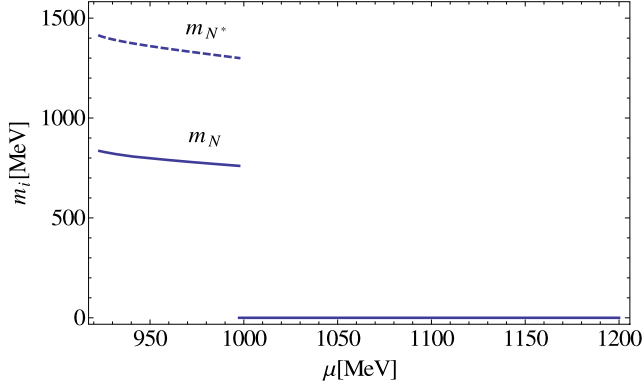


Fig. 4. Masses of the nucleons (solid line) and their chiral partners (dashed line) as function of the chemical potential.

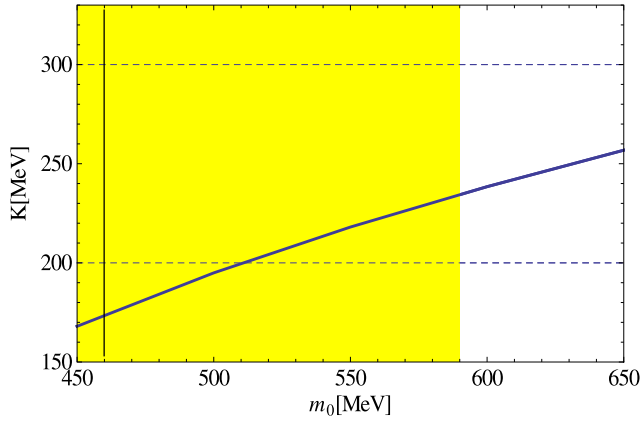


Fig. 5. Compressibility as a function of m_0 (solid line) and allowed range for K as obtained by phenomenology (dashed lines). The constraint on the compressibility, together with vacuum decays properties, fixes m_0 to be in the range 500–600 MeV. The result (460 ± 130) MeV is shown (gray (yellow online) band).

transition decreases for large values of m_0 ; within the allowed range of m_0 the density jump at the phase transition is anyway large: $\rho_2/\rho_1 = 2\text{--}4$. Remarkably, recent lattice results [37] show for a temperature $T \sim 0.8T_c$ a rather broad range of density for the mixed phase, $\rho_2/\rho_1 \sim 2$. Such studies are presently limited for temperatures $T \gtrsim 0.8T_c$, therefore a quantitative comparison with our results is premature. Nevertheless, the lattice simulations seem to indicate that the interval for the mixed increase further when diminishing the temperature T . Future detailed analysis on this issue is surely interesting.

5. Conclusions

In this work we have used the chiral model developed in Ref. [11] to study nuclear matter properties. The model contains, in addition to the standard quark–antiquark fields with (pseudo)scalar and (axial-)vector quantum numbers, also a light scalar–isoscalar tetraquark field. In the framework of the mirror assignment the latter can be easily coupled to the nucleon by

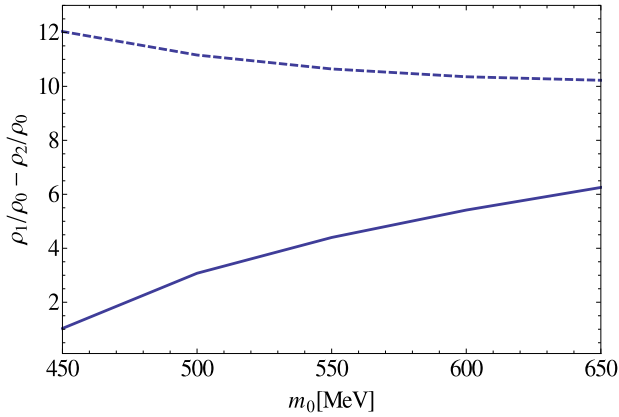


Fig. 6. Critical densities associated with the chiral phase transition as functions of m_0 . ρ_1 (solid line) corresponds to the onset of the phase transition and ρ_2 (dashed line) to the end (the densities are referred to the saturation density ρ_0). The first order phase transition becomes weaker for large values of m_0 .

using the general requirement of dilatation invariant interactions and contributes to the nucleon–nucleon interaction, see Fig. 1.

Similarly to the results of Ref. [27] at finite temperature and vanishing density, we find that also at nonzero density a light tetraquark field has a strong influence on the medium properties of the system due to the interplay of two condensates, the tetraquark and the chiral (quark–antiquark) condensates. Interestingly, the described scenario and also the values of the coupling constants are in agreement with the work of Ref. [20], where two scalar fields are needed to describe nucleon–nucleon scattering data. As described in Ref. [31], in such a scenario nuclear matter is a prerogative of our world with three colors ($N_c = 3$), but would cease to exist as soon as a larger number of colors is considered.

An important parameter of the model is m_0 , which describes the contribution to the nucleon mass which does not stem from the chiral condensate and which, in the present study, is saturated by the tetraquark condensate. Its value has been fixed in the vacuum as $m_0 = 460 \pm 130$ MeV. It is then remarkable that the compressibility is in agreement with the experiment in a compatible range of m_0 (see Fig. 5), thus showing that vacuum results and nonzero density properties can be understood within the same theoretical model.

Further studies along the direction of the present work can be performed: (i) the contribution of the gluon condensate to m_0 and of the dilaton/glueball field to the nucleon–nucleon interaction should be included. Although the glueball is heavier (~ 1.5 GeV) and should not affect the interaction of two nucleons, its inclusion is important in a theoretical framework which is based on the dilatation invariance of QCD and on its anomalous breaking. (ii) The study of asymmetric nuclear matter and its application to neutron stars is also important since, recently, very massive neutron stars have been discovered which can give useful constraints for the stiffness of the equation of state. Moreover, also the symmetry energy of nuclear matter at high density is a crucial quantity that we can investigate by introducing in our model the scalar isovector $a_0(980)$, as done in Ref. [38], which in our assignment is also a tetraquark. (iii) The extension of the model to $N_f = 3$ is currently under investigation. The effects of nonzero density and temperature of the complete model with all the relevant degrees of freedoms represents an interesting outlook of the present work.

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Role of the tetraquark in the chiral phase transition

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We investigate the implications of a light tetraquark field on chiral symmetry restoration at nonzero temperature within a simple chirally symmetric model. In order for the chiral phase transition to be crossover, as shown by lattice QCD studies, a strong mixing between scalar quarkonium and tetraquark fields is required. This leads to a light (~ 0.4 GeV), predominantly tetraquark state, and a heavy (~ 1.2 GeV), predominantly quarkonium state in the vacuum, in accordance with recently advocated interpretations of spectroscopy data. The mixing even increases with temperature and leads to an interchange of the roles of the originally heavy, predominantly quarkonium state and the originally light, predominantly tetraquark state. Then, as expected, the scalar quarkonium is a light state when becoming degenerate in mass with the pion as chiral symmetry is restored at nonzero temperature.

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I. INTRODUCTION

In the last 30 years, theoretical and experimental work on the light scalar mesons with mass below ~ 1.8 GeV initiated an intense debate about their nature. The issue is that too many scalar resonances have been identified than can be accommodated in a naive quark-antiquark picture. For instance, in the scalar-isoscalar channel there are five states: $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ [1]. In order to explain their nature, quarkonia, tetraquark, and mesonic molecular assignments, as well as a scalar glueball state with mass around 1.5 GeV suggested by lattice studies of quantum chromodynamics (QCD) [2], have been investigated in a variety of combinations and mixing patterns [3–6]. Yet, a clear answer to the question of which resonance should be dominantly identified as a scalar quarkonium (i.e., quark-antiquark) state is not at hand.

Nowadays evidence for a full nonet of scalars with mass below 1 GeV is mounting; these are the already mentioned isoscalars $f_0(600)$ and $f_0(980)$, as well as the isotriplet $a_0(980)$, and the two isodoublets of $K_0^*(800)$. As proposed long ago by Jaffe [7], a tetraquark assignment for these states can explain some puzzling properties, such as the mass ordering which is reversed compared to the expectation from a quark-antiquark picture, and the strong coupling of $a_0(980)$ and $f_0(980)$ to kaons [8]. Within this context the lightest scalar resonance $f_0(600)$ is interpreted as a tetraquark state $\frac{1}{2}[u, d][\bar{u}, \bar{d}]$, where the commutator indicates an antisymmetric flavor (as well as color) configuration of the diquark. Further indications of a non-quarkonium nature of the scalar states below 1 GeV, and thus of $f_0(600)$, are obtained from a large- N_c study in the framework of unitarized chiral perturbation theory [9] and in the lattice studies of Ref. [10].

If the light scalars are (predominantly) tetraquark states, the question is how to identify the lightest quarkonium

state $\bar{n}n = 1/\sqrt{2}(\bar{u}u + \bar{d}d)$: the broad resonance $f_0(1370)$ is the first candidate. This assignment is also supported by the fact that the scalar quarkonia are p -wave states, and thus expected to lie above 1 GeV together with other p -wave quarkonia such as axial-vector and tensor mesons. The two isoscalars of the quarkonia nonet and the scalar glueball can mix, forming the states $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$; such scenarios have been discussed in Refs. [11]. While it is not clear if $f_0(1500)$ or $f_0(1710)$ carries the largest glueball amount, all the above cited works agree in the assignment of a dominant $\bar{n}n$ component to the resonance $f_0(1370)$.

For N_f massless quark flavors, the QCD Lagrangian [including effects from the $U(1)_A$ anomaly [12]] has a chiral $SU(N_f)_r \times SU(N_f)_l \times U(1)_V$ symmetry, $V = r + l$. If this symmetry is linearly realized, the mass eigenstates of QCD come in degenerate pairs, so-called chiral partners, which have the same quantum numbers except for parity and G-parity; e.g., the chiral partners of the scalar-isoscalar meson are the pseudoscalar isotriplet mesons, i.e., the pions. The chiral symmetry is spontaneously broken to $SU(N_f)_V$ in the vacuum [13], thus lifting the degeneracy of the chiral partners and rendering the pions Goldstone bosons. Since the pion is commonly regarded to be a quark-antiquark state, and if the resonance $f_0(1370)$ is predominantly a quarkonium state, the latter should be considered as the chiral partner of the pion, and not $f_0(600)$, as is usually assumed. Consequently, if the chiral symmetry of QCD is restored above some critical temperature T_c , as predicted by lattice QCD calculations [14], the resonance $f_0(1370)$, and not $f_0(600)$, should become degenerate in mass with the pion. Clarifying this issue is important for the interpretation of data from heavy-ion-collision experiments, whose major goal is to identify signatures for chiral symmetry restoration at nonzero temperature T .

The aim of this paper is to take a first step towards investigating this scenario of chiral symmetry restoration. We employ the toy model discussed in Ref. [6], which is the $N_f = 2$ limit of a more general chiral Lagrangian for $N_f = 3$. This model contains only one tetraquark field in addition to the scalar quarkonium field and the pions. Although the other mentioned scalar-isoscalar resonances $f_0(980)$, $f_0(1500)$, and $f_0(1710)$ are not included at the present stage, this model has all the essential features to analyze the role of the tetraquark and its mixing with the quarkonium at nonzero T . To this end we employ the Cornwall-Jackiw-Tomboulis (CJT) formalism [15] in the Hartree-Fock approximation [16]. We shall show that, as T increases, the $f_0(1370)$ becomes lighter and its tetraquark admixture grows. Within our model calculation, we also find that, for a large range of parameters, there exists a certain temperature $T_s \leq T_c$ above which the state that is predominantly quarkonium becomes lighter than the state that is mostly tetraquark. At and above T_c , the state which is predominantly quarkonium becomes degenerate with the pion, as expected for chiral symmetry restoration.

II. THE MODEL

We consider the pion triplet $\vec{\pi}$, the bare quarkonium field $\varphi \equiv \bar{n}n$, and the bare tetraquark field $\chi \equiv \frac{1}{2}[u, d][\bar{u}, \bar{d}]$. The potential defining our model emerges as the $SU(2)_r \times SU(2)_l$ limit of a more general $SU(3)_r \times SU(3)_l$ chiral invariant Lagrangian studied in Ref. [6] and reads explicitly

$$V = \frac{\lambda}{4}(\varphi^2 + \vec{\pi}^2 - F^2)^2 - \varepsilon\varphi + \frac{1}{2}M_\chi^2\chi^2 - g\chi(\varphi^2 + \vec{\pi}^2), \quad (1)$$

where ε parametrizes explicit chiral symmetry breaking by nonzero quark masses and g , chosen to be ≥ 0 , is the interaction strength of the tetraquark field χ [which is a singlet under $SU(2)_r \times SU(2)_l$] with the quarkonia fields. As we shall see, g also determines the mixing of the scalar fields. When $g \rightarrow 0$ a simple linear sigma model for φ and $\vec{\pi}$ is left. In fact, the field χ , with mass M_χ , decouples in this limit. The minimum of the potential (1) is, to order $O(\varepsilon)$, assumed for

$$\varphi_0 \simeq \frac{F}{\sqrt{1 - 2g^2/(\lambda M_\chi^2)}} + \frac{\varepsilon}{2\lambda F^2}, \quad \chi_0 = \frac{g}{M_\chi^2} \varphi_0^2, \quad (2)$$

and $\vec{\pi} = 0$. The $\bar{n}n$ condensate φ_0 is identified with the pion decay constant $f_\pi = 92.4$ MeV. Note that the vacuum expectation value (vev) χ_0 is proportional to φ_0^2 . Thus, the tetraquark condensate χ_0 is induced by the spontaneous symmetry breaking in the quarkonium sector. After shifting the fields $\varphi \rightarrow \varphi_0 + \varphi$ and $\chi \rightarrow \chi_0 + \chi$ and expanding the potential around the minimum, we obtain up to second order in the fields

$$V = \frac{1}{2}(\chi, \varphi) \begin{pmatrix} M_\chi^2 & -2g\varphi_0 \\ -2g\varphi_0 & M_\varphi^2 \end{pmatrix} \begin{pmatrix} \chi \\ \varphi \end{pmatrix} + \frac{1}{2}M_\pi^2\vec{\pi}^2 + \dots, \quad (3)$$

where $M_\varphi^2 = \varphi_0^2(3\lambda - \frac{2g^2}{M_\chi^2}) - \lambda F^2$, $M_\pi^2 = \frac{\varepsilon}{\varphi_0}$. The value $M_\pi = 0.139$ GeV is used. Because of the off diagonal terms in the mass matrix in Eq. (3), the fields φ and χ are not mass eigenstates of the potential V . The latter, denoted by (H, S) , are obtained after an $SO(2)$ rotation of the fields (φ, χ)

$$\begin{pmatrix} H \\ S \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} \chi \\ \varphi \end{pmatrix}, \quad (4)$$

where $\theta_0 = \frac{1}{2} \arctan \frac{4g\varphi_0}{M_\varphi^2 - M_\chi^2}$. The tree-level masses of H and S are

$$M_H^2 = M_\chi^2 \cos^2\theta_0 + M_\varphi^2 \sin^2\theta_0 - 2g\varphi_0 \sin(2\theta_0),$$

$$M_S^2 = M_\varphi^2 \cos^2\theta_0 + M_\chi^2 \sin^2\theta_0 + 2g\varphi_0 \sin(2\theta_0).$$

Assuming $-\pi/4 \leq \theta_0 \leq \pi/4$, the state H is then predominantly tetraquark and S is predominantly quarkonium. As discussed above, we shall identify the state H with the resonance $f_0(600)$ and the state S with $f_0(1370)$. A natural choice is then that the pure tetraquark should be lighter than the pure quarkonium, i.e., $M_\chi < M_\varphi$. Using the fact that the trace and the determinant of the mass matrices before and after the $SO(2)$ rotation are equal, we obtain $(M_S^2 - M_H^2)^2 = (M_\varphi^2 - M_\chi^2)^2 + (4g\varphi_0)^2$, implying that for $g > 0$ the masses of the states H and S repel each other: $M_H < M_\chi < M_\varphi < M_S$. We also obtain the constraint $|M_S^2 - M_H^2| \geq 4g\varphi_0$.

As a side remark, we mention that the tree-level decay width of $f_0(600)$ is larger than 300 MeV when the mass lies above 0.6 GeV and when the mixing is large. However, we refrain from a more elaborate study of vacuum properties, since a realistic description of the latter requires the inclusion of other scalar states and of (axial-)vector mesons [17–19] and we concentrate on the behavior at nonzero T .

III. RESULTS

In order to study chiral symmetry restoration at nonzero T we employ the CJT-formalism in the Hartree-Fock approximation; for details see Ref. [16]. As a result, the masses $M_H(T)$, $M_S(T)$, $M_\pi(T)$, and the mixing angle $\theta(T)$ become functions of T . Moreover, both scalar-isoscalar fields attain T -dependent vev's, $\varphi_0 \rightarrow \varphi(T)$ for the quarkonium and $\chi_0 \rightarrow \chi(T)$ for the tetraquark state, respectively, with $\varphi(0) = \varphi_0$ and $\chi(0) = \chi_0$; see Eq. (2).

We first study the order of the chiral phase transition and the associated T_c as a function of the model parameters g , $M_H \equiv M_H(0)$, and $M_S \equiv M_S(0)$. Figure 1(a) shows the phase diagram in the g - M_S plane for fixed $M_H = 0.4$ GeV which is close to the value of Ref. [20], while Fig. 1(b) depicts the g - M_S plane for fixed $M_S = 1.2$ GeV which is in the experimentally established range of values for $f_0(1370)$ [1].

One observes in Fig. 1(a) that, in the limit $g \rightarrow 0$ in which S is a pure quarkonium, the transition is crossover below a value $M_S \simeq 0.948$ GeV and of first order above this value. The fact that a heavy chiral partner of the pion

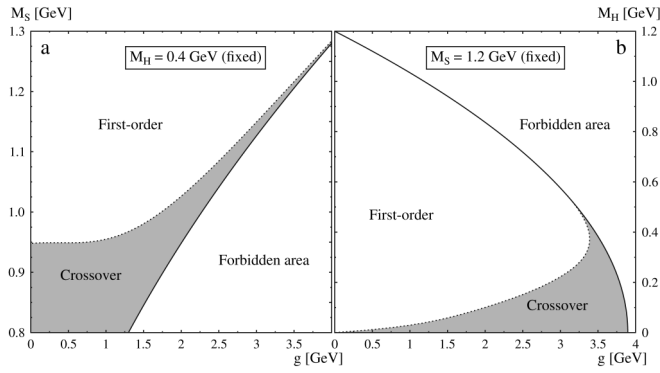


FIG. 1. Order of the phase transition as a function of the parameters of the model. The forbidden area violates the constraint $|M_S^2 - M_H^2| \geq 4g\varphi_0$. On the border line between the first-order and the crossover transitions a second-order phase transition is realized.

induces a first-order chiral phase transition has been discussed previously; see e.g. Ref. [18]. This means that, in a linear sigma model without tetraquark degrees of freedom, a heavy (i.e., mass larger 1 GeV) chiral partner of the pion is excluded by lattice QCD calculations [14], which indicate a crossover transition. Including tetraquarks dramatically changes this conclusion: as one observes in Fig. 1(a), for increasing g the region of crossover transitions extends towards larger values of M_S . Thus, the scenario outlined in the Introduction, in which $H \equiv f_0(600)$ and $S \equiv f_0(1370)$, can accommodate for a crossover transition if g , and thus the mixing between quarkonium and tetraquark, is sufficiently large. Note that the range of g values for which the transition is crossover, narrows substantially when M_S increases. Along the line of second-order phase transitions in Fig. 1(a), T_c sizably decreases for increasing g , for instance from $T_c \simeq 241$ MeV at $g = 0$ to $T_c \simeq 186$ MeV at $g = 3$ GeV and $T_c \simeq 173$ MeV at $g = 4$ GeV. We observe in Fig. 1(b) that a crossover transition occurs only for small values of M_H . The crossover region widens when g increases. In order to accommodate a value ~ 0.4 GeV [20], a large value of g is required. Along the line of second-order transitions, T_c first decreases, and then increases for increasing g . The minimum $T_c \simeq 145$ MeV occurs for $g \simeq 2$ GeV.

We now study the T -dependence of masses, condensates [21], and the mixing angle in more detail in the case of $M_H = 0.4$ GeV and $M_S = 1.2$ GeV (in the range quoted by Refs. [1,9,20]; a mass $M_H \sim 0.4$ GeV, although leading to a too small tree-level decay width due to lack of phase space, allows for a nice illustrative description of the qualitative features of the nonzero T). Also, we set the coupling strength $g = 3.4$ GeV, in order to obtain a crossover phase transition in agreement with lattice QCD calculations [14]. These parameter values lead (together with values $\varphi_0 = f_\pi = 92.4$ MeV and $M_\pi = 0.139$ GeV) to $M_\chi = 0.82$ GeV, $M_\varphi = 0.96$ GeV, and $\lambda = 52.85$.

The condensates $\varphi(T)$ and $\chi(T)$ are shown in Fig. 2(a). The quark condensate $\varphi(T)$ drops at T_c and then ap-

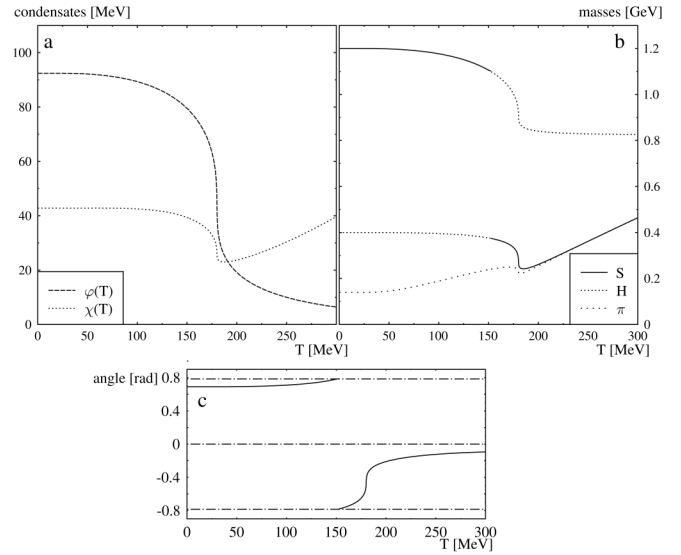


FIG. 2. Condensates (panel a), masses (panel b), and mixing angle (panel c) as functions of T .

proaches zero, signaling the restoration of chiral symmetry. The tetraquark condensate $\chi(T)$ first drops together with $\varphi(T)$, but increases above T_c : this is due to the fact that in the equation determining $\chi(T)$, the growth of the S and H tadpole contributions with T has to be balanced by an increase of $\chi(T)$; this could be different if we include additional terms $\sim \chi^4$ in the potential (1). In any case, the increase of $\chi(T)$ affects the behavior of the masses or of other physical quantities only slightly. Note that the field χ is a singlet under chiral transformations in the $N_f = 2$ case and therefore the nonzero value of the condensate does not imply a breaking of chiral symmetry at high T .

The T -dependent masses $M_H(T)$, $M_S(T)$, $M_\pi(T)$ of the particles H , S , and π are shown in Fig. 2(b). The solid line corresponds to $M_S(T)$, the mass of the state which is predominantly quarkonium [$|\theta(T)| < \pi/4$], and the dotted line to $M_H(T)$, the mass of the state which is predominantly tetraquark. At $T_s \simeq 160$ MeV, both masses behave discontinuously and the states interchange their roles: for $T < T_s$, the state S is the heavier scalar and H the lighter one, and for $T > T_s$, the state S is lighter than H . Above T_s the state S becomes degenerate with the pions as in the sigma model without tetraquark. Note that, before becoming degenerate with the pion, the thermal mass of the lightest state (identified with H for $T \leq T_s$ and with S above it) slightly decreases; see Ref. [19] for comparison.

In Fig. 2(c), the mixing angle $\theta(T)$ is plotted. At T_s , $\theta(T)$ is discontinuous, which leads to the discontinuity in the masses noted above: it jumps suddenly from $\pi/4$ to $-\pi/4$, $\lim_{T \rightarrow T_s^\pm} \theta(T) = \mp \frac{\pi}{4}$. At T_s the mixing is maximal: the two physical states H and S have the same amount (50%) of quarkonium and tetraquark. Note that, at T_s , the field φ and χ are degenerate in mass, which explains the maximal value of the mixing angle. As a last remark we note that the relative magnitude of T_s and T_c ($T_s < T_c$ as in our example or vice versa) depends on the choice of the

parameters. When increasing the mixing strength g the temperature T_s decreases faster than T_c , thus realizing the ordering $T_s < T_c$ in which the jump occurs at smaller temperatures than chiral symmetry restoration.

IV. CONCLUSIONS

In this paper, we proposed a novel scenario for chiral symmetry restoration at nonzero T , in which two scalar-isoscalar states, a tetraquark and bare quarkonium field, are considered. The mixing of the latter two generates two physical states which can be associated with the resonances $f_0(600)$ and $f_0(1370)$. When the tetraquark mass is smaller than the quarkonium mass, as supported by various spectroscopic studies in the vacuum, the state $f_0(600)$ is predominantly tetraquark and $f_0(1370)$ is predominantly quarkonium. This scenario has been studied by employing a simple model which includes only these two scalar resonances and the pion triplet.

A remarkable aspect of our results is that the tetraquark-quarkonium mixing generates a softer first-order phase transition or, depending on the coupling strength, even a crossover transition. While in the standard linear sigma model ($g = 0$) a heavy chiral partner of the pion (with mass exceeding 1 GeV) always leads to a first-order phase transition, this is not necessarily the case when tetraquark-quarkonium mixing is considered: for sufficiently large coupling strength g , the chiral transition is crossover, just like in lattice QCD studies [14]. We also demonstrated that

the mixing between quarkonium and tetraquark states increases with increasing temperature, and, in most cases, reaches its maximal value of 45° at a temperature T_s where the physical states consist of an equal amount of quarkonium and tetraquark. For $T > T_s$ the physical states interchange their roles: the lighter state is predominantly quarkonium and the heavier predominantly tetraquark. Further increasing T leads to the standard scenario of chiral symmetry restoration, where the scalar quarkonium becomes degenerate in mass with its chiral partner, the pion. Thus, our approach can possibly solve an inconsistency between low-energy spectroscopy, where a nonquarkonium structure for $f_0(600)$ is favored, and studies at nonzero temperature, where the scalar partner of the pion should be sufficiently light (~ 0.6 GeV) in order for the chiral symmetry restoring transition to be crossover.

Since the present work is a first explorative study on the relevance of the tetraquark at nonzero T , we omitted other scalar-isoscalar states such as $f_0(980)$, $f_0(1500)$, and $f_0(1710)$, which would naturally appear in an $SU(3)$ -symmetric model with two scalar nonets and a glueball state. Also (axial-)vector mesons should be considered [17,18]. All these fields are important in a more realistic framework which aims to describe at the same time vacuum phenomenology and nonzero T properties. We regard the results of this paper as a motivation to undertake this more ambitious step in the near future.

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