TETRAQUARKS

What is a tetraquark?

A bound state of a diquark and an antidiquark.

What is a diquark?

A bound state of two quarks. (similarly for an anti-diquark...)

$$|qq\rangle = \underbrace{|L=0\rangle|S=0\rangle}_{\text{ground state}}\underbrace{|\text{flavor}: \bar{N}_f\rangle|\text{color}: \bar{3}_c\rangle}_{\text{what does it mean?}}$$

(groundstate)

Let us start from color.

R, G, B ...

If we have two objects (two quarks), then the following is true:

- one cannot build a colorless object (at least 3 quarks are needed).

- There are $N_c \times N_c = 3\times3 = 9$ possibilities for a diquark in the color space:

  RR, RG, RB, ...

Out of group theory we can identify 3 antisymmetric states

$$[R, G] = RG - GR$$

$$[R, B] = RB - BR$$

$$[G, B] = GB - BG$$

and 6 symmetric contributions:

$$\{R, R\}, \{G, G\}, \{B, B\}$$

$$\{R, G\}, \{R, B\}, \{G, B\}$$

In flavor-space we can do the same:

$$[u, d]$$

$$[u, s]$$

$$[d, s]$$

and

$$\{u, d\}, \{u, s\}, \{d, s\}, \{u, u\}, \{d, d\}, \{s, s\}$$

N.b:

For $N_f = 2$:

antisymmetric configuration : $[u,d]$

Symmetric configuration : $\{u,u\}, \{u,d\}, \{d,d\}$

For $N_f = 1$:

No antisymm. config.

One symm. config: $\{u,u\} = 2uu$.

In general for N_f

$$\frac{N_f(N_f-1)}{2}$$ antisymm. conf.

$$\frac{N_f(N_f+1)}{2}$$ symm. conf.

Back to $|qq\rangle$:

$$|qq\rangle_A = |L=0\rangle|S=0\rangle\,|\text{flavor: antisym.}\rangle\,|\text{color}: \bar{3}_c\rangle$$

$\bar{3}_F$

$\uparrow\downarrow - \downarrow\uparrow$ antisym. antisym.

Exchange operator:

$$(-1)^L \cdot (-1)^{S+1} \cdot (-1)\cdot(-1) = -1 \;\#$$

OK.

Pauli is respected.

This is also called by Jaffe the "good" diquark....

$$|qq\rangle = |L=0\rangle|S=0\rangle\,|\text{flavor: symm}\rangle \cdot |\text{color: symm}\rangle$$

Exchange operator:

$$(-1)^L \cdot (-1)^{S+1} \cdot (+1)(+1) = -1$$

It is still ok.

However, all models and calculations (instanton-based, lattice, NJL, 1-gluon exchange) show that the "good" diquark is more stable than the bad diquark.

We work then with the "good" diquark only.

They are easier: $\frac{N_F(N_F-1)}{2} < \frac{N_F(N_F+1)}{2}$, ...

Now, what is a (good) tetraquark? A bound states of a (good) diquark and a (good) antidiquark.

First point: color. It must be <u>white</u>. There is only one possibility:

$$|color\rangle = [R,B]\cdot[\bar{R},\bar{B}] + [G,B]\cdot[\bar{G},\bar{B}] + [G,R]\cdot[\bar{G},\bar{R}]$$

This is the only 'white' configuration existing.

Why is it like that? Group theory...

$N_c = \underline{3}$ is particular.

$[R,B]$ transforms as $\bar{G}$ $\qquad [\bar{R},\bar{B}]$ t.a. G

$[R,G]$ transform as $\bar{B}$ $\qquad$ ...

$[G,B]$ ..... as $\bar{R}$ $\qquad$ ...

Then, the previous object is (for what concerns group theory) analogous to

$\bar{G}G + \bar{R}R + \bar{B}B$, which is obviously invariant, Now, for what concerns flavors we have

$$\left(\frac{N_f \cdot (N_f - 1)}{2}\right)^2$$

possibilities.

$N_f = 1 \longmapsto 0$

$N_f = 2 \longmapsto 1 \qquad |\alpha\rangle = [u,d]\cdot[\bar{u},\bar{d}]$.

(Adam's thesis...)

$N_f = 3 \longmapsto 9 \longrightarrow$ PARTICULAR, A full nonet set of $\bar{q}q$.

$+$

$$\begin{cases} [u,s]\cdot[\bar{d},\bar{s}] = a_{[4q]} & (u\bar{d} \text{ with hidden } s \text{ content}) \\ [\bar{u},\bar{s}]\,[d,s] = a^{-}_{[4q]} & (\bar{u}d \ \ldots) \\ [u,s][\bar{u},\bar{s}] - [d,s]\cdot[\bar{d},\bar{s}] = a^{(0)}_{[4q]} & (u\bar{u} - \bar{d}d) \end{cases}$$

$$[u,s]\cdot[\bar{u},\bar{s}] + [d,s]\cdot[\bar{d},\bar{s}] = f_0 \qquad (\bar{u}u + \bar{d}d) \text{ with } s\ldots$$

$+$

$$\begin{cases} [u,d][\bar{d},\bar{s}] = K_{[4q]}(u\bar{s}) \text{ with } d\text{-hidden}\ldots \\ \ldots \\ \ldots \\ \ldots \end{cases}$$

Feel true:

$$[u,\bar{u}]\cdot[d,\bar{d}] = \sigma_{[49]}$$

Mass ordering:

$$\boxed{M_\sigma < M_K < M_\rho = M_{a_0}}$$

Remember for $\bar{q}q$ object:

$$\begin{cases} u\bar{d} = a_0^+ \\ \cdots \end{cases}$$

$$\bar{u}u + \bar{d}d = \sigma$$

$$u\bar{s} = K^+$$

$\vdots$

$$\bar{s}s = \rho$$

You then have

$$M_\sigma = M_{a_0} < M_K < M_\rho$$

The tetraquark system = reversed mass ordering.

In PDG

$f_0(600)$

$K(800)$

$f_0(980)$
$=$
$a_0(980)$

exactly what the tetraquark ordering predicts.

$f_0(600) = [u,d] \cdot [\bar{u},\bar{d}]$

$I = 1/2 \quad K(800) = [u,d][\bar{d},\bar{s}], \ldots$

$I = 1 \quad a_0(980) = [u,s] \cdot [\bar{d},\bar{s}], \ldots$

$f_0(980) = [u,s] \cdot [\bar{u},\bar{s}] + [d,s][\bar{d},\bar{s}]$

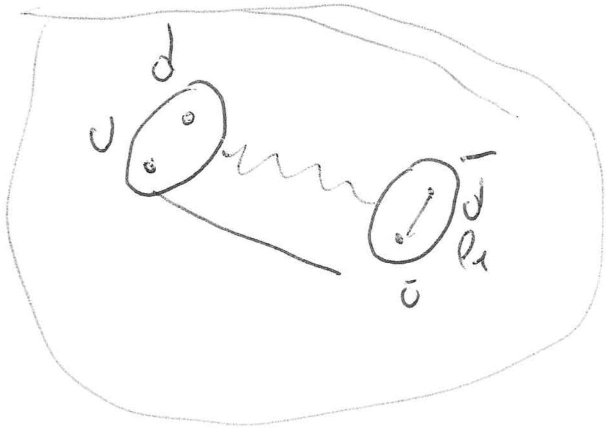
what if you try with quarkonium?
$a_0(980) = u\bar{d}$. $f_0(980) = \bar{u}u + \bar{d}d$. But then $\bar{s}s$ should be heavier... the next state would be $f_0(1710)$... but then, what is $f_0(600)$?
Kaon as well....
It does not work!!!

Tetraquark or molecular state?

$f_0(500)$

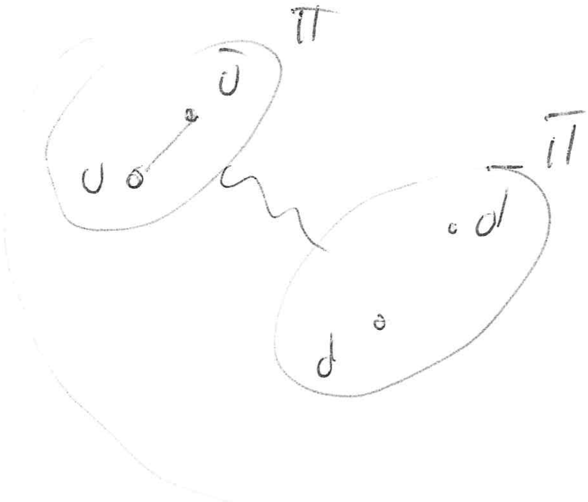
Tetraquark: $[u,d] \cdot [\bar{u},\bar{d}]$

Molecular state: $\pi - \pi$



tetraquark

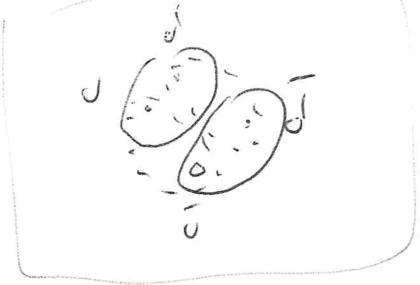
$l_1 << l_2$



Molecular state..

$l_2 << l_1$

Clearly:



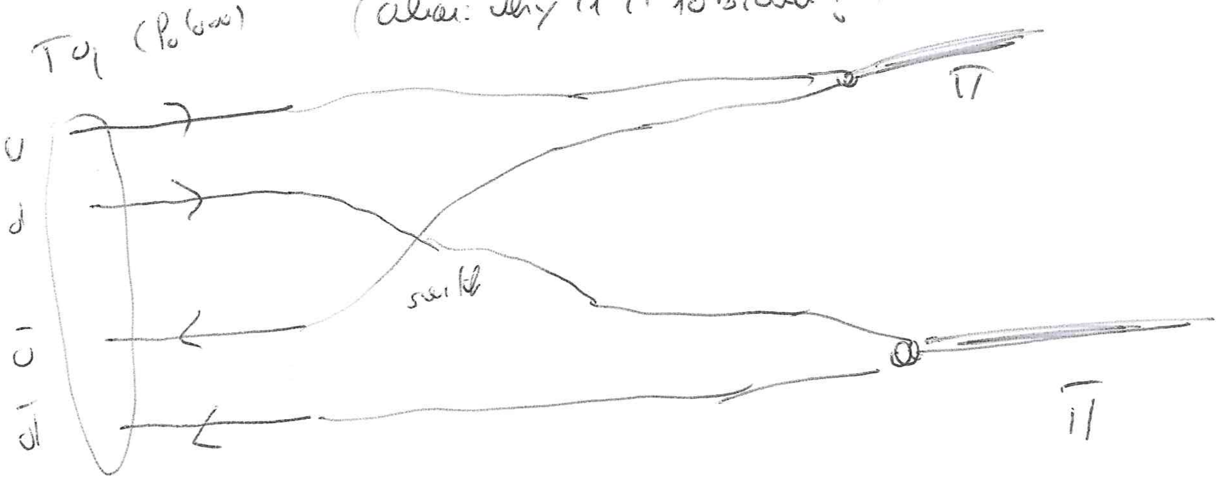
$l_1 \sim l_2$ ...
problem.

Difficult to distinguish....

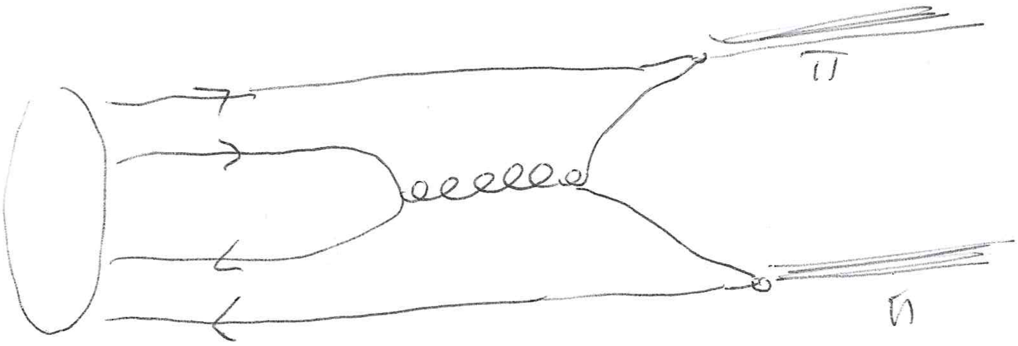
However, why do we love the bump in the cross section?

Decay of a tetraquark

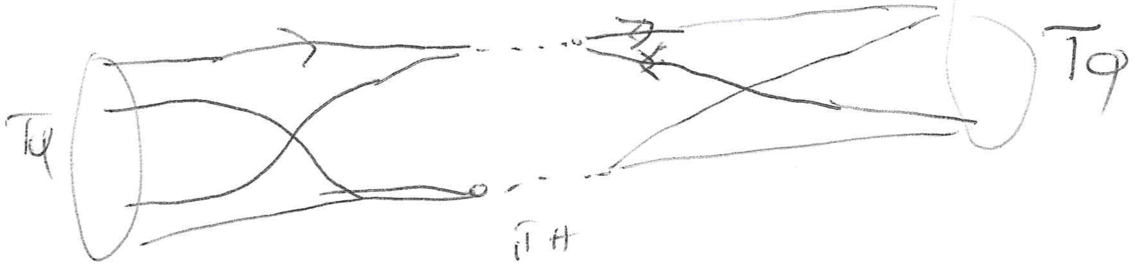
(also: why is it so broad?)



Huge probability... especially if close ----



Second possibility, smaller because of OZI-RULE (one emiss.) which is indeed a consequence of large N_c -



Now, suppose we could eliminate the tetraquark state....
would the resonance still exist? It seems that there is no solution... Draw:

Is $\pi\pi$ interaction enough?

There is a clear why one has to do it.

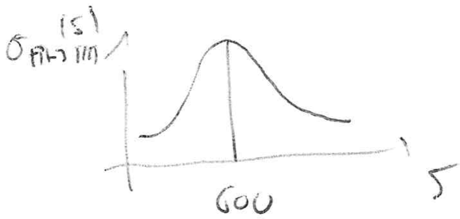
Let us take, for instance, an sigma model. We then calculate

it contains
- by def. only
$\bar{q}q$ states
(and not glueballs...)

Scalars above 1 GeV of course...

In this way if we obtain a quasi-bound state, that is a molecular state.

In fact, in the (generalized) σ model the pions are "point-like" and a bump in the cross-section is indeed only the molecular state exist.

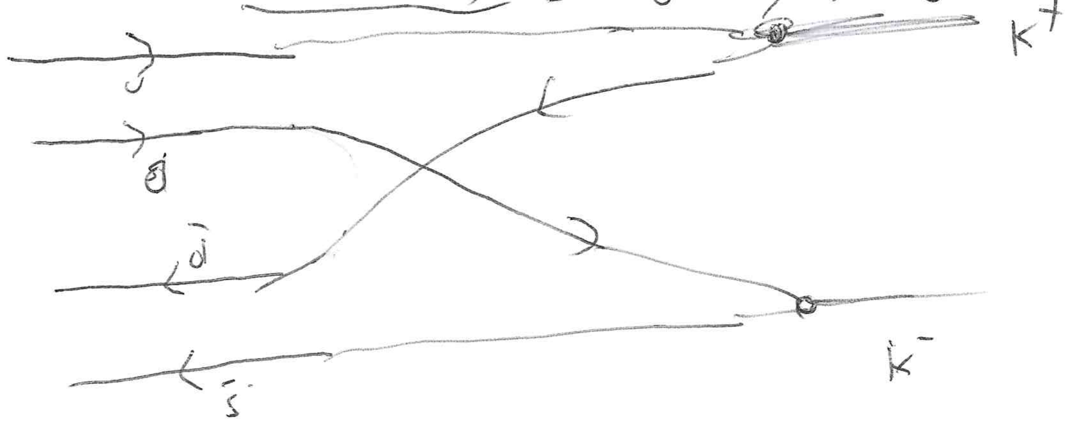


of course, resummed... with loops. Do we get the bump or not?

→ if yes: forget about the Eq...

↳ if not: You need the Eq from the very beginning....

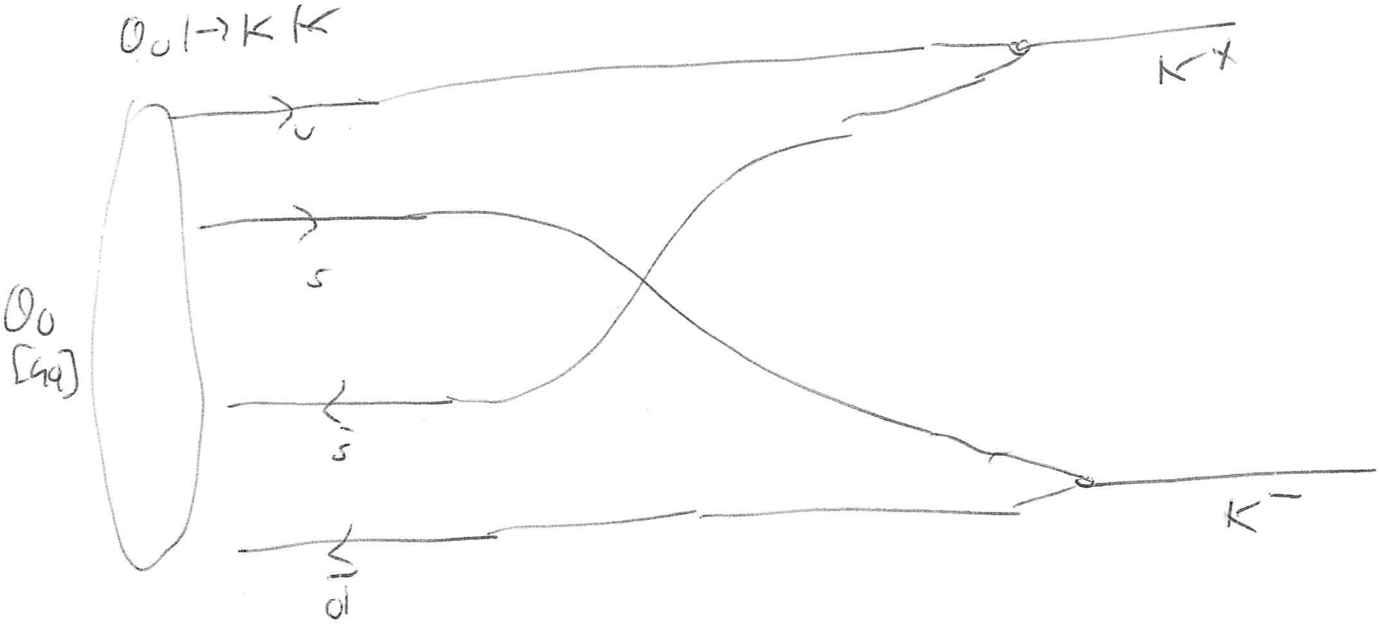
Kaon decay = also very good



Also very good... (mine: does it exist? yeah.. J/ψ does...)

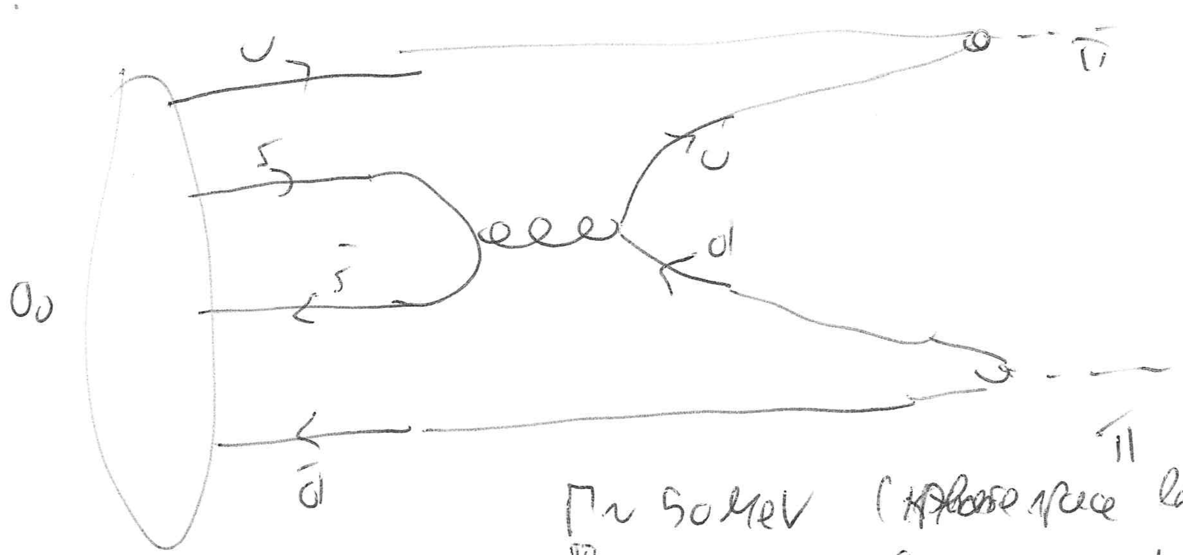
f_0 and a_0 decay:

$a_0 \to \bar{K}K$



However, $M_{f_0} \lesssim 2M_K$..
The decay is small because the phase space is very small. This is similar to the $f_0(600)$ and $K(800)$....

It is, let's say, a particular situation.



$\Gamma_{\pi\pi} \sim 50\,\text{MeV}$ (phase space large, amplitude small)
$\Gamma_{KK} \sim 50\,\text{MeV}$ (" " small, amplitude large)

This is the suppressed channel... However, it is crucial.

There is actually a rather philosophical point.

If the $\pi\pi$ channel would not exist, then it would be $\Gamma_{\pi\pi}$:
$(M_\pi \sim 1\,\text{GeV})$
not zero

$M_{f_0} < 2M_K \to \Gamma_{KK} = 0$.

It would be a stable resonance in the hadronic world...

However, $\Gamma_{\pi\pi}$ is not very large but $\neq 0$. This induces a width. Then, ...

there is a nonzero probability that $M_{f_0} > 2M_K$...

Then, the scalar decays into $K\bar{K}$ can take place.

The same for $f_0(980)$...

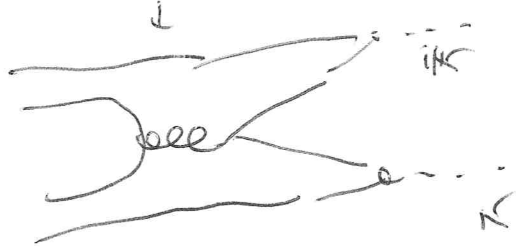
A very strange phenomenon.

(My research...)

The same in the ρ_0-channel...

Indeed:

$$A_{\rho_0 \mapsto \bar{K}K} \supset A_{a_0 \mapsto \bar{K}K}$$



especially strong for ρ_0...

$$A_{\rho_0 \mapsto \bar{K}K} = A^{(1)}_{\rho_0 \mapsto \bar{K}K} + A^{(2)}_{\rho_0 \mapsto \bar{K}K}$$

$$\parallel \qquad \mapsto \ldots$$

$$A_{a_0 \mapsto \bar{K}K} = A^{(1)}_{a_0 \mapsto \bar{K}K} + A^{(2)}_{a_0 \mapsto \bar{K}K}$$

(Then use my result...)

All phenomenology: ok.

Another important point → to study all this in the chiral model.

General conclusions: Scalar are special... much more to come but broad likewise...

Exploration in the $\bar{c}c$-region. One of the reasons of PANDA

$f_0(600)$

Important at nonzero T and ρ.

Nonzero T: it was often considered as the chiral partner of the pion...

Thus, here, seems to be wrong.

(By these considerations and many works... also the one of Denis)

First work: by Achim.

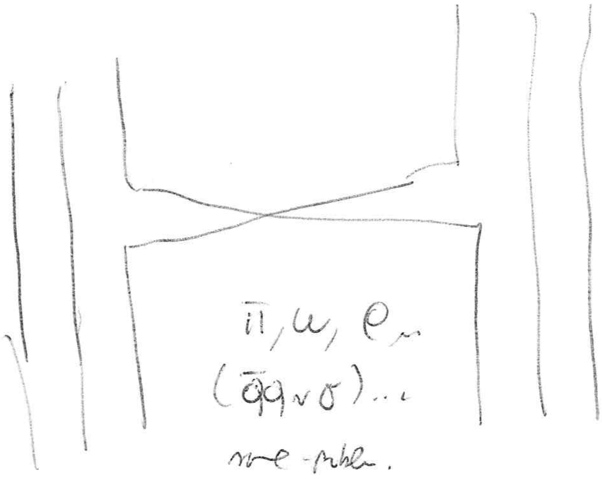
Here, future studies should be performed.

Nonzero μ: The resonance $f_0(600)$ is also crucial for the nucleon-nucleon interaction

and therefore also for nonzero density study.

Actually, a big ... is expected and quite natural...

It is indeed strange that it has not been considered up to now.



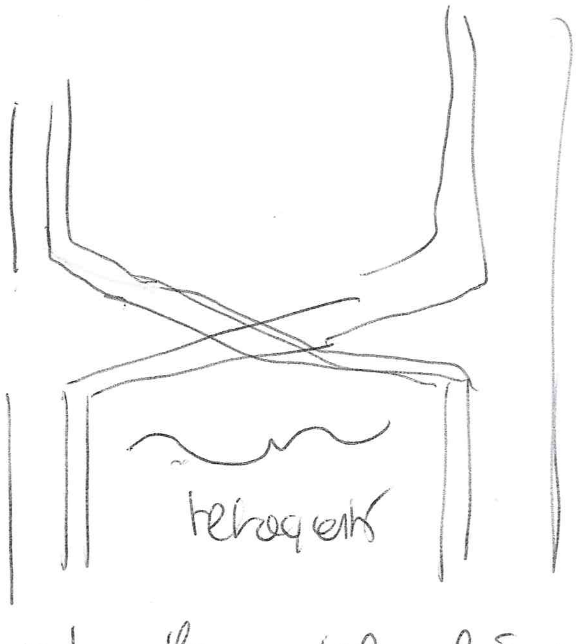
usual picture

However:

diquark-quark bound state.

[depends on diquark-quark similar to q̄q ...
a baryon is in that case not so much ≠ from a
meson....]

But then:



Interesting point: the model of S...

Tetraquark and large-N_c

- Tq do not exist for large N_c ...
(in the sense of 4)

only two $(\bar{q}q)\cdot(\bar{q}q)$ separated states (i.e. two white $\bar{q}q$ mesons) would exist.

It is however possible to study another limit in which the tq does not disappear.

We started from the good diquark.... it is an object which transforms as an antiquark,

$$d_a = \varepsilon_{abc}\, q^b q^c \qquad a,b,c = 1,2,3 = R,G,B$$

antisymm....

Now, consider the large N_c-world.

$$d_{a_1} = \varepsilon_{a_1 a_2 \ldots a_{N_c}} \underbrace{q^{a_2} q^{a_3} \cdots q^{N_c}}_{N_c - 1 \text{ quarks}}$$

$a_1, \ldots, a_{N_c} = 1, \ldots, N_c$.

There are N_c d-objects.... $a_1 = 1, \ldots, N_c$.

d_{a_1} is a $(N_c - 1)$ object which is the gener. of the diquark for large N_c.

Tetraquark:

$$\mathcal{N} = \sum_a \bar{d}_a^{\dagger} d_a \qquad d_a = \varepsilon_{abc}\, q^b q^c \qquad \bar{d}_a = \varepsilon_{abc}\, \bar{q}^b \bar{q}^c$$

In the large N_c limit we have similarly:

$$\mathcal{N} = \sum_{a_1}^{\dots} \bar{d}_{a_1} d_{a_1} \dots$$

This is an object with $(N_c-1)q$ and $(N_c-1)\bar{q}$.

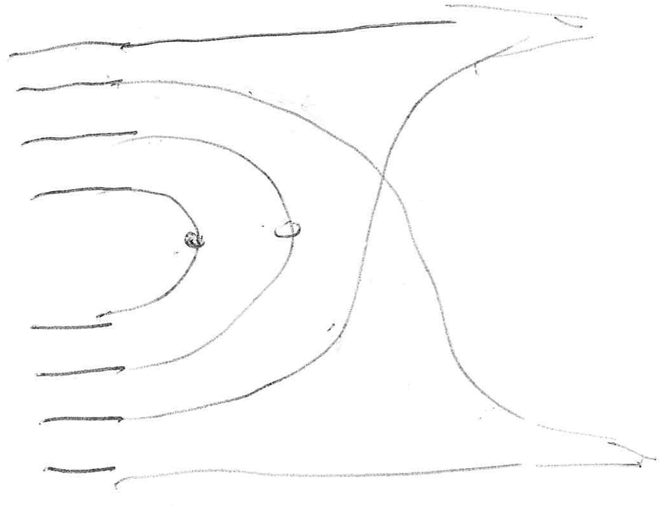
It is obviously white. It is called dibaryon.

$$M_{\mathcal{N}} \sim 2N_c - 2 \sim N_c$$

How is the decay into mesons? Suppressed,

in fact, from $2N_c-2$ to 4

$N_c = 5$



2 annihilation...

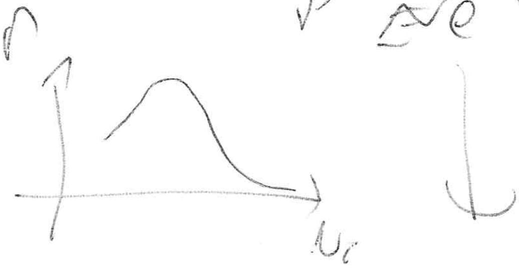
In general $(N_c-3) \sim N_c$ ann.

But then the prob. is ...

$\Gamma \sim p^{N_c}$ where p is the prob. of the single ... $\quad p<1$

$p^{N_c} \sim e^{-\lambda N_c}$

$$\Gamma \sim \frac{\sqrt{\frac{M_X^2}{4} - M_B^2}}{8\pi M_X^2} \cdot e^{-\lambda N_c}$$



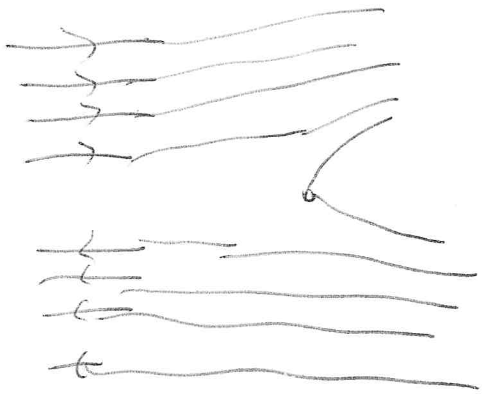
$\left(p = e^{\log p} \right.$

$\left. p^{N_c} = e^{\log p \cdot N_c} = e^{-\lambda N_c} \right)$

Tach. instability...

Then, the decay into mesons is suppressed. However, a baryon is done with quarks.

Then:



$\propto p$

$\Gamma_{GDTB} \longmapsto \bar{B}B \propto N_c^0$
(GEN-TQ)

Then, we know how to put the tq in the large N_c context.

Even more: the tq in this sense surely exist in the large N_c limit. Then, this also explains the particular value of the "good" diq and "good" tq,