Quarks

u d s ...

$\vdots$ $\vdots$ $\vdots$

u_R d_R s_R

u_G d_G s_G

u_B d_B s_B

$$B = 1/3$$

(for each of them)

$$q_{a,i}$$

$a = R, G, B$

$i = u, d, s$

m_u, m_d, m_s are the three "masses"
$\begin{cases} m_u, m_d \sim 5\ \text{MeV} \\ m_s \sim 100\ \text{MeV} \end{cases}$

caution: there is no pole corresponding to these values.

SSB $m_u^* \simeq m_d^* \simeq 300\ \text{MeV}$, $m_s^* \simeq 500\ \text{MeV}$ effective (or constituent) mass.

Quantum numbers of quarks

- SUMMARIZING TABLE

QUARK FLAVOR (FERMION-SPIN 1/2 FIELDS	COLOR	BARYON NUMBER	CHARGE	I	I_3	Strangeness	Bare Mass (MeV)	Const. Mass (MeV)
u	R, G, B	1/3	2/3	1/2	1/2	0	$2.3^{+0.7}_{-0.5}$	~300
d	R, G, B	1/3	-1/3	1/2	-1/2	0	$4.8^{+0.7}_{-0.5}$	~300
s	R, G, B	1/3	-1/3	0	0	-1	95 ± 5	~500

Achtung: no of these is a mass in the "electron meaning".

Bare mass: param. in the Lag.

Const. mass: value found in constituent quark model and NJL(-type) model.

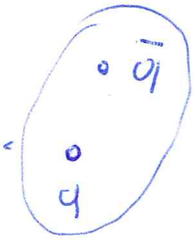
Mesons

Quantum numbers and phenomenological descriptions

Mesons: Bosons which interact strongly $\equiv$ Hadronic bosons

Hadrons with baryon number $= 0$

Conventional mesons are quark-antiquark states.

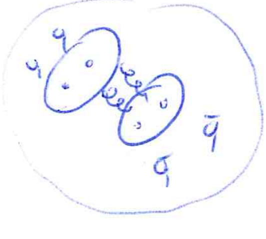


Constituent or quarks with a large mass ~ 300 MeV.

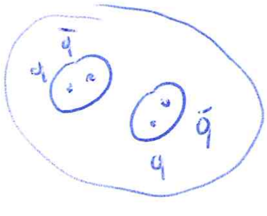
Other possibilities are:

- glueballs

- tetraquarks

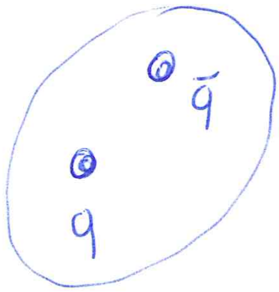


- Molecular states



- other dynamical generated states due to quantum fluctuations

We first concentrate on conventional $q\bar{q}$ mesons made by a quark and an antiquark.



The quantum numbers for such a system are (in a non-relat. language and beyond)

- total spin S
- angular momentum L

} total angular momentum J

- radial quantum numbers

of particular importance there are also

- Parity $(\bar{x} \mapsto -\bar{x})$
- Charge conjugation (Particle $\longleftrightarrow$ Antiparticle)

... and, of course, color!!!

$|\text{color}\rangle$

But the color is the easiest.

$$|\text{color}\rangle = \sqrt{\frac{1}{3}}\,|\bar{R}R + \bar{B}B + \bar{G}G\rangle$$

This configuration is white (we come back on details).

and is indeed the only configuration that you can build out of a quark and an antiquark.

Thus, each conventional meson has the very same color wave function.

The total wave function is given by:

$$|\bar{q}q\text{-Meson}\rangle = |\underset{n,\,l}{\text{radial + angular momentum}}\rangle\,|\text{spin}\rangle\,|\text{color}\rangle$$

Recall the (composition of angular momenta

An angular momentum operator $\vec{L}$ fullfills the conditions

$$[L_i, L_j] = i\hbar\, \varepsilon_{ijk} L_k$$

in QM

$$\vec{L} = \vec{r} \times \vec{p}$$

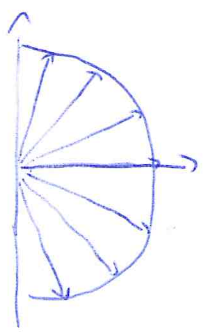
The previous eq. follows from $[x, p] = i\hbar$.

The diagonalization is taken over $\vec{L}^2 = L^2$ and L_z which commute: $[L^2, L_z] = 0$ (then a common basis is possible).

$$\begin{cases} \vec{L}^2 |lm\rangle = \hbar^2 l(l+1) |lm\rangle \\ L_z |lm\rangle = \hbar m |lm\rangle \end{cases}$$

$|\vec{L}| = \hbar\sqrt{l(l+1)}$

$m_{max} = \hbar l < |\vec{L}|$

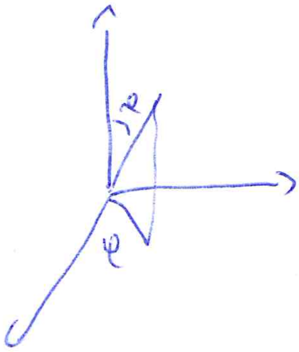


$l = 0, 1, 2, \dots$

$m = -l, \dots, l$

The wave function associated to $|lm\rangle$ are given by:

$$Y_{lm}(\theta, \varphi)$$



$$Y_{\ell m}(\theta,\varphi) = \text{const}\,(\sin\theta)^m \left[\left(\frac{d}{d\cos\theta}\right)^{\ell+m} (\sin\theta)^{2\ell}\right] e^{im\varphi}$$

recall that

$$L_z = -i\hbar \frac{\partial}{\partial\varphi}$$

$\psi = e^{im\varphi}$ is an eigenstate

$m = 0, \pm 1, \pm 2, \dots$ such that $\psi(\varphi) = \psi(\varphi + 2\pi)$

This is a necessary condition for the usual angular momentum.

Explicitly, we have:

$$Y_0^0 = \frac{1}{\sqrt{4\pi}}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta \,, \quad Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\varphi}$$

Now, let us consider two angular momenta together:

$$\vec{L}_1, \quad \vec{L}_2$$

$$[L_{1,i}, L_{1,j}] = i\hbar \varepsilon_{ijk} L_{1,k} \qquad [L_{2,i}, L_{2,j}] = i\hbar \varepsilon_{ijk} L_{2,k}$$

$$[L_{1,i}, L_{2,j}] = 0$$

$\vec{L} = \vec{L}_1 + \vec{L}_2$ is also an operator which fullfills $[L_i, L_j] = i\hbar \varepsilon_{ijk}$

Obviously, a basis can be $|l_1, m_1, l_2, m_2\rangle$ ... but one can construct a basis $|l m\rangle$ with $l = 0, \ldots$
$m = -l, \ldots l$

Now, for fixed l_1, l_2 we can construct $|l m\rangle$ with

$$\left\{\begin{array}{l} l = |l_2 - l_1|, |l_2 - l_1| + 1, \ldots, l_1 + l_2 \\ m = -l, \ldots, l \end{array}\right. \qquad |l m\rangle$$

We discuss later specific examples.

This is "just" a remember.

Spin: the intrinsic rotation

$\vec{S}$ / $[S_i, S_j] = i\hbar\, \varepsilon_{ijk}\, S_k$

But there are crucial differences w.r.t. $\vec{L}$.

- $|s\,m\rangle$ / $\vec{S}^2 |s\,m_s\rangle = \hbar^2 s(s+1)|s\,m_s\rangle$

  s is referred to as the spin of the particle under consideration.

- $s = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$

  $m_s = -s, \ldots, s$

  that is: it can be also semi-integer !!!!!

$\psi(\varphi) = e^{im\varphi}$ / $\psi(\varphi) = \psi(\varphi + 2\pi)$ for $m = 0, \pm 1, \ldots$

but for $m = 1/2$, for instance, we have

$\psi(\varphi) = -\psi(\varphi + 2\pi)$

A full rotation and the w.f. changes sign. This is actually not visible because we always square amplitudes.

Nature does that:

Fermions have semi-integer spin s (and a Fermi-statistic), bosons have integer spin s (and a Bose statistic)

Note that a given fundamental particle has a definite spin s ! it <u>cannot</u> change. It is fixed once for all.

An electron has spin $1/2$. That's it... Its third component $m_s = \pm 1/2$, but the total spin is fixed.

spin of elementary particles

$s=0$ Higgs H (+ ?)

$s = \frac{1}{2}$: $e^-, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau$
u, d, s, c, b, t

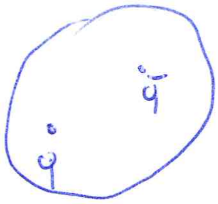
$s=1$ $\gamma, W^{\pm}, Z^0$

$s=2$ Graviton (?)

$$|e^-\rangle \xmapsto{2\pi\text{-rotation}} -|e^-\rangle$$

$$|u\rangle \xmapsto{2\pi\text{-rotation}} -|u\rangle$$

Back to my conventional mesons made of a quark and an antiquark.



Each state has spin 1/2 and can be:

Quark: $|\frac{1}{2}, +\frac{1}{2}\rangle$, $|\frac{1}{2}, -\frac{1}{2}\rangle$ $(\vec{S}_1)_{s=1/2 \text{ always}} \longrightarrow |\uparrow\rangle, |\downarrow\rangle$

Antiquark: $|\frac{1}{2}, \frac{1}{2}\rangle$, $|\frac{1}{2}, -\frac{1}{2}\rangle$ $(\vec{S}_2)$ $s = 1/2$ always $|\uparrow\rangle, |\downarrow\rangle$

A full basis for the spin is given by

$$|\uparrow\uparrow\rangle, \quad |\uparrow\downarrow\rangle, \quad |\downarrow\uparrow\rangle, \quad |\downarrow\downarrow\rangle$$

$\downarrow$ quark $\downarrow$ antiquark

Then, for the meson we construct

$$S = 1, \quad S = 0$$
$$\left(\tfrac{1}{2}+\tfrac{1}{2}\right) \qquad \left(\tfrac{1}{2}-\tfrac{1}{2}\right)$$

TRIPLET
$$\begin{cases} |S=1, S_z = 1\rangle = |\uparrow\uparrow\rangle \\ |S=1, S_z = 0\rangle = \sqrt{\frac{1}{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \\ |S=1, S_z = -1\rangle = |\downarrow\downarrow\rangle \end{cases}$$

SINGLET $|S=1, S_z = 0\rangle = \sqrt{\frac{1}{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

NOTE: the triplet is symmetric under exchange of quark-antiquark

the singlet is antisymmetric " " " " "

Ergo, the total spin of the meson can be either

zero or one!

$$\begin{cases} 2S+1 = 1 & \text{for } S = 0 \\ 2S+1 = 3 & \text{for } S = 1 \end{cases}$$ old relation (but still used!!!!)

$\vec{L}$ and $\vec{J}$

For S there are only two possibilities: $S=0$, $S=1$.

ℓ of the two-body system can be whatever... every rotation is allowed.

$\ell = 0, 1, 2, 3, \ldots$ $\quad$ s, p, d, f, ... (in analogy to the orbitals of the H)

ℓ is also still used in the spectroscopic notation of quark multiplets

(later more on this).

What about J?

$S=0 \rightarrow J=\ell$

$$S=1 \rightarrow \begin{cases} J=\ell-1 \\ J=\ell+1 \end{cases}$$

Examples:

- $L = S = 0 \rightarrow J = 0$

These are the pseudoscalar mesons: $\vec{\pi}, K (= K^{\pm}, K^0, \bar{K}^0), \eta, \eta'$

(Goldstone bosons, ...)

(different quark inside)

$\pi^+ = u\bar{d}, K^+ = u\bar{s}, \ldots$

- $L = 0, S = 1 \mapsto J = 1$

These are the vector mesons: $\vec{\rho}, K^*, \omega, \phi$

- $L = 1, S = 0, \mapsto J = 1$

These are the pseudovector mesons: $\vec{b}_1, K_1, h_1, h_1'$

- $L = 1, S = 1$

$\hookrightarrow \begin{cases} J = 2 \mapsto \text{tensor mesons} & a_2, K_2, f_2, f_2' \\ J = 1 \rightarrow \text{axial-vector mesons} & a_1, K_1, f_1, f_1' \\ J = 0 \mapsto \text{scalar mesons} & \end{cases}$

(huge discussion on these scalars...

these are the scalar partners of the Goldstone bosons. They condense.

of course, we can go further ....

$L=2,\ S=0$

$J=2 \mapsto$ additional tensor states

$L=2,\ S=1$ $\begin{cases} J=1 \mapsto \text{additional vectors} \\ J=2 \mapsto \text{additional tensors} \\ J=3 \mapsto J=3 \dots \end{cases}$

$L=4 \begin{cases} S=0 \\ S=1 \end{cases}$

$L=5 \begin{cases} S=0 \\ S=1 \end{cases}$

...

...

...

there is in theory no end... now, we shall stop here.

For our purposes, in a chiral theory one usually stops at $L=1$.

Flavour wave function

$\vec{\pi}$, K, η, η'

what does it mean exactly? why 1?

Actually there are 9 states:

$\vec{\pi}$	3
K	4
η	1
η'	1
tot	9

$\bar{u}, \bar{d}, \bar{s}$

u, d, s

$$I=1 \left\{ \begin{aligned} \pi^+ &= u\bar{d} \\ \pi^- &= \bar{u}d \\ \pi^0 &= \sqrt{\tfrac{1}{2}}(\bar{u}u - \bar{d}d) \end{aligned} \right.$$

$$\boxed{|\pi^+\rangle = \#|n=1, l=0\rangle |u\bar{d}\rangle |\uparrow\downarrow - \downarrow\uparrow\rangle |\bar{R}R + \bar{B}B + \bar{G}G\rangle}$$

$$\tfrac{1}{2} \left\{ \begin{aligned} K^+ &= u\bar{s} \\ K^- &= \bar{u}s \\ K^0 &= d\bar{s} \\ \bar{K}^0 &= \bar{d}s \end{aligned} \right.$$

$$0 \left\{ \begin{aligned} \eta_N &= \sqrt{\tfrac{1}{2}}(\bar{u}u + \bar{d}d) \\ \eta_S &= \bar{s}s \end{aligned} \right.$$

$$\eta = \cos\theta\, \eta_N + \sin\theta\, \eta_S$$

$$\eta' = -\sin\theta\, \eta_N + \cos\theta\, \eta_S$$

$\theta \simeq -40^\circ$

(large!! ANOMALY...)

(nb: $\sqrt{2}\,\eta_N + \eta_S$ is a flavour singlet...)

Note about isospin and strangeness

Isospin:

$$\begin{pmatrix} u \\ d \end{pmatrix}$$

Under a SU(2) transf. I transform $\begin{pmatrix} u \\ d \end{pmatrix} \mapsto U \begin{pmatrix} u \\ d \end{pmatrix}$

The isospin triplet state $\vec{\pi}$ rotates in itself... $\vec{\pi} \mapsto R\vec{\pi}$

But $\sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ is isospin invariant. So has nothing to do with isosp.

K-states are isodoublets.

$I_{\pi} = 1$; $I_K = 1/2$; $I_{\eta, \eta'} = 0$

$\hookrightarrow |I=1, I_3=+1\rangle = |\pi^+\rangle$, $|I=1, I_3=0\rangle = |\pi^0\rangle$, $|I=1, I_3=-1\rangle = |\pi^-\rangle$

Strangeness

s-quark has by convention -1.

$K^+ = u\bar{s}$ has strangeness $+1$.

but

$\eta_s = \bar{s}s$ has strangeness zero $(+1-1)=0$. One speaks of hidden strangeness.

The very same thing takes place for all the other nonets that we have mentioned....

For the vector meson nonet we have:

$$\begin{cases} \rho^+ = u\bar{d} \\ \rho^- = \bar{u}d \\ \rho^0 = \sqrt{\frac{1}{2}}\,(\bar{u}u - \bar{d}d) \end{cases} \qquad \boxed{I_\rho = +1} \quad \begin{cases} |\rho^+\rangle = |I = 1,\ I_3 = 1\rangle \\ |\rho^0\rangle = |I = 1,\ I_3 = 0\rangle \\ |\rho^-\rangle = |I = 1,\ I_3 = -1\rangle \end{cases} \qquad S = 0$$

$$\begin{cases} K^{*+} = u\bar{d} \\ K^{*-} = \bar{u}d \\ K^{*0} = s\bar{d} \\ \bar{K}^{*0} = \bar{s}d \end{cases} \qquad I_{K^*} = 1/2, \quad |S| = 1$$

$$\begin{cases} \omega = \sqrt{\frac{1}{2}}\,(\bar{u}u + \bar{d}d) & I_\omega = 0 & S = 0 \\ \phi = \bar{s}s & I_\phi = 0 & S = 0 \text{ (hidden)} \end{cases}$$

Radial quantum number

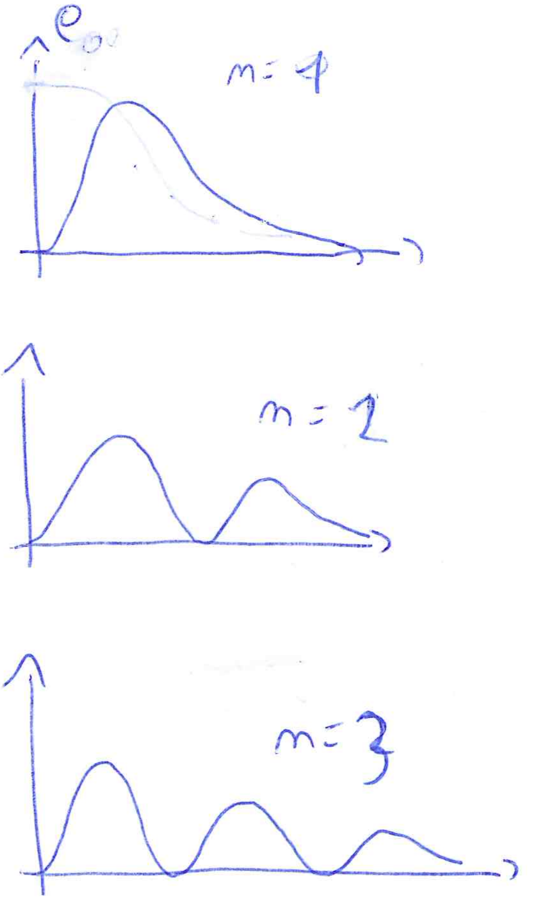
In a central problem with a central potential we write the eigenkets of the hamiltonian as

$|n\,l\,m\rangle \qquad \psi(\vec{r}) = \psi_n(r)\, Y_{lm}(\theta, \varphi)$

$\downarrow$

radial quantum numbers

it is the nr of zeros of the wave function $|r\psi(r)|^2 = \rho_{nl}(r)$



In the H-atom

$$E_{nl} = - \frac{\text{const}}{(n+l+1)^2}$$

depends on n and l (and not on m as expected).

In the H-atom one substitutes n with the principal quantum number

$$N = n+l+1 = 1, 2, 3, \ldots$$

$$[\, l = 0, \ldots, N-1 \,]$$

$$\boxed{E_N = - \frac{\text{const}}{N^2}}$$

But this is not always possible...

In the case of mesons we still refer to the "radial" quantum number n.

Example:

For each choice of L and S seen before we can make an ∞ of copies of them by increasing n.

In all previous cases we tacitly assumed that $n = 1$...

but that does not need to be always the case.

One can have excited states, which are more and more massive.

$n=1$ $\quad L = S = 0 \qquad \pi \quad K \quad \eta \quad \eta'$

$n=2$ $\quad L = S = 0 \qquad \underbrace{\pi(1300)\ K(1400)\ \eta(1295)\ \eta(1475)}_{\text{excited pseudoscalars}}$

$n=1$ $\quad L=0,\ S=1 \qquad \rho,\ K^*,\ \omega,\ \phi$

$n=2$ $\quad L=0,\ S=1 \qquad \rho(1450),\ K^*(1410),\ \omega(1420),\ \phi(1680)$

Summarising:

$(n-1)\ L, S \mapsto J$

The notation for the spectroscopy of mesons is then given by:

$n\ {}^{2S+1}\ell_J$ where $\ell = \overset{0}{s}, \overset{1}{p}, \overset{2}{d}, \overset{3}{f}, \overset{4}{g}, \overset{5}{h}$

$\uparrow$
$a_0(2450)$

$1\ {}^1S_0$ π, K, η, η'

$1\ {}^3S_1$ ρ, K^*, ω, ϕ

$1\ {}^3P_0$ $a_0(1450), K_0^*(1430), f_0(1370), f_0(1500)/f_0(1710)$

...

$2\ {}^1S_0$ $\pi(1300)\ \ldots$

$2\ {}^3S_1$ $\rho(1450)\ \ldots$

The modern (and for me) better notation is J^{PC} ... but for that we need PC.

How, in our case with a fermion or antifermion

the parity is

$$\boxed{P = (-1)^{\ell+1}}$$

Extra (-1) due to intrinsic opposite parity of a fermion and antifermion.

Convention $\begin{cases} \to \text{Positive for fermions} \\ \to \text{negative for antifermions} \end{cases}$

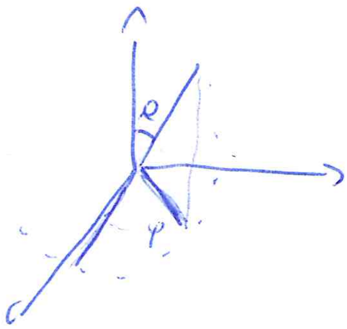
In order to understand this properly we need the Dirac eq... at the present level one has to accept it.

Parity

It is not yet over... we still have parity and charge conjugation. These quantum numbers can be studied rigorously by using quark fields...

Still, we can already start now by using our analogy with the nonrelativistic systems.

$$\psi(r,\vartheta,\varphi) = \psi_{n\ell}(r)\, Y_{\ell m}(\vartheta,\varphi)$$



The parity transformation $\vec{x} \mapsto -\vec{x}$ is equivalent to

$r \mapsto r$
$\vartheta \mapsto \vartheta + \pi$
$\varphi \mapsto$

$$Y_{\ell m}(\vartheta+\pi,\varphi) = (-1)^{\ell}\, Y_{\ell m}(\vartheta,\varphi)$$

Ergo

$$P = (-1)^{\ell}$$

is the parity for a level of the H-atom (or whatever analog. system) charge conjugation

C

C = transform particle $\longleftrightarrow$ antiparticle

$|\text{meson}\rangle = |n, \ell\rangle \, |\text{spin}\rangle \, |\text{color}\rangle$

By exchanging quark and antiquark we get:

$$(-1)^{\ell+1} \cdot (-1)^{S+1}$$

(it is like parity but without (-1) extra)

Ergo:

$$\boxed{C = (-1)^{\ell+S}}$$

Achtung:

The mesonic state |meson> is an eigenstate of C only if the state is chargeless.

For instance, $0^{-+} \to \pi, K, \eta, \eta'$

C = +1 means that for $|\pi^0\rangle$, $|\eta\rangle$, $|\eta'\rangle$ we have

$C|\pi^0\rangle = +|\pi^0\rangle$ $\pi^0 = \sqrt{\frac{1}{2}}(\bar{u}u - \bar{d}d)$

$C|\eta\rangle = +|\eta\rangle$ $\eta =$

$C|\eta'\rangle = +|\eta'\rangle$

but particle antiparticle

$C|\pi^+\rangle = +|\pi^-\rangle$ $\pi^+ = u\bar{d} \longmapsto \bar{u}d = \pi^-$

$C|\pi^-\rangle = +|\pi^+\rangle$

$C|K^0\rangle = |\bar{K}^0\rangle$ $K^0 = d\bar{s}$

$C|\bar{K}^0\rangle = |K^0\rangle$ $\bar{K}^0 = \bar{d}s$

$C|K^+\rangle = |K^-\rangle$

$C|K^-\rangle = |K^+\rangle$

A similar situation holds for the other nonets.

Let us consider still the vector nonet

1^{--}

$C|\rho^0\rangle = -|\rho^0\rangle$

$C|\omega\rangle = -|\omega\rangle$

$C|\phi\rangle = -|\phi\rangle$

but

$C|\rho^+\rangle = -|\rho^-\rangle$

$C|K^-\rangle = -|K^+\rangle$

$C|K^{*0}\rangle = -|\bar{K}^{*0}\rangle$

G-parity

The charge-conjugation has been also enlarged by considering the so-called G-parity.

$$G = C e^{i\pi I_2}$$

$\to$ 2$^{\text{nd}}$ component of isospin

It is easy to show that all $|\pi^+\rangle, |\pi^-\rangle, |\pi^0\rangle$ are eigenstates because $e^{i\pi I_2}$ rotates $|\pi^+\rangle$ in $|\pi^-\rangle$.

In particular, one has: $e^{i\pi I_2}|\pi^+\rangle = -|\pi^-\rangle$

$$\begin{cases} G|\pi^0\rangle = +|\pi^0\rangle \\ G|\pi^+\rangle = -|\pi^+\rangle \\ G|\pi^-\rangle = -|\pi^-\rangle \end{cases}$$

So, for the whole multiplet one may speak of G-parity instead of C... note that there is a extra minus.

In general:

$$\boxed{G = (-1)^{C+L+I} = C(-1)^{I}}$$

Note, it makes sense only for integer I (but not for kaons).

Historical comment... but still in PDG

Ergo, the spectroscopic relat. notation is J^{PC}

$n=1$, $L=S=0$	$1\ {}^1S_0$	$J^{PC}=0^{-+}$	π, K, η, η'
$L=0, S=1$	$1\ {}^3S_1$	$J^{PC}=1^{--}$	ρ, K^*, ω, ϕ
$L=1, S=0$	$1\ {}^1P_1$	$J^{PC}=1^{+-}$	b_1, K_1, h_1, h_1'
$L=1, S=1$	$1\ {}^3P_0$	$J^{PC}=0^{++}$	a_0, K_0^*, f_0, f_0'
	$1\ {}^3P_1$	$J^{PC}=1^{++}$	a_1, K_1, f_1, f_1'
	$1\ {}^3P_2$	$J^{PC}=2^{++}$	a_2, K_2, f_2, f_2'

For I = 1 and I = 1 states (but charged)
one finds:

$\pi^{\pm}$ $I^G(J^P) = 1^-(0^-)$

π^0 $I^G(J^{PC}) = 1^-(0^{-+})$

...

let us go through PDG ...

Exotic

There are some combinations of J^{PC} which are not possible for a $\bar{q}q$ pair

For instance:

$$\boxed{0^{+-}}$$

is not possible for the rules shown above.

Namely:

$P = (-1)^{L+1}$ $\quad L = 1, 3, \dots$

but if $J = 0$ we are obliged to chose $L = 1$

$L = 1$, $S = 1$ coupled to $J = 0 \dots$ but then $C = +1$.

Another example:

$J^{PC} = 1^{-+}$

$P = (-1)^{L+1} \qquad L = 0, 2, 4 \ldots$

$J = 1$ means that $S = 1$

$L = 0, S = 1 \mapsto J = 1$ but $C = (-1)^{L+S} = -1$

$L = 2, S = 1 \mapsto J = 1$ but $C = (-1)^{2+1} = -1$

1^{-+} is not possible for ordinary mesons.

Experimental candidates:

$\pi_1(1400), \pi_1(1600)$ are 1^{-+} states. $(I = 1)$

In order to describe them:

- tetraquark
- hybrid: gluon$^{\text{constituent}}$ $+ \bar{q}q$
- glueballs (but only for $I = 0$ ...).