Recall with

$N_f = 1$ and χPT

· $N_f = 1$, quark u only

· χPT can be introduced in this simplified framework !!!!

$N_f = 1$: a simple but instructive limit + [illegible]

Let us be a bit more precise but with one flavor only:

$$\left(\sigma, \pi \equiv \eta \equiv \bar{u}u\right)$$

$$V = V_{dil}(G) - \frac{a}{2} G^2 \left(\sigma^2 + \pi^2\right) + \frac{\lambda}{4}\left(\sigma^2 + \pi^2\right)^2 - \underbrace{\varepsilon \sigma}_{\text{expl. breaking (only with } \sigma \text{... parity)}}$$

In this case the transf

Chiral transf.

$$\phi = \sigma + i\pi$$

$$U_V(1): \quad \phi \mapsto U \phi U^\dagger = \phi.$$

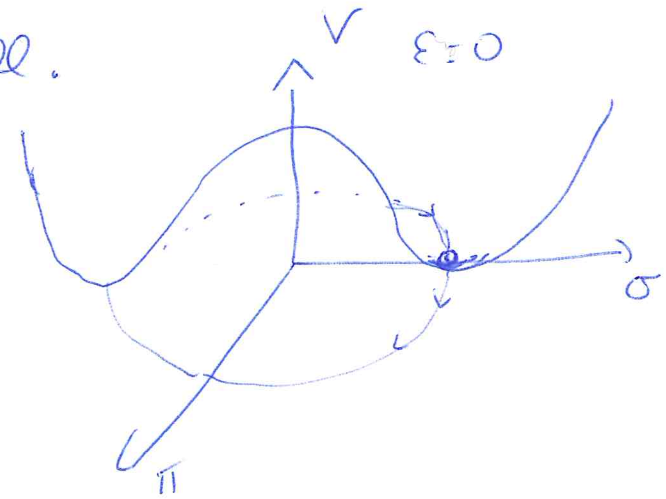
$$U_A(1): \quad \phi \mapsto U_A \phi U_A = e^{i\alpha} \phi e^{i\alpha} = e^{2i\alpha}(\phi)$$

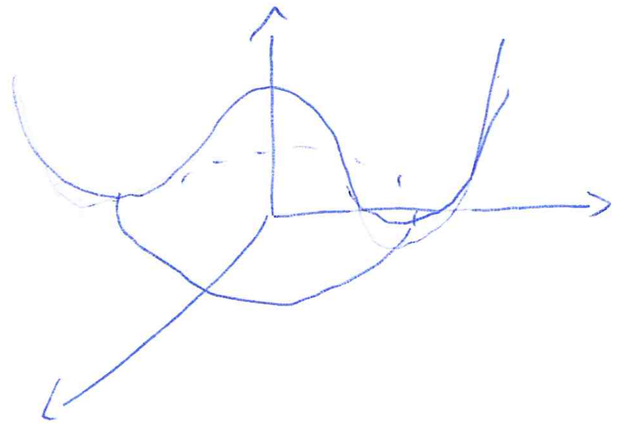
it is completely equivalent to

$$\begin{pmatrix} \pi \\ \sigma \end{pmatrix} \mapsto \begin{pmatrix} \pi' \\ \sigma' \end{pmatrix} = \begin{pmatrix} \cos(2\alpha) & \sin(2\alpha) \\ -\sin(2\alpha) & \cos(2\alpha) \end{pmatrix} \begin{pmatrix} \pi \\ \sigma \end{pmatrix}$$

$\pi \longleftrightarrow \sigma$
rotate one into each other
chiral transf. with parity change.

Let us then 'fix' $G = G_0 = \Lambda_G$ and forget about the glueball.





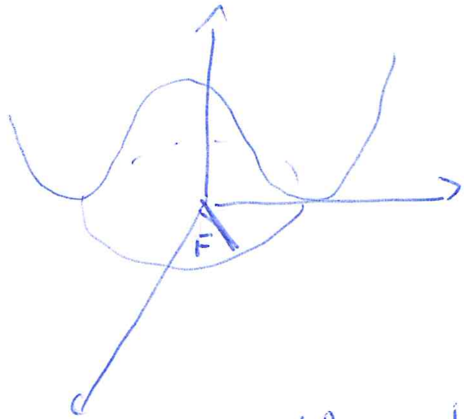
For $\varepsilon = 0$ there are <u>many</u> minima... a full circle of them.

$$V_\chi = -\frac{a G_0^2}{2}(\sigma^2+\pi^2) + \frac{\lambda}{4}(\sigma^2+\pi^2)^2 =$$

$$= \frac{\lambda}{4}\left[(\sigma^2+\pi^2) - \frac{a G_0^2}{\lambda}\right]^2 + \underset{\sim\sim}{\text{const}}$$

$$= \frac{\lambda}{4}(\sigma^2+\pi^2)^2 - 2\frac{\lambda}{4}\frac{a G_0^2}{\lambda}(\sigma^2+\pi^2) + \ldots$$

$$V = \frac{\lambda}{4}\left[\sigma^2 + \pi^2 - F^2\right]^2$$



F is the radius of the "chiral circle"

Which point is chosen? Which vacuum is realised?

In principle, each one is equivalent.

$$\begin{cases} \sigma = F\cos\vartheta_0 \\ \pi = F\sin\vartheta_0 \end{cases} \qquad \vartheta_0 \in (0, 2\pi)$$

Each one is "ok"....

Indeed, in this case each ϑ_0 is ok, and one could have a condensation of σ and π.

However, it is enough to have (a ~~very~~ even small) $\varepsilon > 0$ to obtain a well defined and unique minimum for $(\sigma \neq 0, \pi = 0)$.

$\sigma = F, \pi = 0 \quad (\varepsilon = 0^+)$.

By looking at the figure:

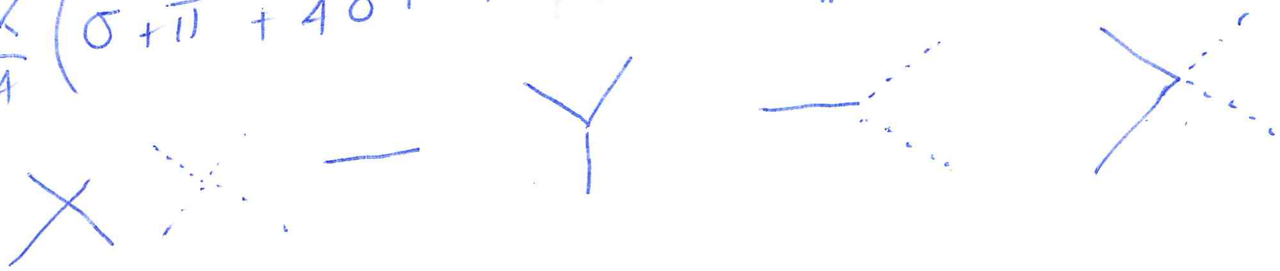
$m_\sigma \neq 0$, $m_\pi = 0$.

Let us verify this by an explicit calculation:

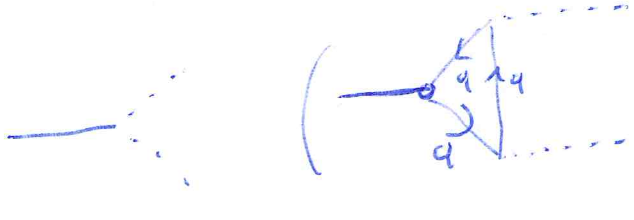
$\sigma \mapsto F + \sigma$

$$V = \frac{\lambda}{4}\left(\sigma^2 + \cancel{F^2} + 2\sigma F + \pi^2 - \cancel{F^2}\right)^2$$

$$= \frac{\lambda}{4}\left(\sigma^4 + \pi^4 + 4\sigma^2 F^2 + 4F\sigma^3 + 4F\sigma\pi^2 + 2\sigma^2\pi^2\right)$$



$$\begin{cases} M_\sigma^2 = 2\lambda F^2 \\ M_\pi^2 = 0 \end{cases}$$



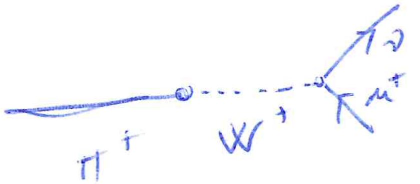
$\mathcal{L}_{\sigma\pi^2} = \lambda F \sigma \pi^2$ $\qquad \Gamma_{\sigma \mapsto \pi\pi} = \frac{|\vec{k}|}{8\pi M_\sigma^2}\left[\sqrt{2}\,\lambda F\right]^2$

The model shows how interactions arise after spont. symmetry breaking.

$$\sigma_0 = \phi = F$$

$$J^A_\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \pi)} \delta\pi = (\partial_\mu \pi)\cdot\phi = f_\pi (\partial_\mu \pi)$$

$$\phi = F = f_\pi .$$



$$\boxed{\mathcal{L}_{\text{weak}} = g_W J^A_\mu W^\mu + \dots}$$

$\varepsilon \propto m_u^2$.

When ε is non-negligible we have:

$$V = \frac{\lambda}{4}(\sigma^2+\pi^2-F^2)^2 - \varepsilon\sigma$$

$\sigma = \phi \neq F$.

$$\begin{cases} \dfrac{\partial V}{\partial \sigma} = \lambda\sigma(\sigma^2-F^2) - \varepsilon = 0 \\ \dfrac{\partial^2 V}{\partial \sigma^2} = \lambda(\sigma^2-F^2)+2\lambda\sigma^2 = \boxed{M_\sigma^2 \ \ (\sigma = F,\ M_\sigma^2 = 2\lambda F^2)} \end{cases}$$

(set $\pi = 0$)

$$\begin{cases} \dfrac{\partial V}{\partial \pi} = \lambda\pi(\sigma^2+\pi^2-F^2) \\ \dfrac{\partial^2 V}{\partial \pi^2} = \lambda(\sigma^2+\pi^2-F^2) + 2\lambda\pi^2 \end{cases}$$

(Do not $\sigma = 0$...)

$\Rightarrow \sigma = F,\ \pi = 0$

$M_\pi^2 = 0$!!!

But then:

$$\left.\frac{\partial^2 V}{\partial \pi^2}\right|_{\substack{\sigma=\phi \\ \pi=0}} = \lambda(\phi^2 - F^2) = \frac{\varepsilon}{\phi}$$

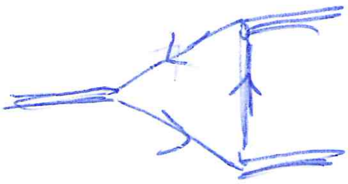
$$\boxed{M_\pi^2 = \frac{\varepsilon}{\phi}}$$ → Goldstone theorem and origin of mass...

$$\boxed{M_\sigma^2 = \lambda(\phi^2 - F^2) + 2\lambda\phi^2 = 2\lambda\phi^2 + M_\pi^2}$$

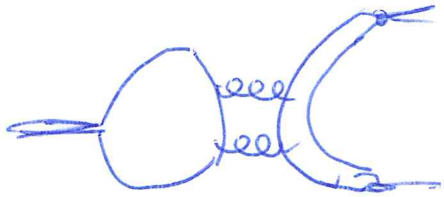
Note: GOR Relation:

In the "big model" it is all like that...

SIMPLY, MORE TERMS and MORE DECAYS APPEAR.

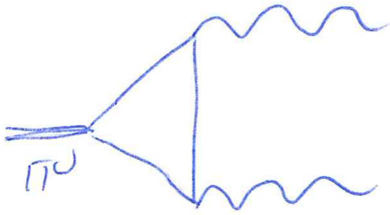


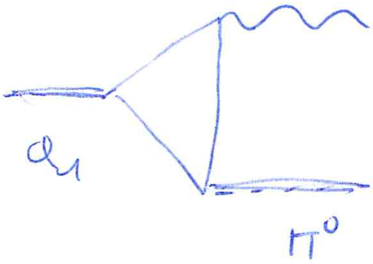
STRONG DECAY
(DOMINANT)

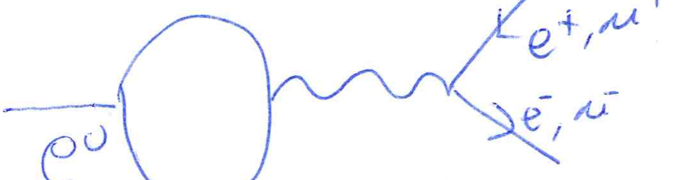


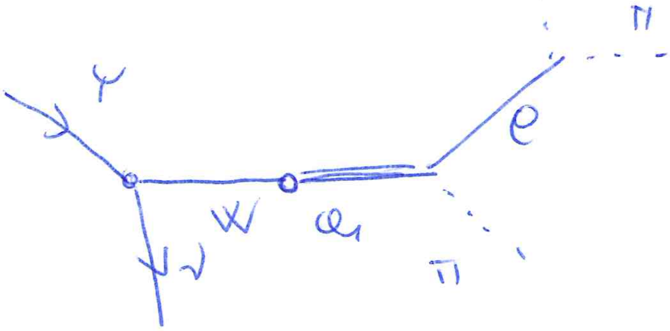
STRONG DECAY
(SUBDOMINANT)

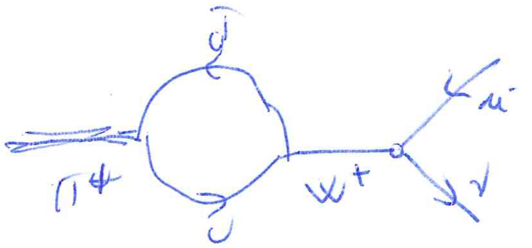
OZI-RULE
(ALIAS: large-N_c)







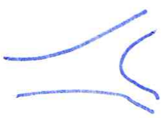




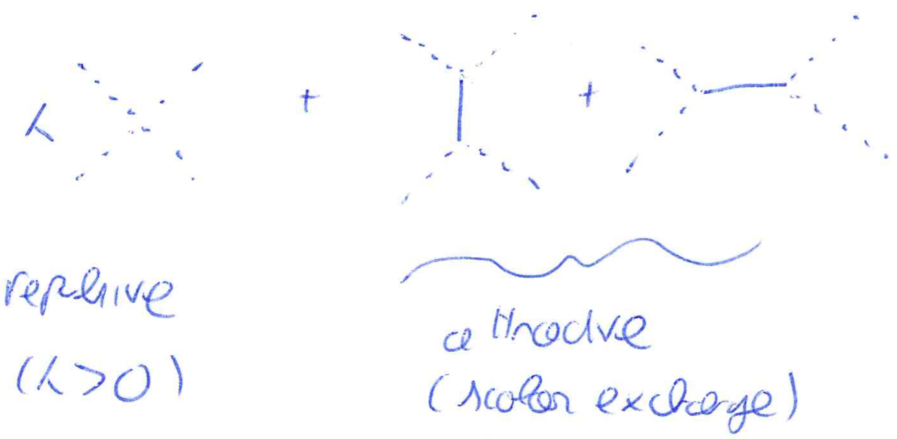
General (but not always)

$$\boxed{\Gamma_{strong} > \Gamma_{em} > \Gamma_{weak}}$$

Discussion of the PDG and the different channel.

Discussion of the J/ψ, for which  is forbidden.

ππ scattering (at tree-level!)



They practically cancel each other...

$T \cong 0$ for $m_\pi \to 0$ (on shell, for soft pions!!!)

chiral symmetry ensures this!!!

The smallness of ππ scattering (and similarly of πN scattering) is a consequence of chiral symmetry.

This is very well

→ Generalization to N_f

All this can be generalised to each N_f...

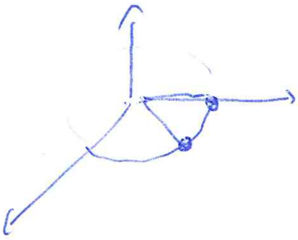
In particular, the existence of Goldstone bosons.

$$A_\mu^a = \bar{q}\gamma^\mu t^a q$$

$$Q_A^a = \int d^3x\, A_0^a = \int \bar{q}\gamma^0 t^a q$$

$|0\rangle$ is not such as $Q^a|0\rangle = 0$...

$$Q^a|0\rangle = |0'\rangle \neq 0$$



But: $[H, Q_A^a] = 0$!

Then:

$$H\left(Q_A^a|0\rangle\right) = Q^a H|0\rangle = 0 \;!$$

$Q_A^a|0\rangle$ is then a state with zero mass...

this is the Goldstone boson.

$a = 1, \dots, N_f^2 - 1 = 8$ (π, K, η)

$a = 0$ No → anomaly $[H, Q_A^0] \neq 0$!!! → η' is not a Goldstone...

<u>Polar coordinates:</u>

$$\begin{cases} \sigma = \rho \cos\left(\dfrac{\varphi}{\phi}\right) \\ \\ \pi = \rho \sin\left(\dfrac{\varphi}{\phi}\right) \end{cases} \qquad \begin{cases} \rho = \sqrt{\sigma^2 + \pi^2} \\ \\ \dfrac{\varphi}{\phi} = \arctan\left(\dfrac{\pi}{\sigma}\right) \end{cases}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi)^2 - \frac{\lambda}{4}\left(\sigma^2 + \pi^2 - F^2\right)^2 + \varepsilon\sigma$$

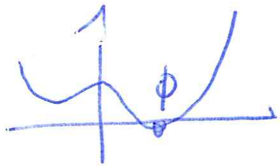
How does $\mathcal{L}$ look like as function of ρ and φ?

$$\mathcal{L} = \frac{1}{2}\left[\partial_\mu \rho \cdot \cos\frac{\varphi}{\phi} - \rho\frac{\partial_\mu \varphi}{\phi} \cdot \sin\frac{\varphi}{\phi}\right]^2 + \frac{1}{2}\left[\partial_\mu \rho \cdot \sin\frac{\varphi}{\phi} + \rho\frac{\partial_\mu \varphi}{\phi}\cos\frac{\varphi}{\phi}\right]^2$$

$$- \frac{\lambda}{4}\left[\rho^2 - F^2\right]^2 + \varepsilon\rho\left(1 - \frac{\varphi^2}{\phi^2} + \dots\right)$$

$$= \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}\left(\frac{\rho}{\phi}\right)^2(\partial_\mu \varphi)^2 - \frac{\lambda}{4}\left[\rho^2 - F^2\right]^2 + \varepsilon\rho - \frac{\varepsilon\rho}{\phi^2}\varphi^2$$

$$V(\rho) = \frac{\lambda}{4}\left[\rho^2 - F^2\right]^2 + \varepsilon \rho$$



$$\rho \mapsto \phi + \rho$$

$$V(\rho) = \frac{1}{2} m_\rho^2 \rho^2 + \# \rho^3 + \# \rho^4 \;:$$

$$\boxed{m_\rho^2 = m_\sigma^2}$$ (indeed ρ is "sigma", just as φ is Π...)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}(\partial_\mu \varphi)^2 + \frac{1}{2}\,\frac{2\rho\phi}{\phi^2}(\partial_\mu\varphi)^2 + \frac{1}{2}\frac{\rho^2}{\phi^2}(\partial_\mu\varphi)^2 +$$

$$\frac{1}{2} m_\sigma^2 \rho^2 + \# \rho^3 + \# \rho^4 - \frac{\varepsilon}{\phi}\varphi^2 - \frac{\varepsilon}{\phi^2}\rho\varphi^2$$

$$\underbrace{m_\varphi^2 = \frac{\varepsilon}{\phi} = m_\pi^2}$$ (just as before...)

Note, for ε very small $\phi = F$:

$$\mathcal{L}_{\rho\varphi^2} = \frac{\rho}{\phi}(\partial_\mu\varphi)^2 \qquad \Gamma_{\rho\to\varphi\varphi} = \frac{|k_1|}{8\pi M_\sigma^2}\cdot\left[\frac{\sqrt{2}}{\phi^2}\cdot(k_1\cdot k_2)\right]^2 = \Gamma_{\sigma\to\pi\pi}$$

Decay width

In fact:

$$k_1 \cdot k_2 = \frac{M_e^2}{2}$$

$$M_e^2 = 2\lambda F^2$$

→ It's a subtle point... when including loop things change (ren. theory → non-ren. one)

Still under debate

Derivative: more complicated

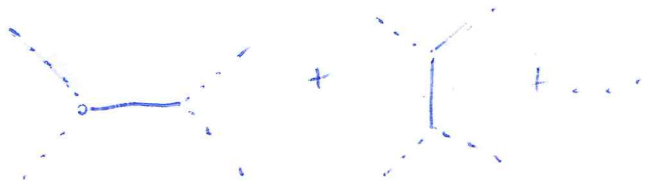
ergo:

$$\Gamma_{e\to\varphi\varphi} = \frac{|k|}{8\pi M_0^2}\left[\frac{2}{F^2}\cdot\lambda^2 F^4\right]$$

✓

Scattering

Note, to lowest order: $e^2(\partial_\mu\varphi)^2$



$(k_1k_2)(k_3k_4)$

Because of the derivatives we see that $T \simeq 0$ for $m_i \to 0$...

Here, with the angular variable, we immediately see that!!!

So, the different representation may be useful for some aspects and more disadvantageous for others!

(This is a general one)

chiral (axial) transformation:

$$\phi = \sigma + i\pi = \rho\cos\left(\frac{\varphi}{F}\right) + i\sin\left(\frac{\varphi}{F}\right) = \rho e^{i\varphi/\phi}$$

$$\phi \mapsto U_A\left[\rho e^{i\pi/\phi}\right]U_A = e^{i2\alpha}\rho e^{i\varphi/\phi} =$$

$$= \rho e^{i(\pi + 2\alpha F)/F}$$

↳ chiral invariant

Now, let us use the fact that M_ρ is heavy and M_φ light:

Send λ to ∞ by keeping F fixed.

~~The~~ ρ field can be integrated out...

It can be integrated out!!!!!

(in the extreme limit)

$\rho = F =$ const. No fluctuation!!! Too heavy...)

$M_\rho = \infty$ or extreme limit

Nonlinear model

$$\Phi = U_\chi = e^{i\varphi/\phi}$$

$$\boxed{U_\chi \mapsto U_L U_\chi U_R^+}$$

If we integrate out ψ because it is heavy

we get an interacting effective theory with φ only

$$\mathcal{L}_{eff}(\varphi)!!!$$

This is not an easy task in view of the

interactions involved...

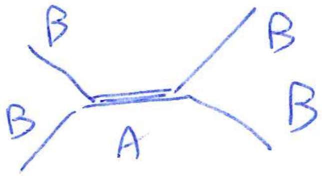
First, let us recall what does it mean to

integrate out... and then discuss why

this is actually not needed!!!!

Digression: Integrating out....

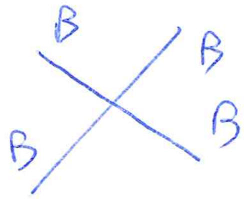
$$\mathcal{L}_{int} = g A B^2$$



$$T \propto \frac{g^2}{p^2 - M_A^2}$$

if M_A is very large... at first order

$$T \propto -\frac{g^2}{M_A^2} = -\lambda$$



$$\lambda = +\frac{g^2}{M_A^2}$$

$$\mathcal{L}_{eff} = -\lambda B^4$$

(and this is only at first order....)

You can go further by writing

$$T \propto \frac{g^2}{M_A^2\left(1-\frac{p^2}{M_A^2}\right)} = \frac{g^2}{M_A^2}\sum_{k=0}^{N}\left(\frac{p^2}{M_A^2}\right)^2$$

$N = 1$

$$T(p^2) = \frac{g^2}{M_A^2} + \frac{g^2}{M_A^4}p^2$$

$$\mathcal{L}_{eff}(B) = \frac{g^2}{M_A^2}B^4 + \frac{g^2}{M_A^4}B^2(-\Box B^2)$$

$$= \frac{g^2}{M_A^2} - 2\frac{g^2}{M_A^4}(B\partial_\mu B)^2$$

and so on and so forth...

Now the same should be done for very large M_σ...

then out of $\mathcal{L}(\varphi)$ you get $\mathcal{L}_{eff}(\varphi)$ only...

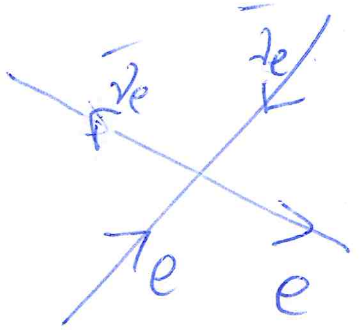
It is indeed a complicated task!!!!!!

Prominent example of "low-energy effective theory":

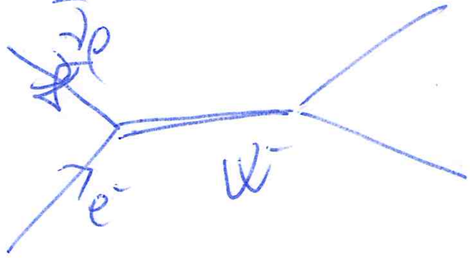
the Fermi theory !!!

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} \left(\bar{\psi}_{\nu_e} \gamma^\mu (1+\gamma^5) \psi \right)^2$$

$$[G_F] = [E^{-2}]$$



actually arises from



$$G_F \propto \frac{g_{weak}^2}{M_W^2} \ll 1 !!$$

very small because M_W is heavy !!!!!

Back to our problem:

If we 'integrate out' ρ because it is heavy, we get an interacting theory with φ only!

$\mathcal{L}(\varphi)$.

Now, suppose that you do not know what is the original Lagrangian...

... But the starting point is - from the very beginning - the quantity

$$U_\alpha = e^{i\varphi/\phi} \quad \text{with} \quad U_\alpha \mapsto U_L \psi_\alpha U_R^+$$

$$= e^{2i\alpha_A} \psi_\alpha$$

Note, and this is the crucial point, a very analogous form of U_χ is present for $N_f = 3$:

$$U_\chi = e^{i\varphi^a t^a/\phi} \qquad \phi = f_\pi$$

$$U_\chi \mapsto U_L U_\chi U_R^\dagger$$

$\varphi^a \equiv$ generalized "angular coordinates"

$$\hat{\varphi} = \varphi^a t^a \equiv P$$

(Just as φ and π ...)

Now, we have the same transformation of our old Φ

$$\begin{cases} \Phi = S + i P \mapsto U_L \Phi U_R^{+} \\ U_{\pi} = e^{i \varphi^a t^a / f_\pi} \mapsto U_L U_\pi U_R^{+} \end{cases}$$

but the Lagrangians are different... (of course, related (they should) upon integrating out the S-fields from $\mathcal{L}_\Phi$.

Note:

$\mathrm{Tr}[U_\pi^{+} U_\pi]$

of course... chiral invariant... but: $U_\pi^{+} U_\pi = 1$!!!

This is a trivial thing!!!

Completely $\neq$ from $\mathrm{Tr}[\Phi^{+}\Phi]$.

Here (and just as with our "trivial" A-B example) we have to include derivatives.

The easiest non-trivial object that you can write down with U_N is:

$$\frac{F^2}{4} \mathrm{Tr}\left[\partial_\mu U_N^\dagger \partial^\mu U_N\right]$$

in order to get $\mathcal{L}_{kin} = \frac{1}{2}(\partial_\mu \varphi^a)^2$ $\qquad \mathcal{L}_2 = \frac{\Phi^2}{4} \mathrm{Tr}\left[\partial_\mu U_N^\dagger \partial^\mu U_N\right]$

Generically related on cos of relation...

[Note → with strict chiral invariance → NO MASS TERM IS POSSIBLE

Breaking of symmetry is then encoded in:

$$\mathcal{L}_{esB} = \frac{\Phi^2}{4} \mathrm{Tr}\left[\mathcal{M}(U+U^\dagger)\right] \qquad \mathcal{M} \equiv \mathrm{diag}\{\mathcal{M}_0, \mathcal{M}_0\}$$

Two parameters: $\Phi, \mathcal{M}_0$

Gauge fields:

4 derivatives ...

$$\mathcal{L}_4 = L_1 \left(\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right]\right)^2 +$$

$$L_2 \,\mathrm{Tr}\left[\partial_\mu U^\dagger \partial_\nu U\right] \mathrm{Tr}\left[\partial^\mu U^\dagger \partial^\nu U\right]$$

$$+ L_3 \,\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger\right]$$

$$+ \dots$$

low energy constants ...

they cannot be determined at this level ... not by symmetry ... but only from the experiment ...

- loop and power counting
- very well for ππ (and πN) scattering (unbreakable)

General discussion

- comparison and
- Advantages and disadvantages ... again, it depends on what ~~one~~ we want to do
- Religious views and similar things
- work of Florian