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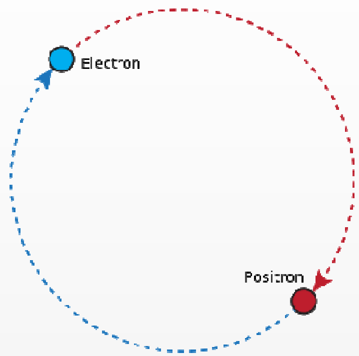
The mystery of hadron masses

Francesco Giacosa

Positronium mass



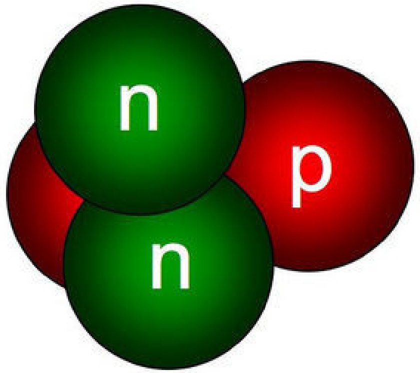
$$m_e = 0.511 \ \text{MeV}$$

$$m_{\text{Positronium}} = 2m_e - 6.8 \cdot 10^{-6} \ \text{MeV}$$

$$m_{\text{Positronium}} \approx 2m_e$$

Mass of the α particle

Nucleus of a Helium-atom



$$m_\alpha = 3.727379240 \text{ GeV}$$

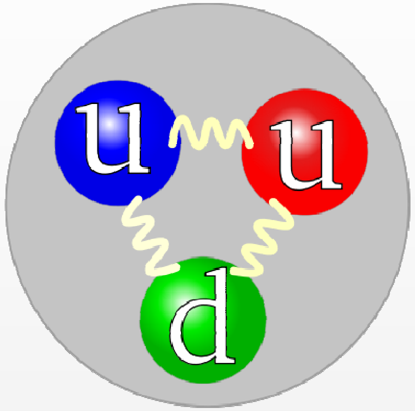
$$m_p = 0.93827 \text{ GeV} \;\; ; \;\; m_n = 0.93956 \text{ GeV}$$

$$m_\alpha \approx 2m_p + 2m_n$$

$$m_\alpha = (2m_p + 2m_n) - 28.2956 \text{ MeV}$$

Proton

$$m_p = 938.27\,\mathrm{MeV}$$

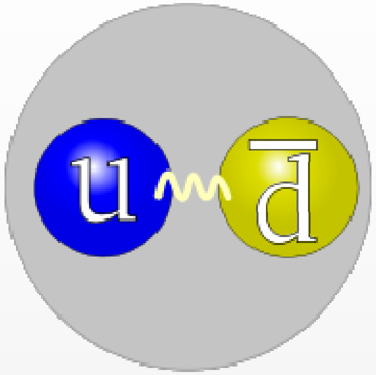


$$m_u = 2.3^{+0.7}_{-0.5}\ \mathrm{MeV}$$

$$m_d = 4.8^{+0.7}_{-0.5}\ \mathrm{MeV}$$

$$m_p >> 2m_u + m_d \approx 10\ \mathrm{MeV}$$

The ρ and the π mesons



Rho-meson $m_{\rho^+} = 775 \ \text{MeV}$

Pion $m_{\pi^+} = 139 \ \text{MeV}$

$$m_u + m_d \approx 7 \ \text{MeV}$$

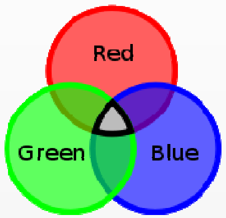
The Lagrangian of QCD and its symmetries

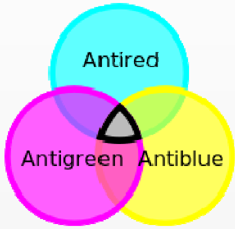
Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

Francesco Giacosa

Fields of the QCD Lagrangian

Quark: u,d,s R,G,B





$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix} ; \quad i = u,d,s$$

8 type of gluons ($R\bar{G}$,$B\bar{G}$,…)

$$A_\mu^a \; ; \quad a = 1,..., 8$$

SU(3)color: local gauge group

$$U(x) \in SU(3) \rightarrow U^{+}U = 1\,,\ \det U = 1. \qquad U = e^{it^a\theta^a}$$

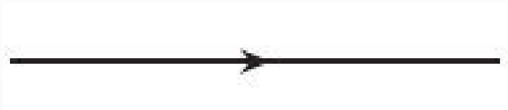
$$q_i(x) \rightarrow U(x)q_i(x) \qquad A_\mu = A_\mu^a t^a \rightarrow U(x)A_\mu U^{+}(x) - \frac{i}{g_0}U(x)\partial_\mu U^{+}(x)$$

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i(i\gamma^\mu D_\mu - m_i)q_i - \frac{1}{4}G_{\mu\nu}^a G^{a,\mu\nu}$$

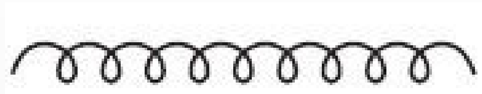
$$D_\mu = \partial_\mu - ig_0 A_\mu^a t^a$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_0 f^{abc} A_\mu^b A_\nu^c,\ \ a, b, c = 1, ..., 8$$

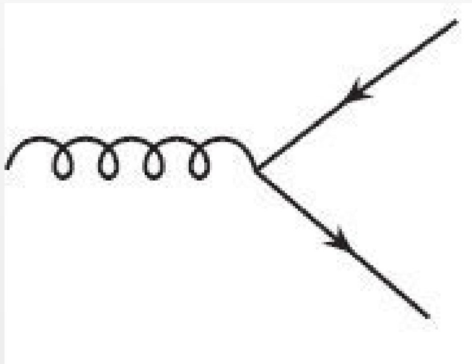
Feynman diagrams of QCD



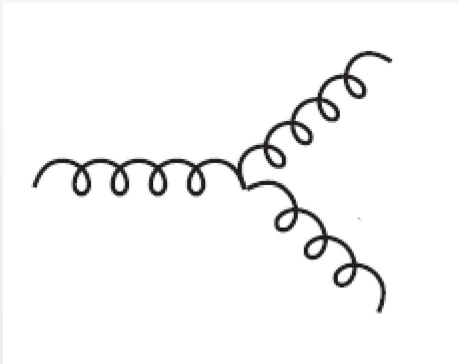
Quark



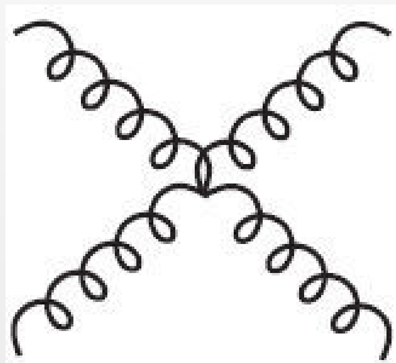
Gluon



Gluon-quark-antiquark vertex



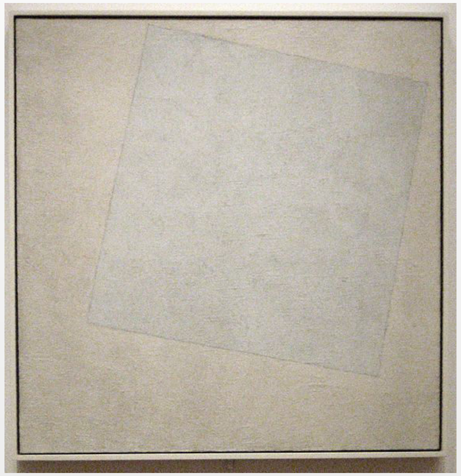
3-gluon vertex



4-gluon vertex

No ‚colored‘ state has been seen.

Confinement: physical states (hadrons) are white.



Painting of K. Malevich: ‚white on white‘.

Immediate question: bound state of gluons?

Trace anomaly

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}, \; D_\mu = \partial_\mu - i g_0 A^a_\mu t^a$$

Chiral limit: $m_i = 0$

$x^\mu \rightarrow x'^\mu = \lambda^{-1} x^\mu$ is a classical symmetry broken by quantum fluctuations (trace anomaly)

$$g_0 \overset{\text{Renormierung}}{\longrightarrow} g(\mu)$$

$$\partial_\mu J^\mu = T^\mu_\mu = \frac{\beta(g)}{4g} G^a_{\mu\nu} G^{a,\mu\nu} \neq 0 \qquad \beta(g) = \mu \frac{\partial g}{\partial \mu}$$

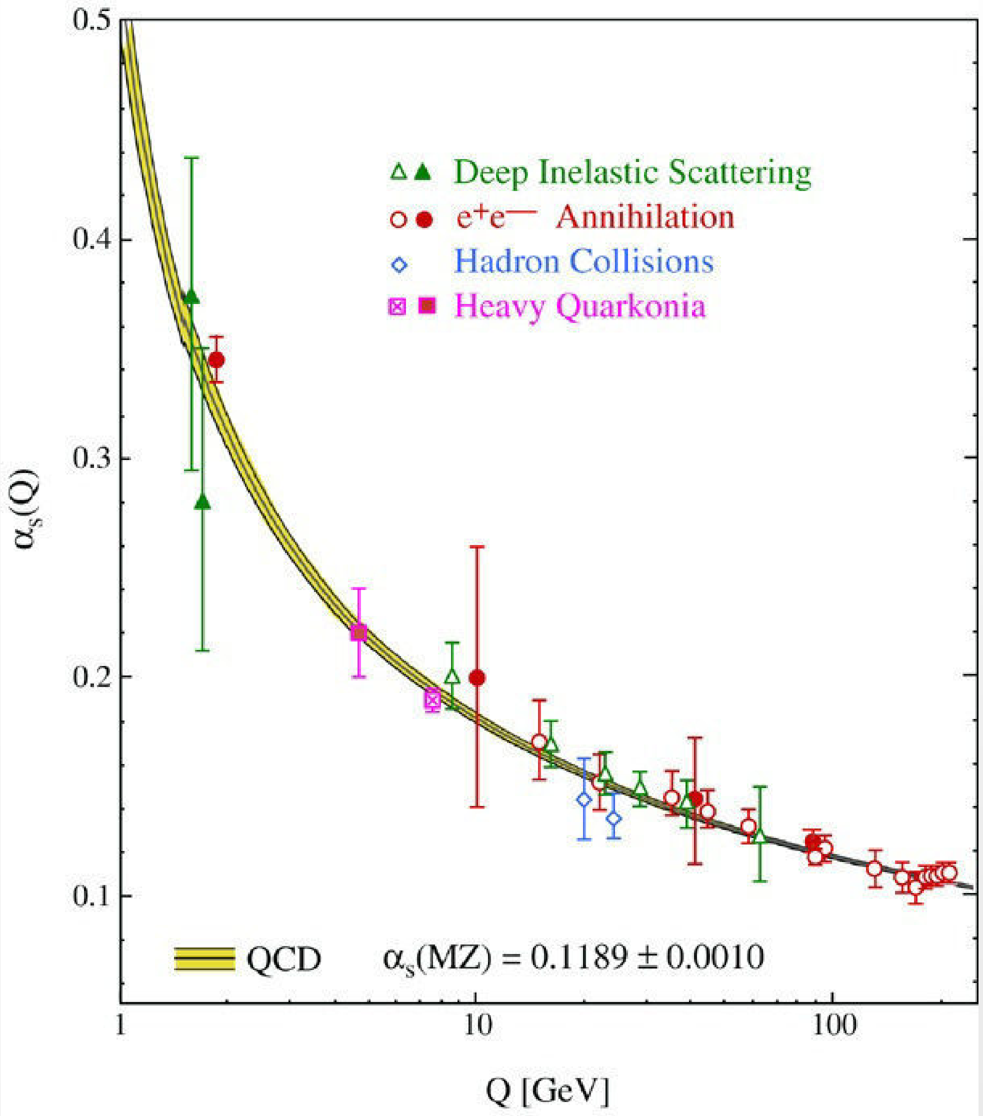
$$g^2(\mu) = \frac{1}{2b \log \frac{\mu}{\Lambda_{YM}}}$$

$$\Lambda_{YM} \approx 250 \quad \text{MeV}$$

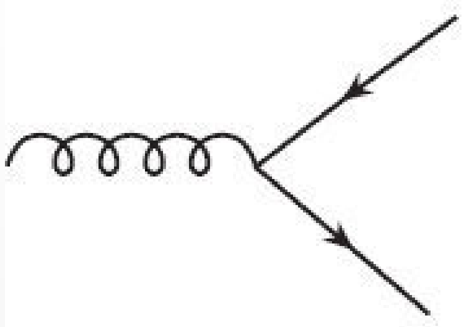
Dimensional transmutation

Trace anomaly and running coupling

$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Flavor symmetry



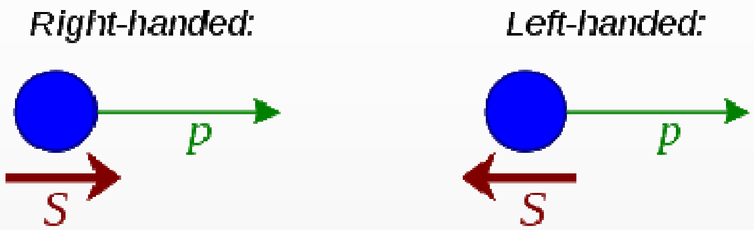
Gluon-quark-antiquark vertex.

It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

$$\mathrm{U} \in \mathrm{U}(3)_{\mathrm{V}} \rightarrow \mathrm{U}^{+}\mathrm{U} = 1$$

Chiral symmetry/1



$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L}$$

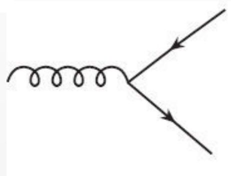
$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \rightarrow (U_R)_{ij}\, q_{j,R} + (U_L)_{ij}\, q_{j,L}$$

$$U_R \subset U(3)_R \; ; \; U_L \subset U(3)_L$$

Chiral symmetry/2

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}, \; D_\mu = \partial_\mu - i g_0 A^a_\mu t^a$$



$$\overline{q}_i i\gamma^\mu D_\mu q_i = \overline{q}_{i,R} i\gamma^\mu D_\mu q_{i,R} + \overline{q}_{i,L} i\gamma^\mu D_\mu q_{i,L}$$ is chirally invariant

$$m_i \overline{q}_i q_i = m_i \overline{q}_{i,R} q_{i,L} + m_i \overline{q}_{i,L} q_{i,R}$$ is **not** chirally invariant

In the chiral limit (m_i=0) chiral symmetry is exact

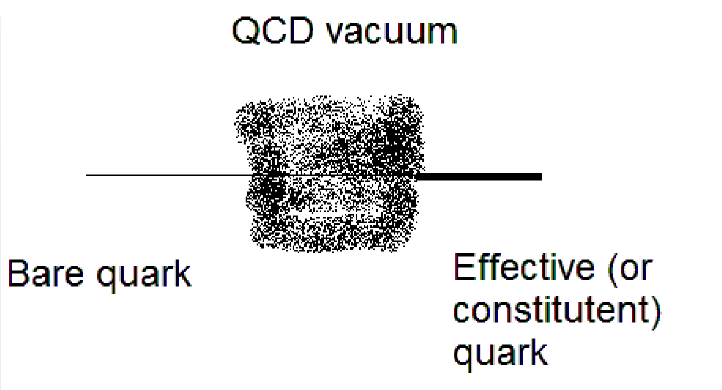
Spontaneous breaking of chiral symmetry

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

$$SU(3)_R \times SU(3)_L \to SU(3)_{V=R+L} \quad \Rightarrow \quad q_i \to U_{ij} q_j$$

$$\left\langle \bar{q}_i q_i \right\rangle = \left\langle \bar{q}_{i,R} q_{i,L} + \bar{q}_{i,L} q_{i,R} \right\rangle \neq 0$$

$$m \simeq 5 \text{ MeV} \longrightarrow m^* \simeq 300 \text{ MeV} \gg m$$



Masses revisited

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$$m^* \approx 300\,\mathrm{MeV}$$

$$m_p \approx 3m^*$$

$$m_\rho \approx 2m^*$$

$$m_\pi << 2m^*$$

Pion: (quasi) Goldstone boson. $m_\pi^2 \propto (m_u + m_d)\langle \bar{q}q \rangle$

Symmetries of QCD: summary

$SU(3)_{color}$: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (trace anomaly) and by small quark masses

$SU(3)_R \times SU(3)_L$: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is spontaneously broken to $U(3)_{V=R+L}$

$U(1)_{A=R-L}$: holds at a classical level, but is also broken by quantum fluctuations (chiral anomaly)