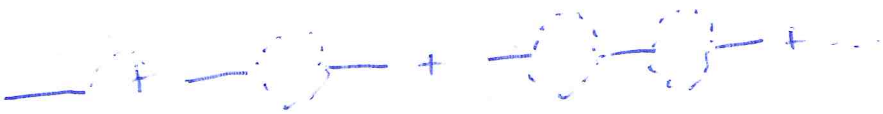
Recall of the basic results in a scalar QFT

$$\mathcal{L} = \frac{1}{2}(\partial_\mu S)^2 - \frac{M_0^2}{2} S^2 + \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{m^2}{2}\varphi^2 + g S \varphi^2$$

Tree-level decay width

$$\begin{cases} \Gamma_{S \mapsto \varphi\varphi} = \dfrac{\sqrt{\frac{M_0^2}{4} - m^2}}{8\pi M_0^2} \cdot \dfrac{1}{2} |-iM|^2 \, \theta(M_0 - 2m) \; ; \quad -iM = 2ig \\ P(t) = e^{-\Gamma_{S \mapsto \varphi\varphi} t} \end{cases}$$

One can go further by studying the propagator of S

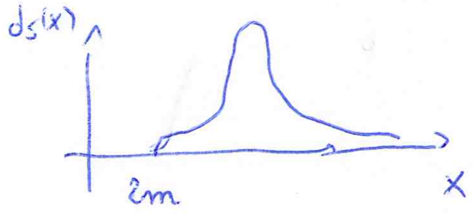


$$\Delta_S(p^2) = \frac{1}{p^2 - M_0^2 + (\sqrt{2}g)^2 \Sigma(p^2) + i\varepsilon}$$

$d_S(x) = \delta(x - M_0)$ for $g \mapsto 0$

$p = (x, \vec{0})$ rest-frame ($x \equiv$ "running mass")

$$d_S(x) = \frac{2x}{\pi} \lim_{\varepsilon \mapsto 0} \left[\operatorname{Im} \Delta_S(x) \right]$$



$$a(t) = \int_0^\infty d_S(x)\, e^{-ix} \longrightarrow P(t)$$

$\Sigma(x)$ is divergent.. M_0 "bare mass" to M "renormalized mass".

Indeed, you may recalculate $\Gamma_{S \mapsto \varphi\varphi}$ using the mass M.

Källén-Lehmann representation:

$$\Delta_S(p^2) = \frac{1}{p^2 - M_0^2 + g^2\Sigma(p^2) + i\varepsilon} = \underbrace{\int_0^\infty dx \, \frac{d_S(x)}{p^2 - x^2 + i\varepsilon}}$$

sum of many free propagators, weighted by $d_S(x)dx$.

$$=$$

$$d_S(x) = \frac{2x}{\pi} \frac{(\sqrt{2}g)^2 \operatorname{Im}\Sigma(x)}{(x^2 - M_0^2 + (\sqrt{2}g)^2\operatorname{Re}\Sigma)^2 + ((\sqrt{2}g)^2\operatorname{Im}\Sigma)^2}$$

optical theorem:

$$\boxed{(\sqrt{2}g)^2 \operatorname{Im}\Sigma(x) = x\,\Gamma(x)}$$

$$(\sqrt{2}g)\,\Gamma(x) = \underbrace{\frac{\sqrt{\frac{x^2}{4} - m^2}}{8\pi x^2} [\sqrt{2}g]^2}\,\theta(x-2m)$$

i.e. decay with a 'running mass'.

How do I get the usual B-W limit?

$$d_S(x) = \frac{2x}{\pi} \frac{x\,\Gamma(x)}{(x^2 - M_0^2 + (\sqrt{2}g)^2\operatorname{Re}\Sigma)^2 + (x\,\Gamma(x))^2}$$

$$\simeq N \frac{2M}{\pi} \frac{M\,\Gamma(M)}{(x^2 - M^2)^2 + (M\,\Gamma(M))^2}\,\theta(x-2m)$$

$\downarrow$

I need an extra-normalisation...

$$d_5^{BW,rel}(x) = N \frac{\vartheta(x-2m)}{(x^2-M^2)^2+(M\Gamma)^2} = N \frac{\vartheta(x-2m)}{(x-M)^2(x+M)^2+(M\Gamma)^2} =$$

$$\simeq \frac{N}{(2M)^2} \frac{\vartheta(x-2m)}{(x-M)^2+(\Gamma/2)^2} \simeq \tilde{N} \frac{1}{(x-M)^2+(\Gamma/2)^2} = d_5^{BW,nr}(x) .$$

There are <u>not</u> only scalar particles...

actually, to be precise in the SM there is only one (spinless) scalar field: the Higgs.

In QCD we have scalar and pseudoscalar fields, which emerge as composite objects ($\bar{q}q$)

π, K, η, η'
All the f_0 states, ...

Many Bosons of the SM are vectors:

- γ (A_μ)
- $W^\pm, Z^0$ (W_μ)
- gluons (A_μ^a)

... and in QCD ⟼ emerging vector and axial-vector fields ($\vec{\rho}, \vec{a}_1, \ldots$)

Moreover, an additional field (if we will be able to quantize gravity) is the graviton

$G_{\mu\nu}$

(in QCD ⟼ tensor mesons have similar (but not equal) mathematical properties)

Additionally, there are fermions... but this is another story...

As an example, let us start from a ρ vector meson with mass m_ρ.

Fields: ρ_μ

$$\mathcal{L} = -\frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{m_\rho^2}{2}\rho_\mu\rho^\mu \quad ; \quad \boxed{\rho_{\mu\nu} = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu ;}$$

(Achtung: different sign convention).

For the photon we have:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \longmapsto \text{no mass term, gauge symmetry:}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad A_\mu \longmapsto A_\mu + \partial_\mu \vartheta, \quad \vartheta = \vartheta(x)$$

Eq. of motion:

$$\Box\rho^\mu - \partial^\mu(\partial_\nu\rho^\nu) + m_\rho^2\rho^\mu = 0$$

$$\left(\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) \qquad = 0\right)$$

$\hookrightarrow \partial_\mu$

$$\cancel{\Box(\partial_\mu\rho^\mu)} - \cancel{\Box(\partial_\nu\rho^\nu)} + m_\rho^2\partial_\mu\rho^\mu = 0$$

$$\partial_\mu\rho^\mu = 0 \rightarrow \text{the field must be transversal!} \Rightarrow \boxed{\Box\rho^\mu = 0}$$

(This condition can be introduced as a gauge condition on A_μ, but does not follow from the eqs).

$$e_\mu = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \frac{1}{\sqrt{2\omega}} \sum_{r=1,2,3} \Big\{ \underbrace{a^r_{\vec{k}}}_{\text{annihil. of } e^0} \varepsilon^r_\mu(\vec{k}) e^{-ikx} + \underbrace{a^{r\dagger}_{\vec{k}}}_{\text{creation of } e^0} \varepsilon^r_\mu(\vec{k}) e^{ikx} \Big\}$$

$$\partial_\mu e^\mu = 0$$

$$k^\mu \varepsilon^r_\mu(\vec{k}) = 0$$

$$\boxed{\varepsilon^r_\mu \, \varepsilon^{p,\mu} = -\delta^{rp}}$$ convention

Suppose that: $\vec{k} = (0, 0, |\vec{k}|)$. What are the polarization vectors?

$$\varepsilon^{(1)}_\mu = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \varepsilon^{(2)}_\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{cases} \varepsilon^{(1)^2}_\mu = -1 \\ \varepsilon^{(2)^2}_\mu = -1 \\ \varepsilon^{(1)}_\mu \varepsilon^{(2)\mu} = 0 \\ k_\mu \varepsilon^{(1,2)\mu} = 0 \end{cases}$$

$$\varepsilon^{(3)}_\mu = \begin{pmatrix} A \\ 0 \\ 0 \\ B \end{pmatrix}$$

Let us determine it.... $\varepsilon^{(3)^2}_\mu = A^2 - B^2 = -1$ (norm.)

$$K_\mu \varepsilon^{(3)\mu} = \omega A - |\vec{k}| B = 0$$

$$\omega A = |\vec{k}| B \quad \rightarrow \quad A = \frac{|\vec{k}|}{\omega} B$$

$$\frac{|\vec{k}|^2}{\omega^2} B^2 - B^2 = -1 \qquad B^2 \left(\frac{|\vec{k}|^2}{\omega^2} - 1 \right) = -1$$

$$B^2 = \frac{1}{1 - \frac{|\vec{k}|^2}{\omega^2}} = \frac{\omega^2}{\omega^2 - |\vec{k}|^2} = \frac{\omega^2}{m_e^2}$$

$$B = \frac{\omega}{m_e}$$

$$A = \frac{|\vec{k}|}{\omega} \frac{\omega}{m_e}$$

Ergo:

$$\boxed{\varepsilon_\mu^{(3)} = \begin{pmatrix} |\vec{k}|/m_e \\ 0 \\ 0 \\ \omega/m_e \end{pmatrix}} \mapsto \text{ it is obviously } \perp \text{ to } \varepsilon_\mu^{(1)} \text{ and } \varepsilon_\mu^{(2)}$$

$$\varepsilon_\mu^{(3)^2} = \frac{|\vec{k}|^2}{m_e^2} - \frac{\omega^2}{m_e^2} = \frac{|\vec{k}|^2 - m_e^2 + |\vec{k}|^2}{m_e^2} = -1$$

$$K_\mu \varepsilon^{(3)\mu} = \omega \cdot \frac{|\vec{k}|}{m_e} - |\vec{k}| \frac{\omega}{m_e} = 0 \qquad \text{q.e.d.}$$

Free Propagator of the ρ-field

$$\langle 0|T[\rho_\mu(x)\,\rho_\nu(y)]|0\rangle = \int \frac{d^4k}{(2\pi)^4}\, i\,\Delta_{\mu\nu}(k)\, e^{-ik(x-y)}$$

$$\Delta_{\mu\nu}(k) = \frac{-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_\rho^2}}{k^2-m_\rho^2+i\epsilon} = \frac{\sum_a \epsilon_\mu^a \epsilon_\nu^a}{k^2-m_\rho^2+i\epsilon}$$

$$\sum_a \epsilon_\mu^a \epsilon_\nu^a = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_\rho^2}$$

[Achtung: 'problem' due to the propagator..

$\Delta_{\mu\nu}(k\to\infty) \sim$ const. The prop. does not vanish..

Theory with vector particles is not renorm.

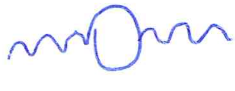
SM → W, Z get their mass from the spont. symm. breaking]

[Remark on the photon:

$$\Delta_{\mu\nu} = \frac{-g_{\mu\nu}+\alpha\frac{k_\mu k_\nu}{k^2}}{k^2+i\epsilon}$$

→ you then realise that $m_\rho \to 0$ would be 'problematic'.

→ α can be whatever... the results should not depend on it (g.i.)



→ one can take $\Delta_{\mu\nu} = \frac{-g_{\mu\nu}}{k^2+i\epsilon}$

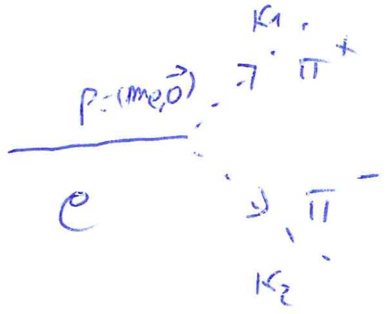
$e \to \pi\pi$

$$\mathcal{L} = -\frac{1}{4} e_{\mu\nu}^2 + \frac{m_e^2}{2} e_\mu^2 + iq\, e_\mu \left(\pi^+ \partial^\mu \pi^- - \pi^- \partial^\mu \pi^+\right)$$

$$+ (\partial_\mu \pi^+)(\partial^\mu \pi^-) - m_\pi^2 \pi^+ \pi^-$$

$[q] = [E^0]$

(true, theory like it's 'renorm').



$$\Gamma = \frac{\sqrt{\frac{m_e^2}{4} - m_\pi^2}}{8\pi m_e^2} \left|\overline{-iM}\right|^2$$

Achtung!!!! Sum over polarisations over final states, average over initial states

$$-iM^a = \varepsilon_\mu^a \left(ik_2^\mu - ik_1^\mu\right) i (iq) = +iq\, \varepsilon_\mu^a \left(k_1^\mu - k_2^\mu\right) = iq\, \varepsilon_\mu^a k^\mu$$

$\boxed{k = k_1 - k_2}$

$$\varphi = \frac{1}{\sqrt{V}} \sum_k \frac{1}{\sqrt{2\omega}} \left(a_k e^{-ikx} + a_k^\dagger e^{ikx}\right)$$

$\langle 0 | a_k$ $\quad$ $+ik_\mu$ for final states...

SUM OVER FINAL STATES, AVERAGE OVER INITIAL STATES

11C

$$\overline{|-iM|}^2 = \frac{1}{3} \sum_a |-iM^a|^2 =$$

$$= \frac{1}{3} \sum_a \varepsilon_\mu^a K^\mu \varepsilon_\nu^a K^\nu g^2 =$$

$$= \frac{1}{3} \left(-g_{\mu\nu} + \frac{p^\mu p^\nu}{m_\rho^2} \right) K^\mu K^\nu g^2$$

$$= \frac{1}{3} \left(-K^2 + \frac{(p \cdot K)^2}{m_\rho^2} \right) g^2$$

$$K = K_1 - K_2$$

$$K^2 = (K_1 - K_2)^2 = K_1^2 + K_2^2 - 2K_1K_2 = 2m_\pi^2 - 2\frac{m_\rho^2 - 2m_\pi^2}{2} =$$

$$= -m_\rho^2 + 4m_\pi^2$$

$$\overline{|-iM|}^2 = \frac{1}{3}(m_\rho^2 - 4m_\pi^2) g^2$$

$$\Gamma_{\rho^0 \mapsto \pi^+\pi^-} = \Gamma_{\rho^- \mapsto \pi^-\pi^0} = \Gamma_{\rho^+ \mapsto \pi^+\pi^0} = \Gamma_{\rho \mapsto \pi\pi} ;$$

$$\bar{\Gamma}_{\rho \mapsto \pi\pi} = \frac{1}{3} \left(\Gamma_{\rho^0 \mapsto \pi\pi} + \Gamma_{\rho^+ \mapsto \pi\pi} + \Gamma_{\rho^- \mapsto \pi\pi} \right)$$

$$\boxed{\Gamma_{\rho\mapsto\pi\pi} = \frac{\sqrt{\frac{m_\rho^2}{4} - m_\pi^2}}{8\pi m_\rho^2}\left(\frac{1}{3}\left(m_\rho^2 - 4m_\pi^2\right)\right) g^2 \,\vartheta(m_\rho - 2m_\pi)}\,.$$

We can put it in a nicer form....

$$\boxed{\Gamma_{\rho\mapsto\pi\pi} = \frac{\left(\frac{m_\rho^2}{4} - m_\pi^2\right)^{3/2}}{6\pi m_\rho^2}\, g^2 \,\vartheta(m_\rho - 2m_\pi)}$$

Achtung: ... different philosophies exist!!!

$m_\rho = 775$ MeV

$m_\pi = 135$ MeV

$\Gamma = 145$ MeV

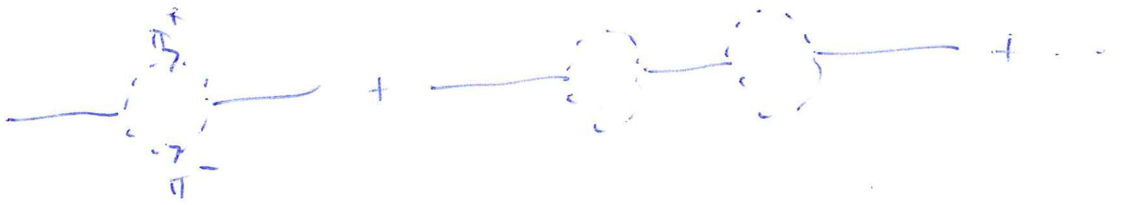
$$\boxed{g \simeq 6}$$

↳ Achtung: it is not small..... other side of hadronic flavor!!!

We know indeed that loop contributions are small!!!

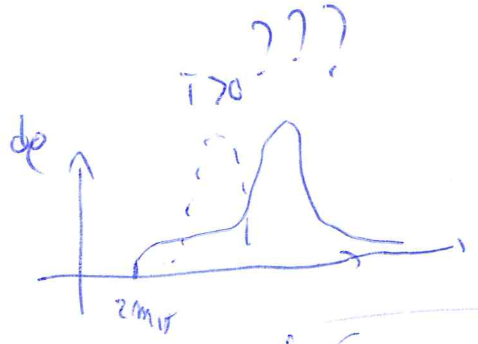
Spectral function of the ρ meson... heuristic discussion

$$\Delta_{\mu\nu} = \frac{-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_\rho^2}}{k^2 - m_\rho^2 + i\varepsilon} = \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_\rho^2}\right)\frac{1}{k^2 - m_\rho^2 + i\varepsilon}$$



$$\rightarrow \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_\rho^2}\right)\underbrace{\frac{1}{k^2 - m_\rho^2 + g^2 \Sigma(k^2) + i\varepsilon}}$$

$$\Downarrow$$



The spectral function is the imaginary part of this piece...

↳ $d_\rho(E)$ → FT of this object

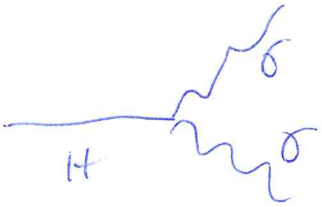
Proofs are complicated... interesting outlook for future studies.

Indeed, the spectral function of the ρ meson is extremely important in hadron physics, not only in the vacuum but also at nonzero T and μ.

↳ discussions about in-medium mass modification of the ρ meson and width and dilepton production.

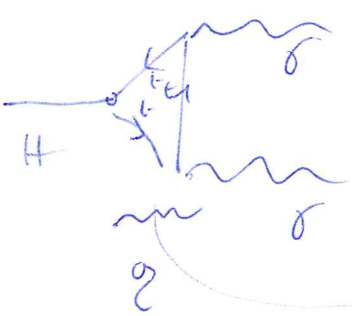
Higgs to photons

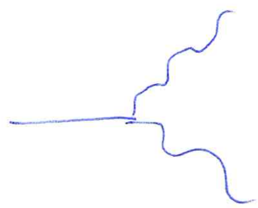
$$\mathcal{L}_1 = g\, H F_{\mu\nu} F^{\mu\nu}$$



$$[g] = [E^{-1}]$$

This interaction is not fundamental. It actually comes from





effective description through a point like coupling.

Without doing calculations, using dimensional reasoning only:

$$\Gamma_{H\to\gamma\gamma} = \#\, g^2 M_H^3 .$$