Before doing explicit calculations, let us consider a general property.

$$\begin{cases} H = \text{full hamiltonian} = H_0 + H_1 \\ |S\rangle / \; H_0|S\rangle = M_0|S\rangle . \end{cases}$$

(H_1 whatever, no matter how complicated

$$a(t) = \langle S| e^{-iHt} |S\rangle = \langle S| 1 - iHt - \frac{1}{2} H^2 t^2 |S\rangle + \dots$$

$$= 1 - it\langle S|H|S\rangle - \frac{1}{2} t^2 \langle S|H^2|S\rangle + \dots =$$

$$a^*(t) = 1 + it\langle S|H|S\rangle - \frac{1}{2} t^2 \langle S|H^2|S\rangle + \dots$$

Note, for a physical system to be meaningful we assume that

$\langle S|H|S\rangle$ and $\langle S|H^2|S\rangle$ are "finite" real number!

Then,

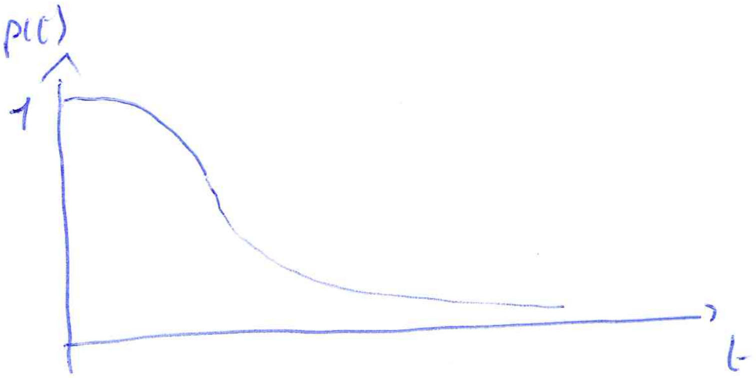
$$p(t) = |a(t)|^2 = a^* a = \left(1 + it\langle H\rangle_S - \frac{t^2}{2}\langle H^2\rangle_S\right)\left(1 - it\langle H\rangle_S - \frac{t^2}{2}\langle H^2\rangle_S\right)$$

$$= 1 + \underbrace{it\langle H\rangle_S} - \frac{t^2}{2}\langle H^2\rangle_S \underbrace{- it\langle H\rangle_S} + t^2\langle H\rangle_S^2 - \frac{t^2}{2}\langle H^2\rangle_S + O(

$$p(t) = 1 - t^2 \left(\langle H^2 \rangle_s - \langle H \rangle_s^2 \right)$$

$$= 1 - \frac{t^2}{\tau_Z^2}$$

Ergo $$\tau_Z = \frac{1}{\sqrt{\langle H^2 \rangle - \langle H \rangle_s^2}}$$



τ_Z is a "finite number...."

Some considerations :

* where does the exp. decay come from?

suppose that $\langle S|H|S\rangle$ is a "complex number" ... of course, this cannot be true from a physical point of view ....

$$\langle S|H|S\rangle = M - i\frac{\Gamma}{2}$$ (M not necess. equal to M_0 ..)

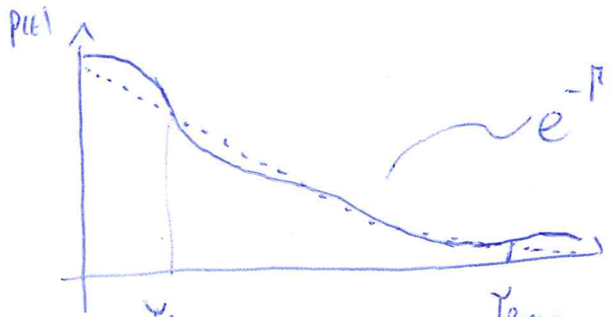
then to first order:

$$\begin{aligned} a(t) &= \langle S|1 - iHt|S\rangle = \\ a^*(t) &= \end{aligned} \begin{cases} 1 - it\left(M - i\frac{\Gamma}{2}\right) = 1 - \frac{\Gamma}{2}t - itM \\ 1 + it\left(M - i\frac{\Gamma}{2}\right) = 1 - \frac{\Gamma}{2}t + itM \end{cases}$$

$$P(t) = a^* a = 1 - \frac{\Gamma}{2}t - itM - \frac{\Gamma}{2}t + itM \simeq 1 - \Gamma t$$

$$\left(e^{-\Gamma t} \simeq 1 - \Gamma t \right)$$

* This is not valid for short times... but for intermediate times "yes".



$e^{-\Gamma t}$ represents a very good approx. for intermediate times.

The function $\gamma(t)$

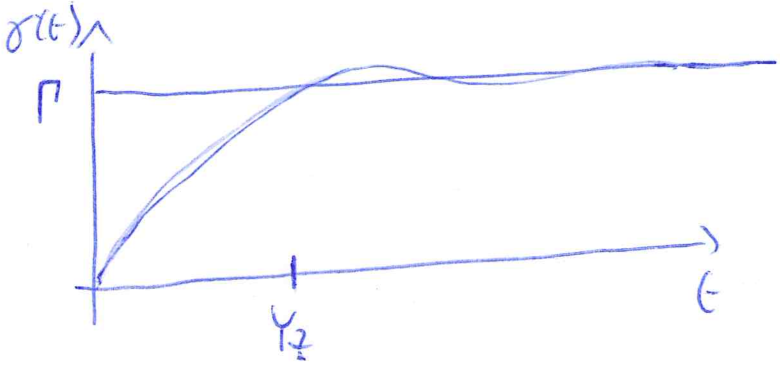
In general, we can write $p(t)$ as

$$p(t) = e^{-\gamma(t)t}$$

$\gamma(t)$ is a function of "t". In the exp. regime we have $\gamma(t) \simeq \Gamma$.

$$\gamma(t) = -\frac{1}{t} \ln p(t)$$

$$p(t) \simeq 1 - \frac{t^2}{\tau_Z^2} \longrightarrow \lim_{t \to 0} \gamma(t) = 0.$$



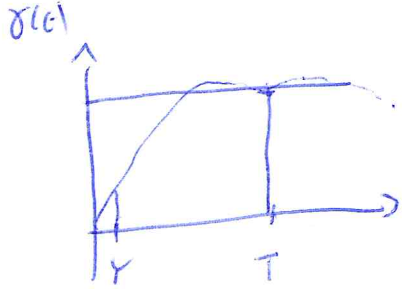
$\gamma(t)$ is just as the "time-dependent" width!

$\gamma(t \ll \tau_Z) \simeq 0$ "zero width"

Measurements (1):

$\hat{I}$

$|s\rangle$



$P(t) = e^{-\gamma(t)t}$

We do a "single measurement" at the time $T >> \tau_Z$

$\gamma(T) \simeq \Gamma$

$P(T) = e^{-\gamma(T)T} \simeq e^{-\Gamma T}$ is the survival probability.

If, on the contrary, we make N measurements:

$0 \quad \tau \quad 2\tau \quad \dots \quad N\tau = N$

$T = N\tau$

$$[P(\tau)]^N = \left[e^{-\gamma(\tau)\tau}\right]^N = e^{-\gamma(\tau)\tau N} = e^{-\gamma(\tau)\cdot T}$$

(Here there is no problem of recombination..)

But $\gamma(\tau)$ is a very small number!!!

That is, $\gamma(\tau)$ very small implies $[P(\tau)]^N = e^{-\gamma(\tau)T} \simeq 1 > e^{-}$

By keeping T fixed, and increasing N (that is, diminishing τ), we get $P(\tau)^N \mapsto 1$ for $N \mapsto \infty$. "Zero effect".

Two considerations:

* We are working with the "ideal", instant. collapse of the wf

In a sense, the "old" collapse rule.

situation can be more complicated than this....
is

Still, the Zeno effect has been seen experimentally. Also in

"real" unstable states, details in the future...

* if $\gamma(t) = \Gamma$ = constant, that is a <u>purely exponential</u>, we

get that no "Zeno" effect takes place:

$$[p(\tau)]^N = p(T) \; !$$

The same...

The non-exp. behavior is necessary for having the Zeno.