Generalization

$$\begin{cases} H_0 = M_0 |S\rangle\langle S| + \omega_1 |\omega_1\rangle\langle\omega_1| + \omega_2 |\omega_2\rangle\langle\omega_2| + \dots + \omega_N |\omega_N\rangle\langle\omega_N| \\ H_1 = g_1 \left(|\omega_1\rangle\langle S| + |S\rangle\langle\omega_1|\right) + \dots + g_N \left(|\omega_N\rangle\langle S| + |S\rangle\langle\omega_N|\right) \end{cases}$$

$$H = H_0 + H_1 = \left(|\omega_1\rangle, \dots, |\omega_N\rangle, |S\rangle\right) \underbrace{\begin{pmatrix} \omega_1 & 0 & & & g_1 \\ 0 & \ddots & & & g_2 \\ & & & & \vdots \\ & & & \omega_N & g_N \\ g_1 & g_2 & \dots & g_N & M_0 \end{pmatrix}}_{\text{diag. this } (N+1)\times(N+1) \text{ matrix}} \begin{pmatrix} \langle\omega_1| \\ \vdots \\ \langle\omega_N| \\ \langle S| \end{pmatrix}$$

The new basis $\{|E_1\rangle, \dots, |E_{N+1}\rangle\}$ diag. H.

In particular, we will have:

$$|S\rangle = \sum_{i=1}^{N+1} c_i |E_i\rangle$$

$$\boxed{a(t) = \langle S| e^{-iHt} |S\rangle = \sum_{i=1}^{N+1} |c_i|^2 e^{-iE_i t}}$$

$$\boxed{p(t) = |a(t)|^2}$$

We can 'reduce' the time-evolution problem to a mixing problem.
Note, how, the issue becomes more and more complicated by increasing N.

One has to determine c_i, E_i, ...

Although the problem is 'in principle' such as $N=1$, it is very difficult to obtain analytic and explicit expressions.

Numerical analyses become necessary.

How, as long as N is finite, there is always a "Poincaré" time such that

$p(\tau_P) = 1$!

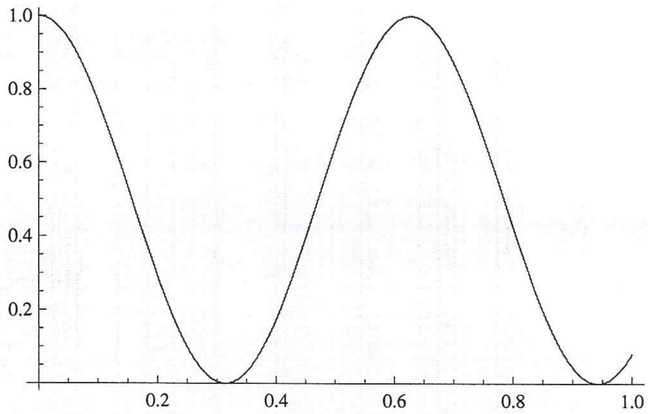
The system is always periodic and will always come back to its starting point.

```
Emin := 0;
Emax := 10;

Nt := 1;
```

$$\texttt{Ep[n\_]} := \texttt{Emin} + \frac{\texttt{n}}{\texttt{Nt}} * (\texttt{Emax} - \texttt{Emin})$$

$$\texttt{a[t\_]} := \sum_{n=0}^{Nt} \frac{1}{\texttt{Nt} + 1} * \texttt{Exp[-}i * \texttt{Ep[n]} * \texttt{t]}$$

```
p[t_] := Abs[a[t]]^2

Plot[p[t], {t, 0, 1}]
```

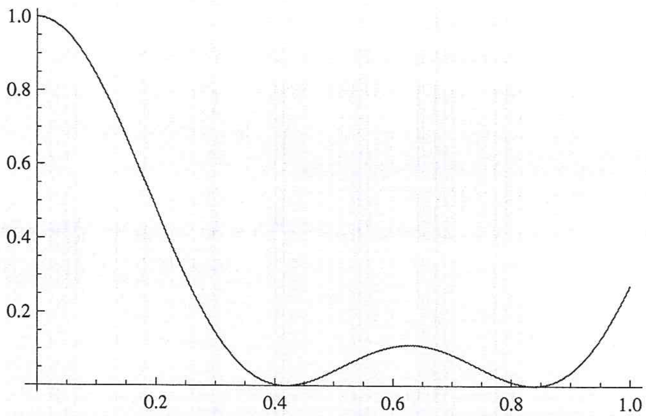


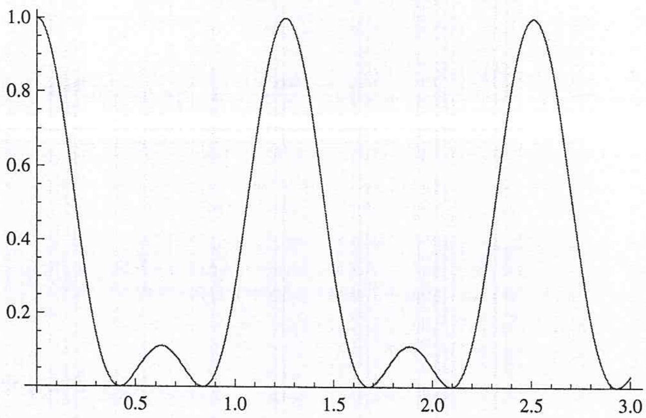
```
Emin := 0;
Emax := 10;

Nt := 2;
```

$$\texttt{Ep[n\_]} := \texttt{Emin} + \frac{\texttt{n}}{\texttt{Nt}} * (\texttt{Emax} - \texttt{Emin})$$

$$\texttt{a[t\_]} := \sum_{\texttt{n}=0}^{\texttt{Nt}} \frac{1}{\texttt{Nt} + 1} * \texttt{Exp}[-\,i * \texttt{Ep[n]} * \texttt{t}]$$

```
p[t_] := Abs[a[t]]^2

Plot[p[t], {t, 0, 1}]
Plot[p[t], {t, 0, 3}]
```



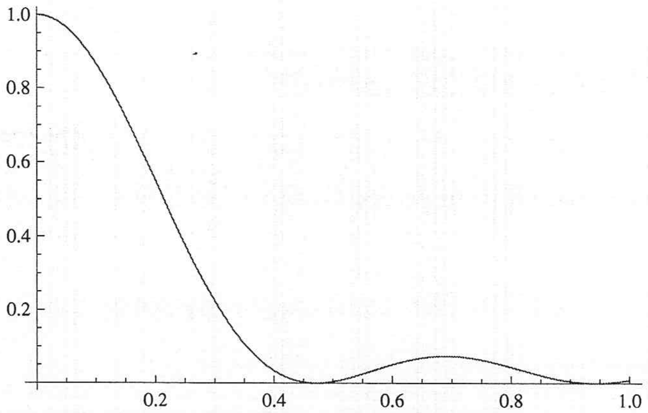


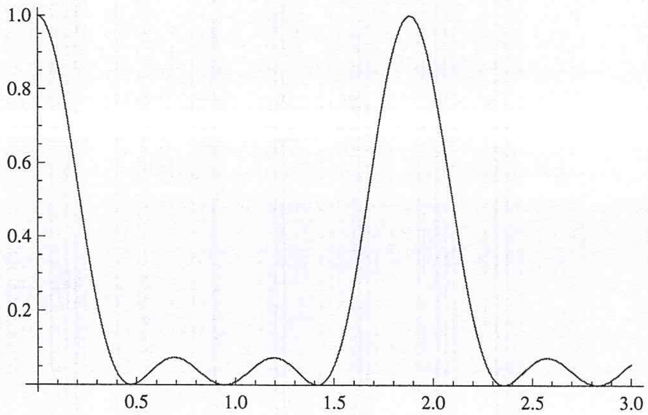
```
Emin := 0;
Emax := 10;

Nt := 3;
```

$$Ep[n\_] := Emin + \frac{n}{Nt} * (Emax - Emin)$$

$$a[t\_] := \sum_{n=0}^{Nt} \frac{1}{Nt + 1} * Exp[-i * Ep[n] * t]$$

```
p[t_] := Abs[a[t]]^2

Plot[p[t], {t, 0, 1}]
Plot[p[t], {t, 0, 3}]
```



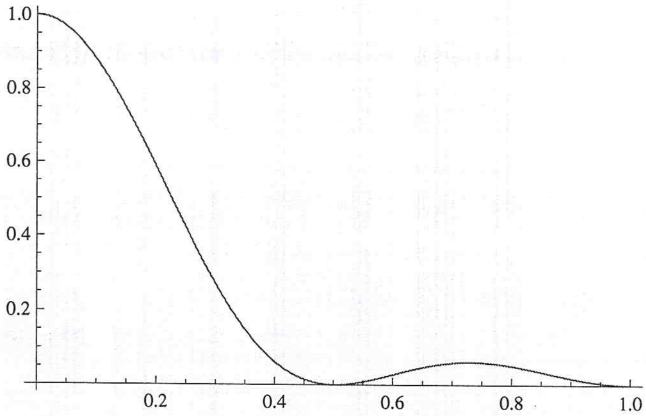


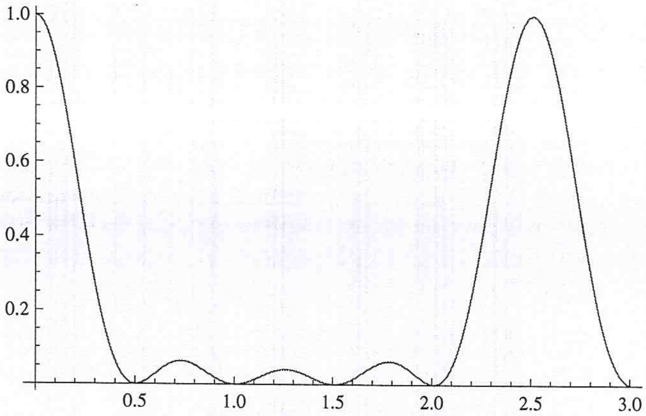
```
Emin := 0;
Emax := 10;

Nt := 4;
```

$$Ep[n\_] := Emin + \frac{n}{Nt} * (Emax - Emin)$$

$$a[t\_] := \sum_{n=0}^{Nt} \frac{1}{Nt + 1} * Exp[-i * Ep[n] * t]$$

```
p[t_] := Abs[a[t]]^2

Plot[p[t], {t, 0, 1}]
Plot[p[t], {t, 0, 3}]
```



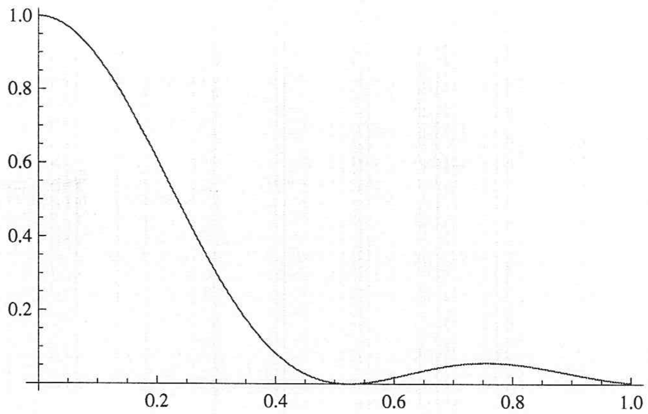


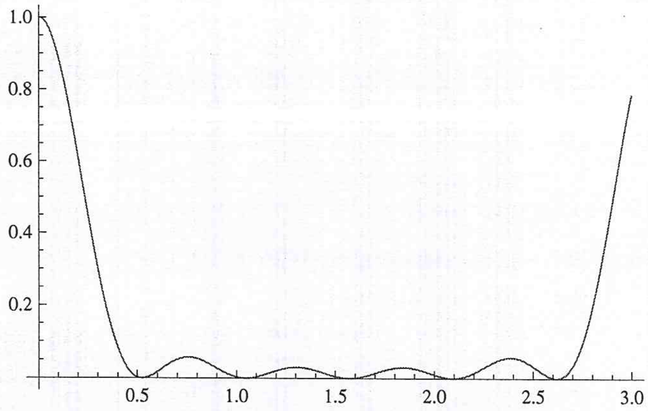
```
Emin := 0;
Emax := 10;

Nt := 5;
```

$$Ep[n\_] := Emin + \frac{n}{Nt} * (Emax - Emin)$$

$$a[t\_] := \sum_{n=0}^{Nt} \frac{1}{Nt + 1} * Exp[-i * Ep[n] * t]$$

```
p[t_] := Abs[a[t]]^2

Plot[p[t], {t, 0, 1}]
Plot[p[t], {t, 0, 3}, PlotRange → All]
```

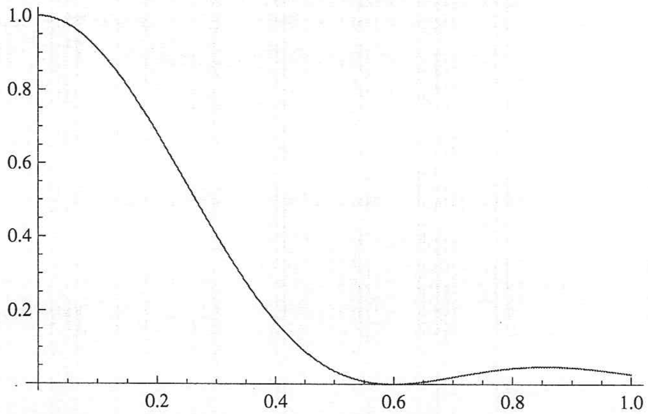


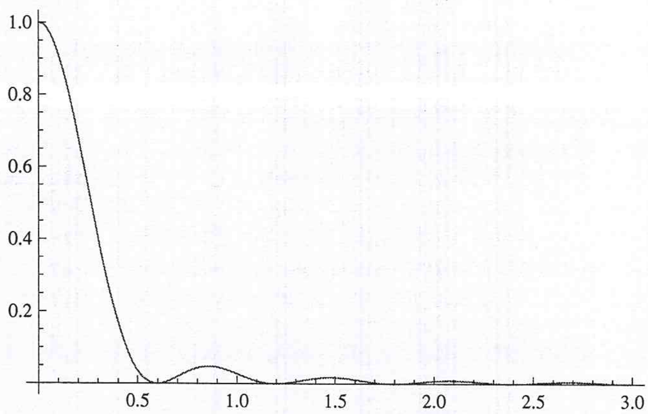


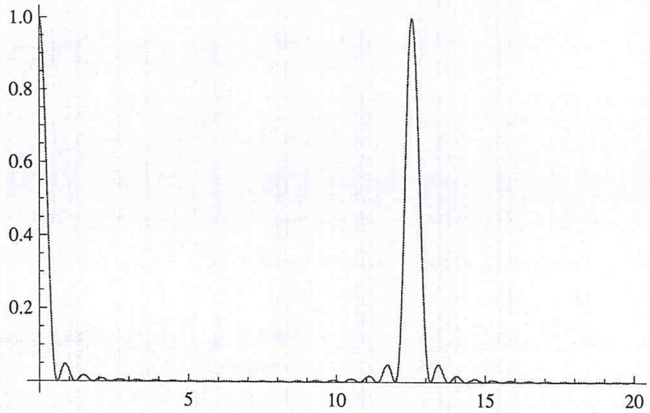
```
Emin := 0;
Emax := 10;

Nt := 20;
Ep[n_] := Emin + n/Nt * (Emax - Emin)
```

$$a[t\_] := \sum_{n=0}^{Nt} \frac{1}{Nt + 1} * \text{Exp}[-i * \text{Ep}[n] * t]$$

```
p[t_] := Abs[a[t]]^2

Plot[p[t], {t, 0, 1}]
Plot[p[t], {t, 0, 3}, PlotRange → All]
Plot[p[t], {t, 0, 20}, PlotRange → All]
```





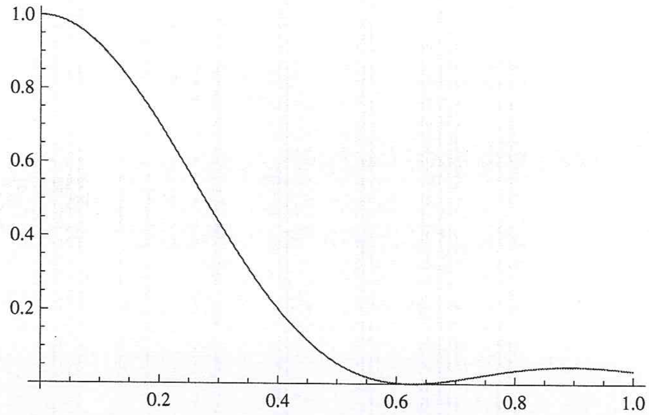


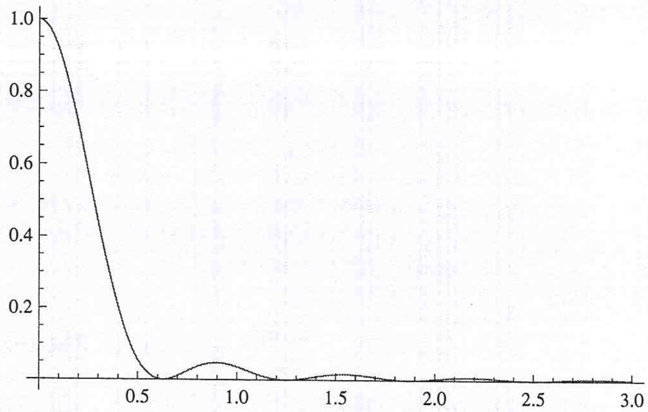
```
Emin := 0;
Emax := 10;

Nt := 200;
```

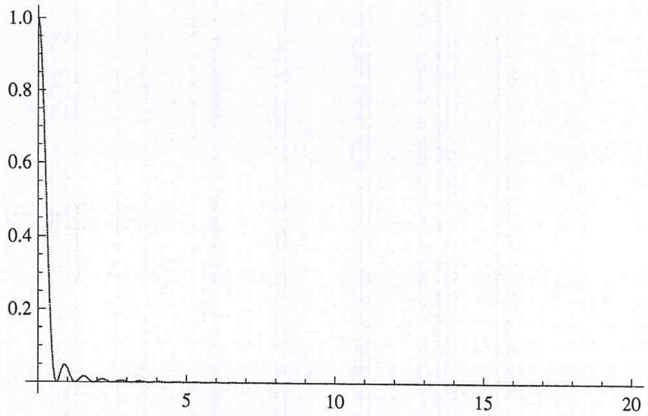
$$\text{Ep}[n\_] := \text{Emin} + \frac{n}{\text{Nt}} * (\text{Emax} - \text{Emin})$$

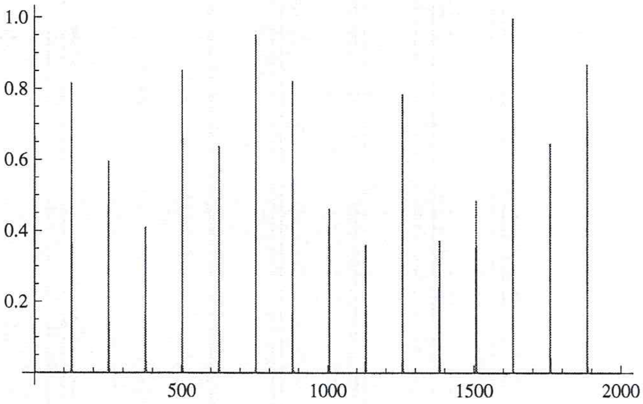
$$a[t\_] := \sum_{n=0}^{\text{Nt}} \frac{1}{\text{Nt} + 1} * \text{Exp}[-i * \text{Ep}[n] * t]$$

```
p[t_] := Abs[a[t]]^2

Plot[p[t], {t, 0, 1}]
Plot[p[t], {t, 0, 3}, PlotRange → All]
Plot[p[t], {t, 0, 20}, PlotRange → All]
Plot[p[t], {t, 0, 2000}, PlotRange → All]
```







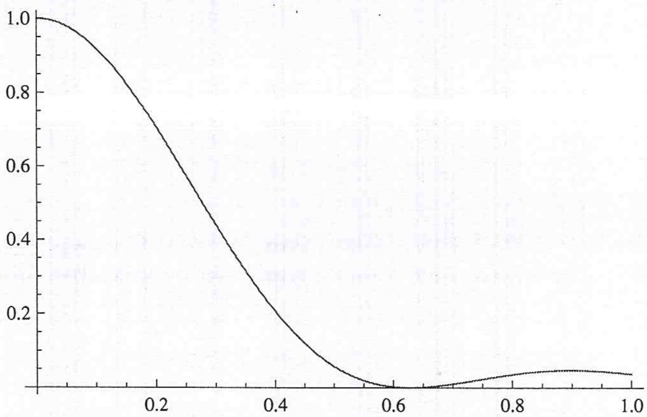


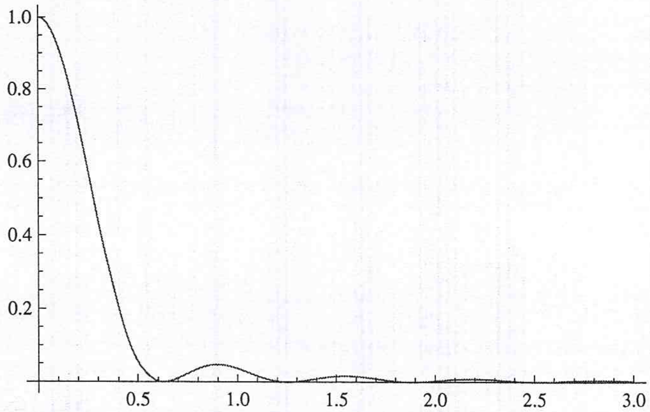

```
Emin := 0;
Emax := 10;

Nt := 10^10;
```

$$Ep[n\_] := Emin + \frac{n}{Nt} * (Emax - Emin)$$

$$a[t\_] := \sum_{n=0}^{Nt} \frac{1}{Nt + 1} * Exp[-i * Ep[n] * t]$$

```
p[t_] := Abs[a[t]]^2

Plot[p[t], {t, 0, 1}]
Plot[p[t], {t, 0, 3}, PlotRange → All]
```



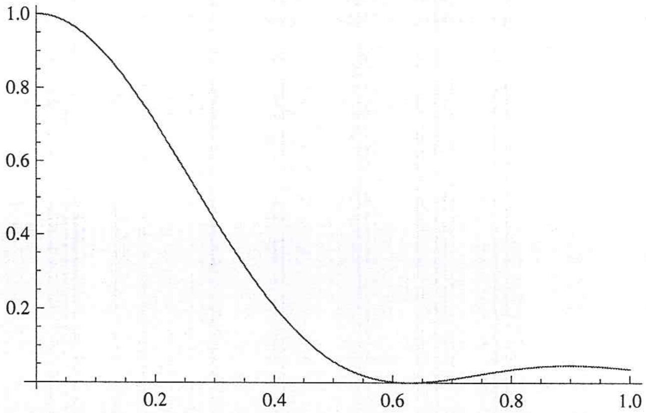


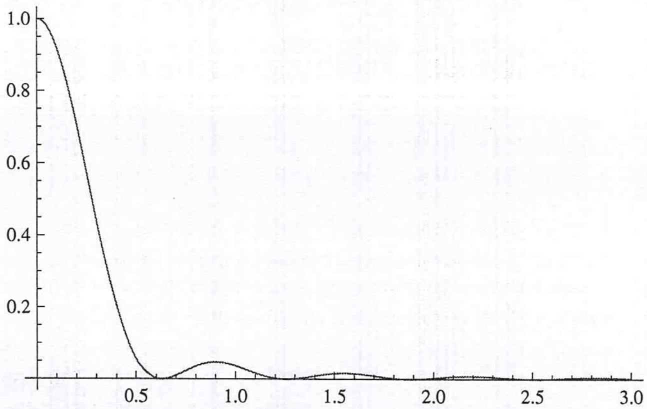
```
Emin := 0;
Emax := 10;

Nt := 10^20;
```

$$Ep[n\_] := Emin + \frac{n}{Nt} * (Emax - Emin)$$

$$a[t\_] := \sum_{n=0}^{Nt} \frac{1}{Nt+1} * Exp[-i * Ep[n] * t]$$

```
p[t_] := Abs[a[t]]^2

Plot[p[t], {t, 0, 1}]
Plot[p[t], {t, 0, 3}, PlotRange → All]
```





```
Show[%195, %187]
```