Prototype of model with "chiral symmetry"

($N_f = 1$) (and to be precise large N_c)

σ, π are two fields.

Chiral symmetry is the transformation corresponding to:

$$\begin{pmatrix} \sigma \\ \pi \end{pmatrix} \mapsto \begin{pmatrix} \sigma' \\ \pi' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \sigma \\ \pi \end{pmatrix}.$$

This is an $O(2)$ rotation.

It corresponds to a chiral transformation at a quark level:

$$\sigma \equiv \bar{q} q$$

$$\pi \equiv \bar{q} i \gamma^5 q$$

$$q \mapsto e^{i\gamma^5\alpha} q \simeq q + i\gamma^5 q \cdot \alpha$$

$$\sigma \mapsto \sigma + 2\alpha \pi + \dots$$

σ and π are called chiral partners.

Now, if this symmetry must be exactly fulfilled by a Lagrangian, we have only the following possibility:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi)^2 - V(\sigma, \pi)$$

$$V(\sigma,\pi) = V(\sigma^2+\pi^2) \; !!!$$

By performing a Taylor expansion up to order 4 we obtain:

(often

$$V = \text{const} + \frac{\mu^2}{2}(\sigma^2+\pi^2) + \frac{\lambda}{4}(\sigma^2+\pi^2)^2 .$$

Note, $\lambda > 0$ to guarantee stability.

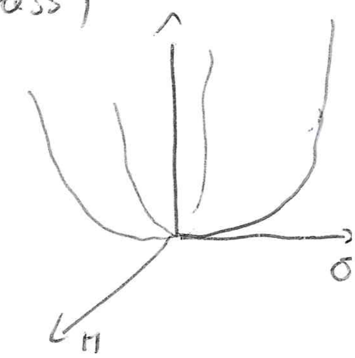
The requirement to stop at order 4:

- old argument $\mapsto$ renormalizability... but this does not need to apply here
- new argument $\mapsto$ dilatation invariance (length dilation, dimensional transmutation)

There are now 3 possibilities for μ^2 ...

$\mu^2 > 0 \rightarrow m_\sigma = m_\pi = \mu$.

They are mass degenerate (chiral partners with the same mass)



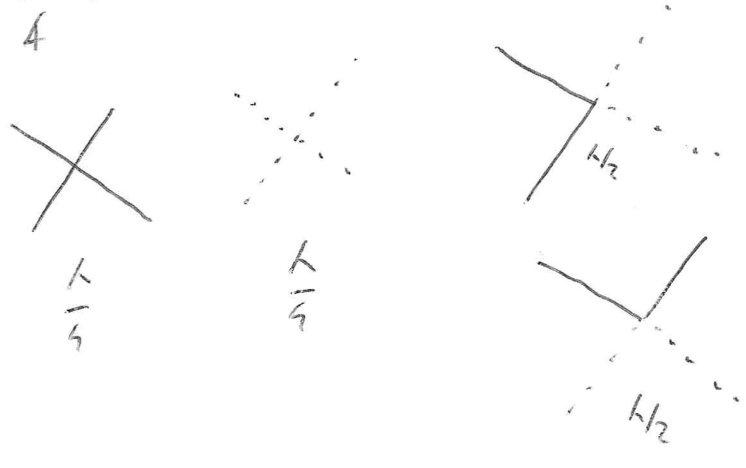
⇝ scattering (repulsive)

$\mu^2 = 0$ $\qquad m_\sigma = m_\pi = \mu = 0$.

⇝ similar situation ... but with massless particle. Thus they is now really invariant under dilatation...

Here there is then a scattering of two particles of mass μ.

$$\frac{\lambda}{4}(\sigma^2+\pi^2)^2 = \frac{\lambda}{4}\left(\sigma^4+\pi^4+2\sigma^2\pi^2\right)$$



The scattering is "fixed".

$\mu^2 < 0$

$m_\sigma = m_\pi = \sqrt{\mu^2} = +i|\mu|$

Imaginary mass!!!!

This is obviously <u>not</u> possible. It's a tachyon.

Let us now recast the potential in a different form.

$\mu^2 = -m_0^2 < 0 \, , \quad m_0^2 > 0$

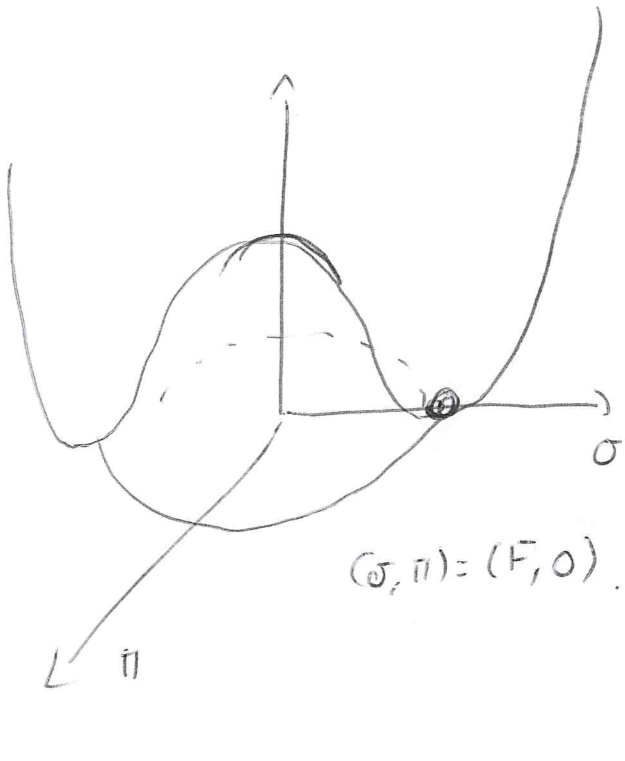
$$V = -\frac{m_0^2}{2}(\sigma^2+\pi^2) + \frac{\lambda}{4}(\sigma^2+\pi^2)^2$$

$$= \frac{\lambda}{4}\left(\sigma^2+\pi^2-F^2\right)^2 + \text{const}$$

Namely, $-\frac{\lambda}{4} \cdot 2F^2(\sigma^2+\pi^2)$

implies that

$$\boxed{m_0^2 = \lambda F^2 > 0}$$



The vacuum does not fulfill the symmetry of the system...

In Nature: σ is a scalar field, π is scalar... only σ can condense.

$$\sigma \mapsto F + \sigma.$$

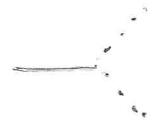
Ergo:

$$V = \frac{\lambda}{4}\left((\sigma+F)^2 + \pi^2 - F^2\right)^2$$

$$= \frac{\lambda}{4}\left(\sigma^2 + \cancel{F^2} + 2F\sigma + \pi^2 - \cancel{F^2}\right)^2$$

$$= \frac{\lambda}{4}\left(\sigma^4 + \pi^4 + 4F^2\sigma^2 + \underline{4\sigma^3 F} + 2\sigma^2\pi^2 + 4F\sigma\pi^2\right)$$

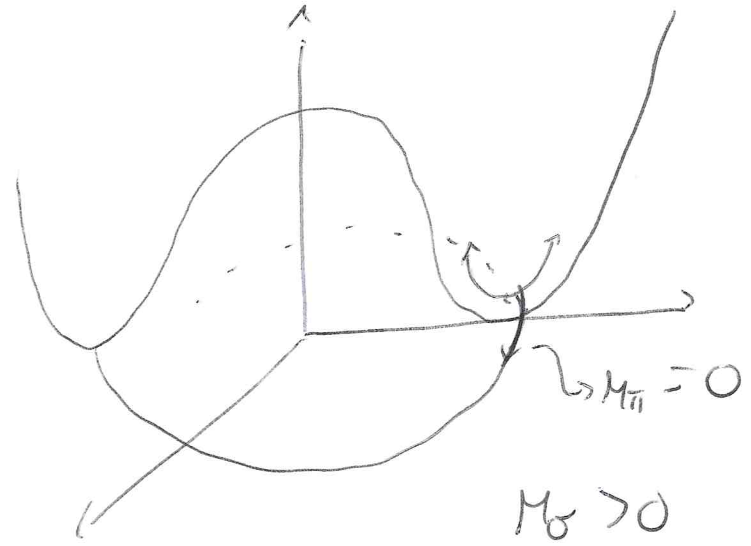
$$= \underbrace{\frac{\lambda}{4}\left(\sigma^4 + \pi^4 + 2\sigma^2\pi^2\right)}_{\text{scattering } 2\to 2} + \underbrace{\lambda F^2\sigma^2}_{\text{mass of the } \sigma} + \lambda\sigma^3 F + \lambda F\sigma\pi^2$$



$$\boxed{M_\sigma^2 = 2\lambda F^2}$$

$\Downarrow$

Decay $\sigma \mapsto \pi\pi$!



σ and π do not have the same mass!!!

The decay is given by:

$$\begin{cases} \Gamma_{\sigma \mapsto \pi\pi} = 2 \dfrac{K}{8\pi M_\sigma^2} \left[\lambda F\right]^2 \\ M_\sigma^2 = 2 \lambda F^2 \end{cases}$$

$F = 92.4$ MeV (weak decay constant of the pion, this is other 'story')

$M_\sigma = 600$ MeV

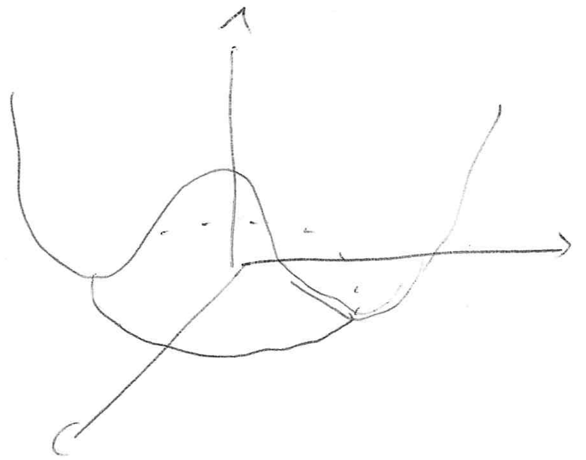
$\Gamma_{\sigma \mapsto \pi\pi} \simeq 250$ MeV

But: if you take into account the fact that there are 3 pions in the real world:

$$\Gamma_{\sigma \mapsto \pi\pi} = 750 \text{ MeV} \quad \text{for } M_\sigma \simeq 600 \text{ MeV} \ldots$$

Then, another thing takes place: explicit symmetry breaking.

$$V = \frac{\lambda}{4} (\sigma^2 + \pi^2 - F^2)^2 - \varepsilon \sigma$$



Note, the symmetry is then broken by $\varepsilon\sigma$.

$$\varepsilon\sigma \mapsto \varepsilon(\sigma \cdot \cos + \pi \sin) \neq \varepsilon\sigma.$$

If you perform the calculation you find that

$M_\pi \geq 0$

(intuitively clear...).

Using the real pion mass:

$$\Gamma_{\sigma \mapsto \pi\pi} = 3 \cdot 220 \text{ MeV} \simeq 660 \text{ MeV}$$

People then considered $f_0(500)$ as this state...

In fact, if you put $\sigma = f_0(1370)$ you get the unrealistic value

$$\Gamma_{\sigma = f_0(1370) \mapsto \pi\pi} = 3 \cdot 2500 \simeq 7500 \text{ MeV}$$

(exp $\simeq$ 200-500 MeV)