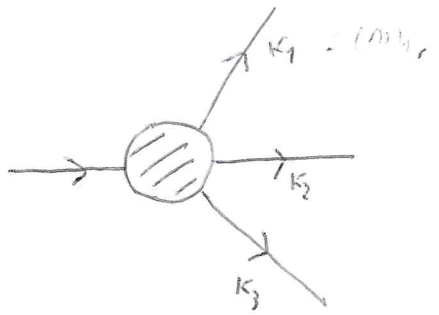
$$P \mapsto \varphi_1 \varphi_2 \varphi_3$$

$$p = (M_P, \vec{0})$$



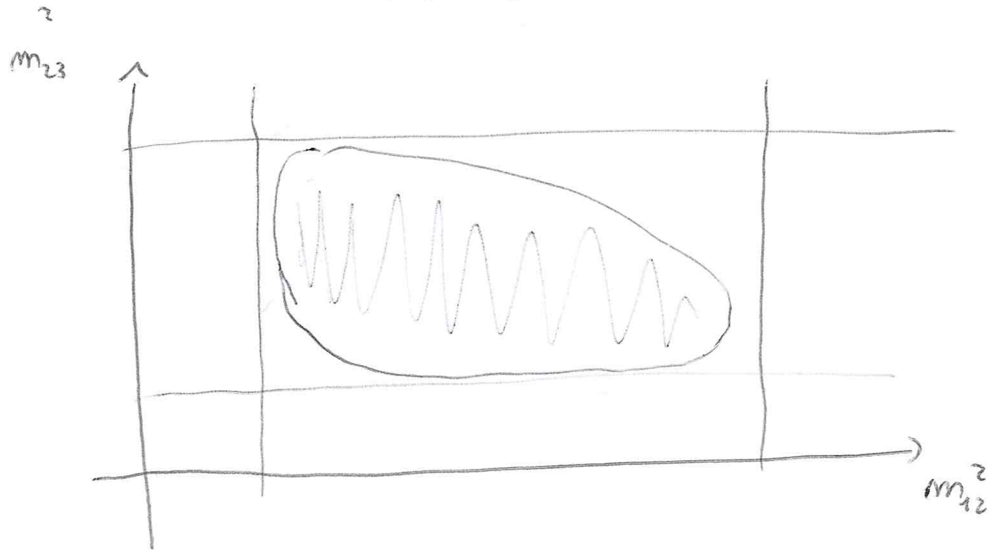
$$k_i = \left(E_i = \sqrt{\vec{k}_i^2 + m_i^2}, \vec{k}_i\right).$$

$$m_{12}^2 = (k_1 + k_2)^2$$

$$m_{23}^2 = (k_2 + k_3)^2$$

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32 M_P^3} \left| i\mathcal{M} \right|^2 dm_{12}^2 \, dm_{23}^2$$

$\hookrightarrow f(m_{12}^2, m_{23}^2)$.

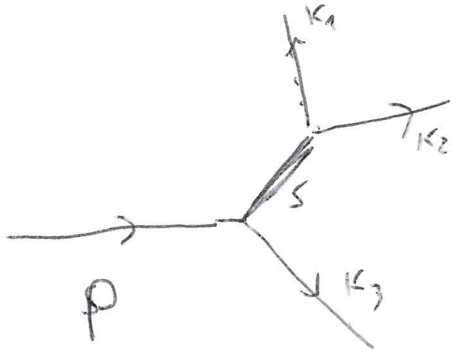


$$(m_1 + m_2)^2 \leq m_{12}^2 \leq (M - m_3)^2$$

$$(m_2 + m_3)^2 \leq m_{23}^2 \leq (M - m_1)^2 .$$

$\Gamma = \int_D d\Gamma$ is the full decay width.

Now, suppose that we have the following situation:



$$\boxed{\mathcal{L}_{int} = \alpha \Phi S \varphi_3 + g S \varphi_1 \varphi_2}$$

S is now an "intermediate" virtual state...

This is a typical situation.

Remind that; for $\mathcal{L}_{int} = g S \varphi_1 \varphi_2$

$$d_S(x) = \frac{2x}{\pi} \operatorname{Im} | \Delta_S(x) | = \frac{2x}{\pi} \frac{x\Gamma_S}{(x^2 - M_0^2 + g^2 \operatorname{Re}\Sigma)^2 + (x\Gamma_S)^2}$$

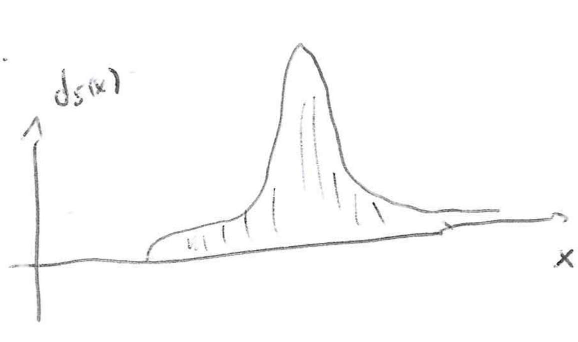
$(x\Gamma_S = g^2 \operatorname{Im}\Sigma)$
optical theorem

$\Gamma_S = \Gamma_{S \mapsto \varphi_1\varphi_2}$.

Γ_S with running x !!!

$d_S(x)$ is the "mass distribution."

$$\int_0^\infty d_S(x)\, dx = 1$$



x running mass

$x\,dx \longmapsto$ is the probability, that S has a mass between

Now, let us see the problem from this "new" perspective.

What do we expect for $d\Gamma$ and Γ?

The decay with of $P \mapsto S\varphi_3$ is easily calculated. For S stable

$$\boxed{\Gamma_{P\mapsto S\varphi_3} = \frac{|\vec{K}_3|}{8\pi M_P^2} |\alpha|^2.}$$

$$|\vec{K}_3| = K(M_P, M_S, m_3) = \frac{1}{2M_P}\sqrt{M_P^4 + 2(M_S^2 - m_3^2)^2 - 2M_P^2(M_S^2 + m_3^2)}$$

But M_S has not a definite mass... namely, S can have a mass between $(0,\infty)$ (better (m_{th},∞)) with the described probability distribution.

~~$M_S \mapsto x$~~ with prob. distribution $d_S(x)$.

But then, which is the probability of decay?

$$d\Gamma = \Gamma_{P\mapsto S\varphi_3}(x)\, d_S(x)\, dx$$

running mass of "x"

$$\Gamma_{P\mapsto S\varphi_3}(x) = \frac{K(M_P, x, m_3)}{8\pi M_P^2}\alpha^2$$

Then, the full decay width is given by the integral

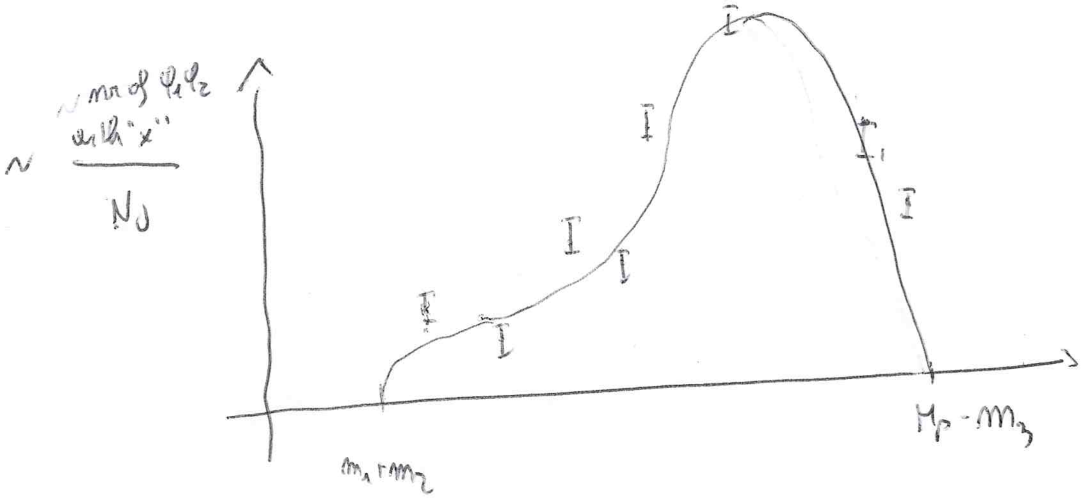
$$\Gamma = \int_0^\infty \Gamma_{P\to S\varphi_3}(x)\, d_S(x)\, dx.$$

→ θ functions are entering....

But:

$x = m_{12}$ is the quantity $\sqrt{(k_1+k_2)^2}$.

From an experiment all outcomes are possible... You measure actually $\frac{d\Gamma}{dx} \equiv \Gamma_{P\to S\varphi_3}(x)\, d_S(x)$



$\Gamma_{P\to S\varphi_3}(x)$ contains $\theta(M_P - x - m_3)$ $\to \begin{cases} x^{MAX} = M_P - m_3 \\ x^{MIN} = m_1 + m_2 \end{cases}$

Note, in interesting limit is:

$$d_S(x) \propto \delta(x - M_S)$$

S is practically stable.

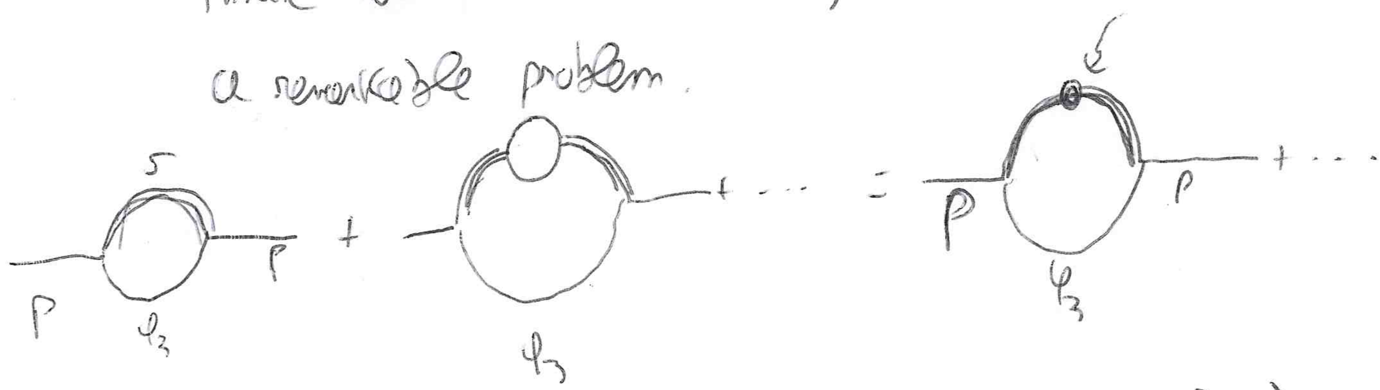
Then

$$\Gamma_{\rho \to \varphi_1 \varphi_2 \varphi_3} \simeq \Gamma_{\rho \to S \varphi_3}(M_S)$$

The 3-body decay

Achtung: this treatment is good if α is small, ie if ρ is long-lived

otherwise, we have to take into account also the finite width corrections of ρ as well. This is a remarkable problem.



$$\sim \int d^4k \, \Delta_S\left(\frac{P}{2}+k\right) \underbrace{\Delta_{\varphi_3}\left(\frac{P}{2}-k\right)}_{\frac{1}{\left(\frac{P}{2}-k\right)^2 - m_3^2}}$$

Cont then

a much more complicated object!!!

The full calculation

$$\Gamma_{P\to q_1 q_2 q_3} = \int_D d\Gamma = \int_D \frac{1}{(2\pi)^3} \frac{1}{32 M_P^3} |-i\mathcal{M}|^2 \, dm_{12}^2 \, dm_{23}^2 .$$

is the full decay width.

Now, let us suppose that $|-i\mathcal{M}|^2$ does not depend on m_{23}^2.

Then, the integral over m_{23}^2 can be performed; at fixed m_{12}^2 we get:

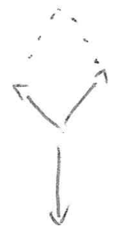
$$(m_{23}^2)_{MAX} - (m_{23}^2)_{MIN} = 4\sqrt{E_2^{*2} - m_2^2}\sqrt{E_3^{*2} - m_3^2}$$

$$\begin{cases} \text{with } E_2^* = \dfrac{(m_{12}^2 - m_1^2 + m_2^2)}{2m_{12}} \\[2ex] E_3^* = \dfrac{M^2 - m_{12}^2 - m_3^2}{2m_{12}} \end{cases}$$

Some lengthy calculations are necessary, but the result is quite simple:

$$\boxed{d\Gamma = \frac{1}{2\pi^3}\,\frac{1}{16 M_p^2}\,\underbrace{|-i\mathcal{M}|^2}_{f(m_{12})}\,|\vec{K}_1^*|\,|\vec{K}_3|\,dm_{12}}$$

momentum of the first particle in the rest frame 1-2.



For instance, if $m_1 = m_2$

$$|\vec{K}_1^*| = \sqrt{\frac{m_{12}^2}{4} - m_1^2}$$

In general:

$$|\vec{K}_1^*| = K(m_{12}, m_1, m_2) =$$

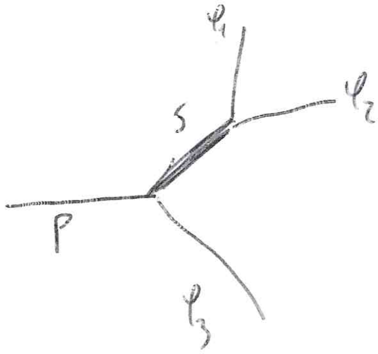
$$= \frac{1}{2m_{12}}\sqrt{m_{12}^4 + (m_1^2 - m_2^2)^2 - 2m_{12}^2(m_1^2 + m_2^2)}$$

Similarly, $|\vec{K}_3|$ is fixed...

$$|\vec{K}_3| = \frac{1}{2M_p}\sqrt{M_p^4 + 2(m_{12}^2 - m_3^2)^2 - 2M_p^2(m_{12}^2 + m_3^2)}$$

$$= K(M_p, m_{12}, m_3)$$

Now, what is $-i\mathcal{M}$ in our case?



Feynman rule:

$$-i\mathcal{M} = \alpha \cdot \frac{i}{m_{12}^2 - M_S^2 + g^2\Sigma}\, g$$

$$|-i\mathcal{M}|^2 = \frac{\alpha^2 g^2}{(m_{12}^2 - M_S^2 + g^2 \mathrm{Re}\Sigma)^2 + (g^2 \mathrm{Im}\Sigma)^2} \qquad m_{12} = x$$

Optical theorem: $g^2 \,\mathrm{Im}\Sigma = x\,\Gamma_{S\to\varphi_1\varphi_2}(x) = x \frac{|\vec{k}_1^*|}{8\pi x^2} g^2$.

Plug it in:

$$d\Gamma = \frac{1}{2\pi^3} \frac{1}{16 M_P^2} \frac{\alpha^2 g^2}{(x^2 - M_S^2 + g^2 \mathrm{Re}\Sigma) + (x\Gamma_{S\to\varphi_1\varphi_2}(x))} |\vec{k}_1^*|\,|\vec{k}_3|$$

Now, we 'group' like terms together; first $\Gamma^{(x)}_{P\mapsto S\varphi_3}$:

$$d\Gamma_{P\mapsto\varphi_1\varphi_2\varphi_3} = \frac{1}{4\pi^2}\,\frac{g^2\,|\vec{k}_1^{\,*}|}{\left(x^2-M_S^2+g^2\,\mathrm{Re}\,\Sigma(x)\right)^2+\left(x\Gamma^{(x)}_{S\mapsto\varphi_1\varphi_2}\right)^2}\,\Gamma^{(x)}_{P\mapsto S\varphi_3}\,dx$$

$$\Gamma^{(x)}_{S\mapsto\varphi_1\varphi_2} = \frac{|\vec{k}_1^{\,*}|}{8\pi x^2}\,g^2$$

$$d\Gamma_{P\mapsto\varphi_1\varphi_2\varphi_3} = \frac{2x}{\pi}\,\frac{x\,\Gamma_{S\mapsto\varphi}}{\left(x^2-M_S^2+g^2\,\mathrm{Re}\,\Sigma\right)^2+\left(x\Gamma^{(x)}_{S\mapsto\varphi_1\varphi_2}\right)^2}\,\Gamma_{P\mapsto S\varphi_3}\,dx$$

$$= d_S(x)\,\Gamma^{(x)}_{P\mapsto S\varphi_3}\,dx$$

q.e.d. ✓