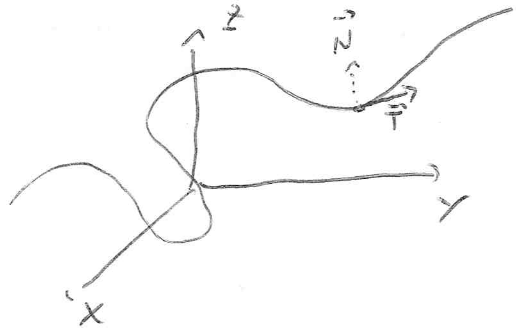
What we did not do ... quick look at it :)

$$\vec{x}(t): \mathbb{R} \mapsto \mathbb{R}^3$$



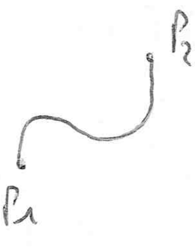
$$\vec{x}(t) = \big(x(t), y(t), z(t)\big)$$

$$\vec{T} = \frac{\frac{d\vec{x}}{dt}}{\left|\frac{d\vec{x}}{dt}\right|}$$

$$\vec{N} = \frac{\frac{d\vec{T}}{dt}}{\left|\frac{d\vec{T}}{dt}\right|}$$

$$K = \frac{\left|\frac{d\vec{T}}{dt}\right|}{\left|\frac{d\vec{x}}{dt}\right|}$$

$=$ $f(x,y,z): \mathbb{R}^3 \mapsto \mathbb{R}$ $\qquad$ Γ is a curve between P_1 and P_2



$$\int_\Gamma f \, ds = ?$$

Find $\vec{x}(t)$ which parametrizes "Γ":

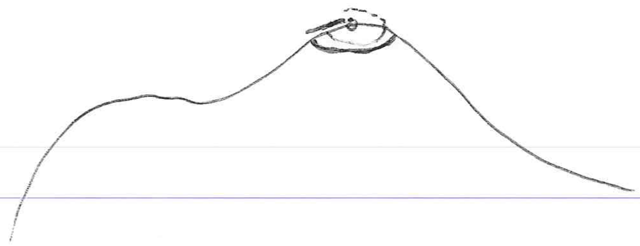
$\vec{x}(t_1) = P_1$

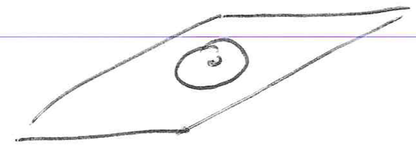
$\vec{x}(t_2) = P_2$

$$\boxed{\int_\Gamma f\,ds = \int_{t_1}^{t_2} f(x(t), y(t), z(t)) \left|\frac{d\vec{x}}{dt}\right| dt}$$

Concept of curvature of a surface

"Ant definition"

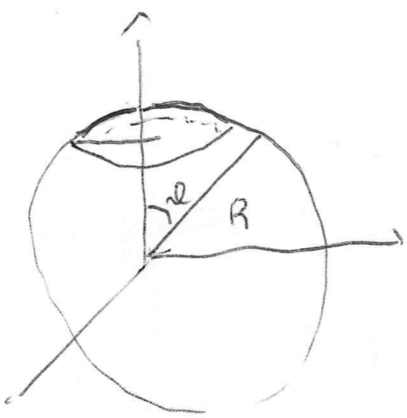




Flat space: $C(r) = 2\pi r$

$K(P) = 0.$

$$K(P) = \lim_{r \to 0} \left(2\pi r - C(r)\right)\frac{3}{\pi r^2}.$$



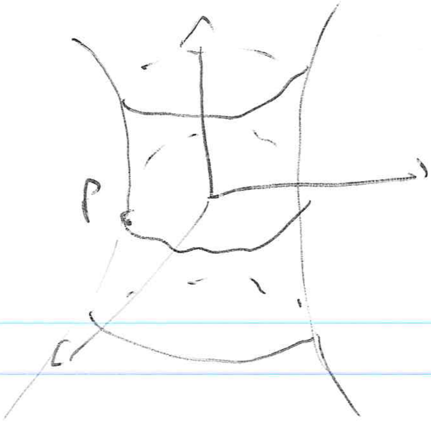
$$\begin{cases} C(r) = 2\pi (R \sin\vartheta) \\ r = R\cdot\vartheta \end{cases}$$

$$K = \lim_{r \to 0} \left(2\pi r - 2\pi R \sin\left(\frac{r}{R}\right)\right)\cdot\frac{3}{\pi r^3} =$$

$$= \lim_{r \to 0} \left(\cancel{2\pi r} \cancel{- 2\pi R \cdot \frac{r}{R}} + 2\pi R\cdot\frac{1}{3!}\frac{r^3}{R^3} + \dots\right)\frac{3}{\pi r^3}$$

$$= \frac{1}{R^2} > 0$$

$x^2 + y^2 - z^2 = R^2$



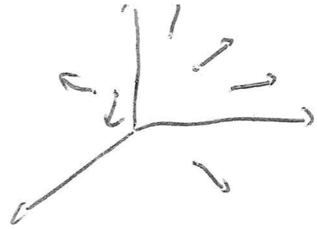
$K < 0$

$K > 0$ "sphere like"

$K = 0$ flat

$K < 0$ "hyp. like"

Vector fields: $\vec{E}(x,y,z) : \mathbb{R}^3 \mapsto \mathbb{R}^3$



$$\vec{E}(x,y,z) = \left(E_x(x,y,z),\ E_y(x,y,z),\ E_z(x,y,z) \right)$$

def: $\vec{\nabla}\cdot\vec{E} = \partial_x E_x + \partial_y E_y + \partial_z E_z$

Integral on a surface S

$$\int_S \vec{E}\, d\vec{S} = \int \vec{E}\cdot\vec{n}\cdot dS$$

Example: $\vec{E} = \frac{K}{r^2}\frac{\vec{r}}{r}$

$$\int_{\odot} \vec{E}\cdot d\vec{S} = 4\pi R^2 \cdot \frac{K}{R^2} = 4\pi K.$$

A very important theorem (Gauß):

$$\int_V \vec{\nabla}\cdot\vec{E}\, d^3X = \int_{S=\partial V} (\vec{\nabla}\cdot\vec{E})\vec{n}\, dS$$

P. S: surface integral

A surface can be expressed as

$$\vec{x}(u,v) = r_x(u,v)\,\vec{e}_x + r_y(u,v)\,\vec{e}_y + r_z(u,v)\,\vec{e}_z =$$

$$= (r_x(u,v), r_y(u,v), r_z(u,v)) \text{ and } u,v \in D.$$

The infinitesimal area is

$$d\sigma = \left| \frac{\partial \vec{x}}{\partial u} \wedge \frac{\partial \vec{x}}{\partial v} \right| du\,dv$$

Ergo:

$$\int_S \vec{E} \cdot d\vec{S} = \int_D \vec{E}(r_x(u,v), r_y(u,v), r_z(u,v)) \cdot \left(\frac{\partial \vec{x}}{\partial u} \wedge \frac{\partial \vec{x}}{\partial v} \right) du\,dv$$

This is an integral in two dimensions of the type that we have studied.

A particular example of curve is obtained for

$u, v = x, y$ and

$$\vec{x}(x,y) = (x, y, f(x,y))$$

$\Rightarrow$ simplification of formulae ...

$$\vec{\nabla} \times \vec{E} = \left(\partial_y E_z - \partial_z E_y , - \partial_x E_z + \partial_z E_x , \partial_x E_y - \partial_y E_x \right)$$

$$= \det \begin{pmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial_x & \partial_y & \partial_z \\ E_x & E_y & E_z \end{pmatrix}$$

$$\int_{\Gamma \text{ closed}} \vec{E} \cdot d\vec{\ell} = \int_S (\nabla \times \vec{E}) \cdot d\vec{S} \qquad \text{(stokes theorem)}$$

$$\boxed{\partial S = \Gamma}$$