Saturation, coherence and exclusive final states

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Supported by grant: LIDER/02/35/L-2/10/NCBiR/2011
LHC as a scanner of gluon

central-central i.e. not so dense-not so dense

forward-central i.e. dilute – not so dense

forward-forward i.e. dilute -dense

\[ \phi(x,k) \]
QCD at high energies – high energy factorization

\[
\frac{d\sigma}{dy_1 dy_2 d^2 p_1 d^2 p_2} = \sum_{a,b,c,d} \int \frac{d^2 k_{1t}}{\pi} \frac{d^2 k_{2t}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 S)^2} |M_{ab\rightarrow cd}|^2 \delta^2(\vec{k}_{1t} + \vec{k}_{2t} - \vec{p}_{1t} - \vec{p}_{2t}) \\
\times \mathcal{F}_{a/A}(x_1, k_{1t}^2, \mu^2) \mathcal{F}_{b/B}(x_2, k_{2t}^2, \mu^2) \frac{1}{1 + \delta_{cd}}
\]

\[
k_1^\mu = x_1 P_1^\mu + \bar{x}_1 P_2^\mu + k_{1t}^\mu \quad k_2^\mu = x_2 P_2^\mu + \bar{x}_2 P_1^\mu + k_{2t}^\mu
\]

\[
\bar{x}_1 = \frac{k_1^2}{S x_1} \quad \bar{x}_2 = \frac{k_2^2}{S x_2}
\]

\[
|M_{ab\rightarrow cd}|^2 = \frac{2 x_1 k_1^{\mu_1} k_{1t}^{\nu_1} 2 x_2 k_2^{\mu_2} k_{2t}^{\nu_2}}{k_1^2 k_2^2} M_{ab\rightarrow cd \mu_1 \nu_1} M_{ab\rightarrow cd \mu_2 \nu_2}
\]

Originally derived for quarks in final state. Lipatov provided general framework.

Recently new approach consistent with Lipatov’s action allowed for formulation and numerical calculation of any tree level amplitude with off-shell gluons in initial state

Van Hameren, Kotko, KK ’12
Generalized to p-A
Dominguez, Huan, Marquet, Xiao ’10

Parton densities “do not talk” to one another

Decreasing longitudinal momentum fractions of off-shell partons

Gribov, Levin, Ryskin ’81
Ciafaloni, Catani, Hautman ’93
The BFKL and BK evolutions - solutions

\[ \mathcal{F}(x, k^2) = \mathcal{F}_0(x, k^2) + \alpha_s \int_{x/x_0}^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[ l^2 \mathcal{F}(x/z, l^2) - k^2 \mathcal{F}(x/z, k^2) \right] \left[ \frac{1}{|k^2 - l^2|} + \frac{k^2 \mathcal{F}(x/z, k^2)}{\sqrt{(4l^4 + k^4)}} \right] \]

\[ \mathcal{F}(x, k^2) = \mathcal{F}_0(x, k^2) + \alpha_s \int_{x/x_0}^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[ l^2 \mathcal{F}(x/z, l^2) - k^2 \mathcal{F}(x/z, k^2) \right] \left[ \frac{1}{|k^2 - l^2|} + \frac{k^2 \mathcal{F}(x/z, k^2)}{\sqrt{(4l^4 + k^4)}} \right] \]

\[ \frac{2\alpha_s^2}{N_c R^2} \int_{x_0}^1 \frac{dz}{z} \left[ \left( \int_{k^2}^\infty \frac{dl^2}{l^2} \mathcal{F}(x/z, l^2) \right)^2 + \mathcal{F}(x/z, k^2) \int_{k^2}^\infty \frac{dl^2}{l^2} \ln \left( \frac{l^2}{k^2} \right) \mathcal{F}(x/z, l^2) \right] \]
High energy prescription and forward-central di-jets

\[ \frac{d\sigma}{dy_1 dy_2 dp_{1t} dp_{2t} d\phi} = \sum_{a,c,d} \frac{p_{t1}p_{t2}}{8\pi^2(x_1 x_2 S)^2} |M_{ag\rightarrow cd}|^2 x_1 f_{a/A}(x_1, \mu^2) \phi_{g/B}(x_2, k_{t}^2, \mu^2) \frac{1}{1 + \delta_{cd}} \]

\[ S = 2P_1 \cdot P_2 \]

- Resummmation of logs of x and logs of hard scale
- Knowing well parton densities at larger x, one can get information about low x physics
- Framework goes recently under name "hybride framework"

\[ x_1 = \frac{1}{\sqrt{5}} (p_{t1} e^{y_1} + p_{t2} e^{y_2}) \quad y_1 \sim 0, y_2 \gg 0 \]
\[ \sim 1 \]

\[ x_2 = \frac{1}{\sqrt{5}} (p_{t1} e^{-y_1} + p_{t2} e^{-y_2}) \]
\[ \ll 1 \]

\[ k_1^\mu = x_1 P_1^\mu \]
\[ k_2^\mu = x_2 P_2^\mu + k_t^\mu \]
Pt spectra

Corrections of higher orders
Included. Kin. Constr  DGLAP spf

S.Sapeta. KK ,12
HEF applied to three jets

Van Hameren, Kotko, KK, 13

p-p and p-Pb collisions
CM energy 5 TeV and 7 TeV
\( p_{T1} > p_{T2} > p_{T3} > p_{T\text{cut}} \)
anti-\( k_T \) clustering with \( R = 0.5 \)
collinear PDF \( \Rightarrow \) CTEQ10 NLO set, scale choice \( \mu = a(E_1 + E_2 + E_3) \), where the variation of \( a \) gives the (large) theoretical uncertainty
calculations are made and cross-checked using LxJet and OSCARS

all the jets are in the forward region \( 3.2 < \eta_{1,2,3} < 4.9 \)
\( p_{T\text{cut}} = 20 \text{ GeV} \)

Implemented in MC codes: C++ code LxJet (dijets, trijets), fortran code of A. van Hameren (any process) – OSCARS (Off-shell Currents And Related Stuff))

\(^1\) http://annapurna.ifj.edu.pl/~pkotko/LxJet.html
**CCFM evolution equation - evolution with observer**

Catani, Ciafaloni, Fiorani Marchesin '88

Recent review: Avsar, Iancu '09

\[
\tilde{\xi} > \xi_i > \xi_{i-1} > \ldots > \xi_1 > \xi_0
\]

In \( x \to 1 \) region where emitted gluons are soft, the dominant contribution to the amplitude comes from the angular ordered region.

The same structure for \( x \to 0 \) although the softest emitted gluons are harder than internal.

\[
q_i = \alpha_i p_P + \beta_i p_e + q_{ti} \quad \text{and} \quad s = (p_P + p_e)^2
\]

\[
\eta_i = \frac{1}{2} \ln(\xi_i) \equiv \frac{1}{2} \ln \left( \frac{\beta_i}{\alpha_i} \right) = \ln \left( \frac{|q_i|}{\sqrt{s} \alpha_i} \right) \quad \tan \frac{\theta_i}{2} = \frac{|q_i|}{\sqrt{s} \alpha_i}
\]

\[
\tilde{\xi} = p^2 / (x^2 s)
\]

\[
z_i = x_i / x_{i-1}
\]

\[
dP^0_i = \frac{\alpha_s}{2\pi} dz_i \frac{d^2q_i}{q_i^2} P_{gg}(z_i) \theta(q_i - z_{i-1}q_{i-1})(1 - z_i)
\]

Implement in CASCADE Monte Carlo Jung 02
The KGBJS equation – nonlinear ext. of CCFM

\[ \phi(x, k) = \tilde{\phi}_0(x, k) \]

\[ + \tilde{\alpha}_s \int_{x_0}^{1} \frac{dz}{z} \int \frac{d^2q}{\pi q^2} \theta(q^2 - \mu^2) \Delta_R(z, k, \mu) \]

\[ \left( \phi \left( \frac{x}{z}, k^2 \right) - \frac{q^2}{\pi R^2} \delta(q^2 - k^2) \phi^2 \left( \frac{x}{z}, q^2 \right) \right) \]

\[ \Delta_R(z, k, \mu) = \exp \left( -\tilde{\alpha}_s \log \frac{1}{z} \log \frac{k^2}{\mu^2} \right) \]

\[ \mathcal{E}(x, k^2, p) = \mathcal{E}_0(x, k^2, p) \]

\[ + \tilde{\alpha}_s \int_{x_0}^{1} \frac{dz}{z} \int \frac{d^2\hat{q}}{\pi \hat{q}^2} \theta(p - z\hat{q}) \Delta_{ns}(z, k, \hat{q}) \]

\[ \left( \mathcal{E} \left( \frac{x}{z}, k^2, \hat{q} \right) - \frac{\hat{q}^2}{\pi R^2} \delta(\hat{q}^2 - k^2) \mathcal{E}^2 \left( \frac{x}{z}, \hat{q}^2, \hat{q} \right) \right) \]

\[ \Delta_{ns} = \exp \left( -\tilde{\alpha}_s \ln \frac{1}{z} \ln \frac{k^2}{zq^2} \right) \quad \text{for} \quad k^2 > zq^2 \]
Saturation scale in KGBJS

Relative differences between linear and nonlinear

\[ \beta(x, k, p) = \frac{|E_{CCFM}(x, k, p) - E_{KGBJS}(x, k, p)|}{E_{CCFM}(x, k, p)} \]


Toton, KK '13
Gluon density at the large coupling values

weak coupling

\[ \chi(\gamma) = 2\psi(1) - \psi(1 - \gamma) - \psi(\gamma) \]

\[ f(x, k^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu (k^2)^{1/2+\nu} \bar{f}(x_0, 1/2 + i\nu) x^{-\alpha_s(1/2+i\nu)} \]

Re(\(\bar{\alpha_s} \chi(1/2 + i\nu)\))

\(-3\) \(-2\) \(-1\) \(1\) \(2\) \(3\)

Re(\(\chi_{\text{eff}}(1/2 + i\nu \bar{\alpha_s})\))

\(-4\) \(-2\) \(0\) \(2\) \(4\)

Strong coupling

critical point dominates at large coupling

Pochinski, '02
Stasto '07

Surowka, KK '13
Gluon density at the large coupling values

Nonlinear nonlinear equation valid at strong coupling limit

$$\partial_Y \Phi(Y, \rho) = \frac{1}{2} \lambda'_{\alpha \beta} \partial_\rho \Phi(Y, \rho) + \frac{1}{2} \lambda'_{\alpha \beta} \partial_\rho \Phi(Y, \rho) + (\lambda_{\alpha \beta} + \lambda'_{\alpha \beta} / 8) \Phi(Y, \rho) - \frac{\bar{\alpha}_s}{\pi R^2} \Phi^2(Y, \rho)$$
Conclusions and outlook

• LHC gives opportunity to test parton densities both when the parton density is probed at low $x$ and at low, medium and large $k_t$ at some external scale.

• The interplay of saturation and coherence leads to new features of saturation scale

• The results for jets give some theoretical hints for saturation