

Bose-Einstein correlations

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Single particle density matrix

$$\tilde{\rho}(\mathbf{x}) \sim \rho(\mathbf{x}; \mathbf{x})$$

$$\rho(\mathbf{x}; \mathbf{x}) \sim \int \rho(\mathbf{p}; \mathbf{p}') e^{-i\mathbf{q}\mathbf{x}} d^3p d^3p';$$

$$\mathbf{q} = \mathbf{p} - \mathbf{p}'$$

Measurable

$$\Omega(\mathbf{p}) \Leftrightarrow \rho(\mathbf{p}; \mathbf{p})$$

Symmetrized product and its invariance

Assuming that the n -particle density matrix can be approximated by the symmetrized product of single particle density matrices

$$\Omega_k(\mathbf{p}_1, \dots, \mathbf{p}_k) \Leftrightarrow \Re [\rho(\mathbf{p}_1; \mathbf{p}_2) \rho(\mathbf{p}_2; \mathbf{p}_3) \dots \rho(\mathbf{p}_k; \mathbf{p}_1)].$$

The predictions are unchanged when

$$\rho(\mathbf{p}_1; \mathbf{p}_2) \Rightarrow \rho_\alpha(\mathbf{p}_1; \mathbf{p}_2) = e^{i\alpha(p_1, p_2)(p_1 - p_2)} \rho(\mathbf{p}_1; \mathbf{p}_2),$$

provided that $\alpha(p_1, p_2)(p_1 - p_2) = f(p_1) - f(p_2)$, where $f = f^*$. E.g.

$$\alpha(p_1, p_2) = a + b(p_1 + p_2)$$

Emission function

$$\rho(\mathbf{p}_1; \mathbf{p}_2) = \int d^4 X S(K, X) e^{iqX}.$$

$$K = \frac{1}{2}(p_1 + p_2), \quad X = \frac{1}{2}(x_1 + x_2).$$

$$\rho_\alpha(\mathbf{p}_1; \mathbf{p}_2) = \int d^4 X S(K, X) e^{iq(X+\alpha)} = \int d^4 X S(K, X - \alpha) e^{iqX}.$$

$$S_\alpha(K, X) = S(K, X - \alpha).$$

X – K correlations

Strategies

- Derive $S(K, X)$ from physics you trust.
- Concentrate on the differences $x_1 - x_2$.
- Constrain α .

$$I(K, X) = \int d^4 X_1 d^4 X_2 S(K, X_1) S(K, X_2) e^{\Delta X q}; \quad \Delta X = X_1 - X_2.$$

$$I(K, X) = \left| \int d^4 X S(K, X) e^{iqX} \right|^2 = |\rho(\mathbf{p}_1; \mathbf{p}_2)|^2,$$

$$I(K, X) = \int d^4 \Delta X \left[\int d^4 X S(K, X + \frac{\Delta X}{2}) S(K, X - \frac{\Delta X}{2}) \right] e^{iq\Delta X}.$$

A model

A. Bialas and K. Zalewski, *Acta Phys. Pol.* **B30**(1999)359.

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A. Bialas et al., *Acta Phys. Pol.* **B32**(2001)2901.

J.D. Bjorken, *Phys. Rev.* **D7**(1973)282.

K. Gottfried, *Phys. Rev. Lett.* **32**(1974)975.

T. Csörgö and J. Zimányi, *Nucl. Phys.* **A517**(1990)588.

$$K^\mu = \lambda X^\mu; \quad K_0^2 - K_{||}^2 = M_T^2; \quad X_0^2 - X_{||}^2 = \tau^2; \quad \Rightarrow \lambda = \frac{M_T}{\tau}.$$

$$S(K, X) = S_{||}(K_0, K_{||}, X_0, X_{||})S_T(\mathbf{K}_T, \mathbf{X}_T, M_T)$$

Transverse emission function I

$$S_T = \exp \left[-\frac{\mathbf{X}_T^2}{2r_T^2} - \frac{(\mathbf{K}_T - \frac{M_T}{\tau} \mathbf{X}_T)^2}{2\delta_T^2} - \frac{\mathbf{K}_T^2}{2\Delta^2} \right].$$

$$S_T = \exp \left[-\frac{\alpha_T^2}{2R_D^2} - \frac{(\mathbf{X}_T - \alpha_T)^2}{2R_\alpha^2} \right].$$

From fit to LEP data: $r_T \approx 0.846$ fm, $\tau \approx 0.9$ fm, $\frac{\Delta}{\delta_T} \approx 1.807$
and

$$\mu \equiv \frac{r_T M_T}{\tau \delta_t} \approx \frac{M_T}{0.248 \text{ GeV}}.$$

Transverse emission function II

$$R_\alpha = \frac{r_T}{\sqrt{1 + \mu^2}} \approx \frac{0.846 \text{ fm}}{\sqrt{1 + \mu^2}}.$$

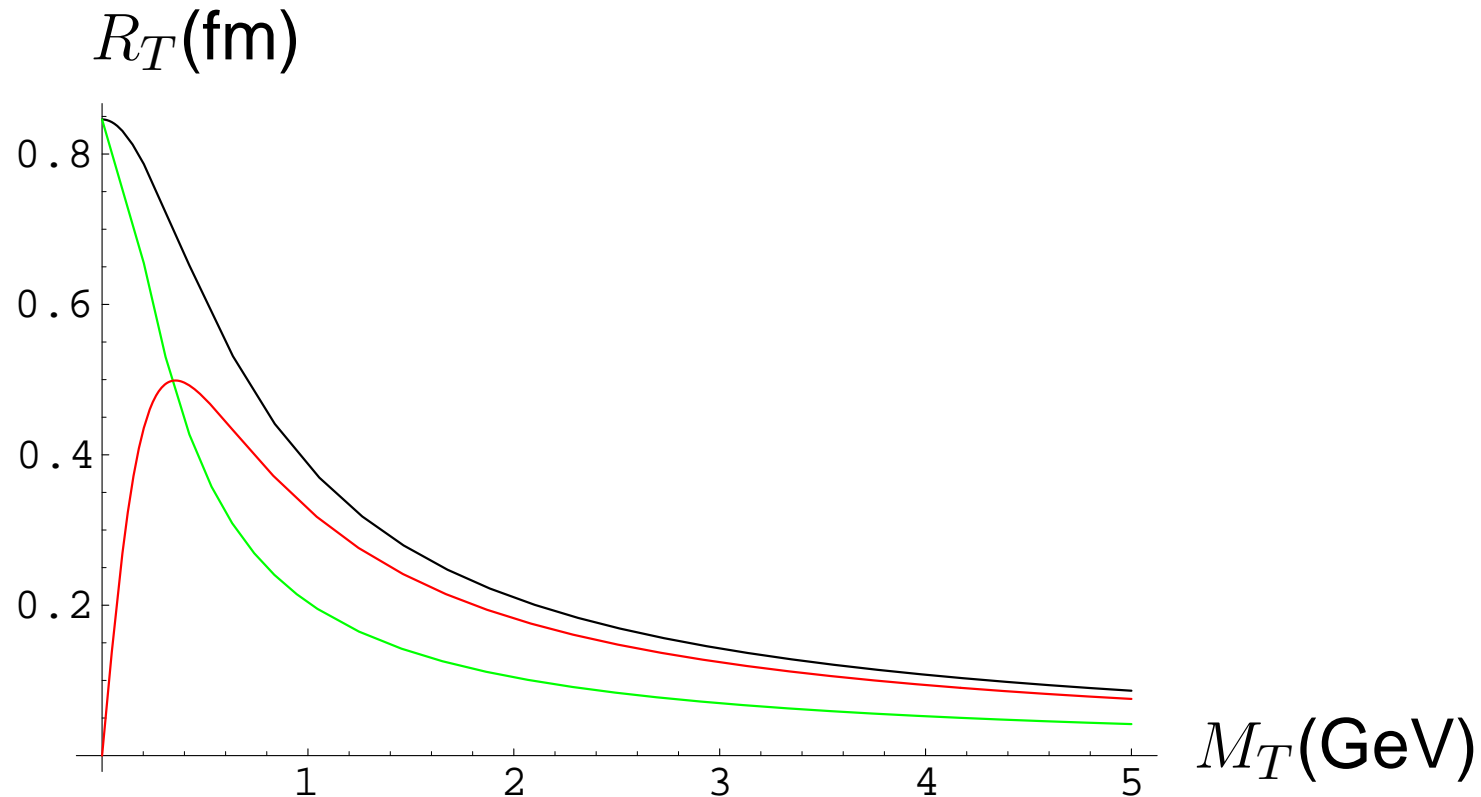
$$\alpha_T = r_T \frac{\mu}{1 + \mu^2} \frac{\mathbf{K}_T}{\delta_T} \approx 0.846 \frac{\mu}{1 + \mu^2} \frac{\mathbf{K}_T}{0.233 \text{ GeV}} \text{ fm}.$$

$$\alpha_T(p_1 - p_2) \sim \mathbf{p}_{1T}^2 - \mathbf{p}_{2T}^2$$

$$R_D = R_\alpha \frac{\mu}{\sqrt{1 + \varepsilon(1 + \mu^2)}}; \quad \varepsilon \equiv \frac{\delta_T^2}{\Delta^2} \approx 0.306.$$

$$R_T(\mu) = \frac{r_T}{\sqrt{1 + \frac{\varepsilon}{1+\varepsilon} \mu^2}} \approx \frac{0.846 \text{ fm}}{\sqrt{1 + 0.234 \mu^2}}.$$

Transverse emission function III



$$R_T = \sqrt{R_\alpha^2 + R_D^2}, \quad R_\alpha \text{ in green, } R_D \text{ in red}$$

Longitudinal emission function I

Shifts leading to

$$X_0^2 - X_{||}^2 \neq \tau^2 \quad \text{forbidden.}$$

Define

$$Y = \frac{1}{2} \log \frac{K_0 + K_{||}}{K_0 - K_{||}}; \quad \eta = \frac{1}{2} \log \frac{X_0 + X_{||}}{X_0 - X_{||}}.$$

Then

$$X_0 = \tau \cosh \eta; \quad X_{||} = \tau \sinh \eta;$$

$$K_0 = M_T \cosh Y; \quad K_{||} = M_T \sinh Y.$$

$$S_{||\alpha_\eta}(M_T, Y, \tau, \eta) = S_{||}(M_T, Y, \tau, \eta - \alpha_\eta)$$

Longitudinal emission function II

$$S_{||} = \exp \left[\frac{1}{2\delta_{||}^2} \left((K_0 - \frac{M_T}{\tau} X_0)^2 - (K_{||} - \frac{M_T}{\tau} X_{||})^2 \right) \right].$$

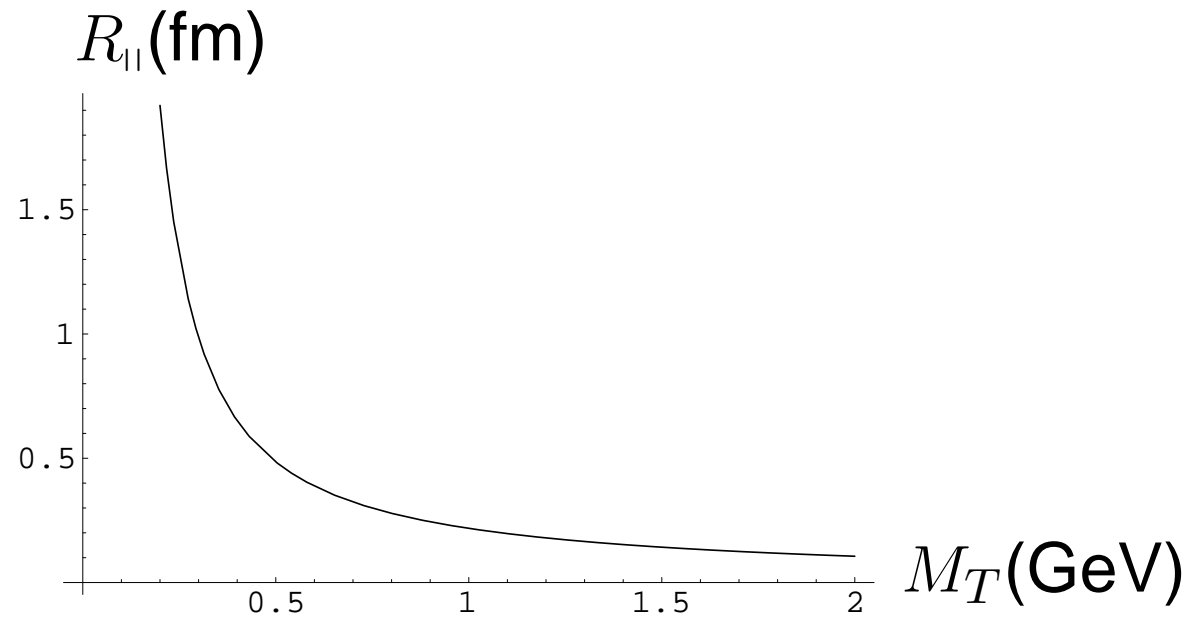
$$S_{||} = \exp \left[\frac{M_T^2}{2\delta_{||}^2} (1 - \cosh(\eta - Y)) \right].$$

$$\alpha_\eta = 0,$$

$$S_{||} = \exp \left[\frac{M_T^2}{2\delta_{||}^2} (1 - \cosh \eta) \right]$$

$$S_{||} = \exp \left[- \left(\frac{\mu\tau}{r_T} \right)^2 \left(\sqrt{1 + \frac{X_{||}^2}{\tau^2}} - 1 \right) \right]$$

Longitudinal emission function III



$$R_{||} \equiv \sqrt{\langle X_{||}^2 \rangle}$$

$$\langle X_{||}^2 \rangle = 0.2025 * \left[\frac{K_3(18.4M_T^2)}{K_1(18.4M_T^2)} - 1 \right]$$