Valon Model for Double Parton Distributions

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Motivation for multi-parton distributions


- Multiple parton scattering at the LHC: will it be possible to see?


Goal: construct dPDF’s which satisfy the formal constraints (GS sum rules) and reproduce phenomenological sPDF’s
“Bottom-up” construction based on educated guessing not unique

A constructive method is based on the Fock expansion of the hadron LC wave function: \((n - 1)\)PDF’s are recursively obtained from \(n\)PDF’s

The wave functions come from models (physics)

Valon model [Hwa, Zahir 1981, Hwa, Yang 2002] = model, where the only correlations come from the momentum conservation

Pion and matching to GRV\(_\pi\)

Nucleon

Evolution and correlations
Assumed factorization

Transverse-longitudinal decoupling (?)

\[ \Gamma_{ij}(x_1, x_2, b_1, b_2, Q_1^2, Q_2^2) = D_{ij}(x_1, x_2, Q_1^2, Q_2^2) f(b_1) f(b_2) \]

... or use the \( b \)-integrated dGPD’s (\( k_T = 0 \))

\[ D_{ij}(x_1, x_2, Q_1^2, Q_2^2) = \int d^2 b_1 d^2 b_2 \Gamma_{ij}(x_1, x_2, b_1, b_2, Q_1^2, Q_2^2) \]

Is phenomenology built by just assuming \( D_{ij}(x_1, x_2) = D_i(x_1) D_j(x_2) \) sufficiently accurate?
(e.g., in Łuszczak, Maciuła, Szczurek 2012)
Gaunt-Stirling sum rules


Fock-space decomposition + conservation laws →

\[
\sum_i \int_0^{1-y} dx \ xD_{ij}(x, y) = (1 - y)D_j(y)
\]

\[
\int_0^{1-y} dx \ D_{i,\text{val}j}(x, y) = \int_0^{1-y} dx \ [D_{ij}(x, y) - \bar{D}_{ij}(x, y)] = (N_{i,\text{val}} - \delta_{ij} + \bar{\delta}_{ij})D_j(y)
\]

- Preserved by the evolution
- Non-trivial to satisfy with the (guessed) functions
Attempts of construction

- Gaunt, Stirling (2011)

\[ D_{ij}(x_1, x_2) = D_i D_j(x_2) \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^2 + n_1 (1 - x_2)^2 + n_2} \]

 dó not satisfy the GS sum rules

- Lewandowska, Golec-Biernat (2014)

\[ D_{ij}(x_1, x_2) = \frac{1}{1 - x_2} D_i \left( \frac{x_1}{1 - x_2} \right) D_j(x_2) \]

\[ D_{qq}(x_1, x_2) = \frac{1}{1 - x_2} \left\{ D_q \left( \frac{x_1}{1 - x_2} \right) - \frac{1}{2} \right\} D_q(x_2) \]

\[ D_{q\bar{q}}(x_1, x_2) = \frac{1}{1 - x_2} \left\{ D_q \left( \frac{x_1}{1 - x_2} \right) + \frac{1}{2} \right\} D_{\bar{q}}(x_2) \]

 moi parton exchange symmetry, negative \( D_{qq} \) at large \( x \)

Problems!
Non-uniqueness of the sPDF constraints

A sample function satisfying GS sum rules (valon model for the nucleon discussed later), $|p\rangle = |uud\rangle$

\begin{align*}
D_{ud}(x,y) &= D_{uu}(x,y) = 240xy(1-x-y)\theta(1-x-y) \\
D_{d}(x) &= \int dy D_{ud}(x,y) = \int dy D_{uu}(x,y)
\end{align*}
Non-uniqueness of the sPDF constraints

A sample function satisfying GS sum rules (valon model for the nucleon discussed later), $|p\rangle = |uud\rangle$

\[ D_{ud}(x,y) = D_{uu}(x,y) = 240xy(1-x-y)\theta(1-x-y) \]

\[ D_u(x) = \int dy D_{ud}(x,y) = \int dy D_{uu}(x,y), \quad 2D_d(x) = \int dx D_{ud}(x,y) \]

\[ (1-x)D_u(x) = \int dy y[D_{ud}(x,y) + D_{uu}(x,y)], \quad (1-x)D_d(x) = \int dy yD_{du}(x,y) \]

(marginal projections)
One solution $\rightarrow$ infinitely many solutions
One solution $\rightarrow$ infinitely many solutions

Explicit construction:

Integrals $\int dy D_{ij}(x, y)$ and $\int dy y D_{ij}(x, y)$ intact! (can be distributed with smeared functions of finite support). This non-uniqueness is obvious: one-particle distributions do not fix the two-particle distribution (correlations)
Top-down from LC wave function

- Light-cone Fock expansion of the proton state in partonic constituents:

\[
|p\rangle = \sum_{N} \sum_{f_1 \ldots f_N} \int dx_1 \ldots dx_N \delta(1 - \sum_{k=1}^{N} x_k) \times \Psi_N(x_1 \ldots x_n; f_1 \ldots f_N) |x_1 \ldots x_N; f_1 \ldots f_N\rangle
\]

(origin of GS sum rules [Gaunt, PhD thesis])

- Model \( \Psi_N \)’s with the constraints from the known sPDF’s,
- Compute the double distributions, \( D_{f_1 f_2}(x_1, x_2) \)
- Run evolution

Simplest assumption: the only correlations come from the longitudinal momentum conservation: \( 1 = x_1 + x_2 + \cdots + x_n \) (we call it generalized valon model):

\[
\psi_N(x_1 \ldots x_n; f_1 \ldots f_N = A \psi_{f_1}(x_1) \ldots \psi_{f_N}(x_N)
\]
Asymptotics

\[ |\psi_N(x_1 \ldots x_n; f_1 \ldots f_N)|^2 = A^2 \phi_{f_1}(x_1) \cdots \phi_{f_N}(x_N) \]

Let the asymptotics of the single-parton functions be

\[ \phi_{f_i}(x) \sim x^{\alpha_{f_i} - 1} \quad \text{at} \quad x \to 0, \]
\[ \phi_{f_i}(x) \sim (1 - x)^{\beta_{f_i}} \quad \text{at} \quad x \to 1, \]

where for integrability \( \alpha_{f_i} > 0 \) and \( \beta_{f_i} > -1 \). Then

\[ D_f(x) \sim x^{\alpha_f - 1} \quad \text{at} \quad x \to 0 \]
\[ D_f(x) \sim (1 - x)^{\beta_f + \alpha_f(1 + \cdots + \alpha_{f_{N-1}})' - 1} \quad \text{at} \quad x \to 1 \]
\[ D_{fh}(x, y) \sim x^{\alpha_f + \alpha_h - 2} \quad \text{at} \quad x, y \to 0 \]
\[ D_{fh}(x, y) \sim \phi_f(x)\phi_h(y)(1 - x - y)^{\alpha_f(1 + \cdots + \alpha_{f_{N-1}})' - 1} \quad \text{at} \quad x + y \to 1 \]

(the large-\( x \) behavior is sensitive to the low-\( x \) behavior of the other components, as in this limit the kinematics “pushes them towards 0)
Scale and evolution

- With increasing scale $Q$ more and more partons at low $x$ are generated, hence more and more Fock components are needed.
- For practical reasons it is then favorable to use the parameterizations at lowest possible $Q$.
- Quark-model scale → not sufficiently many gluons and sea quarks.
- Cannot evolve too far down, negative distributions generated, not perturbative.
Pion from GRV$_\pi$


\[ xv(x) = 0.519 \left(0.381\sqrt{x} + 1\right) (1 - x)^{0.367} x^{0.499} \]
\[ xg(x) = (0.338\sqrt{x} + 0.678) (1 - x)^{0.39} x^{0.482} \]
\[ xq_{\text{sea}}(x) = 0 \quad (Q_0 = 500\text{MeV}) \]

Use momentum fraction as constraints

\[ 2 \int dx \ xv(x) = 0.584, \quad \int dx \ xg(x) = 0.416 \]

Also

\[ \int dx \ v(x) = 1, \quad \int dx \ g(x) = 1.46 \]

hence the average number of gluons is 1.46
Pion from $\text{GRV}_\pi$ (2)

Use the simple ansatz

$$ |\pi^+\rangle = A|\bar{u}d\rangle + B|\bar{u}dgg\rangle $$

The sPDF’s are:

$$ D_{\bar{u}}(x) = A^2|\Psi_{\bar{ud}}(x, 1 - x)|^2 + B^2 \int dx_3 dx_4 |\Psi_{\bar{ud}gg}(x, 1 - x - x_3 - x_4, x_3, x_4)|^2 $$

$$ D_d(x) = D_{\bar{u}}(x) $$

$$ D_g(x) = B^2 \int dx_1 dx_2 \left( |\Psi_{\bar{ud}gg}(x_1, x_2, x, 1 - x - x_1 - x_2)|^2 + |\Psi_{\bar{ud}gg}(x_1, x_2, 1 - x - x_1 - x_2, x)|^2 \right) $$

Conditions

$$ A^2 + B^2 = 1 $$

$$ \int dx \, x[D_{\bar{u}}(x) + D_d(x)] = 0.584 $$

$$ \int dx \, x \, D_g(x) = 0.416 $$

provide constraints for parameters
Generalized valon ansatz

We use

$$|\Psi_{\bar{u}d}(x_1, x_2)|^2 \sim f(x_1; a, b) f(x_2; a, b)$$

$$|\Psi_{\bar{u}dg}(x_1, x_2, x_3, x_4)|^2 \sim f(x_1; \alpha_q, \beta_q) f(x_2; \alpha_q, \beta_q) f(x_3; \alpha_g, \beta_g) f(x_4; \alpha_g, \beta_g)$$

with

$$f(x; \alpha, \beta) = x^{\alpha-1}(1-x)^{\beta}$$

Then with $$a + b = 0.5$$, $$\alpha_q = 0.5$$, $$\beta_q = -0.09$$, $$\alpha_g = 0.48$$, $$\beta_g = -0.09$$ we get

$$A^2 = 0.15$$ and $$B^2 = 0.85$$ – dominance of the component with gluons!
Having fixed the parameters, we may obtain the dPDF’s (and nPDF’s) 
($\bar{u}d$ component generates a singular part, $D_{\bar{u}d} \sim f(x; a, b) f(y; a, b) \delta(1 - x - y)$)
Other dPDF’s of the pion
Correlation

\[ \rho_{ij}(x_1, x_2) = \frac{D_{ij}(x_1, x_2)}{D_i(x_1)D_j(x_2)} - 1 \]
Nucleon

Repeating the construction for the GRV parametrization of the proton $(Q_0 = 480 \text{ MeV})$ would require taking at least

$$|p⟩ = A_{uud}|uud⟩ + \cdots + A_{uud(\bar{q}q)^3}|uud\bar{q})^3⟩ + \cdots + A_{uudgg}|uudgg⟩$$

For now we take the wave function composed just of three valence quarks $|p⟩ = |uud⟩$ [WB, ERA, LC2013 proceedings]

[Kuti, Weisskopf 1971] take the valence quark orbital in the form $ψ(x) \sim x^a$ and include correlations only from the longitudinal momentum conservation [Hwa, Zahir 1980, Hwa, Yang 2002]:

$$|Ψ_{uud}(x_1, x_2, x_3)|^2 δ(1 - x_1 - x_2 - x_3) = Aφ_u(x_1)φ_u(x_2)φ_d(x_3)δ(1 - x_1 - x_2 - x_3)$$
Nucleon in the valon model

Take $\phi_a(x) = x^\alpha$. Then

$$D_{uu}(x_1, x_2) = D_{ud}(x_1, x_2) = \frac{2 \Gamma(3\alpha + 3)}{\Gamma(\alpha + 1)^3} x_1^\alpha x_2^\alpha (1 - x_1 - x_2)^\alpha$$

$$D_u(x_1) = 2D_d(x_1) = \frac{2\Gamma(3\alpha + 3)}{\Gamma(\alpha + 1)\Gamma(2\alpha + 2)} x_1^\alpha (1 - x_1)^{2\alpha + 1}$$

(with $\alpha = 1$ the behavior of $D(x_1)$ at $x_1 \to 1$ conforms to the counting rules [Brodsky, Lepage 1980])
sPDF for valence vs data at $\mu = 2$ GeV

solid $\alpha = 1$, dashed $\alpha = 1/2$, dotted $\alpha = 0$
green NNPDF (no LHC), yellow NNPDF (collider)

At $\mu = 2$ GeV quarks carry 41.6% of the momentum (no gluons) $\rightarrow$ the initial scale is very low, $\mu_0 = 285$ MeV, similarly to the pion case
[Davidson, Arriola 1995, WB, Arriola, Golec-Biernat 2008]
dPDF for valence

\[ x_1 x_2 V(x_1, x_2; \mu) \quad (\alpha = 1/2) \]

Evolution [Kirschner 1979, Shelest, Snigirev, Zinovev 1982]:

\[ \mu = \mu_0 \]

\[ \mu = 800 \text{MeV} \]

\[ \mu = 2 \text{GeV} \]

\[ \mu = 1 \text{TeV} \]
Correlation

\[ \rho_{ij}(x_1, x_2) = \frac{D_{ij}(x_1, x_2)}{D_i(x_1) D_j(x_2)} - 1 \]

Lack of factorization [Snigirev 2003, Korotkikh, Snigirev 2004]
Evolution washes out the correlations at low \( x \)’s
Gluons (only)

[Golec-Biernat, Lewandowska, Serino, Snyder, Staśto, arXiv:1507.08583]

Gluons from the MSTW parameterization, Mellin moment constraints

\[ x_2 = 0.01 \]

\[ x_2 = 0.5 \]

\[ Q^2 = 1 \text{ GeV}^2 \]

\[ Q^2 = 10 \text{ GeV}^2 \]
Summary

- Top-down strategy of constructing multi-parton distributions → formal features guaranteed
- Requires modeling the LC wave functions ← physics
- Phenomenological sPDF’s as constraints
- Many Fock components needed for the popular parameterizations of sPDF’s, even at low scales
- The valon model offers a simple ansatz at the initial low-energy scale that grasps the essential features with just the longitudinal momentum conservation
- Evolution washes out the correlation at low $x_1, x_2$, justifying the product ansatz in that limit