STRANGE PARTICLE PRODUCTION IN A SINGLE-FREEZE-OUT MODEL

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main topic of this talk: transverse-momentum spectra and elliptic flow of multistrange particles produced at RHIC
Thermal model

Koppe (1948), Fermi (1950), Landau, Hagedorn, Rafelski, Letessier, Torrieri, Bjorken, Gorenstein, Gaździcki, Sinyukov, Kostyuk, Heinz, Sollfrank, Braun-Munzinger, Stachel, Redlich, Csörgő, Lörstad, Becattini, Cleymans, Wheaton

\sim e^{-(E-\mu)/T}
our variant

WB + WF, PRL 87 (2001) 272302 ($p_\perp$ spectra of pions, kaons, and protons)
WB + WF, PRC 65 (2002) 064905 ($p_\perp$ spectra of strange particles)
WB + WF + Anna Baran, AIP 660 (HBT radii and $v_2$)
WB + WF + Brigitte Hiller, PRC 68 (2003) 034911 (pion invariant-mass distributions)

single freeze-out model

1. $T_{\text{chem}} = T_{\text{kin}} \equiv T$
2. Complete treatment of resonances
3. Special choice of the freeze-out hypersurface, $\tau = \sqrt{t^2 - x^2 - y^2 - z^2} = \text{const}$
4. Only 4 parameters: $T, \mu_B$ (fixed by the ratios of the particle abundances), invariant time at freeze-out $\tau$ (controls the overall normalization), transverse size $\rho_{\text{max}}$ ($\rho_{\text{max}}/\tau$ controls the slopes of the $p_\perp$ spectra)
5. Hubble-like flow, $u^\mu = \frac{x^\mu}{\tau} = \frac{t}{\tau}(1, \frac{x}{t}, \frac{y}{t}, \frac{z}{t})$

definition of the hypersurface not unique! – see papers by Rafelski and Torrieri

the choice $t = \text{const}, \text{á la Blast-Wave},$ also possible
**Ratios → Transverse-momentum spectra**

**initial step:** standard analysis of the particle ratios gives $T$ and $\mu_B$ (relative normalization), later $\tau$ and $\rho_{\text{max}}$ fitted from the spectra (absolute normalization and shape)

**recent developments:** SHARE - Statistical Hadronization with Resonances, Cracow - Arizona Collaboration supported by the NATO grant, nucl-th/0404083, submitted to Communications in Physics Computing

G. Torrieri, W. Broniowski, J. Letessier, J. Rafelski, and WF

a set of programs in Fortran and Mathematica devoted to the statistical analysis of the ratios of hadron resonances, offered for the community via our web page:

[www.physics.arizona.edu/torrieri/share/share.html](http://www.physics.arizona.edu/torrieri/share/share.html)

**alternative approach:** THERMUS - a thermal model package for ROOT

2 thermal parameters fitted from particle ratios

\[ T \ [\text{MeV}] = 165 \pm 7, \ \mu_B \ [\text{MeV}] = 41 \pm 5, \quad \oplus 130 \ \text{GeV} \]

\[ T \ [\text{MeV}] = 166 \pm 5, \ \mu_B \ [\text{MeV}] = 29 \pm 4, \quad \oplus 200 \ \text{GeV} \]

2 geometric parameters fitted from the spectra of \( \pi^\pm, K^\pm, p, \) and \( \bar{p} \)

spectra of $\Xi$ and $\Omega + \bar{\Omega}$
STAR spectra vs. single freeze-out model

compilation by Patricia Fachini

STAR Preliminary
\( \sqrt{s_{\text{NN}}} = 200 \text{ GeV} \)

\[ \frac{2}{c^2/\text{GeV}^2} \]

\[ 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 10^0 \quad 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \quad 10^8 \quad 10^9 \]

\( \pi^- \quad K^- \quad \bar{p} \)

0-5% (\( \times 10^5 \))
0-10% (\( \times 10^2 \))
40-80%

Single Freeze-out Model
STAR Preliminary
\( \sqrt{s_{NN}} = 200 \text{ GeV} \)

\[ (2\pi p_T)^{-1} \frac{d^2N}{dy dp_T} (c^2/\text{GeV}^2) \]

- \( \Delta^{++} \) 0-5% (x5)
- \( \rho^0 \) 40-80%
- \( K^{*0} \) 0-10% (x0.01)
- \( \phi \) 0-10% (x0.001)
## Ratios including resonances

<table>
<thead>
<tr>
<th></th>
<th>$m_\rho^* = 770$ MeV</th>
<th>$m_\rho^* = 700$ MeV</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ [MeV]</td>
<td>$T = 165.6 \pm 4.5$</td>
<td>$T = 167.6 \pm 4.6$</td>
<td></td>
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<tr>
<td>$\mu_B$ [MeV]</td>
<td>$\mu_B = 28.5 \pm 3.7$</td>
<td>$\mu_B = 28.9 \pm 3.8$</td>
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<tr>
<td>$\eta/\pi^-$</td>
<td>$0.120 \pm 0.001$</td>
<td>$0.112 \pm 0.001$</td>
<td>$0.183 \pm 0.028$ (40-80%)</td>
</tr>
<tr>
<td>$\rho^0/\pi^-$</td>
<td>$0.114 \pm 0.002$</td>
<td>$0.135 \pm 0.001$</td>
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<tr>
<td>$\omega/\pi^-$</td>
<td>$0.108 \pm 0.002$</td>
<td>$0.102 \pm 0.002$</td>
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<tr>
<td>$K^*(892)/\pi^-$</td>
<td>$0.057 \pm 0.002$</td>
<td>$0.054 \pm 0.002$</td>
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<tr>
<td>$\phi/\pi^-$</td>
<td>$0.025 \pm 0.001$</td>
<td>$0.024 \pm 0.001$</td>
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<tr>
<td>$\eta'/\pi^-$</td>
<td>$0.0121 \pm 0.0004$</td>
<td>$0.0115 \pm 0.0003$</td>
<td></td>
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<tr>
<td>$f_0(980)/\pi^-$</td>
<td>$0.0102 \pm 0.0003$</td>
<td>$0.0097 \pm 0.0003$</td>
<td>$0.042 \pm 0.021$ (40-80%)</td>
</tr>
<tr>
<td>$K^*(892)/K^-$</td>
<td>$0.33 \pm 0.01$</td>
<td>$0.33 \pm 0.01$</td>
<td>$0.205 \pm 0.033$ (0-10%)</td>
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<td>$0.219 \pm 0.040$ (10-30%)</td>
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<td>$0.255 \pm 0.046$ (30-50%)</td>
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<td></td>
<td></td>
<td></td>
<td>$0.269 \pm 0.047$ (50-80%)</td>
</tr>
<tr>
<td>$\Lambda(1520)/\Lambda$</td>
<td>$0.061 \pm 0.002$</td>
<td>$0.062 \pm 0.002$</td>
<td>$0.022 \pm 0.010$ (0-7%)</td>
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<td>$0.025 \pm 0.021$ (40-60%)</td>
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<td>$0.062 \pm 0.027$ (60-80%)</td>
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<tr>
<td>$\Sigma(1385)/\Sigma$</td>
<td>$0.484 \pm 0.004$</td>
<td>$0.485 \pm 0.004$</td>
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</tbody>
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see talk about the resonance production by Christina Markert on Sunday
C. Suire, QM2002
Elliptic flow

WB + WF + Anna Baran, Proceedings of the Coimbra Workshop on Hadron Physics, nucl-th/0212053, AIP 660

Ph. D. Thesis by Anna Baran, to be published

When the nuclei collide at non-zero impact parameter, $b \neq 0$, the momentum distribution of the produced particles carries azimuthal asymmetry

$$\frac{dN}{d^2p_{\perp} dy}\bigg|_{y=0} = \frac{dN}{2\pi p_{\perp} dp_{\perp} dy}\bigg|_{y=0} (1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \ldots)$$

Experimentally determined centrality may be used to determine $b$ from the geometric formula

$$c \sim \frac{b^2}{(2R)^2}$$

eccentricity is obtained from the measured values of $R_{\text{side}}(\phi)$, STAR Collaboration,
\[ \epsilon = \frac{\langle y \rangle^2 - \langle x \rangle^2}{\langle y \rangle^2 + \langle x \rangle^2} \]

modification of the freeze-out hypersurface (almond shape)

\[ r_x = \rho_{\text{max}} \sqrt{1 - \epsilon} \cos \phi \]
\[ r_y = \rho_{\text{max}} \sqrt{1 + \epsilon} \sin \phi \]

modification of the flow profile (stronger in-plane)

\[ u_x = \frac{r_x}{N} \sqrt{1 + \delta} \cos \phi \]
\[ u_y = \frac{r_y}{N} \sqrt{1 - \delta} \sin \phi \]
\[ u_z = \frac{r_z}{N} \]
\[ u_t = \frac{t}{N} \]

\( N \) obtained from the normalization condition \( u^\mu u_\mu = 1 \)
results of the fit procedure of $v_2$ for pions, kaons, and protons
single-freeze-out model fit: $T = 165\text{ MeV}$, $\mu_B = 26\text{ MeV}$ (from the ratios), $\tau = 4.04\text{ fm}$, $\rho_{\text{max}} = 3.70\text{ fm}$ (from the spectra), $\epsilon = 0.13$, $\delta = 0.25$ (from $v_2$)
minimum bias (0-80%) data from STAR @ 200 GeV, PRL 92 (2004) 052302

\[ \Lambda + \bar{\Lambda} \]

\[ V_2 \]

\[ p_\perp \text{ [ GeV]} \]

model parameters as above

model parameters as above
again with the same parameters predictions for $\rho$ and $\phi$
Summary

1. Production of strange particles is well described in a thermal model with single freeze-out.

2. Thermodynamic parameters are determined from the ratios of hadron abundances, whereas the expansion parameters are determined from the spectra of pions, kaons, and protons. Then, the predictions about the transverse-momentum spectra and $v_2$ are made for strange particles.

3. Single freeze-out model yields $R_{\text{out}}/R_{\text{side}} \approx 1$, and describes well pion invariant masses and balance functions.
Back-up slides
The phase-shift formula for the density of resonances

\[ \frac{dn}{dM} = f \int \frac{d^3p}{(2\pi)^3} \frac{d\delta_{12}(M)}{\pi dM} \exp \left( \frac{1}{\sqrt{M^2+p^2}} \right) \pm 1 \]

In some works the spectral function of the resonance is used instead of the derivative of the phase shift. For narrow resonances this does not make a difference, since then
\[\frac{d\delta_{12}(M)}{dM} \simeq \pi \delta(M - m_R),\]
and
\[ n^{\text{narrow}} = f \int \frac{d^3p}{(2\pi)^3} \exp \left( \frac{1}{\sqrt{m_R^2+p^2}} \right) \pm 1 \]

For wide resonances, or for effects of tails, the difference between the correct formula and the one with the spectral function is significant.
Concept of the balance functions


\[ B(\delta, Y) = \frac{1}{2} \left\{ \frac{\langle N_{+-}(\delta) \rangle - \langle N_{++}(\delta) \rangle}{\langle N_+ \rangle} + \frac{\langle N_{-+}(\delta) \rangle - \langle N_{--}(\delta) \rangle}{\langle N_- \rangle} \right\} \]

\( N_{+-} \) and \( N_{-+} \) numbers of the unlike-sign pairs

\( N_{++} \) and \( N_{--} \) numbers of the like-sign pairs

two members of a pair fall into the rapidity window \( Y \), their relative rapidity is

\[ \delta = \Delta y = |y_2 - y_1| \]

\( N_+ \) (\( N_- \)) number of positive (negative) particles in the interval \( Y \)
Two contributions for the $\pi^+\pi^-$ balance function

1) **RESONANCE CONTRIBUTION (R)** is determined by the decays of neutral hadronic resonances which have a $\pi^+\pi^-$ pair in the final state

$$K_S, \eta, \eta', \rho^0, \omega, \sigma, f_0$$

2) **NON-RESONANCE CONTRIBUTION (NR)** other possible correlations among the charged pions

In our approach the non-resonance two-particle distribution is determined by the local relative thermal momenta of particles

The pion balance function is constructed as a sum of the two terms

$$B(\delta, Y) = B_R(\delta, Y) + B_{NR}(\delta, Y)$$
Fit to the STAR data

\[ B(\delta) \]

\[ \delta \]

four different centralities: 0-10%, 10-40%, 40-70%, 70-96%

rescaling factors: 0.40, 0.44, 0.51, 0.51 (\( \chi^2 \) fits)

poor man’s way of taking into account the detector efficiency