# Self-similarity in Fluctuations and Clustering of Multiplicities[⋆⋆]

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**Abstract.** In this talk we summarize the main results of our recent publication [1], where we describe the observed by the CERN NA49 experiment a non-monotonic behavior of the fluctuations in multiplicity as a function of centrality of the collision using the concept of correlations between produced particles promoting cluster formation. A new element is a use of log-normal distribution for particles in a cluster which emerges naturally when one considers particle production based on bivariate branching scenario [2].

## 1 Introduction

Fluctuations of physical observables in collisions of ions have become in recent years one of the main topics of interest, as they can provide some important signals for the formation of Quark-Gluon Plasma. The possibility of inclusion of dynamical fluctuations in the study of the energy of phase transition and the search for the critical point of strongly interacting matter has become a motivation for extensive program of fluctuations analyzes at the SPS, LHC as well as at RHIC. The NA49 experiment found a non-monotonic behavior of the fluctuations in multiplicity [3] as a function of centrality of the collision at the highest SPS energy. This intriguing result may be the first sign of the presence of the critical point. Therefore, the NA49 efforts will be continued in the NA61/SHINE experiment [4].

Although there are some models trying to describe the non-monotonic behavior of the scaled variance of multiplicity distribution as a function of collision centrality expressed by the number of nucleons participants for the NA49 result measured in the centre-of-mass rapidity range $1.1 < y_\pi < 2.6$ (rapidity calculated assuming that all particles have pion mass), up to now there is no commonly accepted explanation of this phenomenon. In this talk we discuss multiplicity fluctuations of charged particles produced in nuclear collisions measured event-by-event by the NA49 experiment. In our recent work [1] the observed non-monotonic behaviour of the scaled variance of multiplicity distribution as a function of collision centrality (such effect is not observed in a widely used string-hadronic models of nuclear collisions, see [3] for more details) was described using the concept of correlations between produced particles promoting cluster formation. Here we have extended our analysis for description of the very intriguing result observed recently by the NA61/SHINE experiment at CERN SPS [5]. Namely, the scaled variance of multiplicity distributions of charged particles measured in broad rapidity range $(0 < y_\pi < y_{max})$ is significantly higher in inelastic proton+proton interactions than in the 1% most central Pb+Pb collisions measured by NA49 experiment at the same energy per nucleon. This result is in qualitative disagreement with the predictions of the wounded nucleon

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model [6] assuming that particle production in nucleon+nucleon and nucleus+nucleus collisions is an incoherent superposition of particle production from wounded nucleons. The proper description allow for verification of models of dynamics of particle production and to put relevant constraints on the elementary entropy deposition in the early phase.

## 2 Data on multiplicity fluctuations

The NA49 and NA61/SHINE experiments located at CERN SPS analyzed multiplicity fluctuations of charged particles produced in proton+proton, Be+Be, Ar+Sc and Pb+Pb collisions [3, 7, 8]. Both experiments used scaled variance of multiplicity distribution,

$$\omega(N) = \frac{Var(N)}{\langle N \rangle} = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle} \tag{1}$$

as a measure of multiplicity fluctuations. The NA49 Collaboration published data on multiplicity fluctuations in Pb+Pb reactions as a function of collision centrality [3]. Unexpectedly, the measured scaled variance show very non-trivial centrality dependence. It is close to unity at completely central collisions but it manifests a prominent discrepancy from unity at peripheral interactions. The measurement has been performed at the collision center of mass energy $\sqrt{s_{NN}} = 17.3$ GeV for particles produced in forward hemisphere in the restricted rapidity interval $1.1 < y_\pi < 2.6$ in the center of mass frame. The azimuthal acceptance has also been limited, and about 17% of all produced charged particles have been used in the analysis [3]. Later on NA49 and NA61/SHINE experiments registered multiplicity distributions of negatively charged particles produced in proton+proton and the most central (1%) Be+Be, Ar+Sc and Pb+Pb collisions at the same center of mass energy, but emitted to the full forward hemisphere, $y_\pi > 0$ [7, 8].

We note a substantial difference in the values of scaled variances of multiplicity distributions of charged particles produced in proton+proton and most central Pb+Pb collisions. The scaled variance is significantly larger for inelastic proton+proton interactions than for the 1% most central Pb+Pb collisions. This difference was extensively discussed in Ref. [5] within the WNM and the statistical model (SM) [9] of particle production. The NA61/SHINE and NA49 results clearly contradict predictions of both WNM and SM models [5].

## 3 Clustering of multiplicities

Usually the following bivariate branching process scenario of multiparticle production is considered:

a) In collision of relativistic ions quark-gluon pairs are created.

b) Each quark or gluon can emit a gluon which can convert into a new quark-antiquark pair.

c) Each of the created quarks may again emit a gluon and so on.

Such evolution of the above described cascading process may be treated as a pure birth process. In each vertex there is a certain probability $P(N)$ to produce $N$ particles. Since the probability $P(N)$ is the same in each vertex such process is self-similar [2]. The bivariate branching process has a multiplicative nature, i.e. the number of particles in a given generation is proportional to the number of particles in the previous one.

$$N_i = (1 + \epsilon_i) N_{i-1} \tag{2}$$
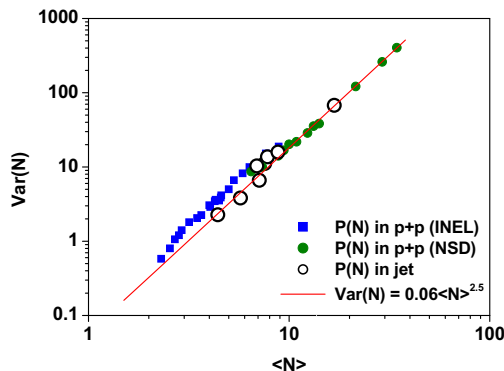
**Figure 1.** Variance of charged particles multiplicity distribution as a function of average charged multiplicity. By squares and circles we indicate data on particle production in proton+proton collisions: squares (inelastic data) are from the compilation for beam energy 3.7-303 GeV presented in [10], full circles (non-single diffractive data) are from the compilation [11]. Open circles are from data on particle production in jets [12–14]. A line shows our fit to the data.

with $\epsilon$ being a random factor. In other words, the final number of particles is a product of a number of random factors:

$$N = N_0 \prod_i (1 + \epsilon_i). \tag{3}$$

Taking logarithms of both sides we see that right hand side of the above equation is a sum of a random components:

$$\ln (N/N_0) = \sum_i \epsilon_i'. \tag{4}$$

where $\epsilon_i' = \ln (1 + \epsilon_i) > 0$ is a random number. Such a sum should be distributed normally due to the Central Limit Theorem. Hence, also a left hand side, i.e. a logarithm of multiplicity should be distributed normally. In other words, multiplicity should be distributed log-normally [2]:

$$P(N) = \frac{1}{\sqrt{2\pi}\sigma} \cdot \frac{1}{N} \exp\left(-\frac{[\ln N - \mu]^2}{2\sigma^2}\right). \tag{5}$$

Formally we get the log-normal (LND) distribution in the limit of a large number of steps. Thus log-normal distribution is continuous. In the case of use of LND for the multiplicities we should apply:

$$P_N = \int_N^{N+1} P(N)\,dN. \tag{6}$$

Figure 1 presents a compilation of values of variances of charged particle multiplicity distributions as a function of average charged multiplicity. Such dependence may be well fitted by a simple formula:

$$Var(N) = 0.06 \cdot \langle N \rangle^{2.5}. \tag{7}$$

The rough formula (7) asserts Taylor's law, $Var(N) = a \cdot \langle N \rangle^b$ with exponent $b > 2$. Such behaviour corresponds a geometrical random walk (as opposed to the ordinary additive random walk) if multiplicity density at each step grows on average (super-critical model) [15]. Comparing multiplicity fluctuations in jets and in minimum bias proton+proton interactions one observes a kind of self-similarity
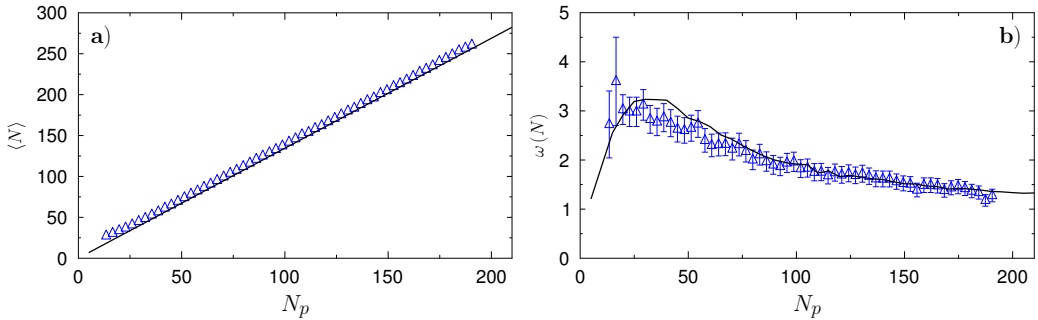
**Figure 2.** Average number of all charged particles (panel a)) and scaled variance of all charged multiplicity distribution (panel b)) of particles produced in Pb+Pb collisions plotted as a function of number of nucleons from projectile nucleus which participate in the collision. By triangles we indicate data of the NA49 experiment [3].
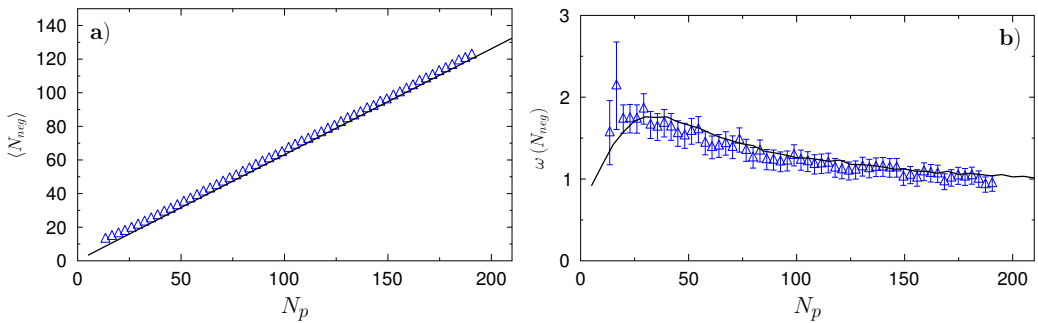


**Figure 3.** The same as in Figure 2 but for negatively charged particles.

of the multiparticle production processes [16]. Regardless of the amount of the available energy, the variance is the same power function of the average multiplicity.

To describe the NA49 data the particle clusterization method discussed in [1] was used. The modification we made for the purpose of this presentation is the following. The multiplicity in a given cluster is calculated according to LND with its variance dependent on $\langle N \rangle$, according to Eq. (7).

## 4 Results

For the simulation of nucleus-nucleus collisions in the framework of Glauber Monte Carlo, we have used an accordingly adopted GLISSANDO package [17]. The parameters of the particle production were adjusted by fitting of experimental data. The resultant centrality dependencies of average all charged multiplicities and corresponding scaled variances of multiplicity distributions are presented in Figure 2. To include experimental acceptance we accepted a fraction of 17% of generated particles. See [1] for detailed discussion of acceptance. Figure 3 presents similar results to those shown in Figure 2 but for negatively charged particles. To obtain corresponding fits we had to adjust only one parameter: the average multiplicity in proton+proton collisions. For the case of negatively charged particles $\langle N_{ch} \rangle = 3.6$. The only difference between scaled variances for negatively and all charged particles is the number of accepted particles. The two-particle correlation function, $\langle v_2 \rangle = (\omega(N) - 1)/\langle N \rangle$ [18] which is not dependent on the experimental acceptance, is roughly the
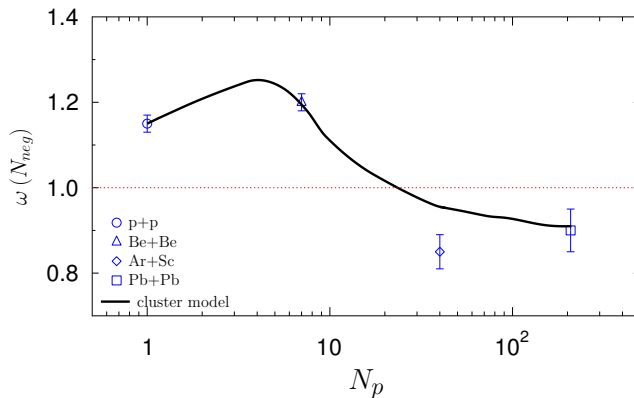
**Figure 4.** Scaled variance of negatively charged multiplicity distribution of particles produced in proton+proton and the most central (1%) Be+Be, Ar+Sc and Pb+Pb collisions, and emitted to the forward hemisphere, $y_\pi > 0$ plotted as a function of number of nucleons from projectile nucleus which participate in the collision. Symbols present data of the NA49 and NA61/SHINE experiments [7, 8]. With the line we show results obtained using our model.

same in both cases. Using similar considerations we have obtained the values for the scaled variance of negatively charged multiplicity distribution produced in proton+proton and the most central (1%) Be+Be, Ar+Sc and Pb+Pb collisions, and emitted to the forward hemisphere, $y_\pi > 0$ [7, 8], see Figure 4. Our model describe most of the NA61/SHINE data (except Ar+Sc collisions) quite reasonable. The observed difference for the colliding system as regards size being comparable to the cluster's size can be caused by a too rough approximation of the cluster size distribution adopted in the model. We note that the previously published results based on the negative binomial distribution gives very similar results [1].

The presence of self-similar cluster phase in the mechanism of particle production, caused by correlations between produced particles, in the natural way allows for reproduction of the effect of the substantially higher value of scaled variance of multiplicity distribution of charged particles produced in proton+proton interactions in comparison to the most central Pb+Pb collisions.

## 5 Concluding remarks

In the present study we used the concept of clusterization in the mechanism of multiparticle production for the description of multiplicity fluctuations observed in relativistic ion collisions at CERN SPS. Our conclusions are as follows:

a) The observed non-monotonic behaviour of the scaled variance of multiplicity distribution as a function of collision centrality can be fully explained by the correlations between produced particles promoting cluster formation Such an effect is not present in a commonly used string-hadronic models of nuclear collisions.

b) The success of the approach lies in treating of all particles included in the cluster-phase as a single system with variance given by Eq. (7) rather than in the details of the probability distributions.

c) The presence of self-similar cluster phase in mechanism of particle production in natural way describes substantially higher value of scaled variance of multiplicity distribution of charged particles produced in proton+proton interactions in comparison to the most central Pb+Pb collisions.

d) Multiplicity clustering (being some kind of implementation of the core-corona model [19, 20]) with self-similar fluctuations (power-law dependence given by Eq. (7) obeys the scaling relationship $Var(N; \lambda \langle N \rangle) = \lambda^{2.5} Var(N; \langle N \rangle))$ provides new insights on non-monotonic behaviour of multiplicity fluctuations.

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