Forward-backward multiplicity correlations in a superposition approach

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[based on AO & W. Broniowski, PRC 88 (2013) 044913, arXiv:1303.5280v2]

Introduction

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Outline

- Three stage superposition model

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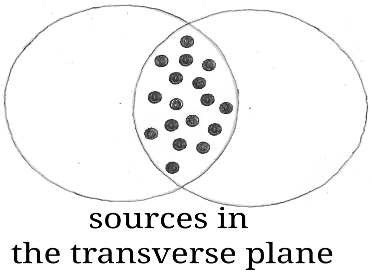
- Comparison to LHC
  1. Glauber model analysis
  2. Model independent analysis

Concept of the sources



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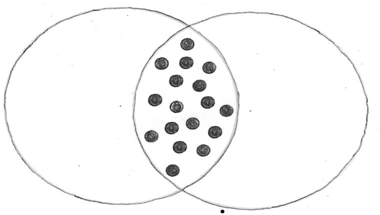

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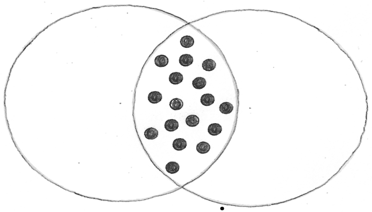

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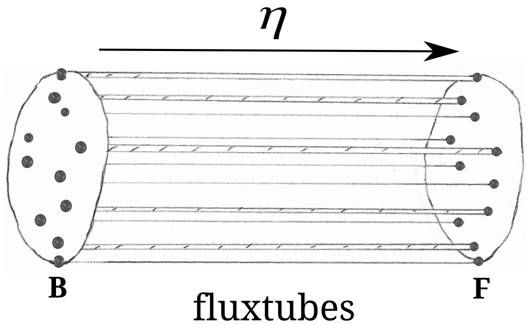

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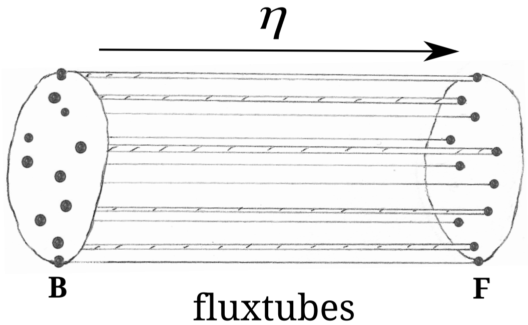

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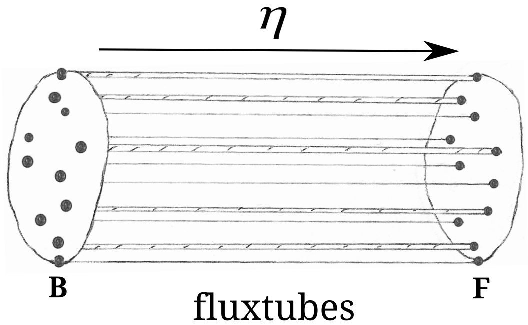

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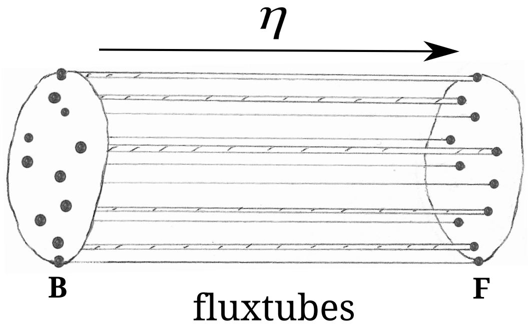

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- We assume well separated F and B rapidity bins
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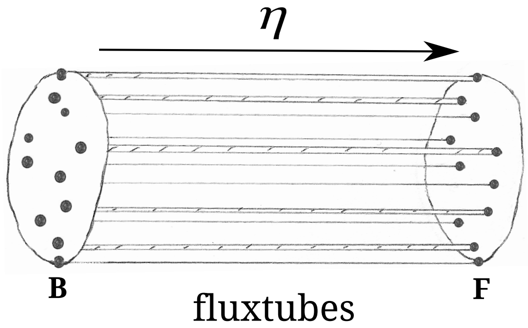

Hypothesis of maximum F-B correlations

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Superposition model

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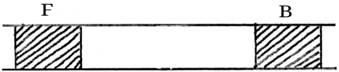

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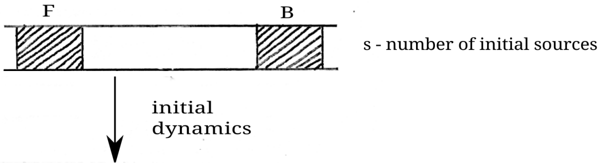

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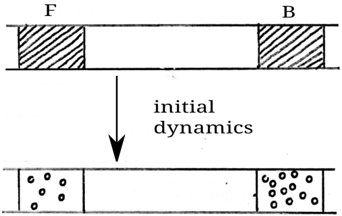

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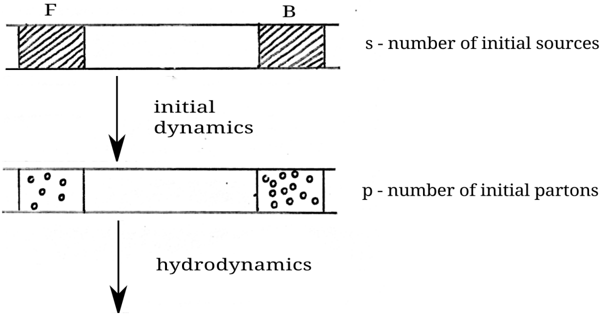

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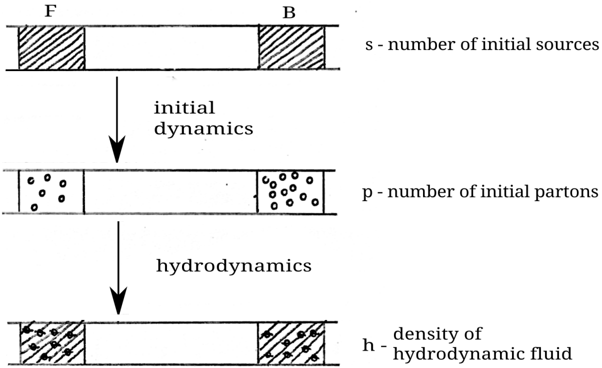

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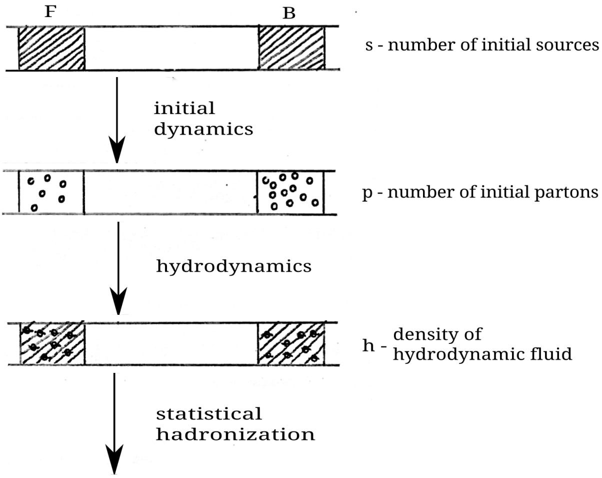

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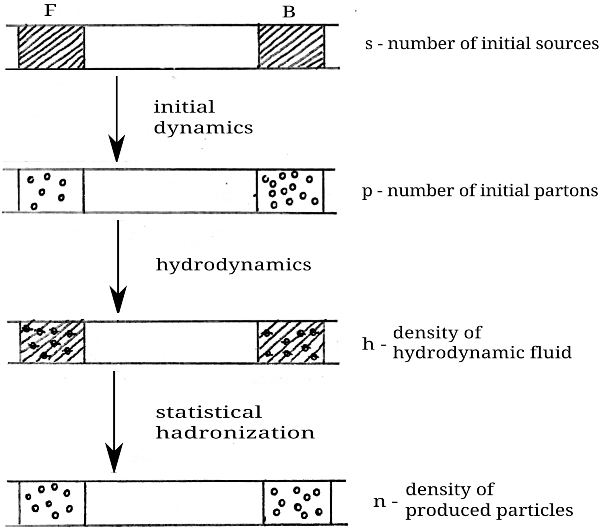

- initial tubes extend along the f-b range
- partons formed from braking fluxtubes
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- overlayed random distribution - freezout of the fluid

Initial phase

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- Distribution of μ is universal $\longrightarrow$ independence from cell location
- Using superposition model $\longrightarrow$ well known formulas

$$s \stackrel{\text{init.production}}{\longrightarrow} p$$

$$\begin{aligned} \langle p_A \rangle &= \langle \mu \rangle \langle s_A \rangle \\ \mathrm{var}(p_A) &= \mathrm{var}(\mu)\langle s_A \rangle + \langle \mu \rangle^2 \mathrm{var}(s_A) \\ \mathrm{cov}(p_F, p_B) &= \langle \mu \rangle^2 \mathrm{cov}(s_F, s_B) \end{aligned}$$

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$$p \xrightarrow{\text{hydro}} h$$

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$\longleftarrow$ Formulas link statistical properties of initial partons and hydrodynamics sources

Statistical hadronization

- Cell emits n hadrons into a region of phase space with some statistical distribution superimposed over h.

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- The importance of $\gamma \longrightarrow$ occurs with variance and covariance

Comparison to LHC

Mixed model

- Wounded nucleon [1] with binary collision [2] → Mixed model [3]

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Methodology and results

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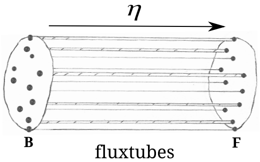

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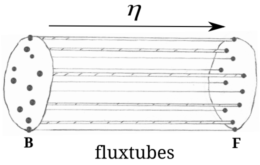

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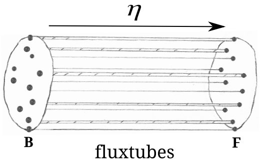

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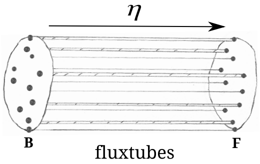

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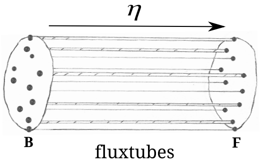
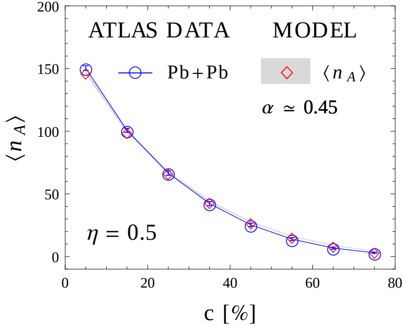
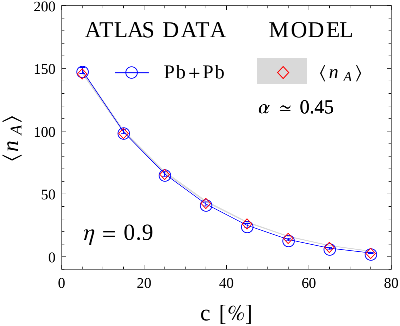
- fit the parameters α, β, γ to the experiment
- calculate theoretical values of statistical properties



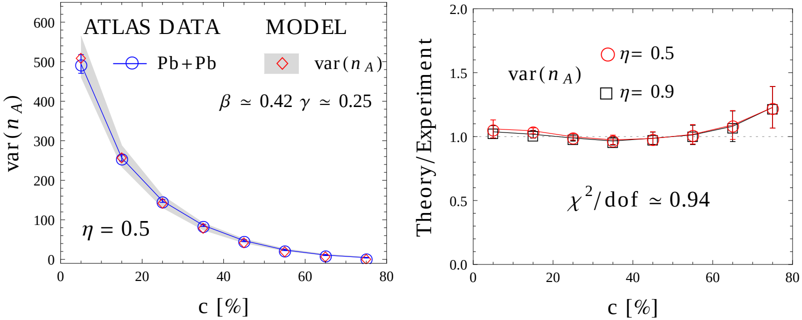
Mean





$$\langle n_A \rangle = \alpha \langle s_A \rangle$$

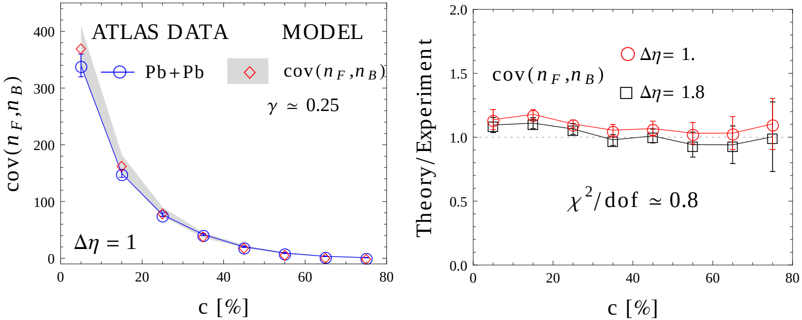
Variance



The α, β, γ practically independent of bin separation $\Delta\eta$

$$\mathrm{var}(n_A) = \beta\langle s_A\rangle + \gamma\mathrm{var}(s_A)$$

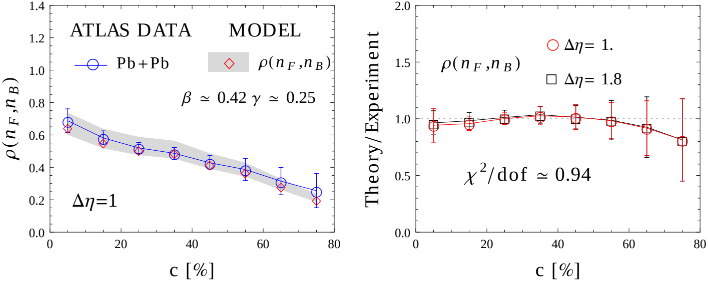
Covariance prediction



We don't fit anything, but use already fitted (test of consistency) γ

$$\mathrm{cov}(n_F, n_B) = \gamma \mathrm{cov}(s_F, s_B) = \rho(s_F, s_B)\gamma \mathrm{var}(s_A)$$
$$\mathrm{cov}(n_F, n_B) = \gamma \mathrm{var}(s_A) \text{ (maximum correlated sources)}$$

Correlation



$$\rho(n_F, n_B) = \frac{\mathrm{cov}(n_F, n_B)}{\mathrm{var}(n_A)} \quad \omega(s_A) = \frac{\mathrm{var}(s_A)}{\langle s_A \rangle}$$

$$\rho(n_F, n_B) = \frac{\rho(s_F, s_B)}{1 + \beta/\gamma\omega(s_A)}$$

Hydrodynamics

We can get information on hydrodynamics from statistical features of mean and covariance

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Consider parameters from model

$$\begin{aligned}\alpha &= t_0 \langle \mu \rangle \langle m \rangle \\ \gamma &= t_1^2 \langle \mu \rangle^2 \langle m \rangle^2\end{aligned}$$

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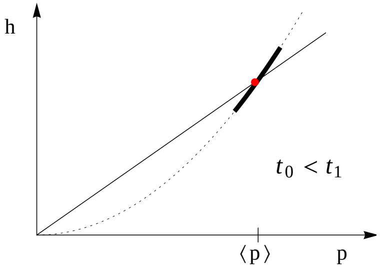
Verification of two possibilities



- Hydrodynamic growth is faster than linear function

We can get information on hydrodynamics from statistical features of mean and covariance

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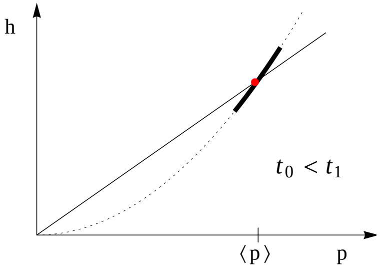
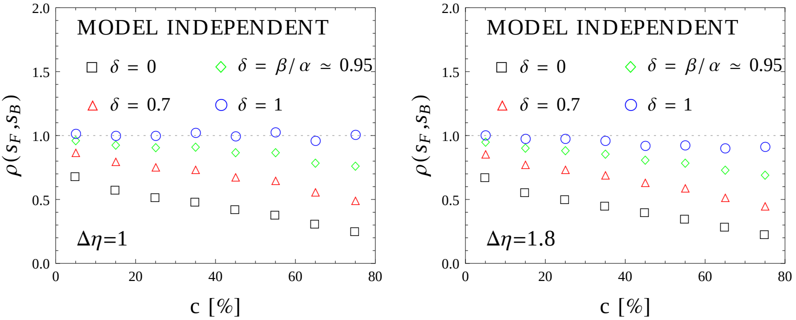


- Hydrodynamic growth is faster than linear function
- Nonlinearity of hydrodynamics

Model independent f-b initial sources correlation

Correlations prediction



- Formula use only measured quantities (no Glauber model)
- Using only one free parameter $\longrightarrow$ maximum correlated sources

$$\delta = \beta/\alpha \text{ (model parameter)}, \quad \rho(s_F, s_B) = \frac{\rho(n_F, n_B)}{1 - \delta/\omega(n_A)}$$

Conclusions

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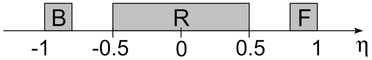
Conclusions

- Simple formulas linking the statistical properties of the F-B correlations in the data and in the original sources have been derived in the three-stage model. Together with the Glauber model for the sources leads to natural description of the early LHC data
- The effect of hydrodynamics may be, under reasonable assumptions, incorporated in terms of just two parameters. Our study shows that the hydrodynamic growth faster than linear function
- The hypothesis of maximal correlation of sources (continuous fluxtubes), $\rho(s_f, s_B) = 1$, works for LHC.

Thank YOU!

Backup slides

STAR methodology



- STAR measurement is affected by correlations to the *reference bin* n_R

STAR measurement

$$\langle n_A \rangle_{n_R} = c_0 + c_1 n_R$$

$$\rho^* (n_F, n_B) = \frac{\rho(n_F, n_B) - R^2}{1 - R^2}$$

$$\omega^* (n_A) = \omega(n_A)\left(1 - R^2\right)$$

$$c_1 = R \frac{\sigma(n_A)}{\sigma(n_R)}, \qquad R = \rho(n_A, n_R)$$

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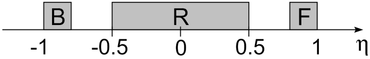

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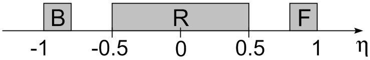

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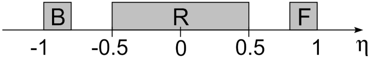

- STAR measurement is affected by correlations to the *reference bin* n_R
- Analysis sets the multiplicity $\longrightarrow$ computes the variance and correlation
- Much more different method
- Complicated formula linking F-B and P-C properties

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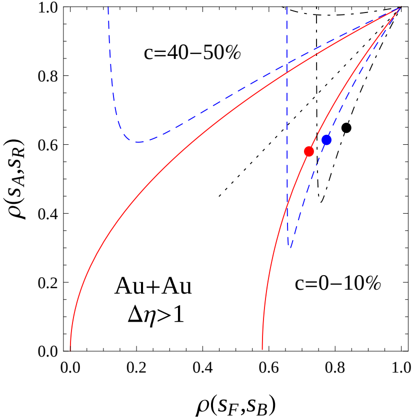
F-b and p-c relation

$$\rho(s_A, s_R)^2 = \frac{\left\{\left[1 - \frac{\delta}{\omega^*(n_A)}\right]\rho(s_F, s_B) - \rho^*(n_F, n_B)\right\}^2}{\left\{1 - \rho^*(n_F, n_B) - \frac{\delta}{\omega^*(n_A)}\right\}\left\{\rho(s_F, s_B) - \rho^*(n_F, n_B) - \frac{\delta}{\omega^*(n_A)}\left[\frac{\langle n_A\rangle}{\langle n_R\rangle}\left(\rho(s_F, s_B) - 1\right) + \rho(s_F, s_B)\right]\right\}}$$

Forward-backward and peripheral-center correlation



- The dots indicate the estimate for $\rho(n_F, n_B) \simeq 0.72$ [2]

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[2] A. Bzdak, Phys. Rev. C 85, 051901 (2012)

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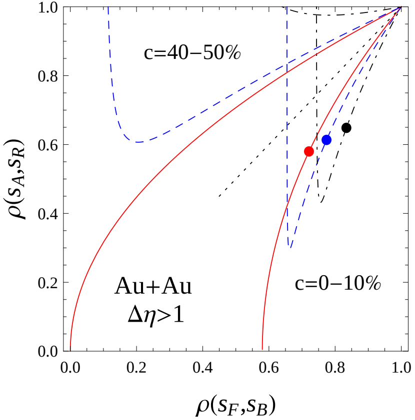

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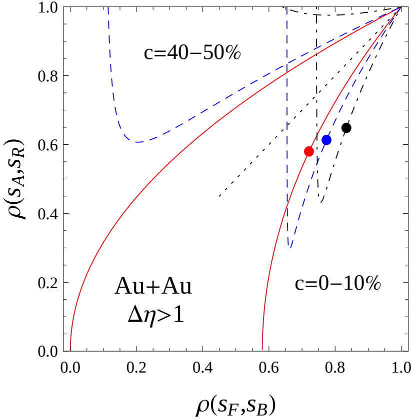

- The dots indicate the estimate for $\rho(n_F, n_B) \simeq 0.72$ [2]
- We consider three δ values and accept the rising parts of the curves
- The most central collisions $\longrightarrow \rho(s_F, s_B) > \rho(s_A, s_R)$. The puzzle [1,2]!

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Mean and Variance

Independent emissions from s sources,

$$n = \sum_{i=1}^{n} m_i,$$

m_i - number of particles produced by the ith source from some distribution ->

$$\langle n \rangle = \langle s \rangle \langle m \rangle.$$

$$\begin{aligned} \mathrm{var}(n) &= \left\langle \sum_{i=1}^{s} (\delta m_i + \langle m \rangle) \sum_{j=1}^{s} (\delta m_j + \langle m \rangle) \right\rangle - (\langle s \rangle \langle m \rangle)^2 \\ &= \left\langle \sum_{i=1}^{s} \delta m_i^2 \right\rangle + \left\langle \sum_{i,j=1, j\neq i}^{s} \delta m_i \delta m_j \right\rangle \\ &+ 2\langle m \rangle \left\langle \sum_{i=1}^{s} \delta m_i \right\rangle + \langle m \rangle^2 \left\langle \sum_{i=1}^{s} \sum_{j=1}^{s} \right\rangle - \langle s \rangle^2 \langle m \rangle^2. \end{aligned}$$

where $\delta m_i = m_i - \langle m \rangle$, with $\langle \delta m_i \rangle = 0$.

$$\mathrm{var}(n) = \langle s \rangle \mathrm{var}(m) + \langle m \rangle^2 \mathrm{var}(s).$$

covariance and correlation

This leads us to simple relation

Next, we look at the covariance between two well-separated bins, which means $\langle m_i m_j \rangle = m^2$, with i and j belonging to two different bins. We have

$$\langle n_1 n_2 \rangle = \left\langle \sum_{i=1}^{s} m_i \sum_{j=1}^{s} m_j \right\rangle = \langle m \rangle^2 \langle s_1 s_2 \rangle,$$

and we get

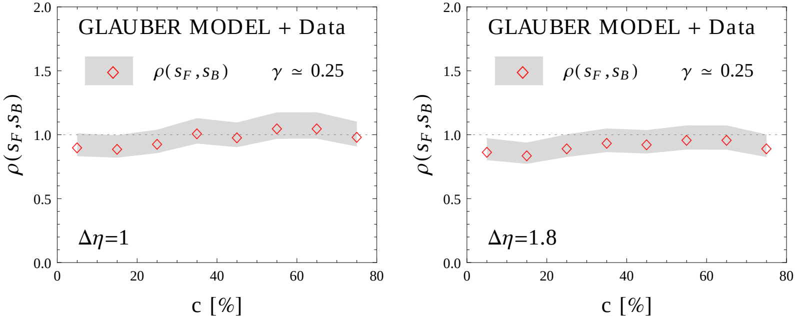
$$\mathrm{cov}(n_1, n_2) = \langle m \rangle^2 \mathrm{cov}(s_1, s_2).$$

For the correlation coefficient it follows that

$$\rho(n_1, n_2) = \frac{\rho(s_1, s_2)}{\sqrt{1 + \frac{\omega(m)}{\langle m \rangle \omega(s_1)}} \sqrt{1 + \frac{\omega(m)}{\langle m \rangle \omega(s_2)}}}.$$

Glauber model f-b initial sources correlations

Correlation prediction



$$\rho(s_F, s_B) = \frac{\mathrm{cov}(n_F, n_B)}{\gamma \mathrm{var}(s_A)}$$