



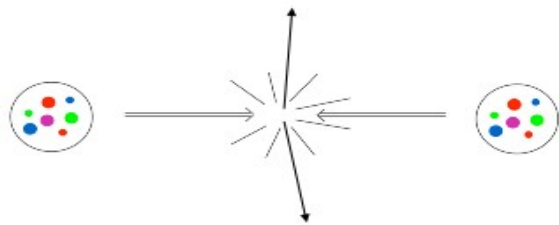
Saturation, coherence and exclusive final states

Krzysztof Kutak

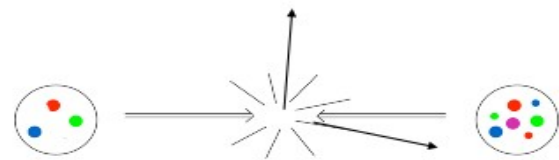


Supported by grant: LIDER/02/35/L-2/10/NCBiR/2011

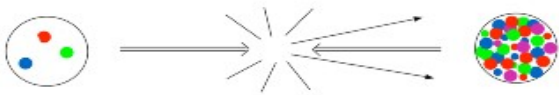
LHC as a scanner of gluon



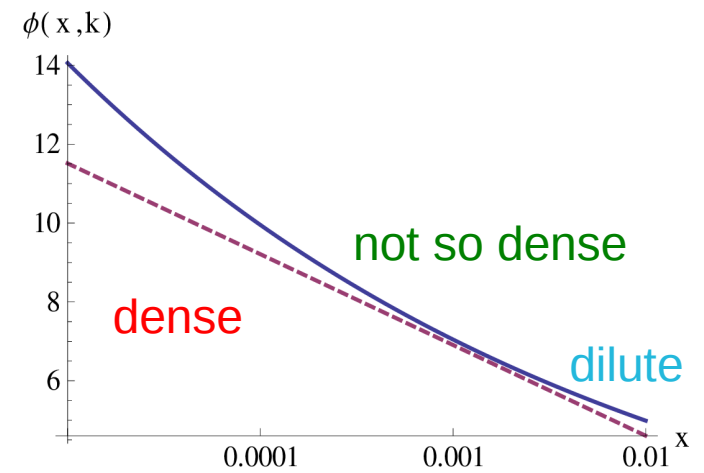
central-central i.e.
not so dense-not so dense



forward-central i.e.
dilute – not so dense



forward-forward i.e.
dilute -dense



QCD at high energies – high energy factorization

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,b,c,d} \int \frac{d^2k_{1t}}{\pi} \frac{d^2k_{2t}}{\pi} \frac{1}{16\pi^2(x_1 x_2 S)^2} |\overline{\mathcal{M}_{ab \rightarrow cd}}|^2 \delta^2(\vec{k}_{1t} + \vec{k}_{2t} - \vec{p}_{1t} - \vec{p}_{2t})$$

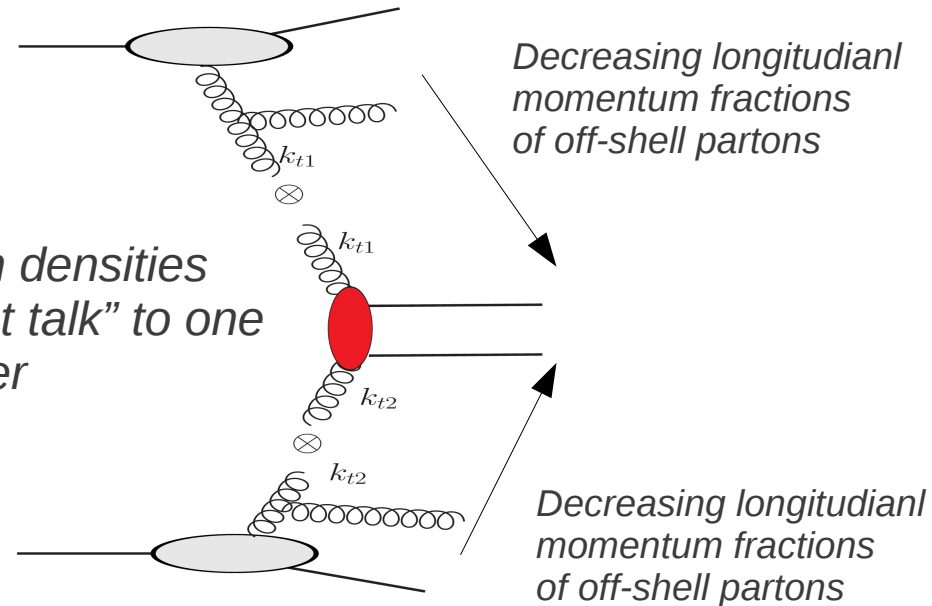
$$\times \mathcal{F}_{a/A}(x_1, k_{1t}^2, \mu^2) \mathcal{F}_{b/B}(x_2, k_{2t}^2, \mu^2) \frac{1}{1 + \delta_{cd}}$$

$$k_1^\mu = x_1 P_1^\mu + \bar{x}_1 P_2^\mu + k_{1t}^\mu \quad k_2^\mu = x_2 P_2^\mu + \bar{x}_2 P_1^\mu + k_{2t}^\mu$$

$$\bar{x}_1 = \frac{k_1^2 + \mathbf{k}^2}{Sx_1} \quad \bar{x}_2 = \frac{k_2^2 + \mathbf{k}^2}{Sx_2}$$

$$|\overline{\mathcal{M}_{ab \rightarrow cd}}|^2 = \frac{2x_1 k_1^{\mu_1} k_1^{\nu_1}}{k_1^2} \frac{2x_2 k_2^{\mu_2} k_2^{\nu_2}}{k_2^2} \mathcal{M}_{ab \rightarrow cd \mu_1 \nu_1} \mathcal{M}_{ab \rightarrow cd \mu_2 \nu_2}^*$$

Parton densities
“do not talk” to one another



Gribov, Levin, Ryskin '81
Ciafaloni, Catani, Hautman '93

Originally derived for quarks in final state.
Lipatov provided general framework.

Recently new approach consistent with Lipatov's action allowed for formulation and numerical calculation of **any tree level amplitude with off-shell gluons in initial state**

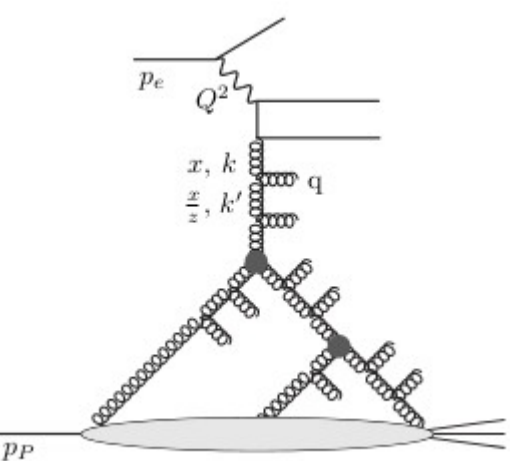
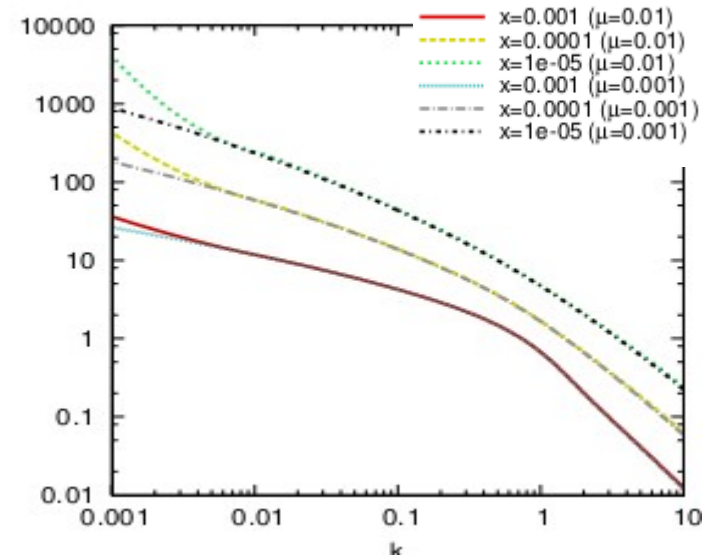
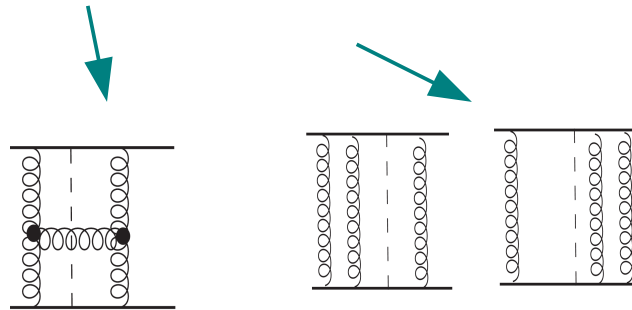
Van Hameren, Kotko, KK '12

Generalized to p-A

Dominguez, Huan, Marquet, Xiao '10

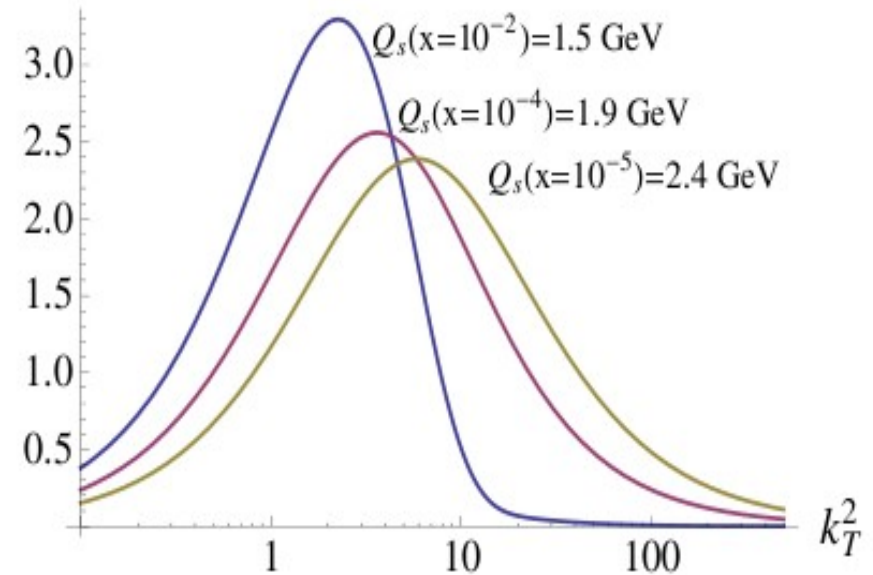
The BFKL and BK evolutions - solutions

$$\mathcal{F}(x, k^2) = \mathcal{F}_0(x, k^2) + \bar{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2 \mathcal{F}(x/z, l^2) - k^2 \mathcal{F}(x/z, k^2)}{|k^2 - l^2|} + \frac{k^2 \mathcal{F}(x/z, k^2)}{\sqrt{(4l^4 + k^4)}} \right]$$



$$\mathcal{F}(x, k^2) = \mathcal{F}_0(x, k^2) + \bar{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2 \mathcal{F}(x/z, l^2) - k^2 \mathcal{F}(x/z, k^2)}{|k^2 - l^2|} + \frac{k^2 \mathcal{F}(x/z, k^2)}{\sqrt{(4l^4 + k^4)}} \right]$$

$$- \frac{2\alpha_s^2 \pi}{N_c R^2} \int_{x/x_0}^1 \frac{dz}{z} \left\{ \left[\int_{k^2}^\infty \frac{dl^2}{l^2} \mathcal{F}(x/z, l^2) \right]^2 + \mathcal{F}(x/z, k^2) \int_{k^2}^\infty \frac{dl^2}{l^2} \ln \left(\frac{l^2}{k^2} \right) \mathcal{F}(x/z, l^2) \right\}$$

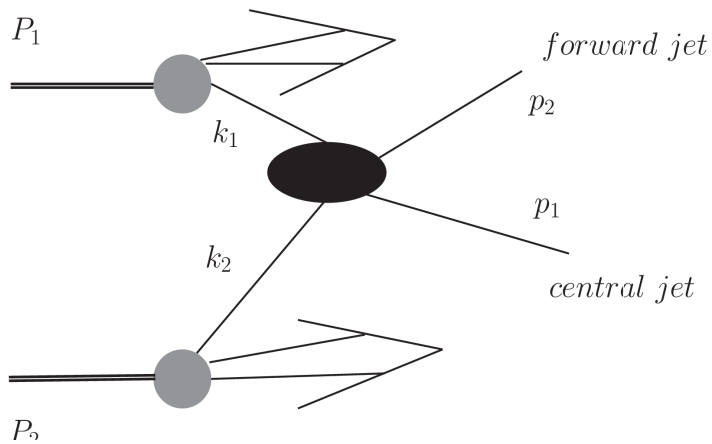


High energy prescription and forward-central di-jets

Deak, Jung, Hautmann Kutak
JHEP 0909:121,2009

$$\frac{d\sigma}{dy_1 dy_2 dp_{1t} dp_{2t} d\phi} = \sum_{a,c,d} \frac{p_{t1} p_{t2}}{8\pi^2 (x_1 x_2 S)^2} |\overline{\mathcal{M}}_{ag \rightarrow cd}|^2 x_1 f_{a/A}(x_1, \mu^2) \phi_{g/B}(x_2, k_t^2, \mu^2) \frac{1}{1 + \delta_{cd}}$$

$$S = 2P_1 \cdot P_2$$



- Resummation of logs of x and logs of hard scale
- Knowing well parton densities at large x one can get information about low x physics
- Framework goes recently under name “ k_T framework”

$$x_1 = \frac{1}{\sqrt{S}} (p_{t1} e^{y_1} + p_{t2} e^{y_2}) \quad \xrightarrow{y_1 \sim 0, y_2 \gg 0} \sim 1$$

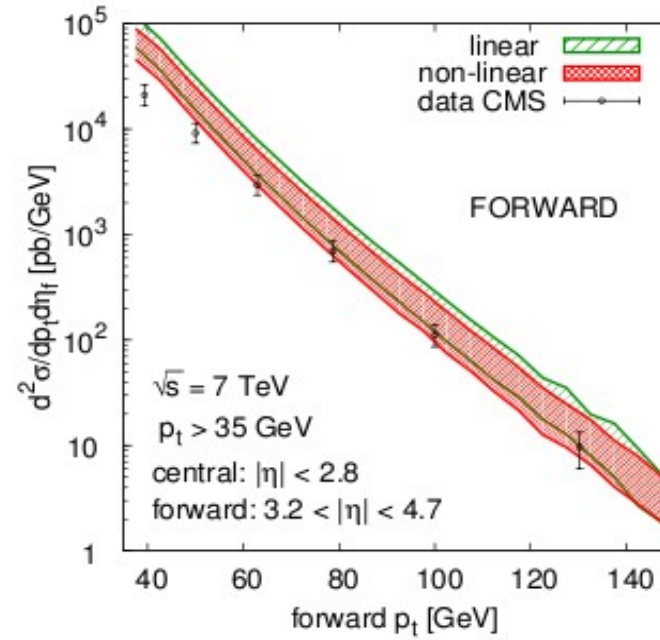
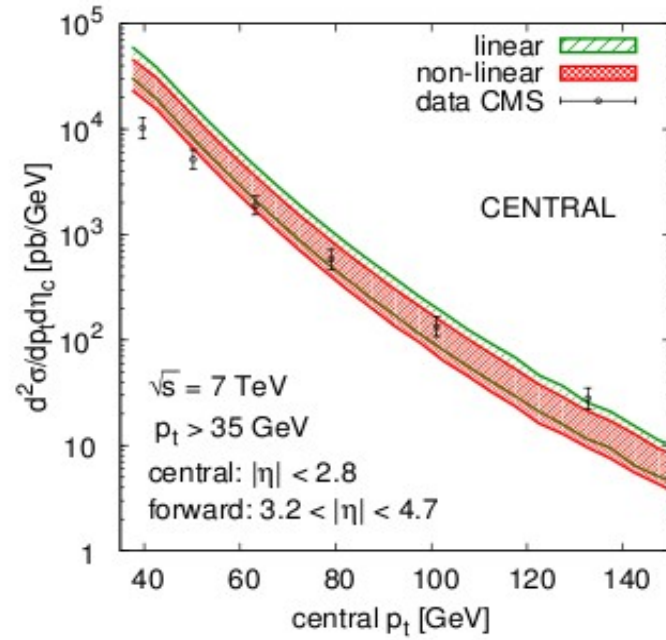
$$x_2 = \frac{1}{\sqrt{S}} (p_{t1} e^{-y_1} + p_{t2} e^{-y_2}) \quad \ll 1$$

$$k_1^\mu = x_1 P_1^\mu$$

$$k_2^\mu = x_2 P_2^\mu + k_t^\mu$$

Pt spectra

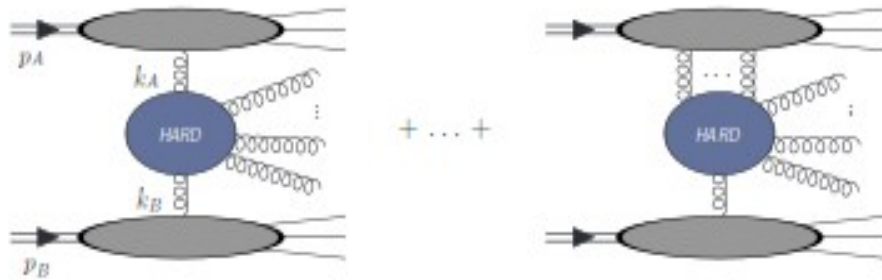
S.Sapeta. KK ,12



Corrections of higher orders
Included. Kin. Constr DGLAP spf

HEF applied to three jets

Van Hameren, Kotko, KK ,13



$$k_A^\mu = x_A p_A^\mu + k_{TA}^\mu, \quad k_B^\mu = x_B p_B^\mu + k_{TB}^\mu \sim x_B p_B^\mu, \quad x_A \ll x_B$$

p-p and p-Pb collisions

CM energy 5 TeV and 7 TeV

$$p_{T1} > p_{T2} > p_{T3} > p_{Tcut}$$

anti- k_T clustering with $R = 0.5$

collinear PDF \Rightarrow CTEQ10 NLO set, scale choice $\mu = a(E_1 + E_2 + E_3)$, where the variation of a gives the (large) theoretical uncertainty

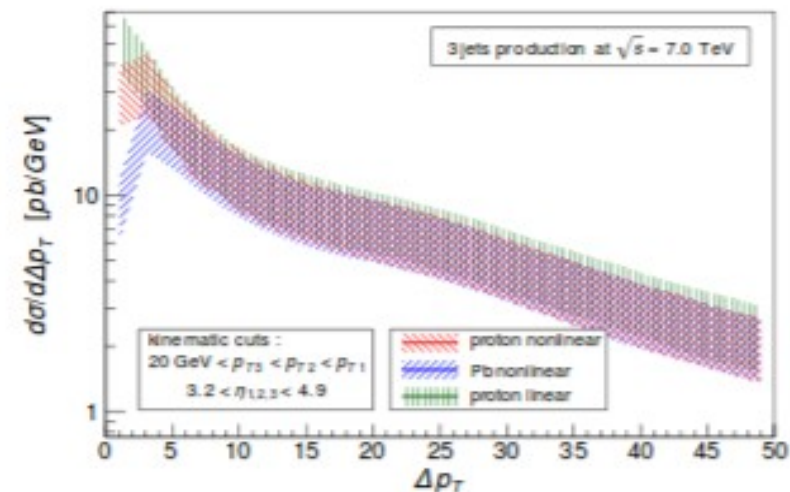
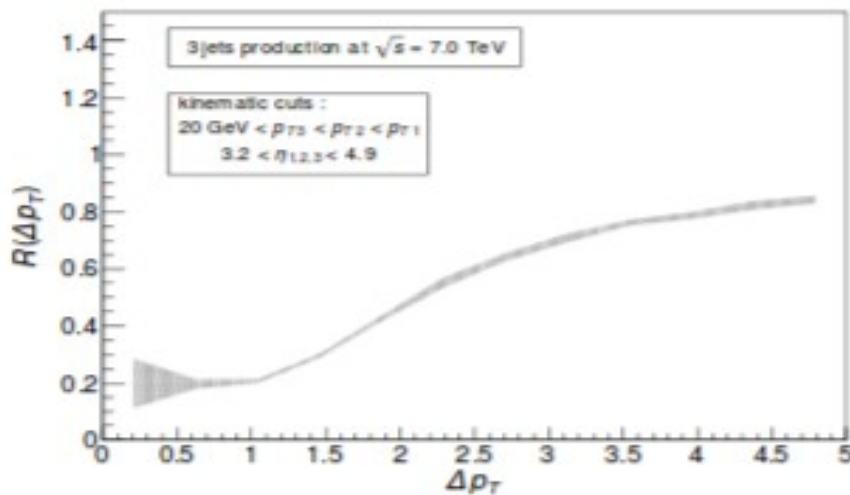
calculations are made and cross-checked using LxJet and OSCARS

all the jets are in the forward region $3.2 < \eta_{1,2,3} < 4.9$

$$p_{Tcut} = 20 \text{ GeV}$$

Implemented in MC codes: C++ code **LxJet** (dijets, trijets), `fortran` code of A. van Hameren (any process) – OSCARS (Off-shell Currents And Related Stuff)

¹ <http://annapurna.ifj.edu.pl/~pkotko/LxJet.html>



CCFM evolution equation - evolution with observer

Catani, Ciafaloni, Fiorani Marchesin '88

Recent review: Avsar, Iancu '09

In $x \rightarrow 1$ region where emitted gluons are soft the dominant contribution to the amplitude comes from the angular ordered region.

$$\bar{\xi} > \xi_i > \xi_{i-1} > \dots > \xi_1 > \xi_0$$

The same structure for $x \rightarrow 0$ although the softest emitted gluons are harder than internal.

$$q_i = \alpha_i p_P + \beta_i p_e + q_{ti}$$

$$s = (p_P + p_e)^2$$

$$\eta_i = \frac{1}{2} \ln(\xi_i) \equiv \frac{1}{2} \ln\left(\frac{\beta_i}{\alpha_i}\right) = \ln\left(\frac{|\mathbf{q}_i|}{\sqrt{s} \alpha_i}\right)$$

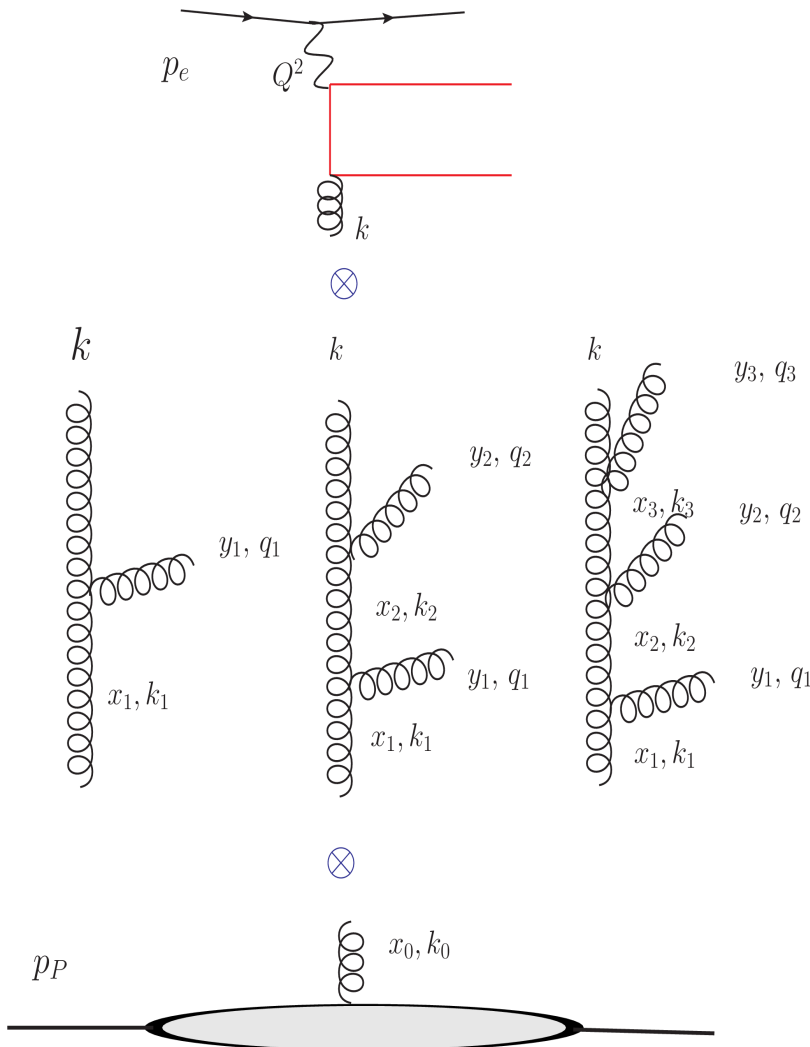
$$\tan \frac{\theta_i}{2} = \frac{|\mathbf{q}_i|}{\sqrt{s} \alpha_i}$$

$$\bar{\xi} = p^2 / (x^2 s)$$

$$z_i = x_i / x_{i-1}$$

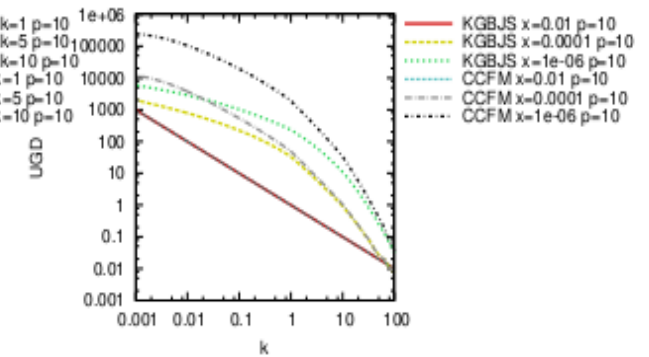
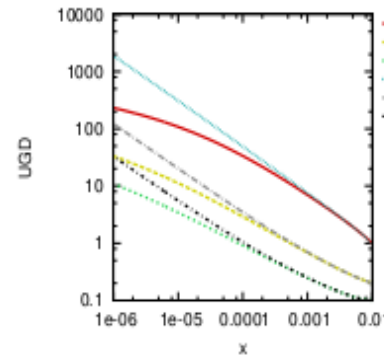
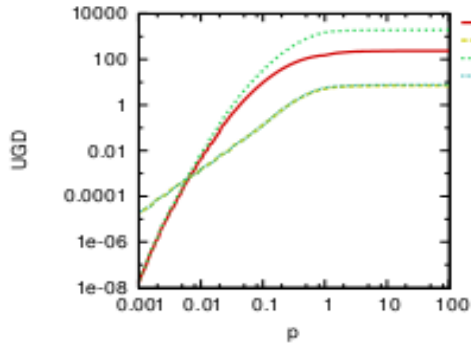
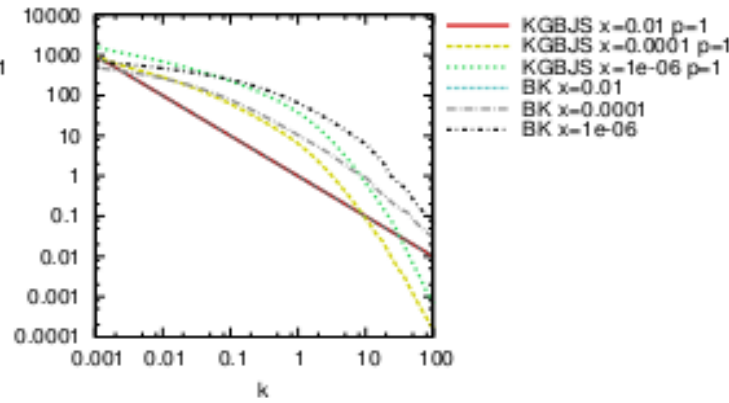
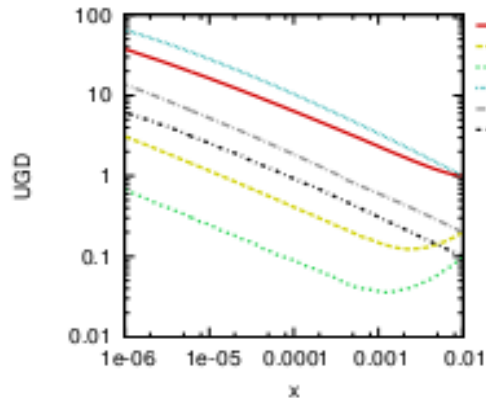
$$dP_i^\theta = \frac{\alpha_s}{2\pi} dz_i \frac{d^2 q_i}{q_i^2} P_{gg}(z_i) \theta(q_i - z_{i-1} q_{i-1}) (1 - z_i)$$

Implemented in CASCADE Monte Carlo Jung **02**



The KGBJS equation – nonlinear ext. of CCFM

Toton, KK '13



$$\begin{aligned} \phi(x, k) &= \tilde{\phi}_0(x, k) \\ &+ \bar{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \theta(q^2 - \mu^2) \Delta_R(z, k, \mu) \\ &\left(\phi\left(\frac{x}{z}, k'^2\right) - \frac{q^2}{\pi R^2} \delta(q^2 - k^2) \phi^2\left(\frac{x}{z}, q^2\right) \right) \end{aligned}$$

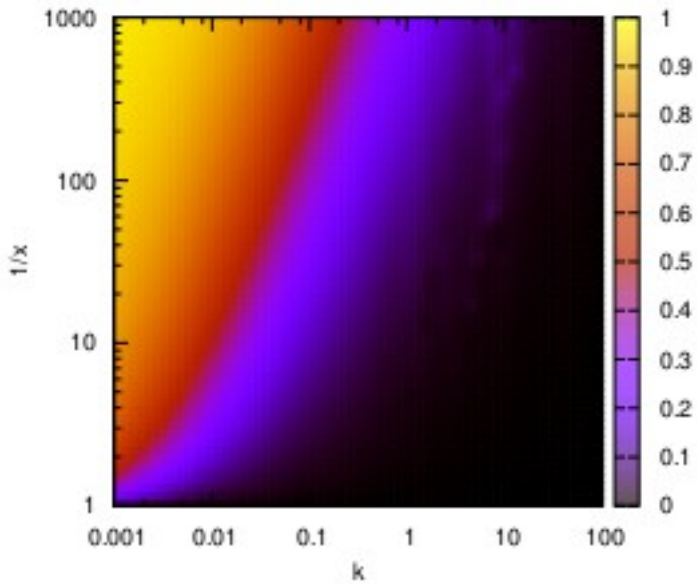
$$\Delta_R(z, k, \mu) = \exp\left(-\bar{\alpha}_s \log \frac{1}{z} \log \frac{k^2}{\mu^2}\right)$$

$$\begin{aligned} \mathcal{E}(x, k^2, p) &= \mathcal{E}_0(x, k^2, p) \\ &+ \bar{\alpha}_s \int_{x/x_0}^1 \frac{dz}{z} \int \frac{d^2\bar{\mathbf{q}}}{\pi \bar{q}^2} \theta(p - z\bar{q}) \Delta_{ns}(z, k, \bar{q}) \\ &\left(\mathcal{E}\left(\frac{x}{z}, k'^2, \bar{q}\right) - \frac{\bar{q}^2}{\pi R^2} \delta(\bar{q}^2 - k^2) \mathcal{E}^2\left(\frac{x}{z}, \bar{q}^2, \bar{q}\right) \right) \end{aligned}$$

$$\Delta_{ns} = \exp\left(-\bar{\alpha}_s \ln \frac{1}{z} \ln \frac{k^2}{zq^2}\right) \quad \text{for } k^2 > zq^2$$

Saturation scale in KGBJS

BK

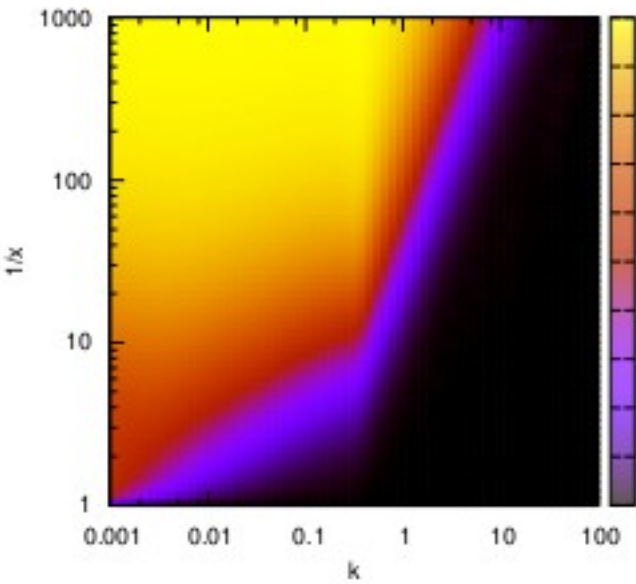


Relative differences between linear and nonlinear

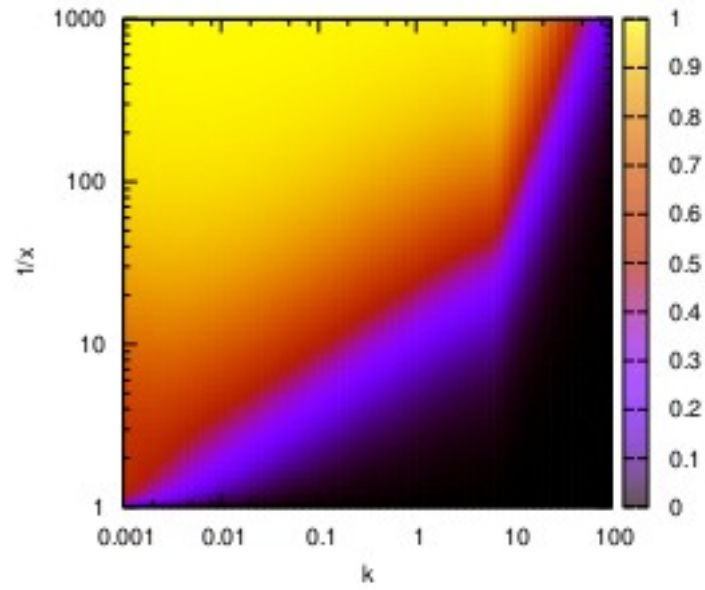
K.K, A. Stasto Eur.Phys.J.C41:343-351,2005

$$\beta(x, k, p) = \frac{|\mathcal{E}_{CCFM}(x, k, p) - \mathcal{E}_{KGBJS}(x, k, p)|}{\mathcal{E}_{CCFM}(x, k, p)}$$

KGBJS



p=1

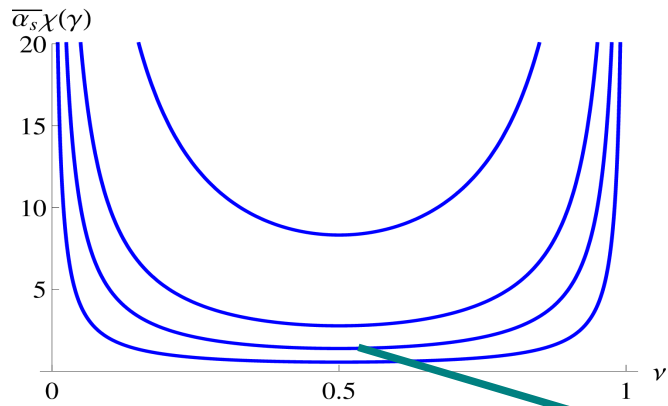


p=10

Toton, KK '13

Gluon density at the large coupling values

weak coupling



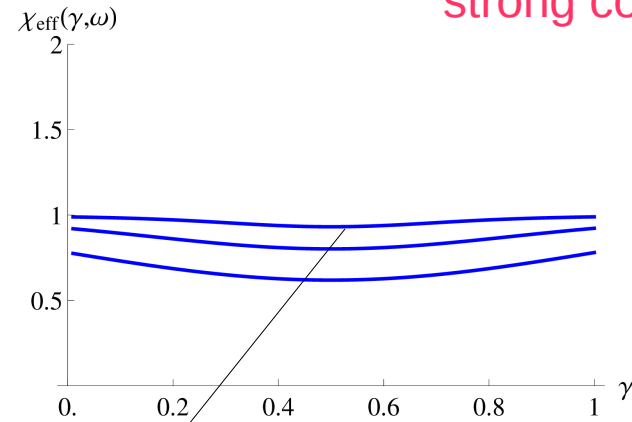
$$\chi(\gamma) = 2\psi(1) - \psi(1-\gamma) - \psi(\gamma)$$

weak coupling

$$f(x, k^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu (k^2)^{1/2+i\nu} \bar{f}(x_0, 1/2+i\nu) x^{-\bar{\alpha}_s \chi(1/2+i\nu)}$$

critical point dominates
at large coupling

strong coupling

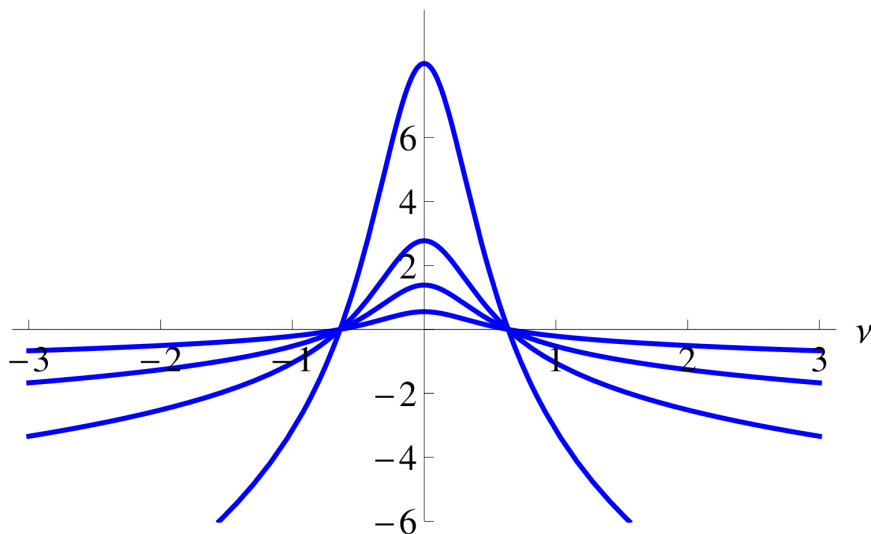


Pochinski '02
Stasto '07

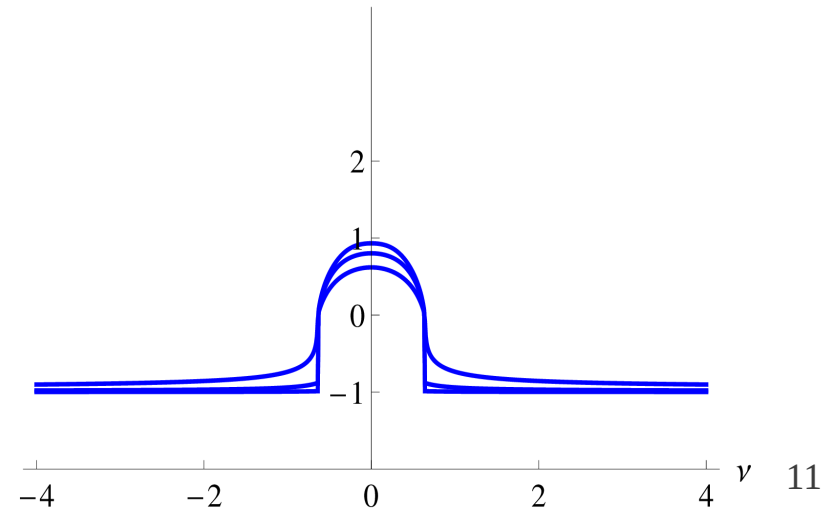
strong coupling

Surowka, KK '13

$\text{Re}(\bar{\alpha}_s \chi(1/2+i\nu))$

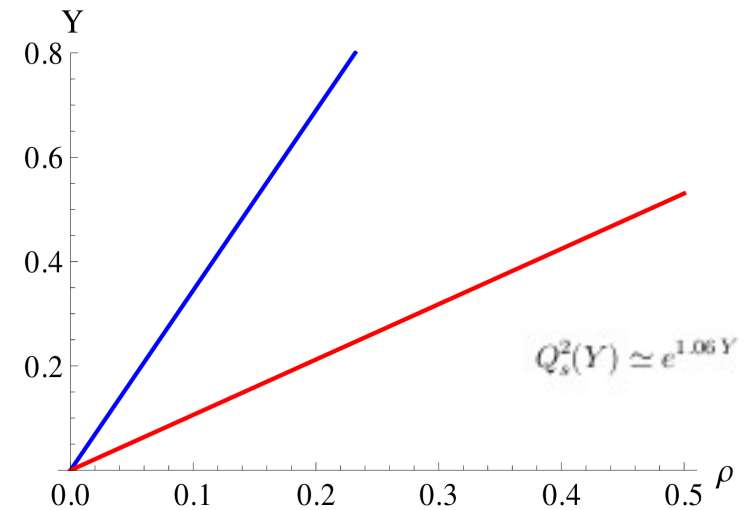
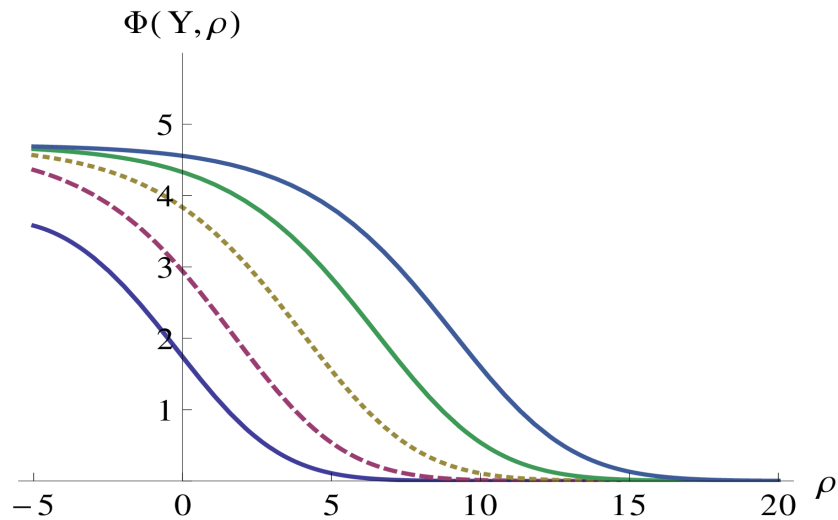


$\text{Re}(\chi_{\text{eff}}(1/2+i\nu, \bar{\alpha}_s))$



Gluon density at the large coupling values

Surowka, KK '13



Nonlinear nonlinear equation valid at strong coupling limit

$$\partial_Y \Phi(Y, \rho) = \frac{1}{2} \lambda'_{st} \partial_\rho^2 \Phi(Y, \rho) + \frac{1}{2} \lambda'_{st} \partial_\rho \Phi(Y, \rho) + (\lambda_{st} + \lambda'_{st}/8) \Phi(Y, \rho) - \frac{\bar{\alpha}_s}{\pi R^2} \Phi^2(Y, \rho)$$

Conclusions and outlook

- *LHC gives opportunity to test parton densities both when the parton density is probed at low x and at low, medium and large kt at some external scale.*
- *The interplay of saturation and coherence leads to new features of saturation scale*
- *The results for jets give some theoretical hints for saturation*