

Mott-Anderson localization & hadron production ¹

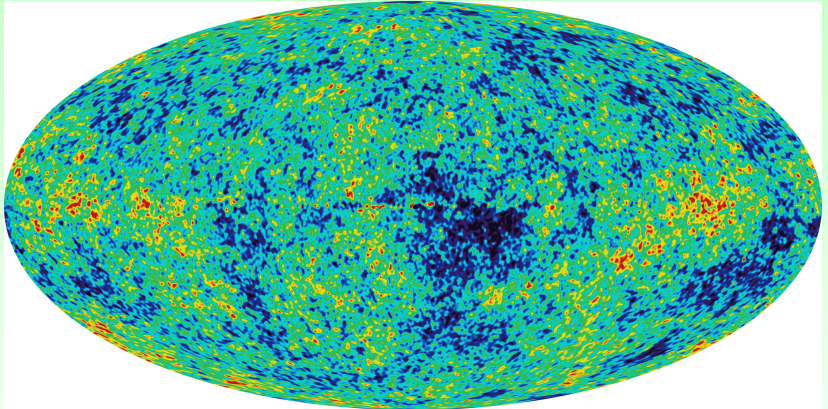
David Blaschke

Institute of Theoretical Physics, University Wrocław, Poland
Bogoliubov Laboratory for Theoretical Physics, JINR Dubna, Russia

Kielce (Poland), December 13 - 15, 2013

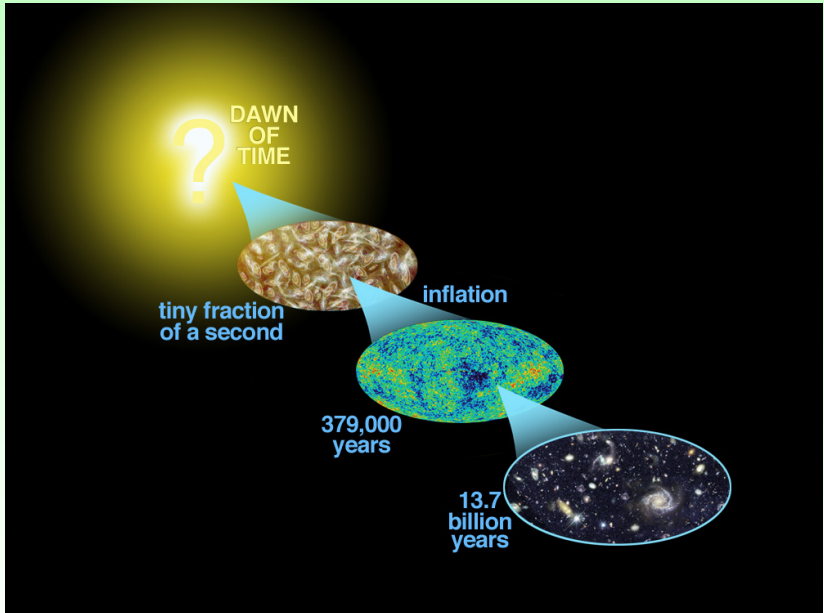
¹Collab.: J. Berdermann, J. Cleymans, D. Prorok, K. Redlich, L. Turko ...

Heavy-Ion Collisions & Hadronic (chemical) Freeze-out

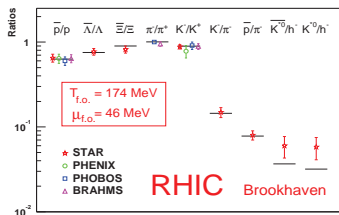
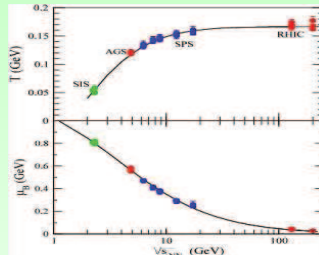
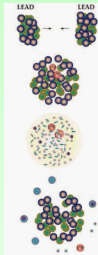
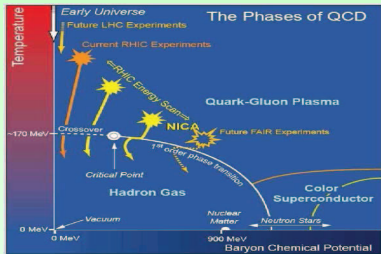


WMAP snapshot of the baby-universe:
Freeze-out of density fluctuations ~ 380000 years after big bang

Heavy-Ion Collisions & Hadronic (chemical) Freeze-out



Heavy-Ion Collisions & Hadronic (chemical) Freeze-out



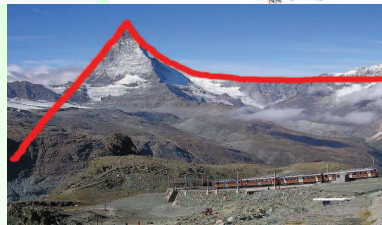
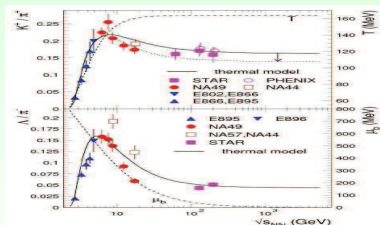
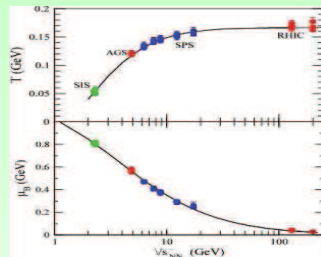
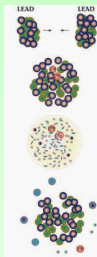
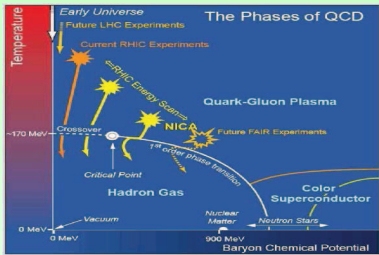
Statistical model describes composition of hadron yields in Heavy-Ion Collisions with few freeze-out parameters.

$$\ln Z[T, V, \{\mu\}] = \pm V \sum_i \frac{g_i}{2\pi^2} \int_0^\infty dp p^2 \ln[1 \pm \lambda_i \exp(-\beta\varepsilon_i(p))]$$

$$\lambda_i(T, \{\mu\}) = \exp[\beta(\mu_B B_i + \mu_S S_i + \mu_Q Q_i)]$$

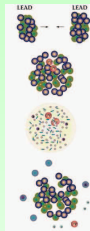
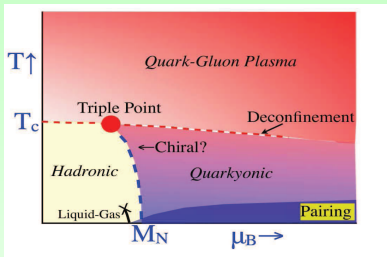
Braun-Munzinger, Redlich, Stachel, in *QGP III* (2003)

Heavy-Ion Collisions & Hadronic (chemical) Freeze-out

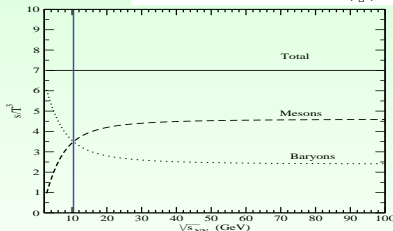
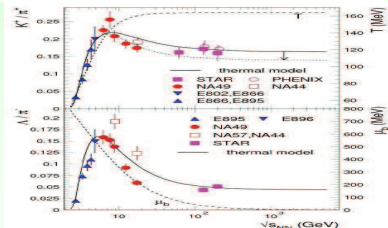
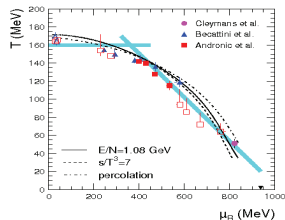


Strange MatterHorn (Pisarski)

Heavy-Ion Collisions & Hadronic (chemical) Freeze-out

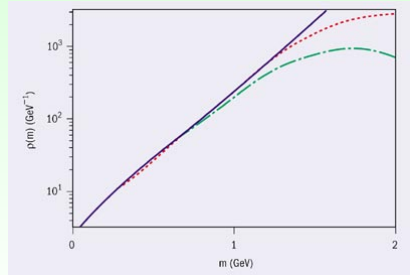
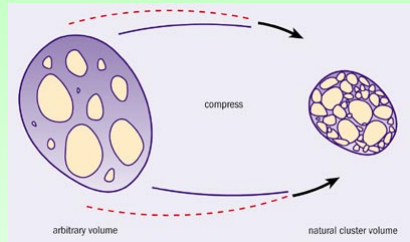


Andronic et al., arxiv:0911.4806

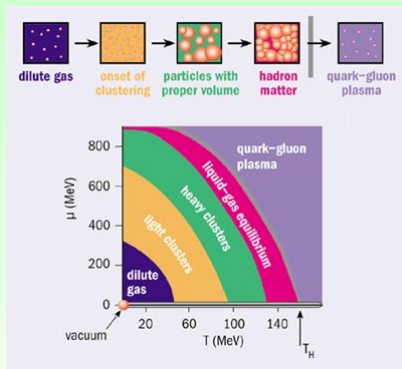


Andronic et al., arxiv:0911.4806; NPA (2010)

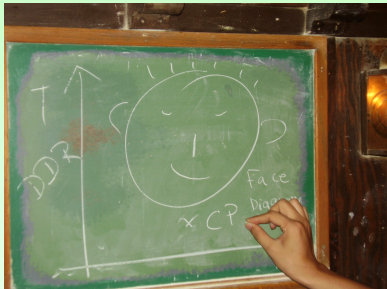
Rolf Hagedorn - Statistical model of particle production



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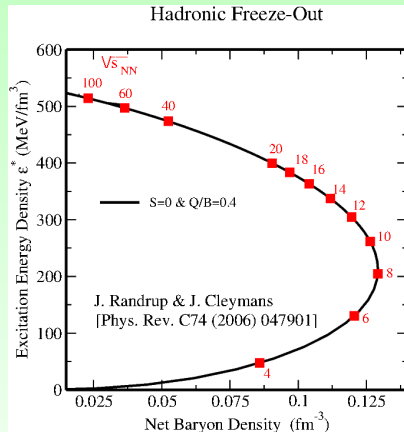
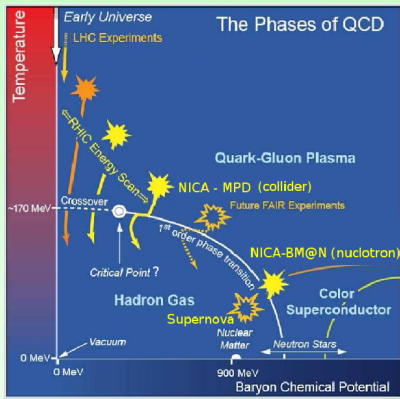
QCD Phase Diagram & Heavy-Ion Collisions



Seattle, August 2008 “Critical Point ...”,

Marlene Nahrgang

QCD Phase Diagram & Heavy-Ion Collisions

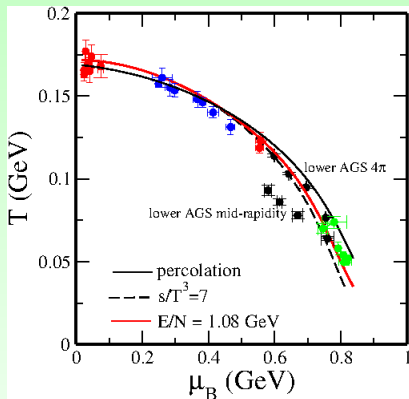


Beam energy scan (BES) programs in the QCD phase diagram

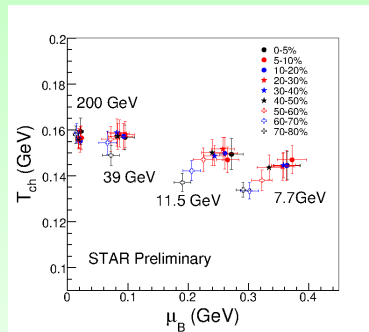
Energy density vs. baryon density at freeze-out for different $\sqrt{s_{NN}}$ (GeV)

Highest baryon densities at freeze-out shall be reached for $\sqrt{s_{NN}} \sim 8$ GeV \rightarrow QGP phase transition ?

Chemical Freeze-out in the QCD Phase Diagram



“Old” freeze-out data from RHIC (red), SPS (blue), AG (black), SIS (green).



“New” freeze-out data from STAR BES @ RHIC. Centrality dependence!

F. Becattini, J. Manninen, M. Gazdzicki, Phys. Rev. C73 (2006) 044905
Lokesh Kumar (STAR Collab.), arxiv:1201.4203 [nucl-ex]

Chemical freeze-out condition

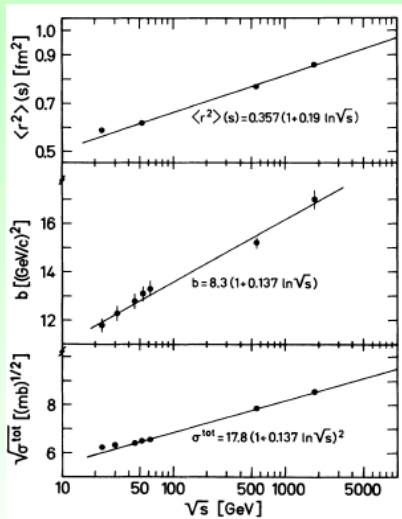
$$\tau_{\text{exp}}(T, \mu) = \tau_{\text{coll}}(T, \mu)$$

$$\tau_{\text{coll}}^{-1}(T, \mu) = \sum_{i,j} \sigma_{ij} n_j$$

$$\sigma_{ij} = \lambda \langle r_i^2 \rangle \langle r_j^2 \rangle$$

D.B. et al., Few Body Systems (2011) arxiv:1109.5391

[hep-ph]



B. Povh, J. Hüfner, PRD 46 (1992) 990

Hadronic radii and chiral condensate

$$r_{\pi}^2(T, \mu) = \frac{3}{4\pi^2} F_{\pi}^{-2}(T, \mu) .$$

$$F_{\pi}^2(T, \mu) = -m_0 \langle \bar{q}q \rangle_{T, \mu} / m_{\pi}^2 .$$

$$r_{\pi}^2(T, \mu) = \frac{3m_{\pi}^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T, \mu}|^{-1} .$$

$$r_N^2(T, \mu) = r_0^2 + r_{\pi}^2(T, \mu) ,$$

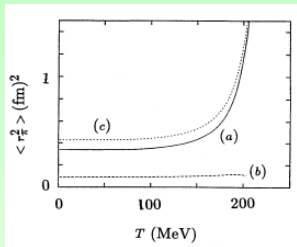
Expansion time from entropy conservation

$$S = s(T, \mu) V(\tau_{\text{exp}}) = \text{const}$$

$$\tau_{\text{exp}}(T, \mu) = a s^{-1/3}(T, \mu) ,$$

D.B., J. Berdermann, J. Cleymans, K. Redlich,

Few Body Systems (2011) [arxiv:1109.5391]



H.-J. Hippe and S. Klevansky, PRC 52 (1995) 2172



Ladenburg (1992)

Clue to the effectiveness: (De)localization !

$$r_{\pi}^2(T, \mu) = \frac{3}{4\pi^2} F_{\pi}^{-2}(T, \mu) .$$

$$F_{\pi}^2(T, \mu) = -m_0 \langle \bar{q}q \rangle_{T, \mu} / m_{\pi}^2 .$$

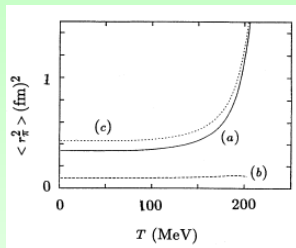
$$r_{\pi}^2(T, \mu) = \frac{3m_{\pi}^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_{T, \mu}|^{-1} .$$

$$r_N^2(T, \mu) = r_0^2 + r_{\pi}^2(T, \mu) ,$$

Effective hadron (de)localization at the chiral restoration transition, a la *Mott-Anderson (de-)localization* of electron wave functions in the insulator-metal transition [Nobel prize (1977)].

D.B., J. Berdermann, J. Cleymans, K. Redlich,

Few Body Systems (2011) [arxiv:1109.5391]



H.-J. Hippe and S. Klevansky, PRC 52 (1995) 2172



Sir N.F. Mott P.W. Anderson

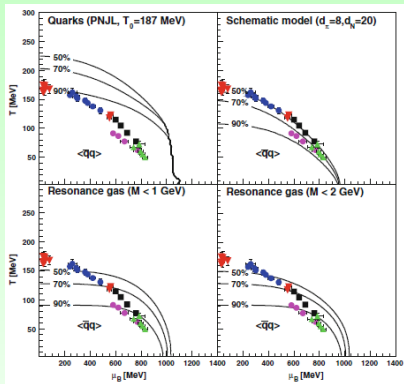
Chiral Condensate in a Hadron Resonance Gas

$$\begin{aligned} \frac{\langle \bar{q}q \rangle}{\langle \bar{q}q \rangle_{\text{vac}}} &= 1 - \frac{m_0}{F_\pi^2 m_\pi^2} \left\{ 4N_c \int \frac{dp p^2}{2\pi^2} \frac{m}{\varepsilon_p} [f_\Phi^+ + f_\Phi^-] \right. \\ &+ \sum_{M=f_0, \omega, \dots} d_M (2 - N_s) \int \frac{dp p^2}{2\pi^2} \frac{m_M}{E_M(p)} f_M(E_M(p)) \\ &+ \left. \sum_{B=N, \Lambda, \dots} d_B (3 - N_s) \int \frac{dp p^2}{2\pi^2} \frac{m_B}{E_B(p)} [f_B^+(E_B(p)) + f_B^-(E_B(p))] \right\} \\ &- \sum_{G=\pi, K, \eta, \eta'} \frac{d_G r_G}{4\pi^2 F_G^2} \int dp \frac{p^2}{E_G(p)} f_G(E_G(p)) \end{aligned}$$

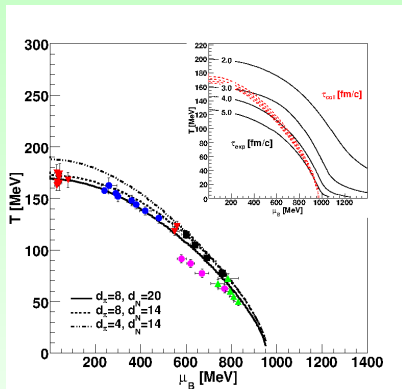
S. Leupold, J. Phys. G (2006)
D.B., J. Berdermann, J. Cleymans,
K. Redlich, Few Body Systems
(2011)



Chemical Freeze-out and Chiral Condensate



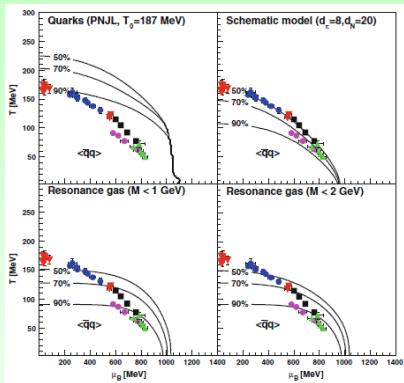
Chemical freeze-out vs. Condensate



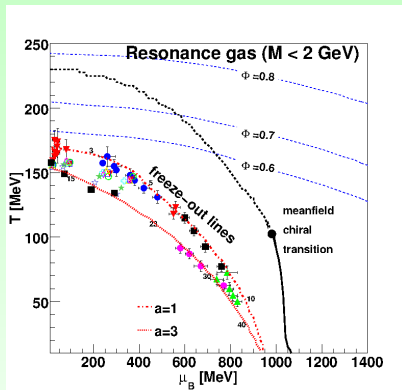
Chemical freeze-out from kinetic condition, schematic model

D.B., J. Berdermann, J. Cleymans, K. Redlich, Few Body Systems (2011)

Chemical Freeze-out and Chiral Condensate



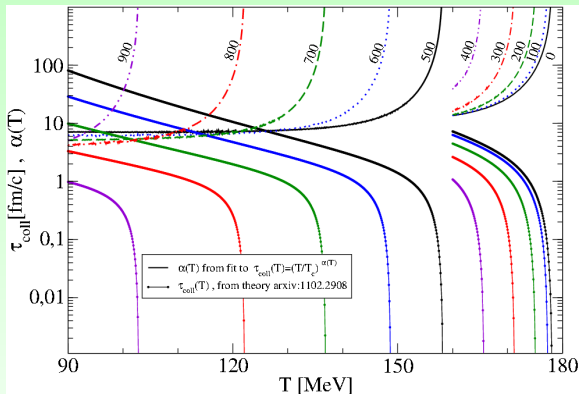
Chemical freeze-out vs. Condensate



Chemical freeze-out from kinetic condition, $a \sim$ inverse system size

D.B., J. Berdermann, J. Cleymans, K. Redlich, in preparation (2013)

Strong T-Dependence of (inelastic) Collision Time



Klabucar, Berdermann (2006)

See: C. Blume in: NICA White Paper (2012)

C. Wetterich, P. Braun-Munzinger, J. Stachel, PLB (2004)

D.B., J. Berdermann, J. Cleymans, K. Redlich, in preparation (2013)

- The model works unreasonably well!
- Improvements are plenty:
 - Hadron mass formulae, e.g. from holographic QCD ...
 - Spectral functions - generalized Beth-Uhlenbeck
 - Thermodynamics ... hydrodynamics .
- Beyond freeze-out towards the deconfined phase: Mott-Hagedorn model

Pion dissociation and Levinson's theorem (A PNJL model case study)

- Gap eqn. & Bethe-Salpeter eqn. in PNJL quark matter
- Mott-Anderson dissociation/delocalization of pions
- Generalized Beth-Uhlenbeck EoS for quark-meson matter
- Levinson theorem & quark-meson thermodynamics

A. Wergieluk, D. Blaschke, Yu. Kalinovsky, A. Friesen, arxiv:1212.5245;
Dubna Report E2-2013=19; Phys. Part. Nucl. Lett. **7** (2013) in press.
D.B., D. Zablocki, M. Buballa, G. Röpke, A. Dubinin, arxiv:1305.3907.v2
A. Dubinin, D. Blaschke, Yu. Kalinovsky; arxiv:1312.0559

The state of the art in January 1994

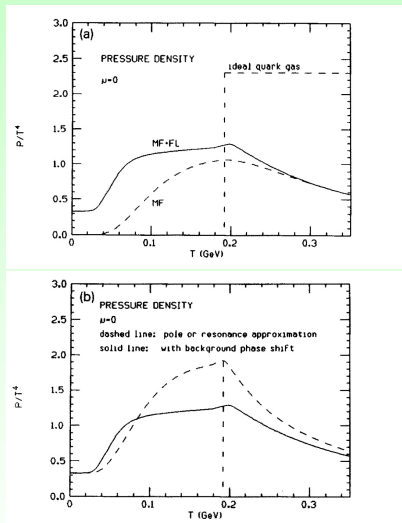
1) The NJL model:

(P. Zhuang, J. Hüfner and
S. P. Klevansky,

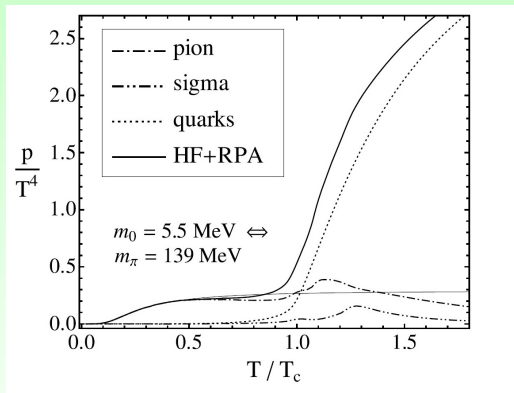
Nucl. Phys. A **576** (1994) 525)



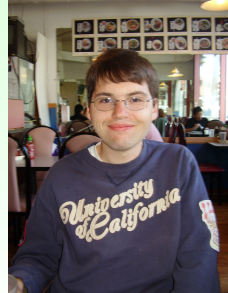
Hüfner, Klevansky, Witzler, D.B.,
Dossenheim (2007)



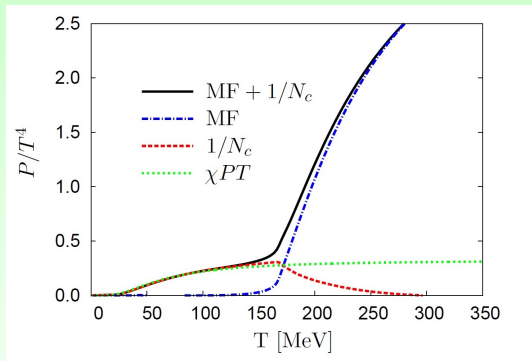
2) The PNJL model:



S. Roessner, T. Hell, C. Ratti and W. Weise,
Nucl. Phys. A **814** (2008) 118; [arXiv:0712.3152]



3) The nonlocal PNJL model:



(A. E. Radzhabov, D. Blaschke, M. Buballa and M. K. Volkov,
Phys. Rev. D **83** (2011) 116004 [arXiv:1012.0664 [hep-ph]])

The PNJL model

Everything begins with a Lagrangian:

$$\mathcal{L}_{PNJL} = \bar{q} (i\gamma^\mu D_\mu - m_0 - \gamma^0 \mu) q + \sum_{M=\sigma', \vec{\pi}'} G_M (\bar{q}\Gamma_M q)^2 - U(\Phi[A]; T),$$

where $D_\mu = \partial_\mu - iA_\mu$,

$$U(\Phi; T) = T^4 \left[-\frac{b_2(T)}{2} \Phi^2 - \frac{b_3}{3} \Phi^3 + \frac{b_4}{4} \Phi^4 \right],$$

$$b_2(T) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3,$$

a_0	a_1	a_2	b_3	b_4	T_0 [MeV]
6.75	-1.95	-7.44	0.75	7.5	208

C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D **73** (2006) 014019,

B.-J. Schaefer, J. M. Pawłowski and J. Wambach, Phys. Rev. D **76** (2007) 074023.



The PNJL model

The partition function in the PNJL model:

$$\mathcal{Z}_{PNJL}[T, V, \mu] = \int \mathcal{D}\bar{q}\mathcal{D}q \exp \left\{ \int_0^\beta d\tau \int_V d^3x \left[\bar{q} (i\gamma^\mu (\partial_\mu - iA_\mu) - m_0 - \gamma^0 \mu) q + \right. \right. \\ \left. \left. + G_S (\bar{q}\Gamma_{\sigma'}q)^2 + G_S (\bar{q}\vec{\Gamma}_{\pi'}q)^2 - U(\Phi[A]; T) \right] \right\}$$

The PNJL model

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$$\mathcal{Z}_{PNJL}[T, V, \mu] = \int \mathcal{D}\sigma' \mathcal{D}\vec{\pi}' \exp \left\{ - \left[\int_0^\beta d\tau \int_V d^3x \left(\frac{\sigma'^2 + \vec{\pi}'^2}{4G_S} + U(\Phi[A]; T) \right) \right] + \text{Tr} \ln [\beta S^{-1}[\sigma', \vec{\pi}']] \right\}$$

The PNJL model

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$$\mathcal{Z}_{PNJL}[T, V, \mu] = \int \mathcal{D}\sigma' \mathcal{D}\vec{\pi}' \exp \left\{ - \left[\int_0^\beta d\tau \int_V d^3x \left(\frac{\sigma'^2 + \vec{\pi}'^2}{4G_S} + U(\Phi[A]; T) \right) \right] + \text{Tr} \ln [\beta S^{-1}[\sigma', \vec{\pi}']] \right\}$$

$$\Omega_{FL}^{(2)}[T, V, \mu] = \frac{T}{V} \ln \left[\det \left(\frac{1}{2G_S} - \Pi_\sigma(q_0, \vec{q}) \right) \right]^{-\frac{1}{2}} + \frac{T}{V} \ln \left[\det \left(\frac{1}{2G_S} - \Pi_\pi(q_0, \vec{q}) \right) \right]^{-\frac{3}{2}}$$

The Generalized Beth-Uhlenbeck approach

$$P_M^{(2)} = -\Omega_M^{(2)}(T, \mu) = \frac{N_M}{2} \int \frac{d^3 q}{(2\pi)^3} \left(\int_0^{+\infty} \frac{d\omega}{\pi} \left[\omega + 2T \ln(1 - e^{-\beta\omega}) \right] \frac{d\Phi_M(\omega, \vec{q})}{d\omega} \right)$$

Mott-effect: bound state (delta function) \rightarrow resonance (spectral broadening)

$$\frac{d\Phi_M(\omega, \vec{q})}{d\omega} \rightarrow \begin{cases} \pi \delta(\omega - E_M), & T < T_{\text{Mott}} \\ \frac{d\phi_R(\omega, \vec{q})}{d\omega}, & T > T_{\text{Mott}} \end{cases}$$

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The Breit-Wigner ansatz for the phase shift derivative:

$$\frac{d\phi_R(s, T)}{ds} = A_R(s, T) = \frac{\pi}{\frac{\pi}{2} + \arctan\left(\frac{\vec{q}^2 + M_M^2}{M_M \Gamma_M}\right)} \frac{M_M \Gamma_M}{(s - M_M^2)^2 + (M_M \Gamma_M)^2}$$

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and the corresponding meson pressure ($\omega = \sqrt{\vec{q}^2 + s}$)

$$P_M(T) = \frac{N_M}{2} \int \frac{d^3q}{(2\pi)^3} \int_{4m^2}^{+\infty} ds \left(\omega + 2T \ln(1 - e^{-\beta\omega}) \right) D_M(s, T)$$

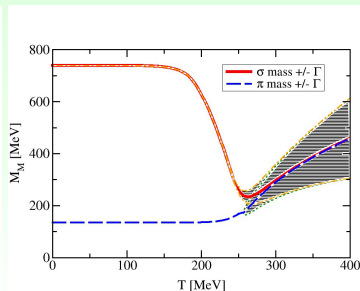
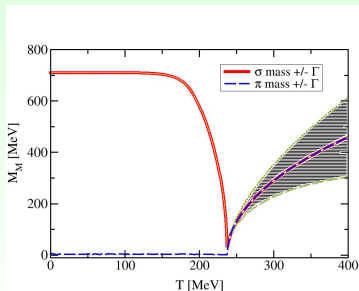
Meson masses with spectral broadening

Separating the real and imaginary part of $\Pi_M(q_0, \vec{0}) = 4N_c N_f I_1 - 2N_c N_f P_M I_2$ results in coupled Bethe-Salpeter equations:

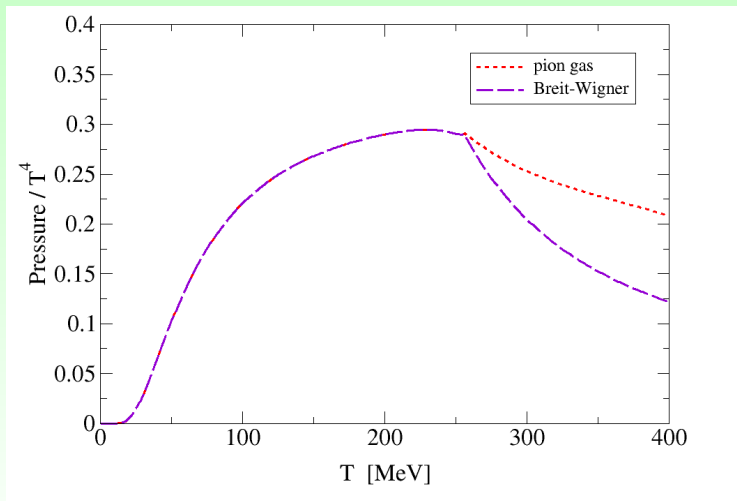
$$M_M^2 - \frac{1}{4}\Gamma_M^2 - \begin{pmatrix} 4m^2 \\ 0 \end{pmatrix} = \frac{\frac{1}{4N_c N_f G_S} - 2I_1}{|I_2(q_0 = M_M - i\frac{1}{2}\Gamma_M)|^2} \cdot \text{Re } I_2(q_0),$$

$$M_M \Gamma_M = \frac{\frac{1}{4N_c N_f G_S} - 2I_1}{|I_2(q_0 = M_M - i\frac{1}{2}\Gamma_M)|^2} \cdot \text{Im } I_2(q_0).$$

See, e.g., D. Blaschke, M. Jaminon, Yu.L. Kalinovsky, *et al.*, NPA **592** (1995) 561



Pion pressure: massive pion gas and Breit-Wigner ansatz



Levinson's theorem

Breit-Wigner ansatz $\rightarrow \phi_R$ is

$$\phi_R(s) = \frac{\pi}{\frac{\pi}{2} - \arctan\left(\frac{4m^2 - M_M^2}{M_M \Gamma_M}\right)} \left(\arctan\left[\frac{s - M_M^2}{M_M \Gamma_M}\right] - \arctan\left[\frac{4m^2 - M_M^2}{M_M \Gamma_M}\right] \right)$$

[it fulfills $\phi_R(s \rightarrow 4m^2) = 0$ and $\phi_R(s \rightarrow \infty) = \pi$]

violates Levinson's theorem which would require

$$\phi(s_{\text{threshold}} = 4m^2) - \phi(\infty) = n\pi = 0,$$

since the number of bound states below threshold vanishes ($n=0$) for $T > \tilde{T}_{\text{Mott}}$

\rightarrow Solution: phase shift corresponding to scattering states missing!

Two contributions to the scattering phase shift: $\Phi_M = \phi_R + \phi_{sc}$

$$\Phi_M = -\arctan\left(\frac{\text{Im}\tilde{I}_2}{\text{Re}\tilde{I}_2}\right) + \arctan\left(\frac{1 - 2G_S\tilde{I}_1}{2G_S|\tilde{I}_2|^2} \cdot \frac{\text{Im}\tilde{I}_2}{P_M + \frac{1 - 2G_S\tilde{I}_1}{2G_S|\tilde{I}_2|^2} \text{Re}\tilde{I}_2}\right).$$

(P. Zhuang, J. Hufner, S. P. Klevansky, Nucl. Phys. A576, 525-552 (1994).)

Zhuang formula - derivation

$$\Phi_M = \phi_R + \phi_{sc} .$$

Zhuang formula - derivation

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We can represent the total scattering phase shift Φ_M as

$$\Phi_M = \frac{i}{2} \ln \frac{1 - 2G_S \Pi_M(\omega + i\eta, \vec{q})}{1 - 2G_S \Pi_M(\omega - i\eta, \vec{q})} .$$

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Using

$$\Pi_M(q_0, \vec{0}) = 4N_c N_f I_1 - 2N_c N_f P_M I_2 = \tilde{I}_1 - P_M \tilde{I}_2,$$

and

$$\frac{i}{2} \ln \left(\frac{1 - ix}{1 + ix} \right) = \arctan x$$

we show that

$$\Phi_M = -\arctan \left[\frac{2G_S P_M \text{Im} \tilde{I}_2}{1 - 2G_S \tilde{I}_1 + 2G_S P_M \text{Re} \tilde{I}_2} \right] .$$

Zhuang formula - derivation

$$\Phi_M = -\arctan \left[\frac{2G_S P_M \text{Im} \tilde{I}_2}{1 - 2G_S \tilde{I}_1 + 2G_S P_M \text{Re} \tilde{I}_2} \right].$$

Zhuang formula - derivation

$$\Phi_M = -\arctan \left[\frac{2G_S P_M \text{Im} \tilde{I}_2}{1 - 2G_S \tilde{I}_1 + 2G_S P_M \text{Re} \tilde{I}_2} \right].$$

(several steps more)

$$\Phi_M = -\arctan \left[\frac{\frac{\text{Im} \tilde{I}_2}{\text{Re} \tilde{I}_2} - \frac{1 - 2G_S \tilde{I}_1}{2G_S |\tilde{I}_2|^2} \cdot \frac{\text{Im} \tilde{I}_2}{P_M + \frac{1 - 2G_S \tilde{I}_1}{2G_S |\tilde{I}_2|^2} \text{Re} \tilde{I}_2}}{1 + \frac{1 - 2G_S \tilde{I}_1}{2G_S |\tilde{I}_2|^2} \cdot \frac{\text{Im} \tilde{I}_2^2}{P_M \text{Re} \tilde{I}_2 + \frac{1 - 2G_S \tilde{I}_1}{2G_S |\tilde{I}_2|^2} \text{Re} \tilde{I}_2^2}} \right].$$

Zhuang formula - derivation

$$\Phi_M = -\arctan \left[\frac{2G_S P_M \text{Im} \tilde{I}_2}{1 - 2G_S \tilde{I}_1 + 2G_S P_M \text{Re} \tilde{I}_2} \right].$$

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Using

$$-(\arctan \alpha \pm \arctan \beta) = -\arctan \left[\frac{\alpha \pm \beta}{1 \mp \alpha \beta} \right]$$

Zhuang formula - derivation

$$\Phi_M = -\arctan \left[\frac{2G_S P_M \operatorname{Im} \tilde{I}_2}{1 - 2G_S \tilde{I}_1 + 2G_S P_M \operatorname{Re} \tilde{I}_2} \right].$$

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$$\Phi_M = -\arctan \left[\frac{\frac{\operatorname{Im} \tilde{I}_2}{\operatorname{Re} \tilde{I}_2} - \frac{1 - 2G_S \tilde{I}_1}{2G_S |\tilde{I}_2|^2} \cdot \frac{\operatorname{Im} \tilde{I}_2}{P_M + \frac{1 - 2G_S \tilde{I}_1}{2G_S |\tilde{I}_2|^2} \operatorname{Re} \tilde{I}_2}}{1 + \frac{1 - 2G_S \tilde{I}_1}{2G_S |\tilde{I}_2|^2} \cdot \frac{\operatorname{Im} \tilde{I}_2^2}{P_M \operatorname{Re} \tilde{I}_2 + \frac{1 - 2G_S \tilde{I}_1}{2G_S |\tilde{I}_2|^2} \operatorname{Re} \tilde{I}_2^2}} \right].$$

Using

$$-(\arctan \alpha \pm \arctan \beta) = -\arctan \left[\frac{\alpha \pm \beta}{1 \mp \alpha\beta} \right]$$

we get

$$\Phi_M = -\arctan \left(\frac{\operatorname{Im} \tilde{I}_2}{\operatorname{Re} \tilde{I}_2} \right) + \arctan \left(\frac{1 - 2G_S \tilde{I}_1}{2G_S |\tilde{I}_2|^2} \cdot \frac{\operatorname{Im} \tilde{I}_2}{P_M + \frac{1 - 2G_S \tilde{I}_1}{2G_S |\tilde{I}_2|^2} \operatorname{Re} \tilde{I}_2} \right).$$

Our approach

Now then

$$\Phi_M = \phi_{sc} + \phi_R = -\arctan\left(\frac{\text{Im}\tilde{I}_2}{\text{Re}\tilde{I}_2}\right) + \arctan\left(\frac{1 - 2G_S\tilde{I}_1}{2G_S|\tilde{I}_2|^2} \cdot \frac{\text{Im}\tilde{I}_2}{P_M + \frac{1 - 2G_S\tilde{I}_1}{2G_S|\tilde{I}_2|^2}\text{Re}\tilde{I}_2}\right)$$

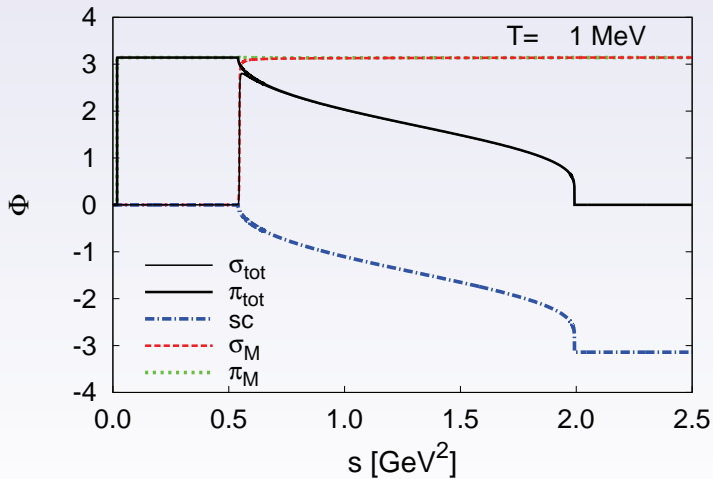
Now then

$$\Phi_M = \phi_{sc} + \phi_R = -\arctan\left(\frac{\text{Im}\tilde{I}_2}{\text{Re}\tilde{I}_2}\right) + \arctan\left(\frac{1 - 2G_S\tilde{I}_1}{2G_S|\tilde{I}_2|^2} \cdot \frac{\text{Im}\tilde{I}_2}{P_M + \frac{1-2G_S\tilde{I}_1}{2G_S|\tilde{I}_2|^2}\text{Re}\tilde{I}_2}\right)$$

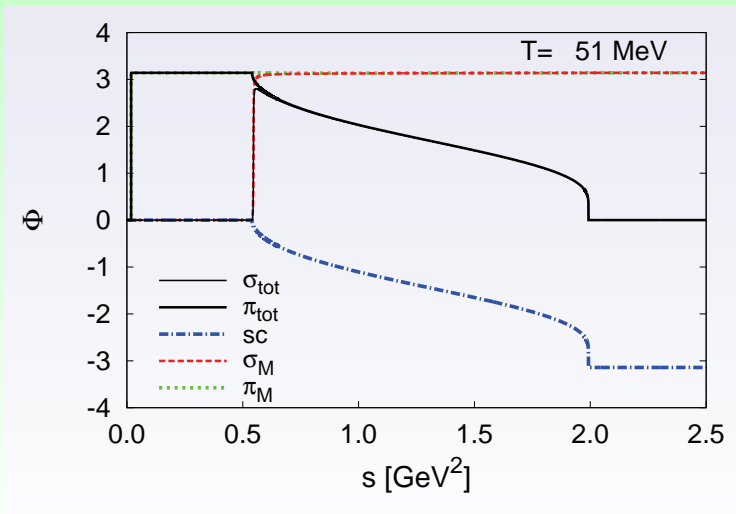
Our analysis is a combined approach:

$$D_M(s) = \frac{1}{\pi} \frac{d\phi_M(s)}{ds} = \begin{cases} \delta(s - M_M^2) + \frac{1}{\pi} \frac{d}{ds} \phi_{sc}(s) , & T < T_{\text{Mott}} , \\ \frac{a_R}{\pi} \frac{\Gamma_M M_M}{(s - M_M^2)^2 + \Gamma_M^2 M_M^2} + \frac{1}{\pi} \frac{d}{ds} \phi_{sc}(s) , & T > T_{\text{Mott}} . \end{cases}$$

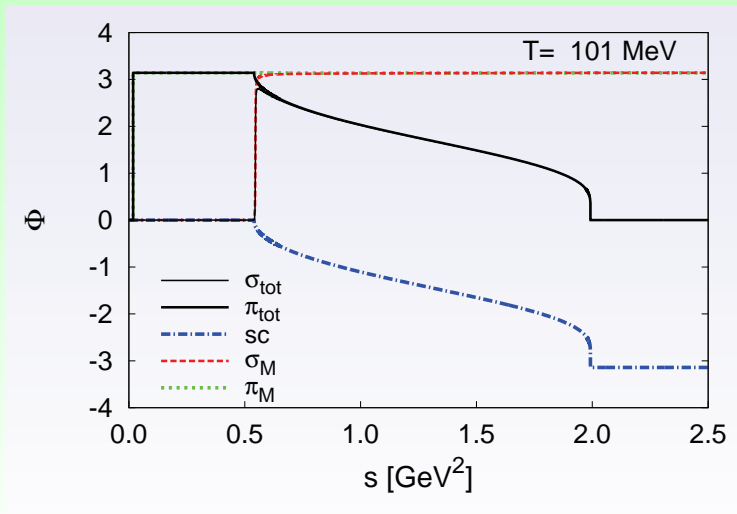
Phase shifts



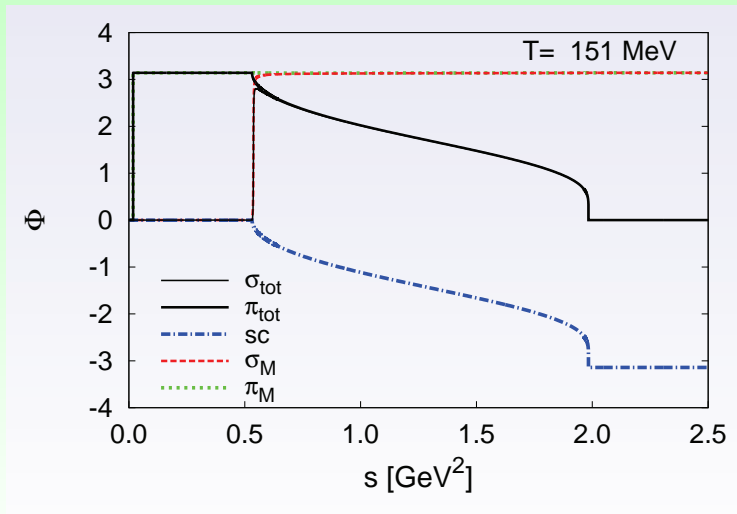
Phase shifts



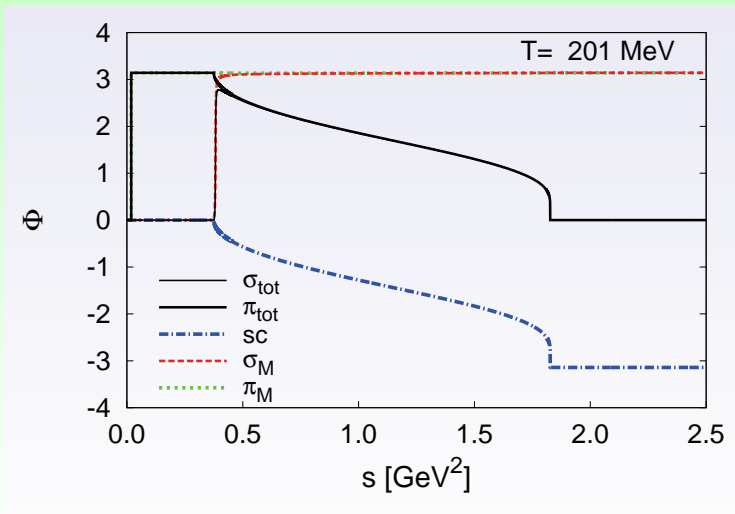
Phase shifts



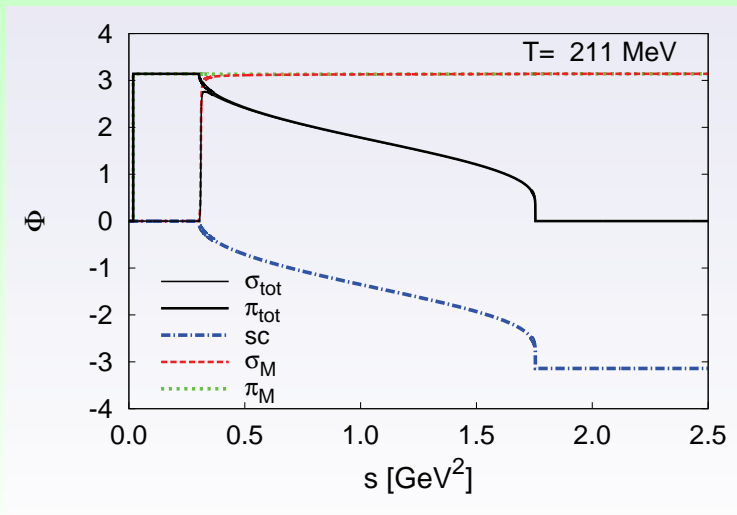
Phase shifts



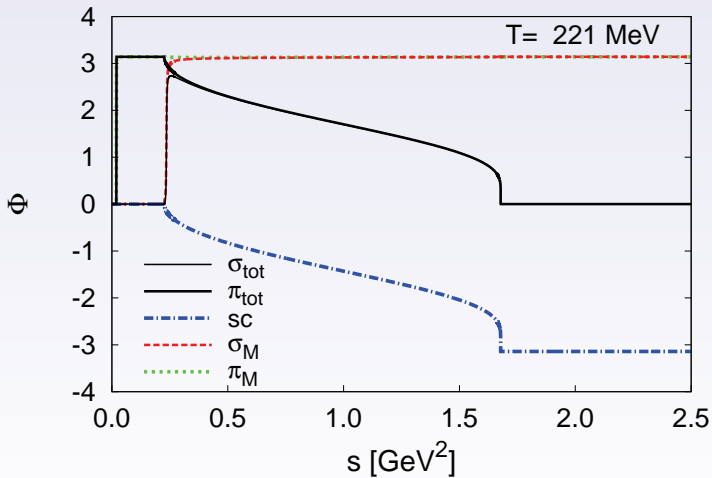
Phase shifts



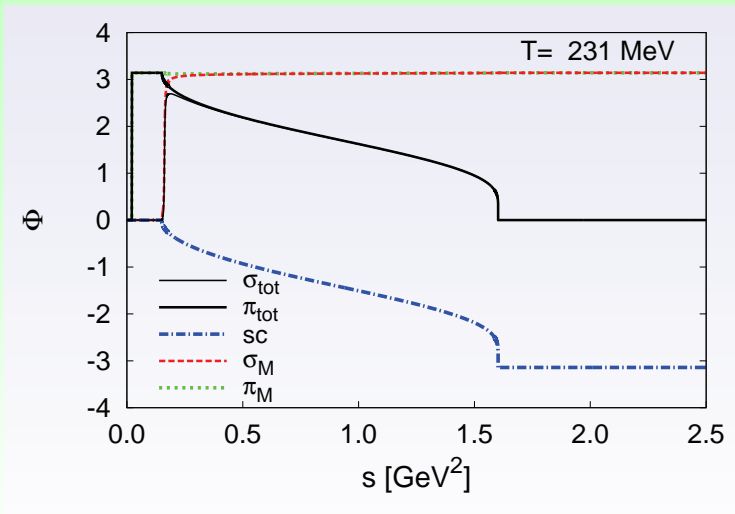
Phase shifts



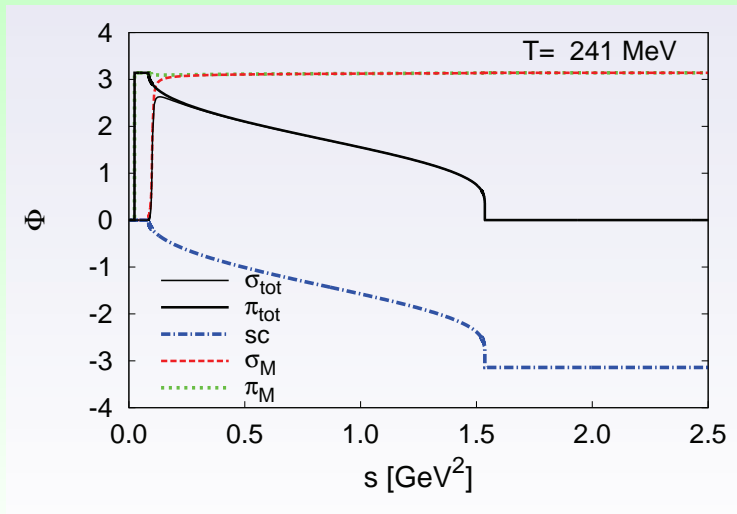
Phase shifts



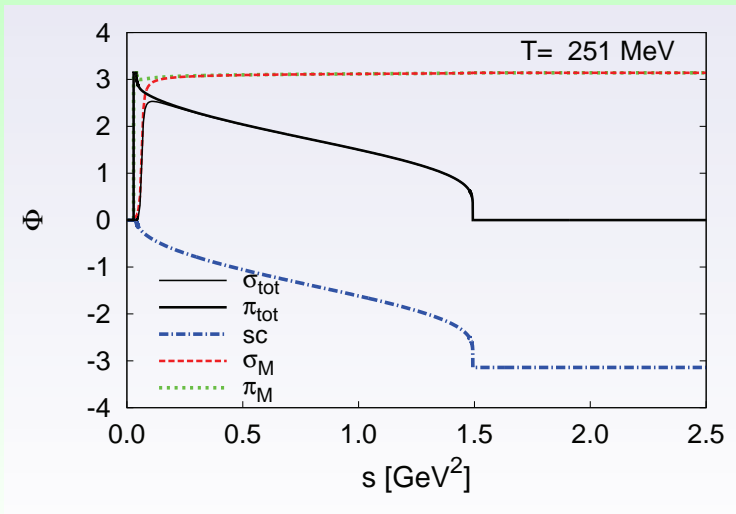
Phase shifts



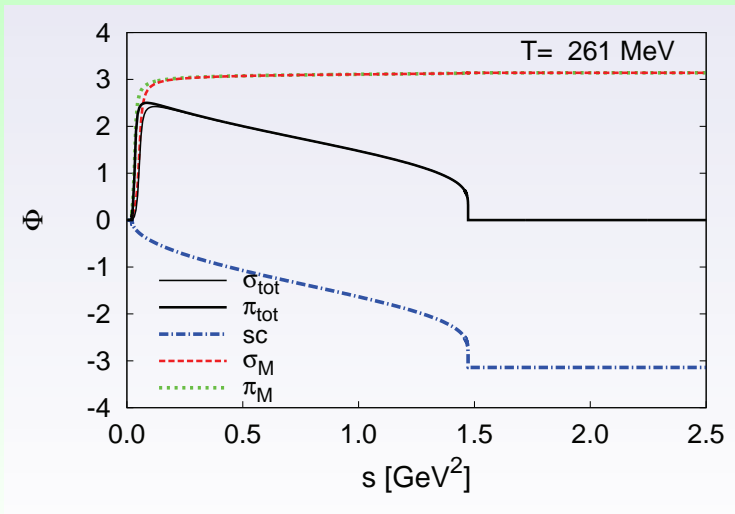
Phase shifts



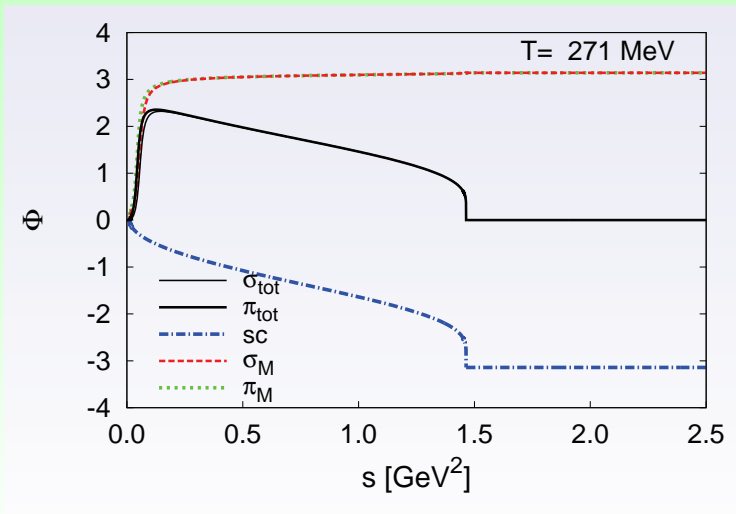
Phase shifts



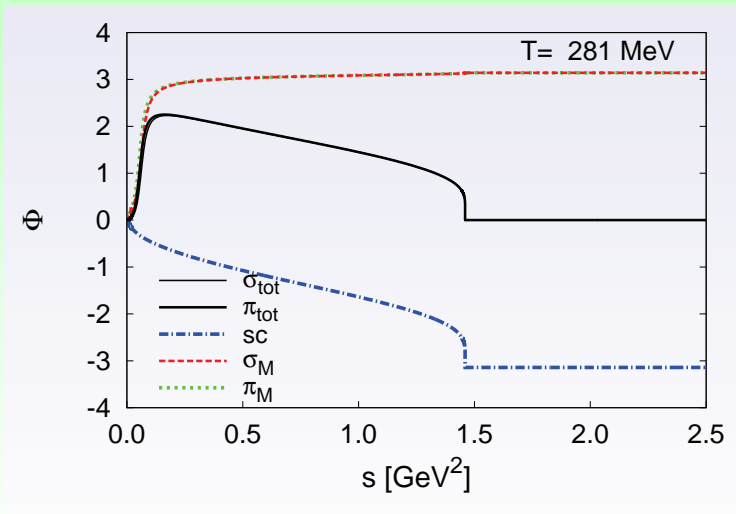
Phase shifts



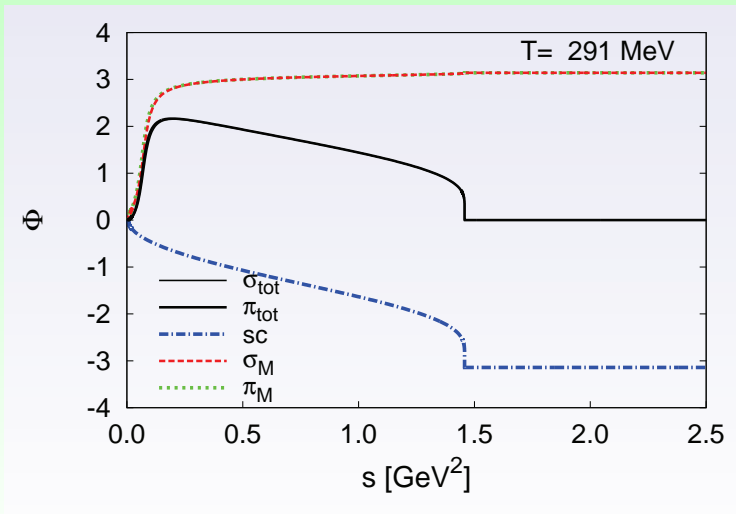
Phase shifts



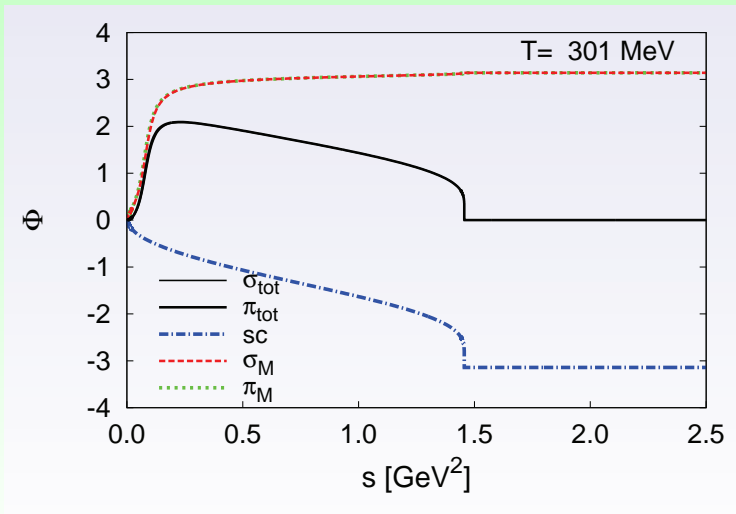
Phase shifts



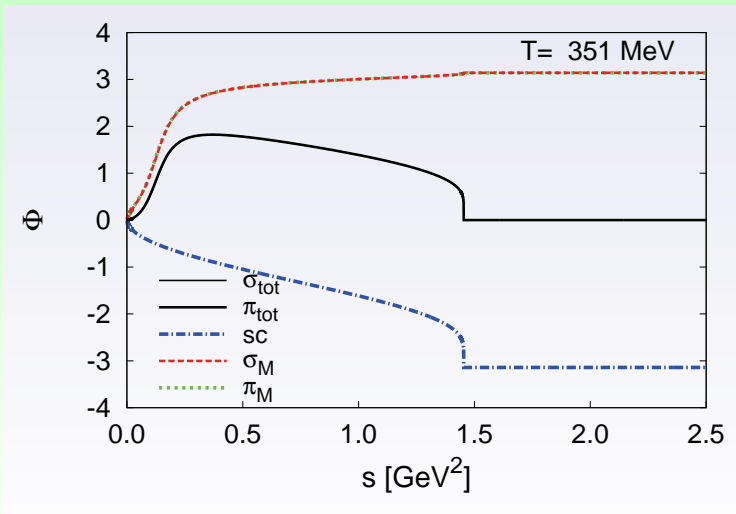
Phase shifts



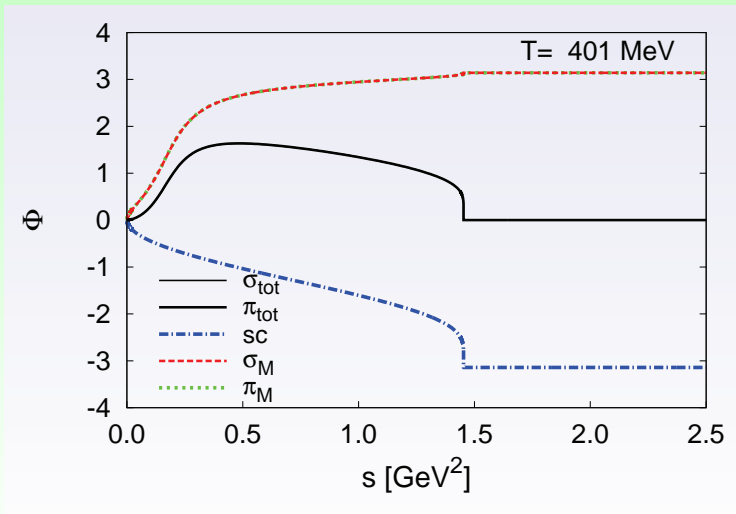
Phase shifts



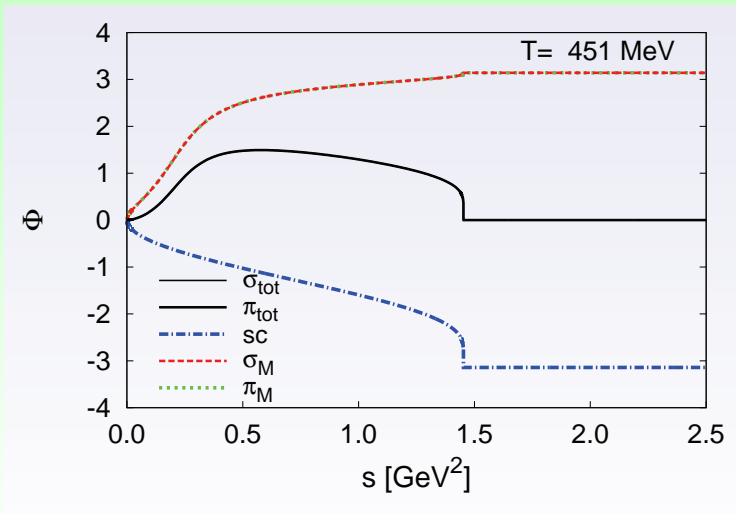
Phase shifts



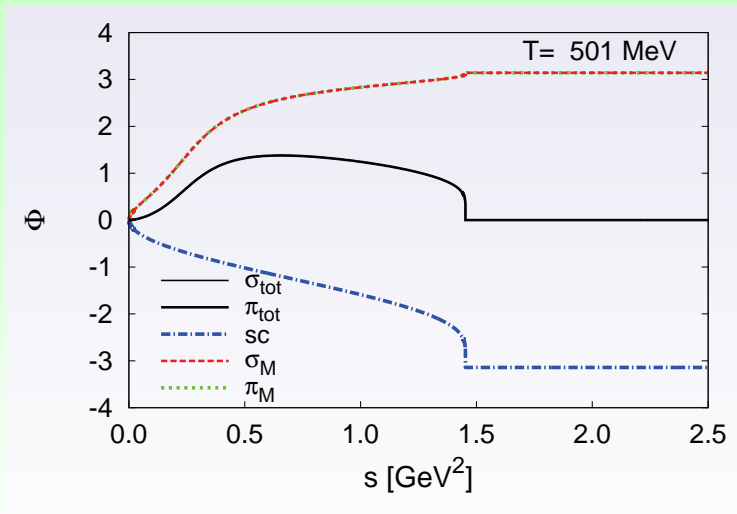
Phase shifts



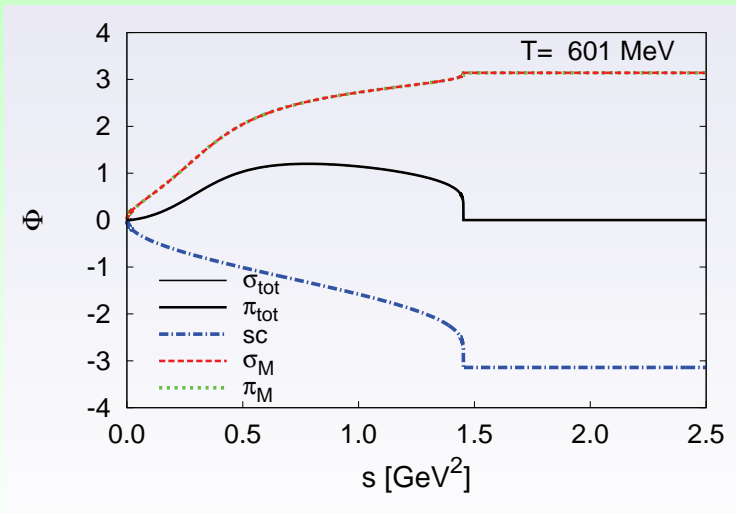
Phase shifts



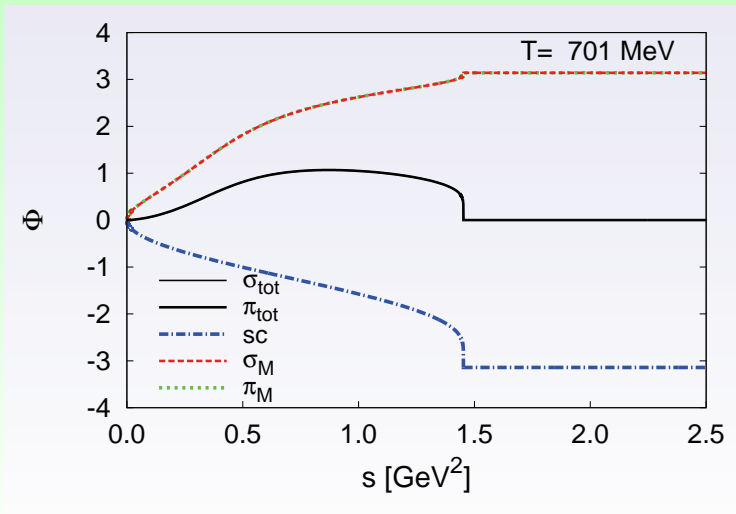
Phase shifts



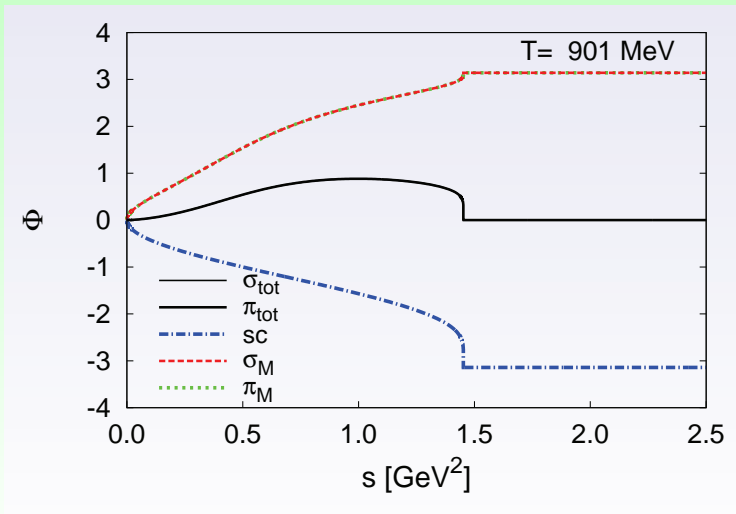
Phase shifts



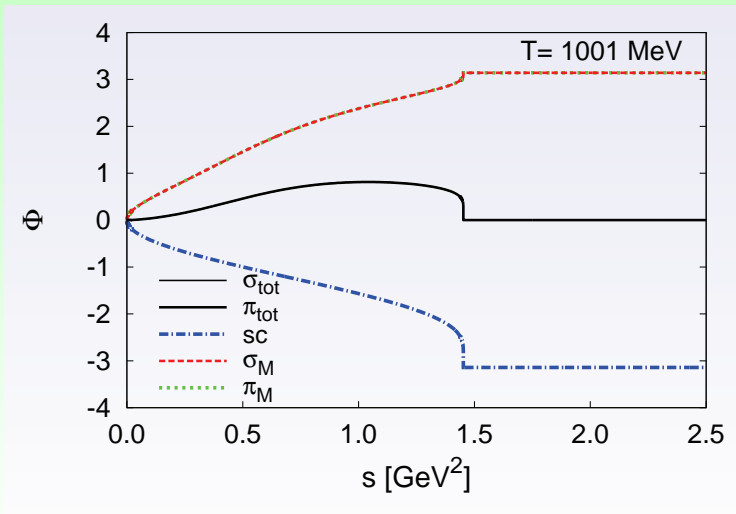
Phase shifts



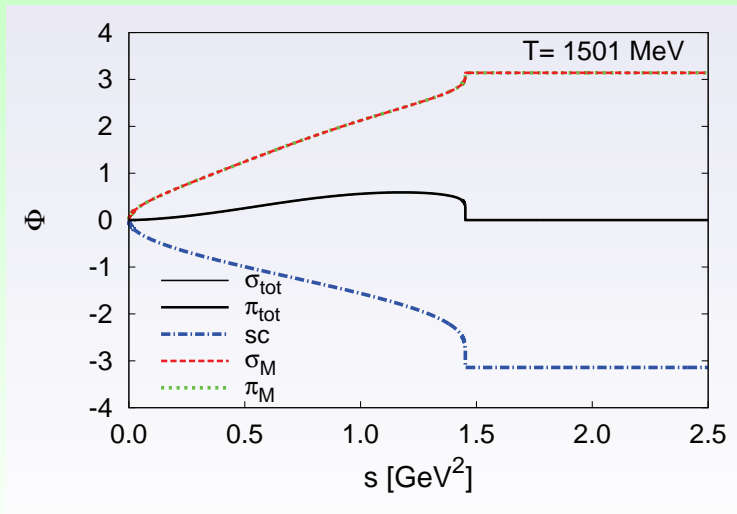
Phase shifts



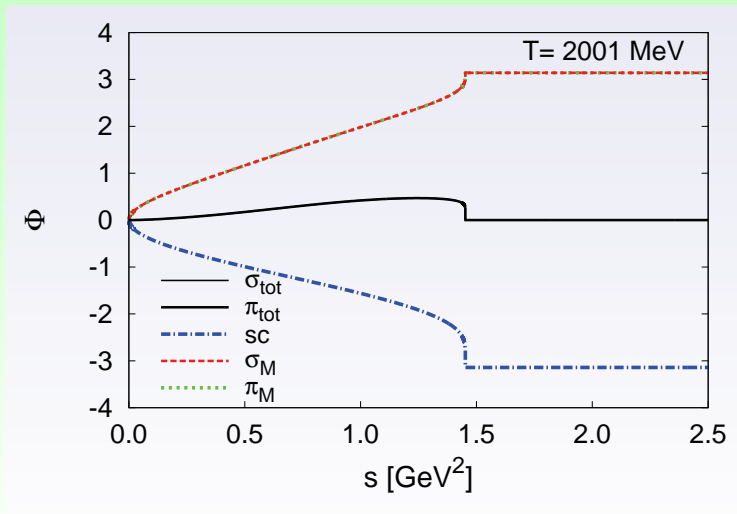
Phase shifts



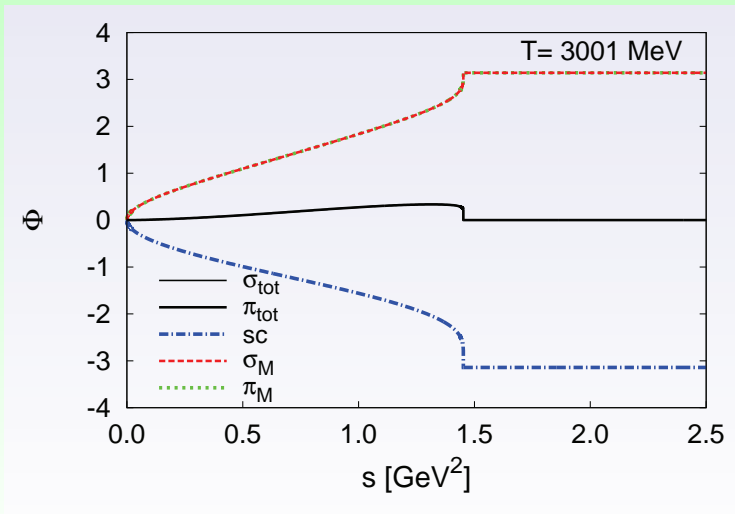
Phase shifts



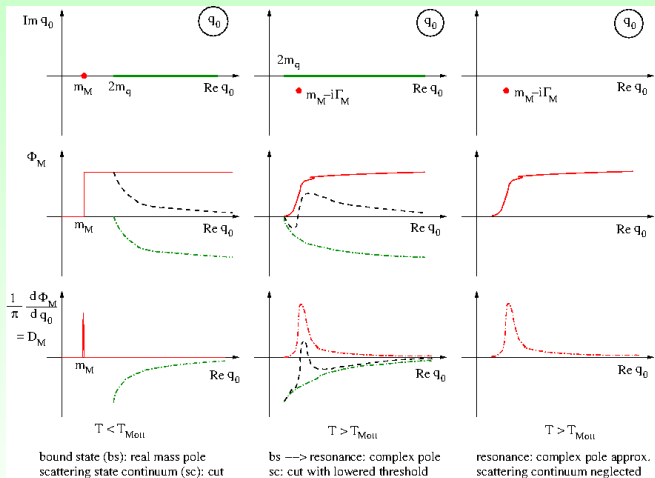
Phase shifts



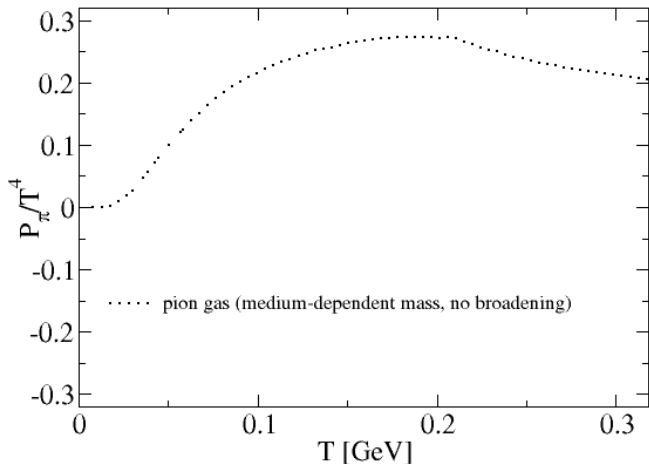
Phase shifts



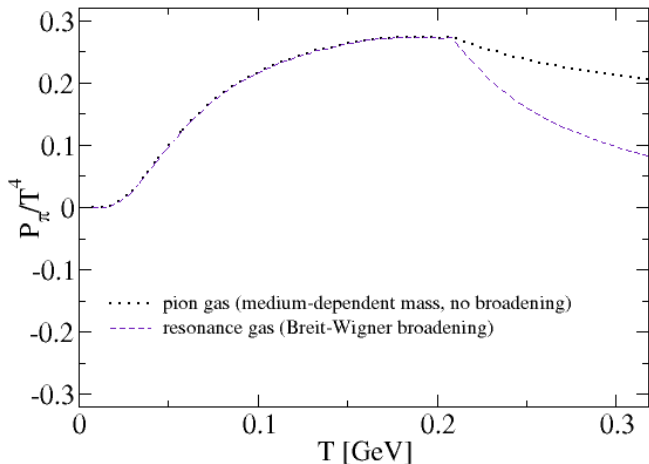
Summary: Levinson's Theorem & analytical properties



Role of scattering continuum (Levinson theorem!) for pressure:

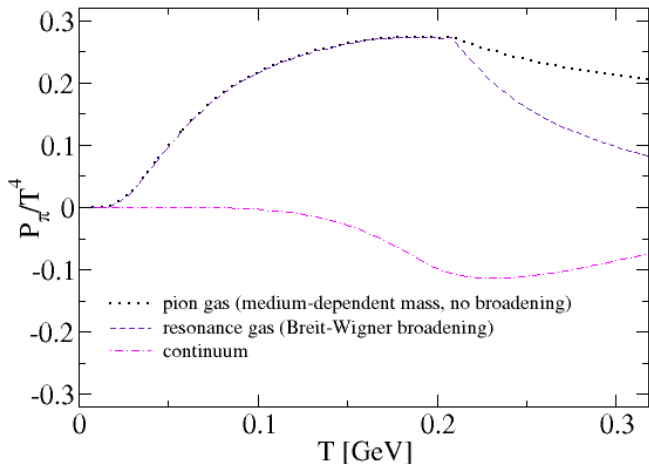


Role of scattering continuum (Levinson theorem!) for pressure:



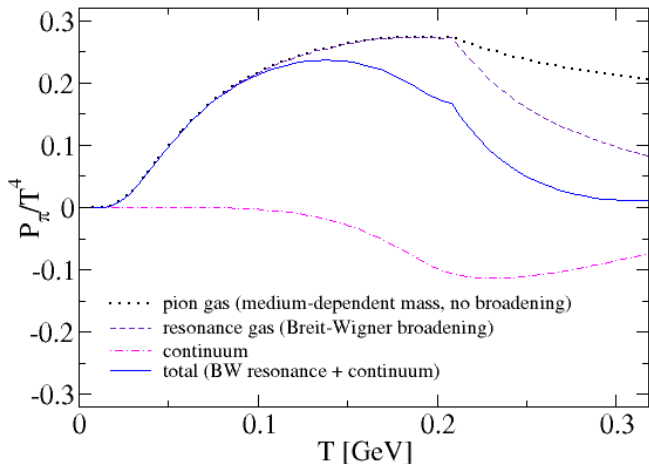
Pion pressure

Role of scattering continuum (Levinson theorem!) for pressure:

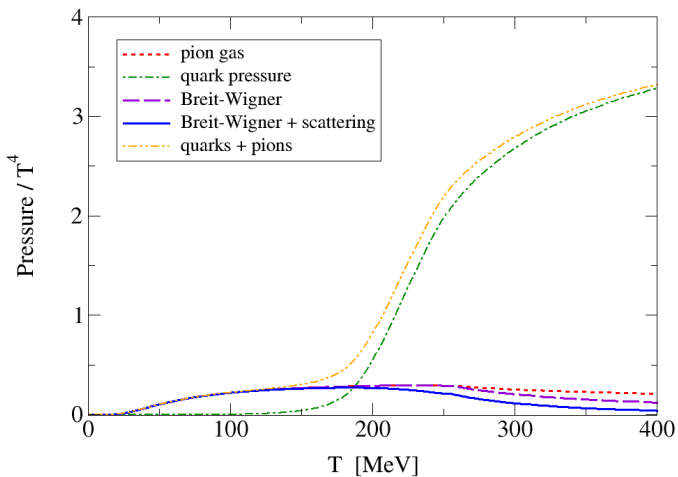


Pion pressure

Role of scattering continuum (Levinson theorem!) for pressure:



Quark + pion pressure



A fantastic result !!



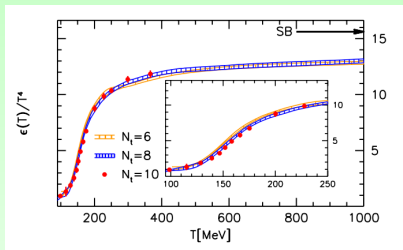
A fantastic result !!



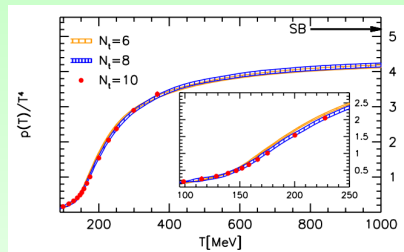
Conclusions - part II

- PNJL model: suitable for describing χ SB and restoration at finite temperature, it describes pions as $q\bar{q}$ bound states and pseudo-Goldstone bosons $\rightarrow m(T), M_M(T), \Gamma_M(T)$
- pressure $P(T)$ for quark mean-field: suppression of quarks for $T < T_c$, correct SB limit
- Gaussian fluctuations in $\sigma, \vec{\pi}$: Generalized Beth-Uhlenbeck
- resonance approximation for pionic mode above T_{Mott} **violates** Levinson's theorem!
- an analytic formula for the continuum states' contribution to the scattering phase shift together with the Breit-Wigner ansatz for the resonance
- resulting phase shift obeys the Levinson theorem
 \rightarrow pressure reduction (ideally to zero) for $T > T_{\text{Mott}}$
- **outlook**: semi-microscopic approach to implement Mott effect for hadrons (here: only pions) consistent with Levinson's theorem into hadron resonance gas (HRG) models

Lattice QCD: Theoretical laboratory of QCD



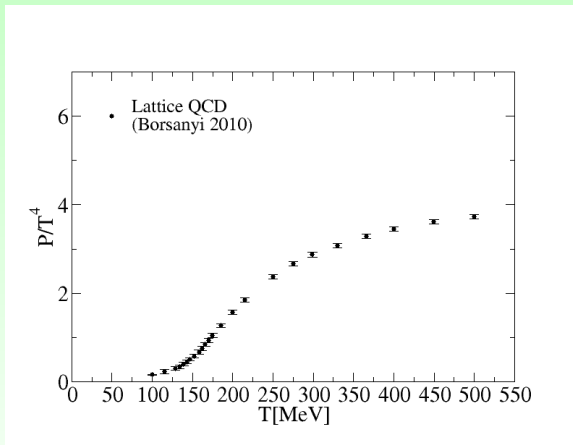
The energy density normalized by T^4 as a function of the temperature on $N_t = 6, 8$ and 10 lattices.



The pressure normalized by T^4 as a function of the temperature on $N_t = 6, 8$ and 10 lattices.

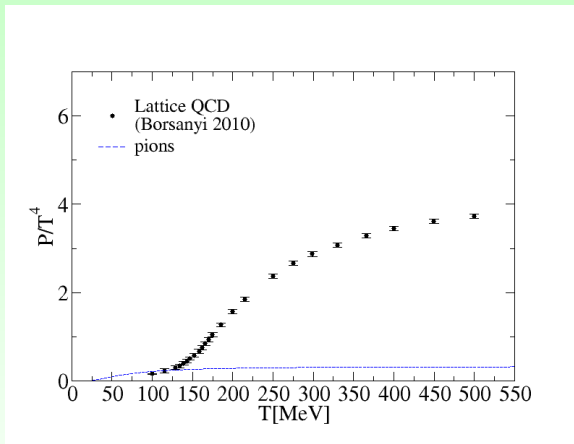
S. Borsanyi *et al.* "The QCD equation of state with dynamical quarks," JHEP **1011**, 077 (2010)

Hagedorn resonance gas: comparison with Lattice QCD



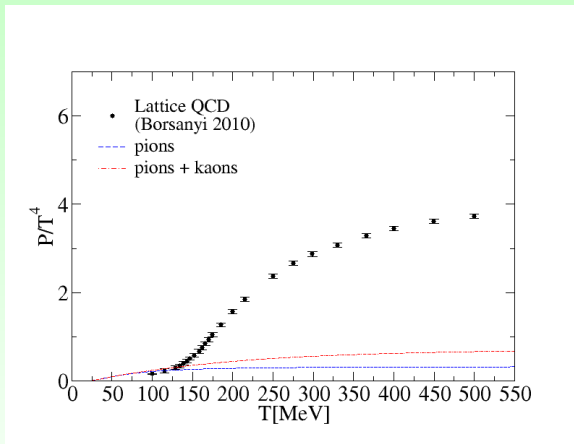
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JHEP **1011**, 077 (2010)

Hagedorn resonance gas: comparison with Lattice QCD



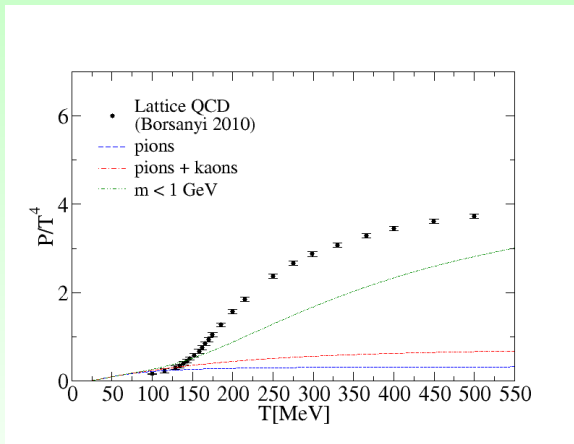
$$P_{\pi}(T) = 3T \int \frac{d^3 p}{(2\pi)^3} \ln \left[1 - e^{-\sqrt{p^2 + m_{\pi}^2}/T} \right]$$

Hagedorn resonance gas: comparison with Lattice QCD



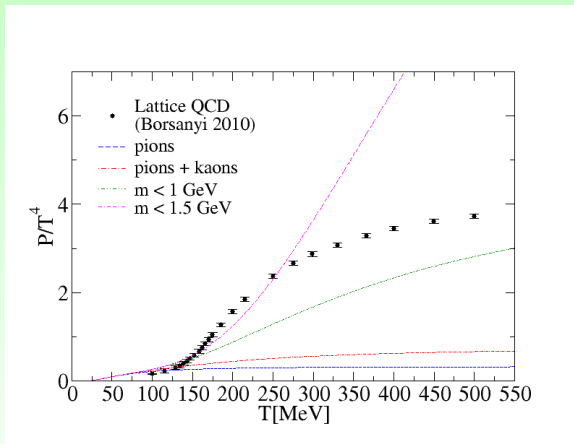
$$P_{\pi+K}(T) = \sum_{i=\pi,K} d_i T \int \frac{d^3 p}{(2\pi)^3} \ln \left[1 - e^{-\sqrt{p^2 + m_i^2}/T} \right]$$

Hagedorn resonance gas: comparison with Lattice QCD



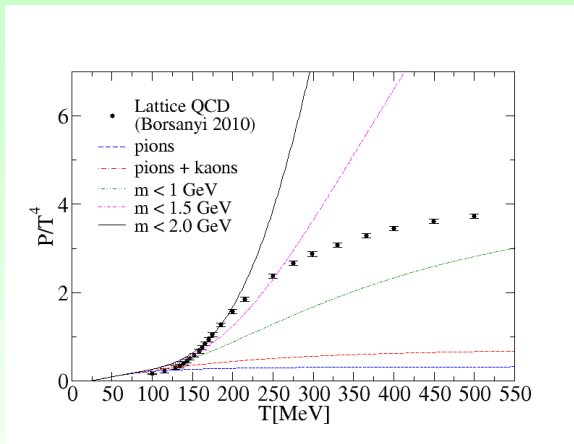
$$P_{\text{HRG}}(T) = \sum_{i, m_i < 1\text{GeV}} \delta_i d_i T \int \frac{d^3 p}{(2\pi)^3} \ln \left\{ 1 + \delta_i e^{-\sqrt{p^2 + m_i^2}/T} \right\}$$

Hagedorn resonance gas: comparison with Lattice QCD



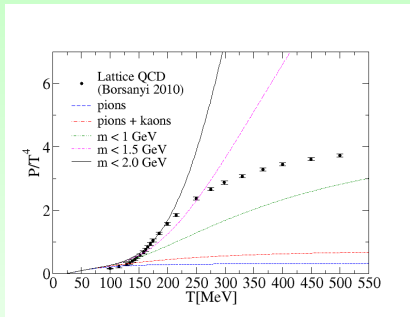
$$P_{\text{HRG}}(T) = \sum_{i, m_i < 1.5 \text{ GeV}} \delta_i d_i T \int \frac{d^3 p}{(2\pi)^3} \ln \left\{ 1 + \delta_i e^{-\sqrt{p^2 + m_i^2}/T} \right\}$$

Hagedorn resonance gas: comparison with Lattice QCD



$$P_{\text{HRG}}(T) = \sum_{i, m_i < 2\text{GeV}} \delta_i d_i T \int \frac{d^3 p}{(2\pi)^3} \ln \left\{ 1 + \delta_i e^{-\sqrt{p^2 + m_i^2}/T} \right\}$$

Hagedorn resonance gas: comparison with Lattice QCD



$$P_{\text{HRG}}(T) = \sum_{i, m_i < 2\text{GeV}} \delta_i d_i T \int \frac{d^3 p}{(2\pi)^3} \ln \left\{ 1 + \delta_i e^{-\sqrt{p^2 + m_i^2}/T} \right\}$$

Courtesy: M. Naskręć (UWr)

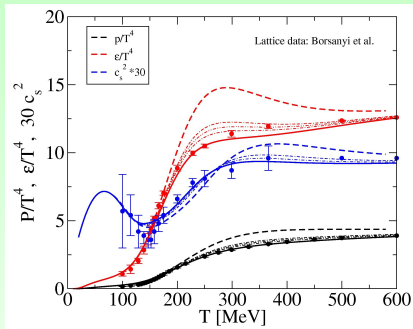
The energy density per degree of freedom with the mass M

$$\begin{aligned}\varepsilon(T, \mu_B, \mu_S) &= \sum_{i: m_i < m_0} g_i \varepsilon_i(T, \mu_i; m_i) \\ &+ \sum_{i: m_i \geq m_0} g_i \int_{m_0^2}^{\infty} d(M^2) A(M, m_i) \varepsilon_i(T, \mu_i; M),\end{aligned}$$

Spectral function

$$\begin{aligned}A(M, m) &= N_M \frac{\Gamma \cdot m}{(M^2 - m^2)^2 + \Gamma^2 \cdot m^2}, \\ \Gamma(T) &= C_\Gamma \left(\frac{m}{T_H}\right)^{N_m} \left(\frac{T}{T_H}\right)^{N_T} \exp\left(\frac{m}{T_H}\right)\end{aligned}$$

Hagedorn resonance gas: hadrons with finite widths



$$P(T) = T \int_0^T dT' \frac{\varepsilon(T')}{T'^2}.$$

N_m in the range from $N_m = 2.5$ (dashed) to $N_m = 3.0$ (solid).

$$C_\Gamma = 10^{-4}$$

$$N_T = 6.5$$

$$T_H = 165 \text{ MeV}$$

$$\Gamma(T) = C_\Gamma \left(\frac{m}{T_H} \right)^{N_m} \left(\frac{T}{T_H} \right)^{N_T} \exp \left(\frac{m}{T_H} \right)$$

D.B. & K. Bugaev, *Fizika B* **13**, 491 (2004); *PPNP* **53**, 197 (2004)

State-dependent hadron resonance width

$$A_i(M, m_i) = N_M \frac{\Gamma_i \cdot m_i}{(M^2 - m_i^2)^2 + \Gamma_i^2 \cdot m_i^2},$$

$$\Gamma_i(T) = \tau_{\text{coll},i}^{-1}(T) = \sum_j \lambda \langle r_i^2 \rangle_T \langle r_j^2 \rangle_T n_j(T)$$

D. B., J. Berdermann, J. Cleymans, K. Redlich, PPN 8, 811 (2011)
[arXiv:1102.2908]

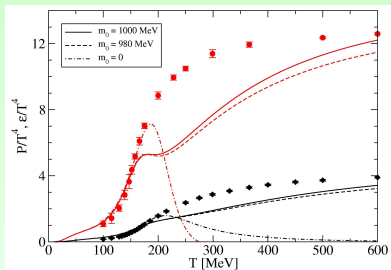
For pions (mesons)

$$r_\pi^2(T, \mu) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_T|^{-1}; \quad \langle \bar{q}q \rangle_T \implies \text{Talk by J. Jankowski}$$

For nucleons (baryons)

$$r_N^2(T, \mu) = r_0^2 + r_\pi^2(T, \mu); \quad r_0 = 0.45 \text{fm pion cloud.}$$

Mott-Hagedorn resonance gas



Quarks and gluons are missing!

Mott-Hagedorn resonance

gas: Pressure and energy density for three values of the mass threshold

$m_0 = 1.0$ GeV (solid lines)

$m_0 = 0.98$ GeV (dashed lines)

and

$m_0 = 0$ (dash-dotted lines)

Systematic expansion of the pressure as the thermodynamical potential in the grand canonical ensemble for a chiral quark model of the PNJL type beyond its mean field description $P_{\text{PNJL,MF}}(T)$ by including perturbative corrections

$$P(T) = P_{\text{MHRG}}(T) + P_{\text{PNJL,MF}}(T) + P_2(T) ,$$

$$P_{\text{MHRG}}(T) = \sum_i \delta_i d_i T \int \frac{d^3 p}{(2\pi)^3} \int dM A_i(M, m_i) T \ln \left\{ 1 + \delta_i e^{-[\sqrt{p^2 + M^2} - \mu_i]/T} \right\} ,$$

Quark and gluon contributions

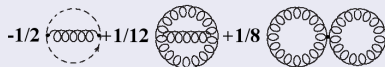
$$P_2(T) = P_2^{\text{quark}}(T) + P_2^{\text{gluon}}(T)$$

Quark and gluon contributions

$P_2^{\text{quark}}(T)$



$P_2^{\text{gluon}}(T)$



Total perturbative QCD correction

$$P_2 = -\frac{8}{\pi} \alpha_s T^4 (I_\Lambda^+ +$$

$$\frac{3}{\pi^2} ((I_\Lambda^+)^2 + (I_\Lambda^-)^2))$$

$$\xrightarrow{\Lambda/T \rightarrow 0} -\frac{3\pi}{2} \alpha_s T^4$$

where

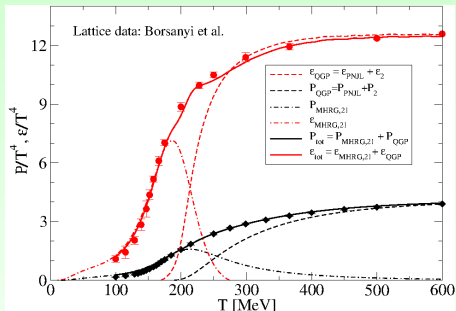
$$I_\Lambda^\pm = \int_{\Lambda/T}^{\infty} \frac{dx x}{e^x \pm 1}$$

· Energy corrections

$$\varepsilon_2(T) = T \frac{dP_2(T)}{dT} - P_2(T) .$$

Quarks, gluons and hadron resonances

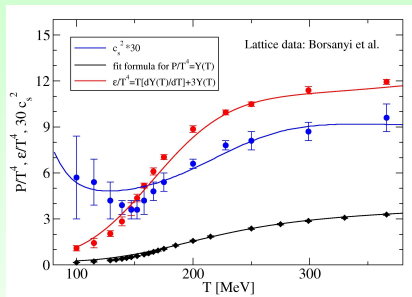
$$P_{\text{MHRG}}(T) = \sum_i \delta_i d_i \int \frac{d^3 p}{(2\pi)^3} \int dM A_i(M, m_i) T \ln \left\{ 1 + \delta_i e^{-[\sqrt{p^2 + M^2} - \mu_i]/T} \right\},$$



- Quark-gluon plasma contributions are described within the improved PNJL model with α_s corrections.
- Heavy hadrons are described within the resonance gas with finite width exhibiting a Mott effect at the coincident chiral and deconfinement transitions.

L. Turko, D. Blaschke, D. Prorok, J. Berdermann, J. Phys. Conf. Ser. **455**, 012056 (2013)

Quarks, gluons and hadron resonances II

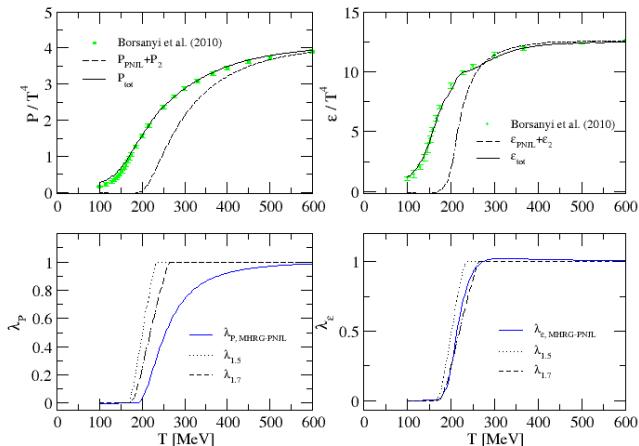


- Contribution restricted to the region around the chiral/deconfinement transition 170-250 MeV
- Fit formula for the pressure

$$P = aT^4 + bT^{4.4} \tanh(cT - d),$$

$$a = 1.0724, \quad b = 0.2254, \\ c = 0.00943, \quad d = 1.6287$$

Application: Parton fraction in the EoS \rightarrow HIC Simulations



Comparison with: M. Nahrgang, J. Aichelin, P.-B. Gossiaux, K. Werner, *Influence of hadronic bound states above T_c on heavy-quark observables* ..., [arxiv:1305.6544]

- An effective model description of QCD thermodynamics at finite temperatures which properly accounts for the fact that in the QCD transition region it is dominated by a tower of hadronic resonances.
- A generalization of the Hagedorn resonance gas thermodynamics which includes the finite lifetime of hadronic resonances in a hot and dense medium

To do

- Join hadron resonance gas with quark-gluon model.
- Calculate kurtosis and compare with lattice QCD.
- Spectral function for all low-lying hadrons from microphysics (PNJL model ...).

Visit (the University of) Wroclaw !



Thank you for invitation and collaboration !

