F. Becattini
University of Florence

Do we understand statistical mechanics?

OUTLINE

- Thermal features in strong interactions in high energy physics: introduction and some selected results
- Thermalization in isolated quantum systems and eigenstate thermalization
- Exclusive rates in e+e-

Kielce, December 14th 2013

Prologue

It has been known for a long time that strong interactions exhibit statistical-thermal features in high energy collisions

1950 : Statistical model of particle production (Fermi)
60's : exponential pT spectra in pp collisions with limited slope
1965 : Hagedorn's bootstrap model

90's: observation of thermal-like abundances in both elementary and heavy ion collisions

> 90's: observation of *local* thermal behaviour (hydrodynamics) in relativistic heavy ion collisions and possibly in pp and p-Pb collisions

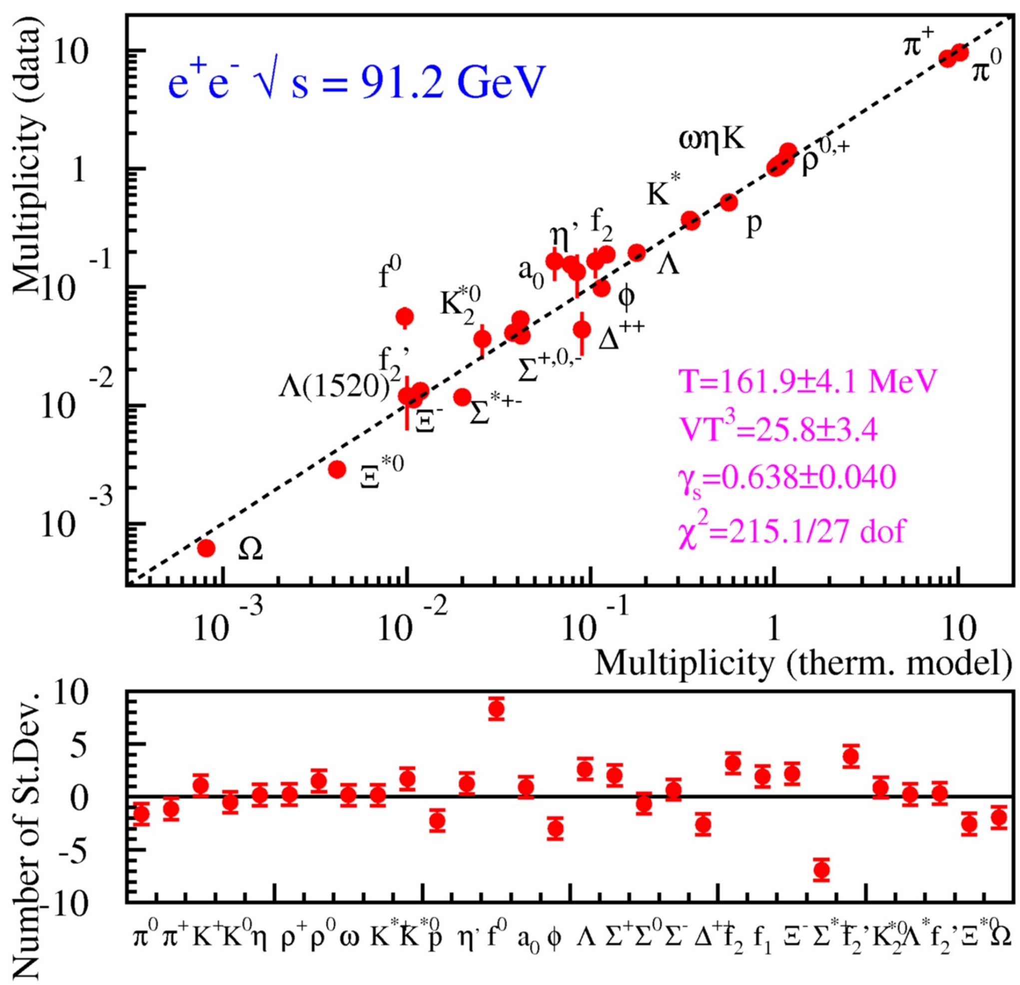
It should be pointed out that thermal features emerge ONLY considering infrared observables, i.e. determined in the low-energy regime of QCD

WHY?

SOME SELECTED RESULTS

e+ e- collisions

F. B., P. Castorina, J. Manninen, H. Satz, Eur. Phys. J. C 56 (2008) 493



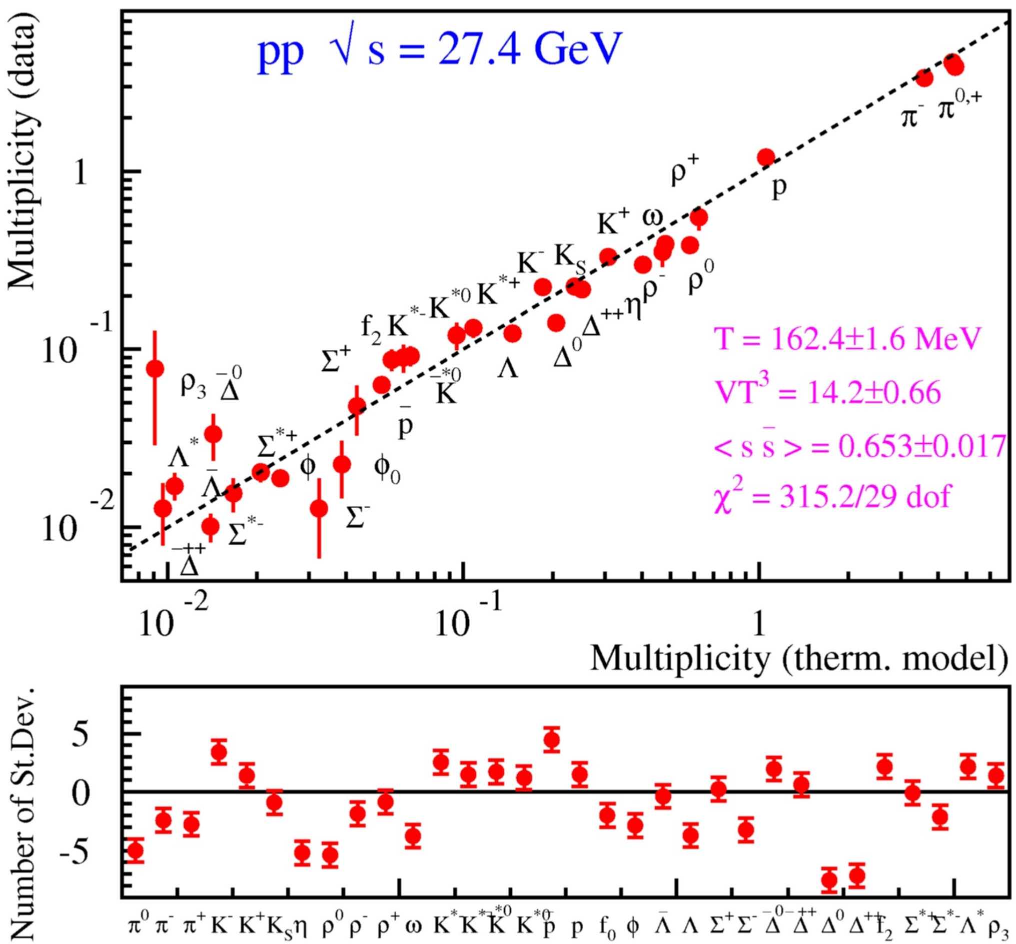
Particle		Experiment (E)	Model (M)	Residual	$(M-E)/E$ [%]
D^0	[28]	0.559 ± 0.022	0.5406	-0.83	-3.2
D^+	[28]	0.238 ± 0.024	0.2235	-0.60	-6.1
D^{*+}	[28–30]	0.2377± 0.0098	0.2279	-1.00	-4.1
D^{*0}	[31]	0.218 ± 0.071	0.2311	0.18	6.0
D_1^0	[32, 33]	0.0173 ± 0.0039	0.01830	0.26	5.8
D_2^{*0}	[32, 33]	0.0484 ± 0.0080	0.02489	-2.94	-48.6
D_s	[28]	0.116 ± 0.036	0.1162	0.006	0.19
D_s^*	[28]	0.069 ± 0.026	0.0674	-0.06	-2.4
D_{s1}	[33, 34]	0.0106 ± 0.0025	0.00575	-1.94	-45.7
D_{s2}^*	[34]	0.0140 ± 0.0062	0.00778	-1.00	-44.5
Λ_c	[28]	0.079 ± 0.022	0.0966	0.80	22.2
$(B^0+B^+)/2$	[35]	0.399 ± 0.011	0.3971	-0.18	-0.49
B_s	[35]	0.098 ± 0.012	0.1084	0.87	10.6
$B^*/B(\text{uds})$	[36–39]	0.749 ± 0.040	0.6943	-1.37	-7.3
$B^{**} \times BR(B^{(*)}\pi)$	[40–42]	0.180 ± 0.025	0.1319	-1.92	-26.7
$(B_2^*+B_1) \times BR(B^{(*)}\pi)$	[41]	0.090 ± 0.018	0.0800	-0.57	-11.4
$B_{s2}^* \times BR(BK)$	[41]	0.0093 ± 0.0024	0.00631	-1.24	-32.1
b-baryon	[35]	0.103 ± 0.018	0.09751	-0.30	-5.3
Ξ_b^-	[35]	0.011 ± 0.006	0.00944	-0.26	-14.2

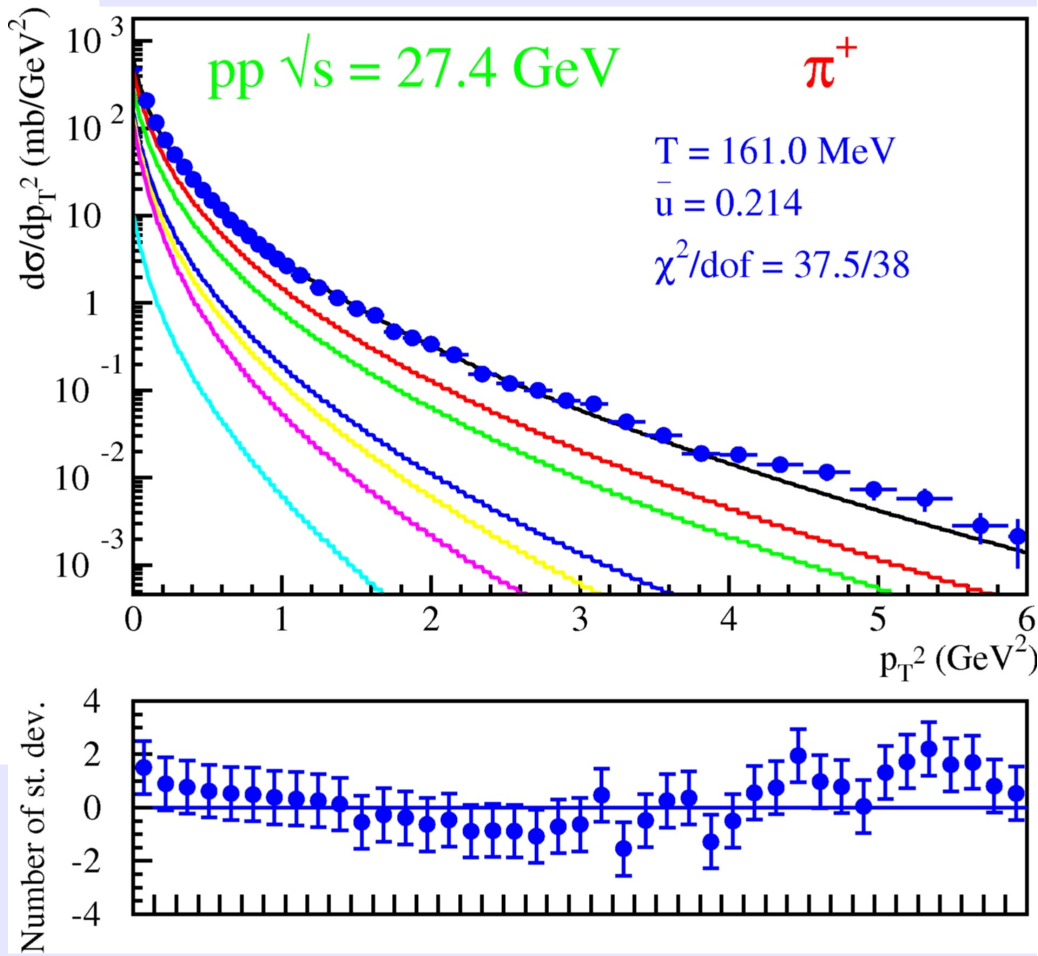
χ^2/dof = 24.6/19 *WITHOUT FREE PARAMETER*

Observed in: F. B., Z. Phys. C 69, 485 (1996)

pp collisions

F. B., G. Passaleva, Eur. Phys. J. C 23 (2002) 551

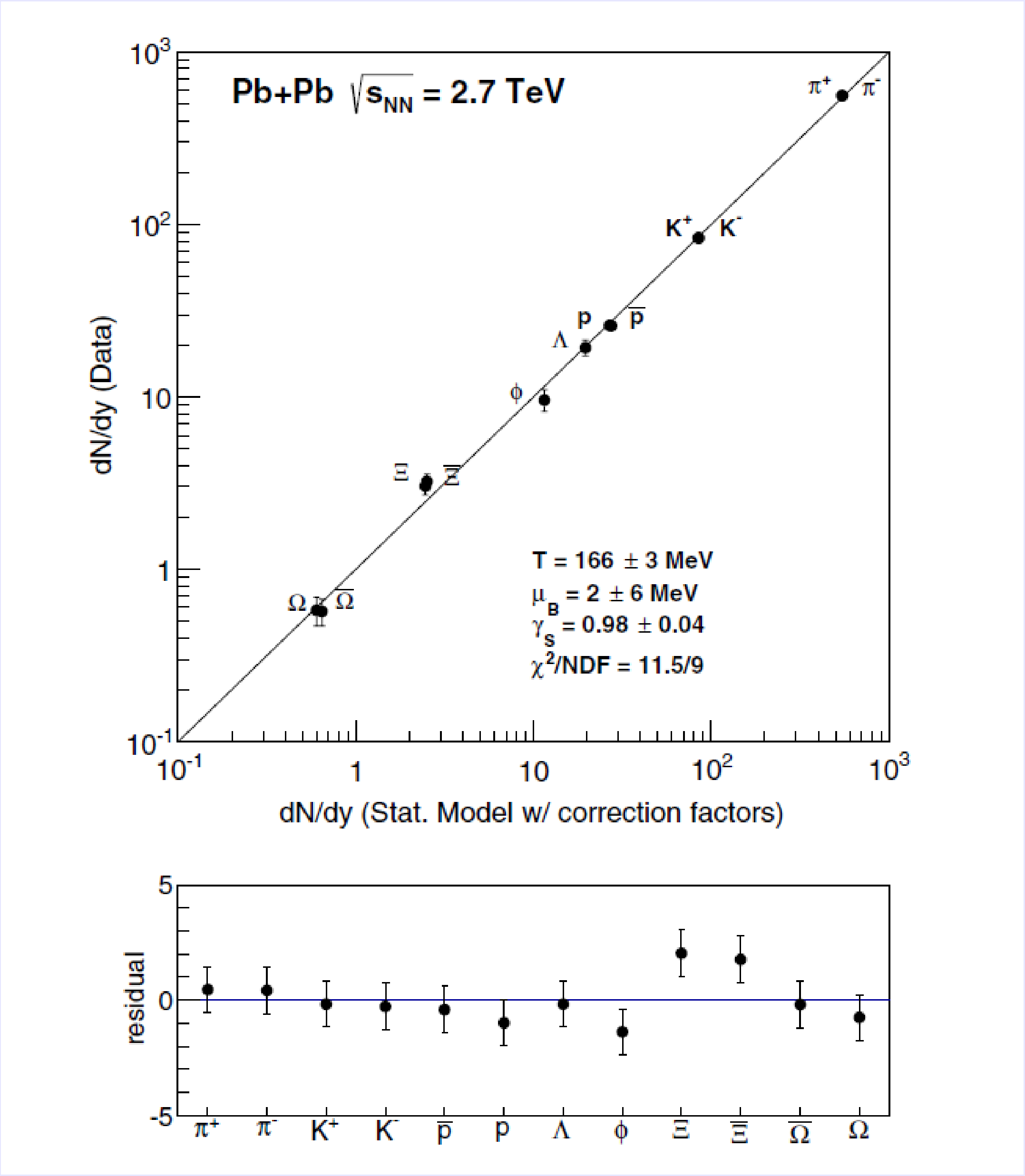




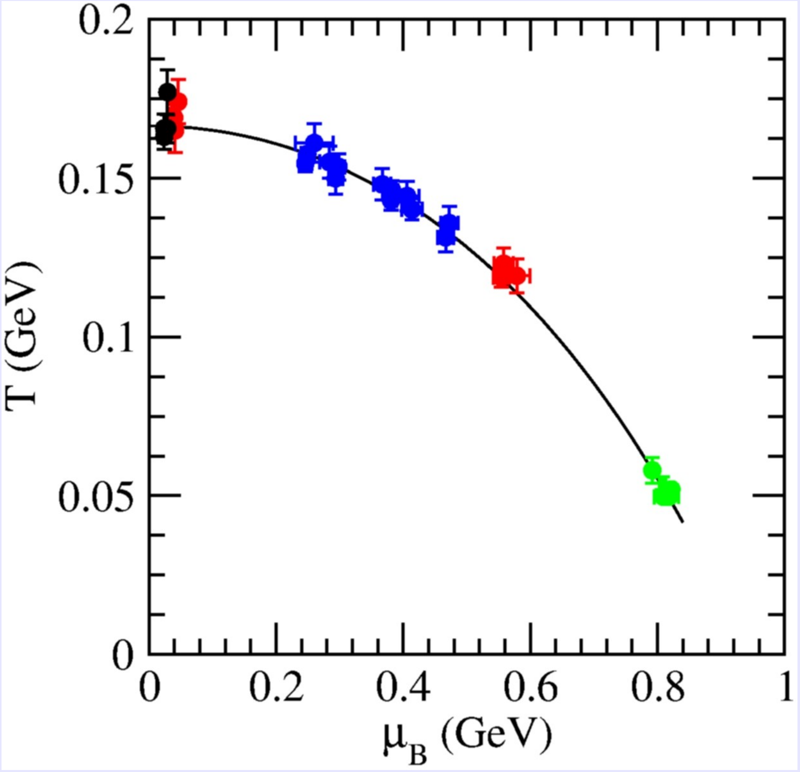
NOTE: no systematic errors

Heavy ion collisions

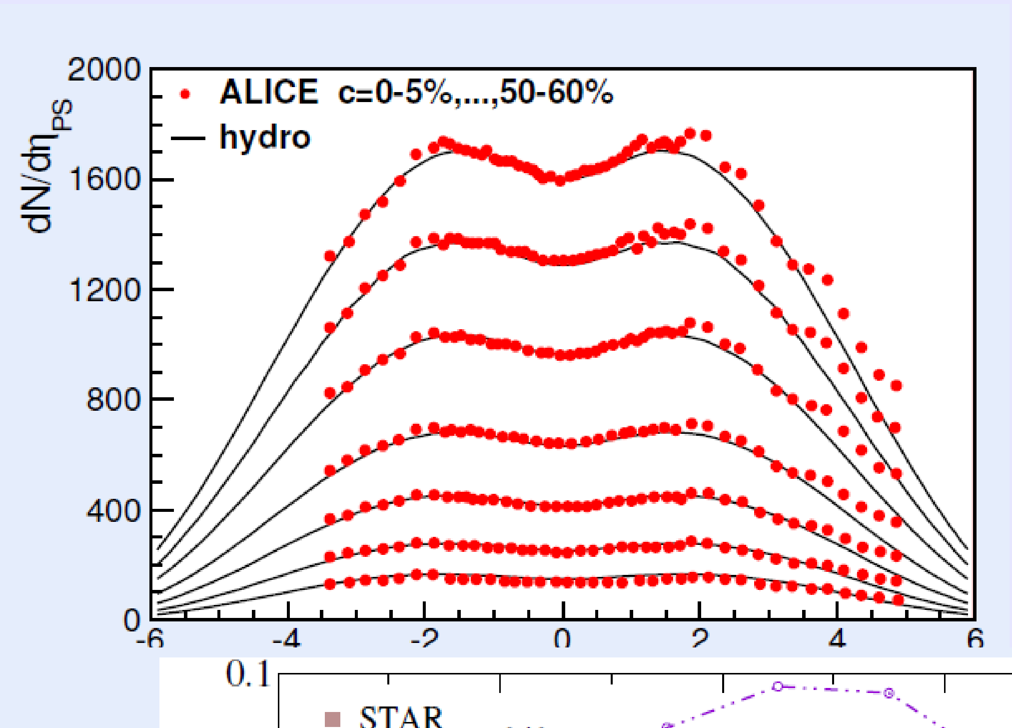
F. B, M. Bleicher, T. Kollegger, T. Schuster, J. Steinheimer and R. Stock, Phys. Rev. Lett. 111(2013) 082302

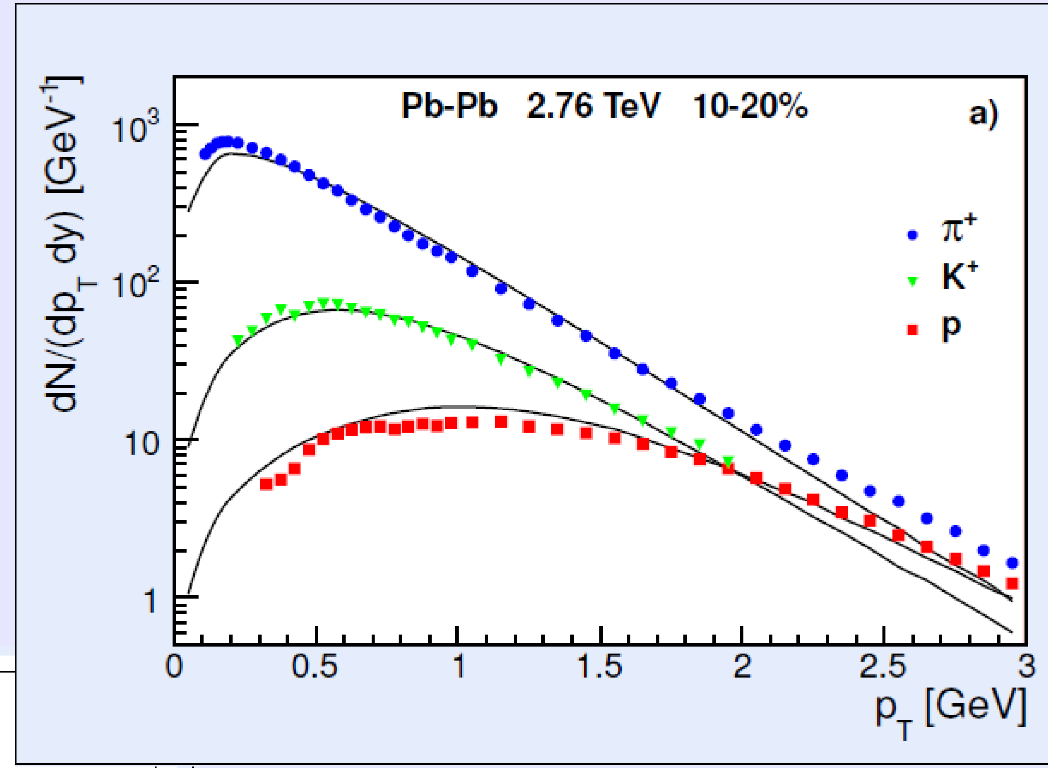


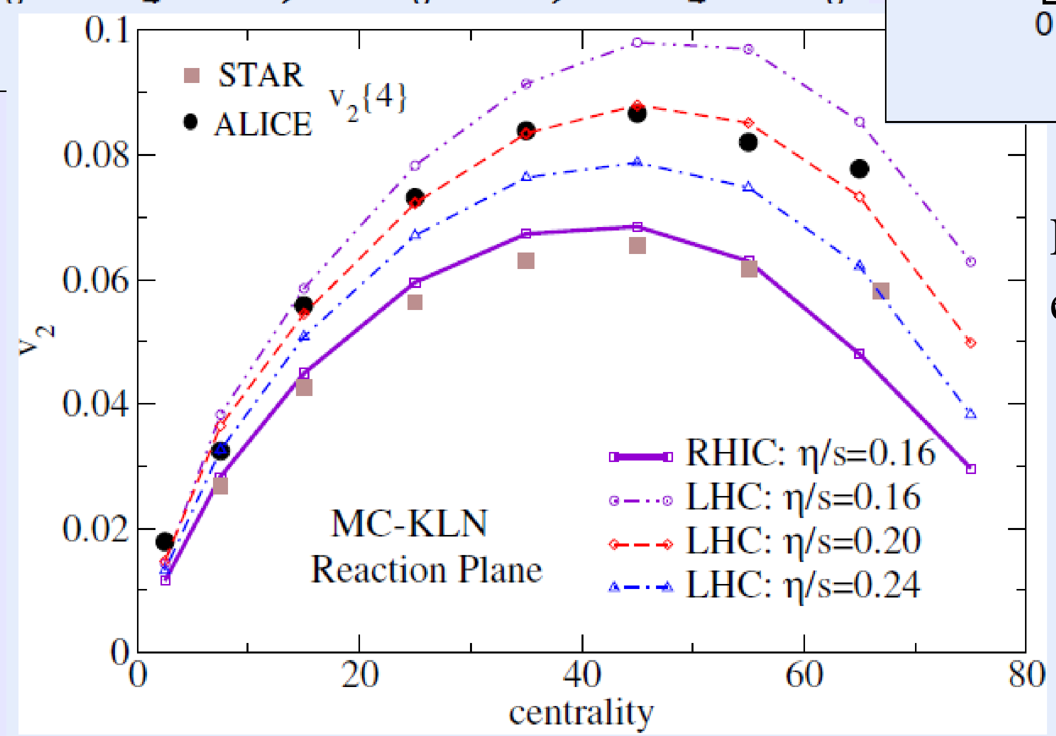
J. Cleymans et al. Phys.Rev. C73 (2006) 034905



Hydrodynamics in relativistic heavy ion collisions

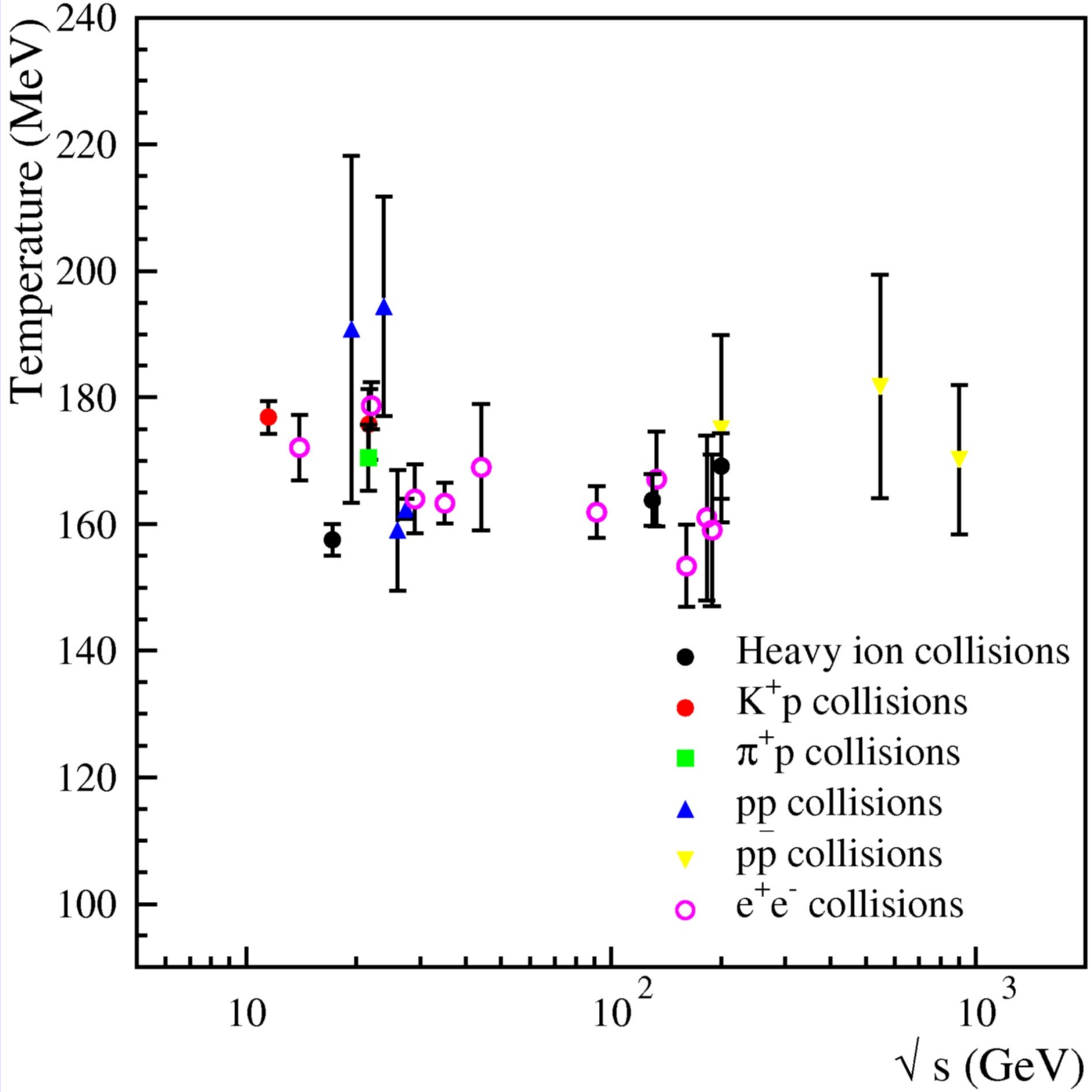






Evidence for LOCAL thermodynamical equilibrium

F. B., R. Fries in Relativistic heavy ion physics, Landolt-Boernstein 1-23 arXiv:0907.1031



Critical energy density at the hadronization?

L. McLerran, "Lectures on RHIC physics", hep-ph 0311028

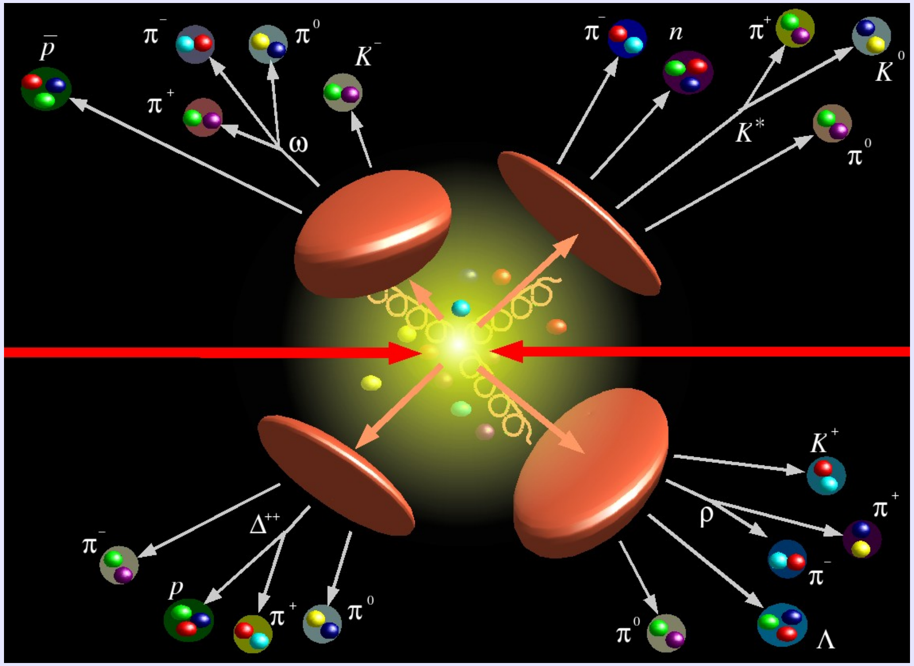
This would appear to be a compelling case for the production of a Quark Gluon Plasma. The problem is that the fits done for heavy ions to particle abundances work even better in e^+e^- collisions. One definitely expects no Quark Gluon Plasma in e^+e^- collisions. There is a deep theoretical question to be understood here: How can thermal models work so well for non-thermal systems? Is there some simple saturation of phase space?

These results should raise some questions.
Yet, few have tried to tackle the issue

H. T. Elze, R. Stock, A. Bialas, S. Hsu, H. Satz, ….

The point, in my view, in this case is not to explain the DETAILS (like professionals tend to do) and to emphasize the differences (elementary vs heavy ion collisions) but to focus on the similarities

The paradox of statistical mechanics in high energy collisions



$$\widehat{\rho} = |\psi\rangle\langle\psi| \longrightarrow U(t) \longrightarrow \widehat{\rho}' = |\psi'\rangle\langle\psi'|$$

A unitary evolution cannot generate a mixed state from a pure state

$$\widehat{\rho} \propto \delta^4(\widehat{P} - P_0) \qquad\qquad \widehat{\rho} = \exp[-\widehat{\beta} \cdot \widehat{P}]$$

A re-foundation for (quantum) statistical mechanics

The evolution is indeed unitary, yet the distributions of a *typical* interacting quantum system are thermal

J. M. Deutsch, *Quantum statistical mechanics in a closed system* Phys. Rev. A 43, 2046 (1991)
M. Srednicki, *Chaos and quantum thermalization* Phys. Rev. E 50, 888 (1994)

An energy eigenstate of a quantum many-body system whose classical limit is chaotic (non integrable) will show thermal microcanonical distributions of few-body observables (*eigenstate thermalization hypothesis ETH*).
The parameters shaping the distributions are the same as obtained through the usual approach (E,V T).

$$\langle E,\alpha|\widehat{N}(p)|E,\alpha\rangle = \frac{\mathrm{d}N}{\mathrm{d}^3p}\Big|_{\mathrm{thermal}}$$

$$\langle E,\alpha'|\widehat{N}(p)|E,\alpha\rangle_{\alpha'\neq\alpha} \ll \langle E,\alpha|\widehat{N}(p)|E,\alpha\rangle$$

ETH is suggested by various results/conjectures in quantum chaos theory and verified numerically for many-body systems which are far from integrability in their parameter space.

A. Krzywicki (hep-ph 0204411) advocated this as the underlying mechanism for the apparent thermal features in hadroproduction in high energy collisions

The Berry conjecture

The high-lying eigenfunctions of a classically chaotic system in configuration space appear to be randomly distributed with a correlation function given by

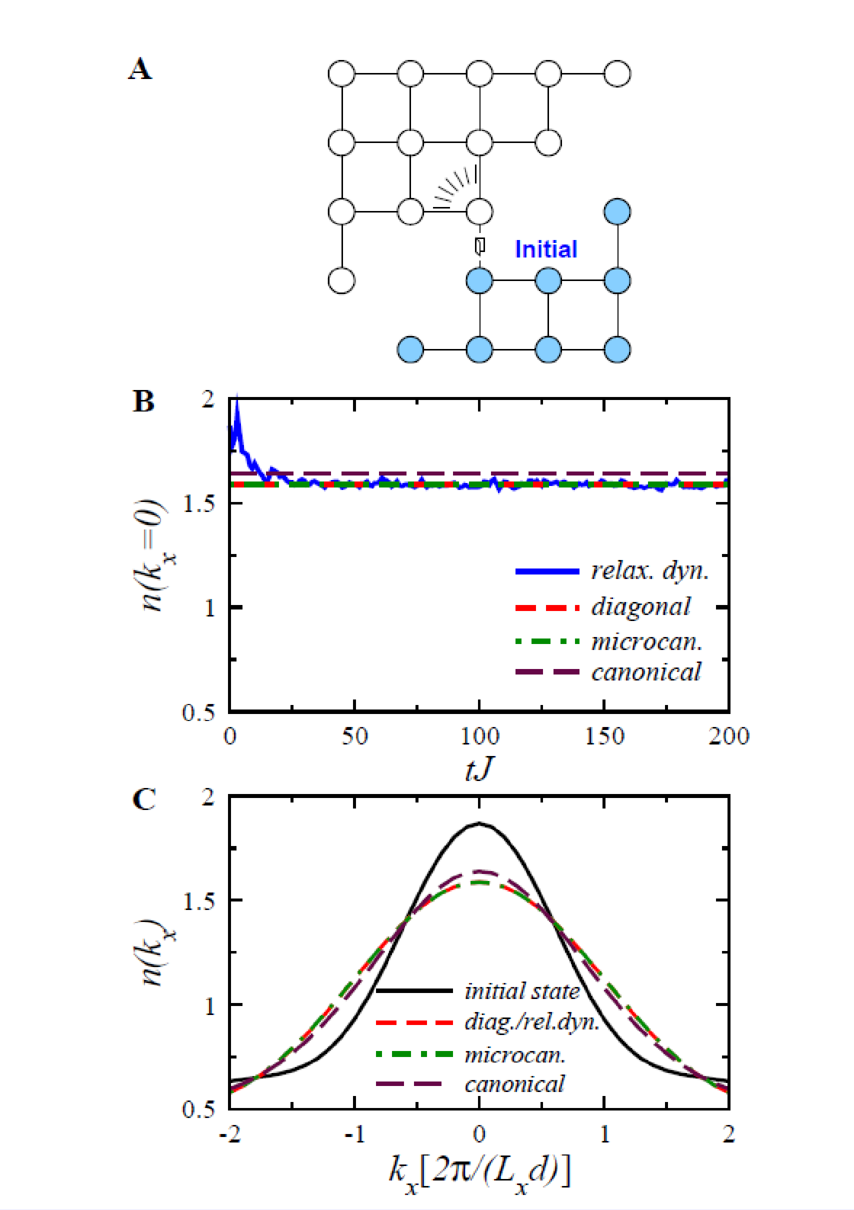
$$\lim_{\alpha\to\infty} \int d\mathbf{X}\ \psi_\alpha(\mathbf{X}+\mathbf{X}_1)\ldots\psi_\alpha(\mathbf{X}+\mathbf{X}_n) = \sum_{\text{pairs}} J(\mathbf{X}_{i_1}-\mathbf{X}_{i_2})\ldots J(\mathbf{X}_{i_{n-1}}-\mathbf{X}_{i_n})\ .$$

$$J(\mathbf{X}) \sim \int d\mathbf{P}\ \exp(i\mathbf{P}\cdot\mathbf{X}/\hbar)\ \delta\big(H(\mathbf{P},\mathbf{X})-U_\alpha\big)\ ,$$

The Berry's conjecture gives us the two necessary ingredients to obtain ETH, that is a microcanonical distribution in momentum space and exponentially (in N) small off-diagonal matrix elements

Recent numerical studies

M. Rigol, V. Dunjko, and M. Olshanii, *Thermalization and its mechanism for generic isolated quantum systems* Nature 452, 854 (2008).



Direct verification of ETH by using 5 bosons with hard core repulsion in a discrete 2-dimensional lattice

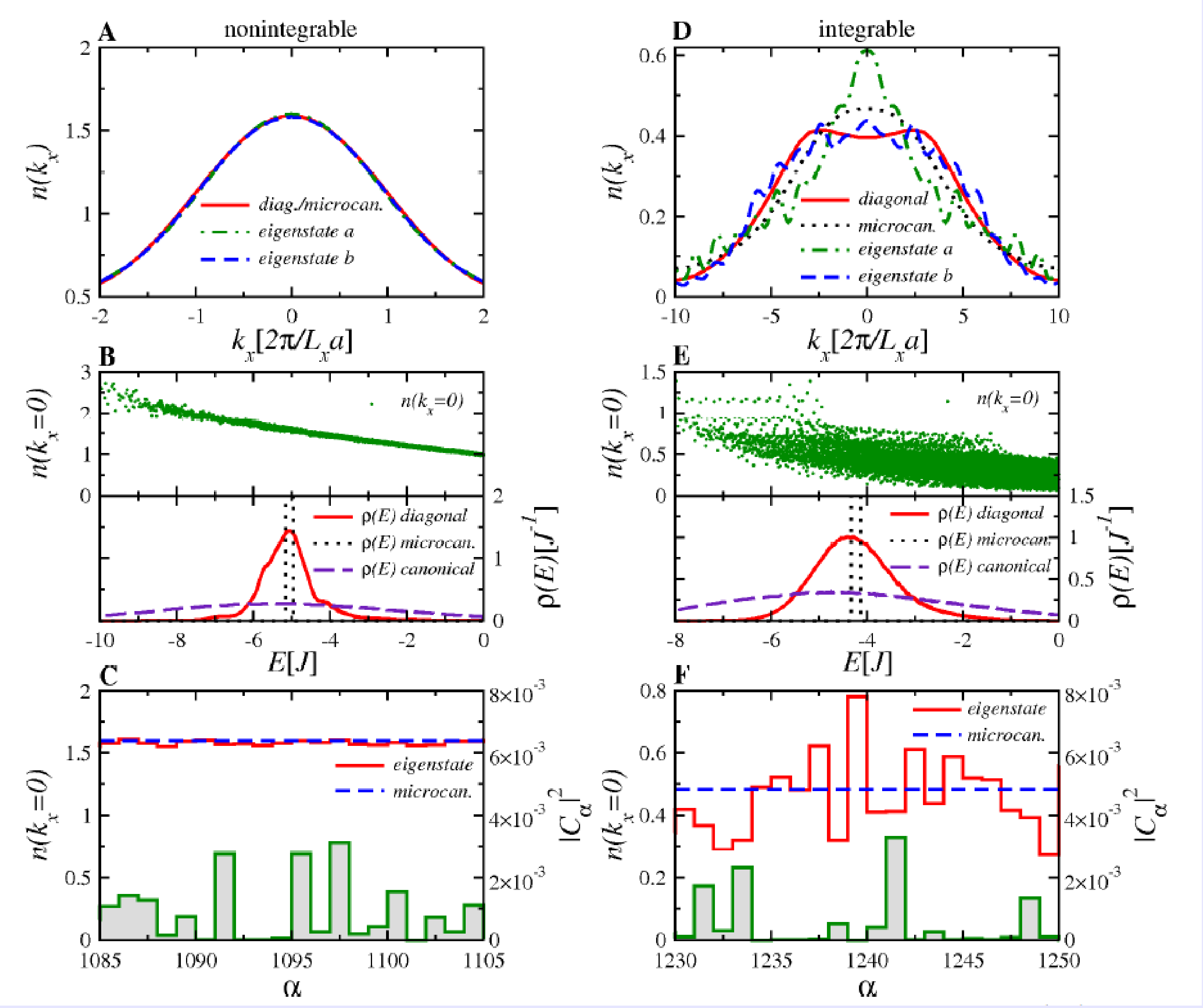
Initial state $$|\psi(0)\rangle \;=\; \sum_\alpha C_\alpha |\Psi_\alpha\rangle$$

Evolved state
$$|\psi(t)\rangle \;=\; e^{-i\hat{H}t}|\psi(0)\rangle \;=\; \sum_\alpha C_\alpha e^{-iE_\alpha t}|\Psi_\alpha\rangle$$

Mean value of an observable at time t
$$\langle \hat{A}(t)\rangle \equiv \langle\psi(t)|\hat{A}|\psi(t)\rangle = \sum_{\alpha,\,\beta} C^\star_\alpha C_\beta e^{i(E_\alpha - E_\beta)t} A_{\alpha\beta}$$
$$A_{\alpha\beta} = \langle \Psi_{\alpha'}|\hat{A}|\Psi_\alpha\rangle$$

Long time mean value (if any)
$$\overline{\langle \hat{A}\rangle} = \sum_\alpha |C_\alpha|^2 A_{\alpha\alpha}\,.$$

M. Rigol convincingly shows that the cause of the apparent equilibration is ETH
The spectrum of particles looks “thermal” for any eigenstate close in energy



$$\langle \Psi_\alpha | \widehat{A} | \Psi_\alpha \rangle = \langle A \rangle_{\text{microcan.}}(E_\alpha).$$

The smallness of time fluctuations bears out the 2nd of the ETH hypotheses namely

$$A^{\text{typical}}_{\substack{\alpha\beta \\ \alpha\neq\beta}} \ll A^{\text{typical}}_{\alpha\alpha}.$$

Consider

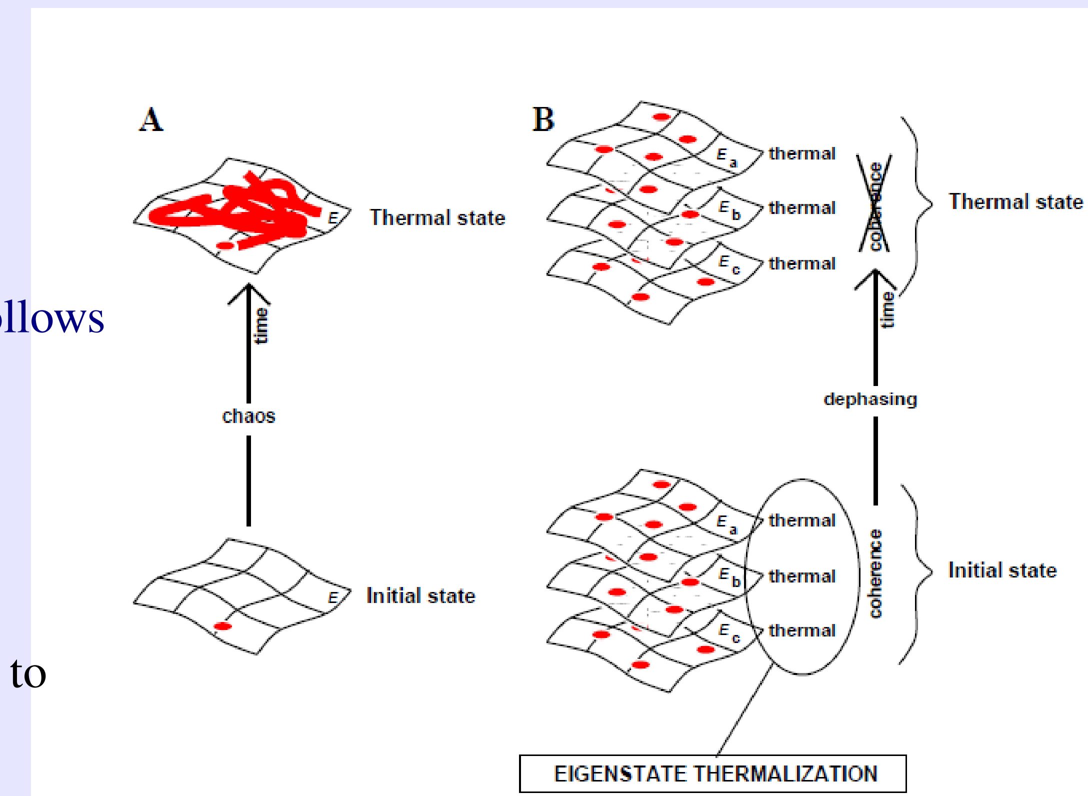
$$\sum_{\alpha\neq\alpha'} C^*_{\alpha'} C_\alpha A_{\alpha'\alpha} e^{i(E'_\alpha - E_\alpha)t}$$

Essentially “random” phases

The sum of N^2 random complex numbers results in a vanishing average, with fluctuations of the order of N times the typical amplitude, i.e.

$$\sqrt{C'_\alpha}\sqrt{C_\alpha}\,|A_{\alpha'\alpha}|$$

Since the observed fluctuations are much
Lower than the average, the top inequality follows



Summary picture of eigenstate
thermalization scenario compared to
Classical thermalization

Can we extend this to hadron production in high energy collisions?

Is classical QCD chaotic? There are strong indications
(T. Biro, C. Gong, B. Muller, H. Markum, R. Pullirsch,...)

Collisions involve scattering states, so one should check that ETH extends to scattering states

$$S|i\rangle \leftrightarrow |\alpha\rangle$$

Certainly in QCD we do not observe a fully thermalized distrubution, yet we observe definite thermal features in many circumstances

However, we have some strong evidence from low energy data

Exclusive channels in e+e- collisions at low energy

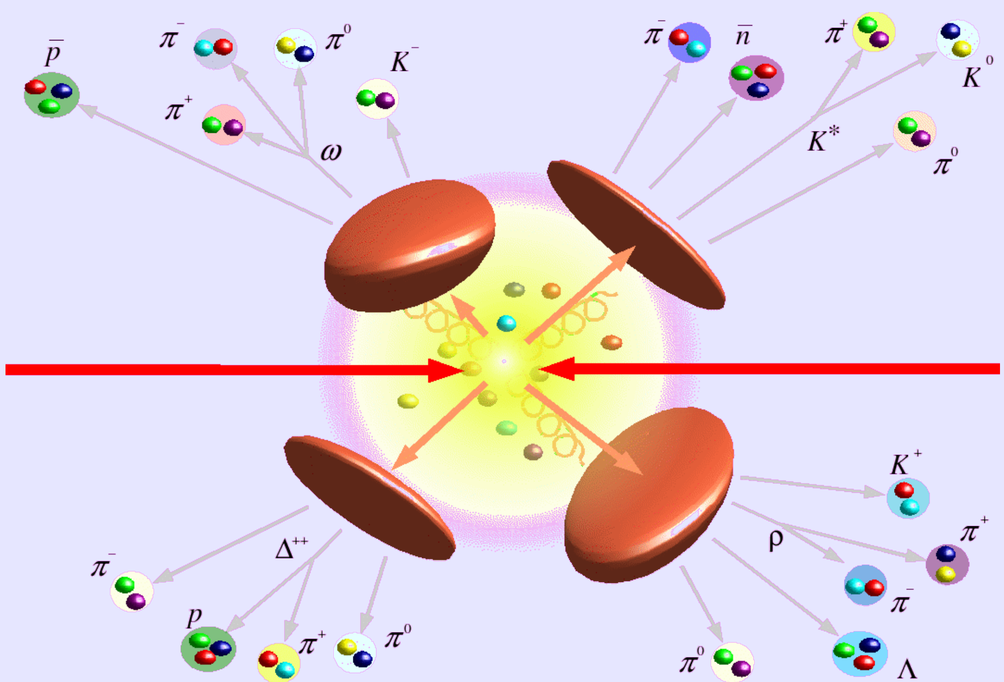
L. Ferroni, F.B., Eur. Phys. J. C 71 (2011) 284

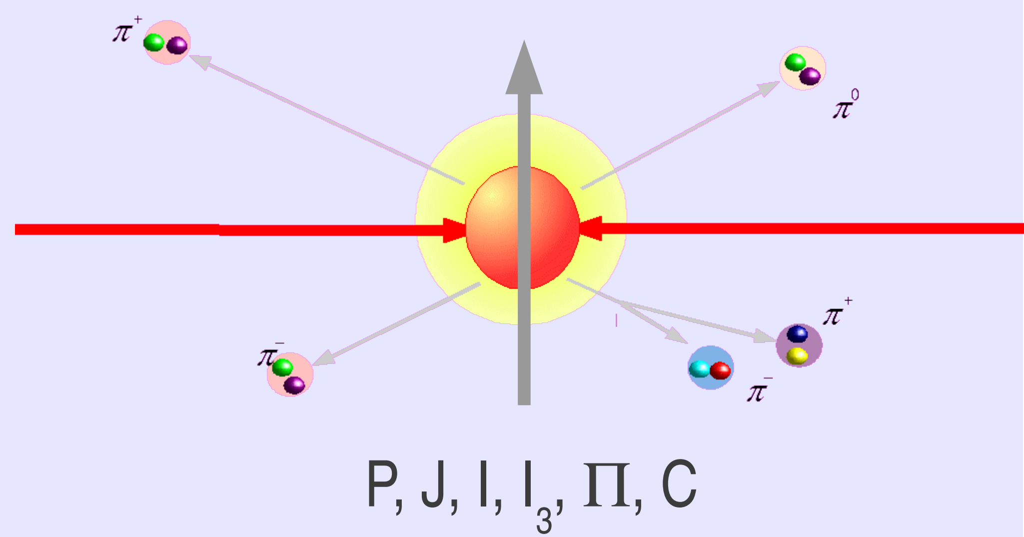
High energy

(Multi-cluster scenario)

low E < 3 GeV

(Single cluster at rest)





ALL conservation laws taken into account with internal symmetry group $SU(2)_{iso} x U(1)_s x U(1)_{baryon}$

$$P_{I,I_3} = |I, I_3\rangle\langle I, I_3|$$

$$P_i = P_{PJ\lambda\pi} P_C P_{I,I_3} P_Q$$

$$P_C = \frac{\hat{I} + C\hat{C}}{2}$$

$$\Omega_{\{N_j\}} = \sum_{\boldsymbol{\rho}} \left[\prod_{j=1}^{K} \chi(\rho_j)^{b_j} \right] \frac{1}{8\pi} \int_0^{4\pi} \mathrm{d}\psi \left[\prod_{j=1}^{k} \frac{1}{N_j!} \prod_{n_j=1}^{N_j} \int \mathrm{d}^3 \mathrm{p}_{n_j} \right]$$
$$\times \delta^4 \left(P - \sum_{n=1}^{N} p_n \right) \sin\frac{\psi}{2} \sin\left[\left(J + \frac{1}{2} \right) \psi \right] \prod_{j=1}^{K} \left[\prod_{l_j=1}^{L_j} \left[\frac{\sin[(S_j + \frac{1}{2}) l_j \psi]}{\sin(\frac{l_j \psi}{2})} \right]^{h_{l_j}(\rho_j)} \right]$$
$$\times \left(\prod_{j=1}^{K} \prod_{l_j=1}^{L_j} F_V^{(s)}(\mathbf{p}_{\rho_j(l_j)} - \mathsf{R}_3^{-1}(\psi)\mathbf{p}_{l_j}) + \Pi\Pi_f \prod_{j=1}^{K} \prod_{l_j=1}^{L_j} F_V^{(s)}(\mathbf{p}_{\rho_j(l_j)} + \mathsf{R}_3^{-1}(\psi)\mathbf{p}_{l_j}) \right)$$
$$\times \left(\mathcal{I}_{\boldsymbol{\rho}}^{\{N_j\}}(I, I_3) \prod_{j=1}^{K} \prod_{l_j=1}^{L_j} \delta_{\alpha_{\rho_j(l_j)} \alpha_{l_j}} + \chi_C^0 \chi_C \overline{\mathcal{I}}_{\boldsymbol{\rho}}^{\{N_j\}}(I, I_3) \prod_{j=1}^{K} \prod_{l_j=1}^{L_j} \delta_{-\alpha_{\rho_j(l_j)} \alpha_{l_j}} \right)$$

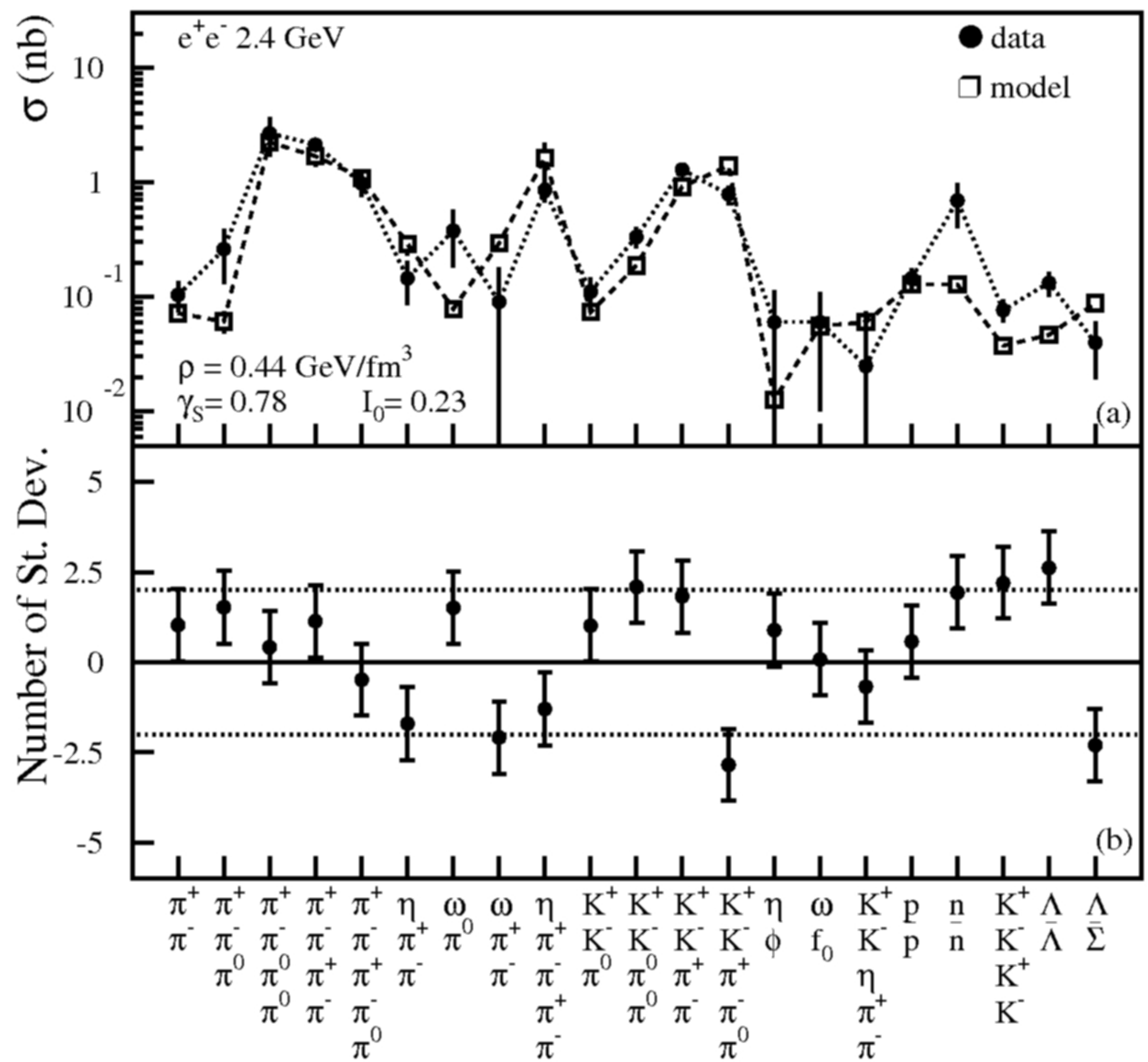
$$\{\langle I^1, I_3^1| \langle I^2, I_3^2| \ldots \langle I^N, I_3^N|\} |I, I_3\rangle\langle I, I_3| \{|I^1, I_3^{1}{}'\rangle |I^2, I_3^{2}{}'\rangle \ldots |I^N, I_3^{N}{}'\rangle\}$$

Quantum statistics included, full conservation of angular momentum and parity

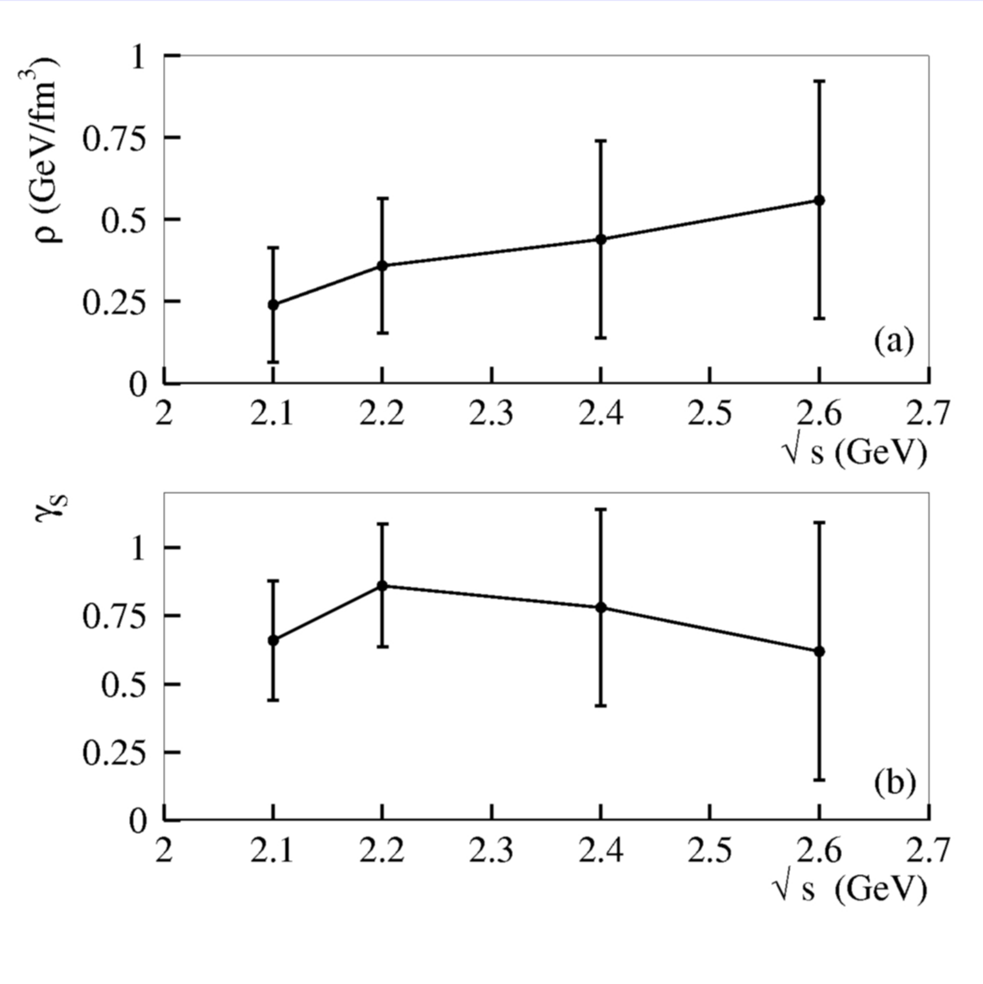
e+e- √s=2.4 GeV

Data from BABAR Experiment - initial state radiation method - and from a compilation by Whalley (DURHAM)

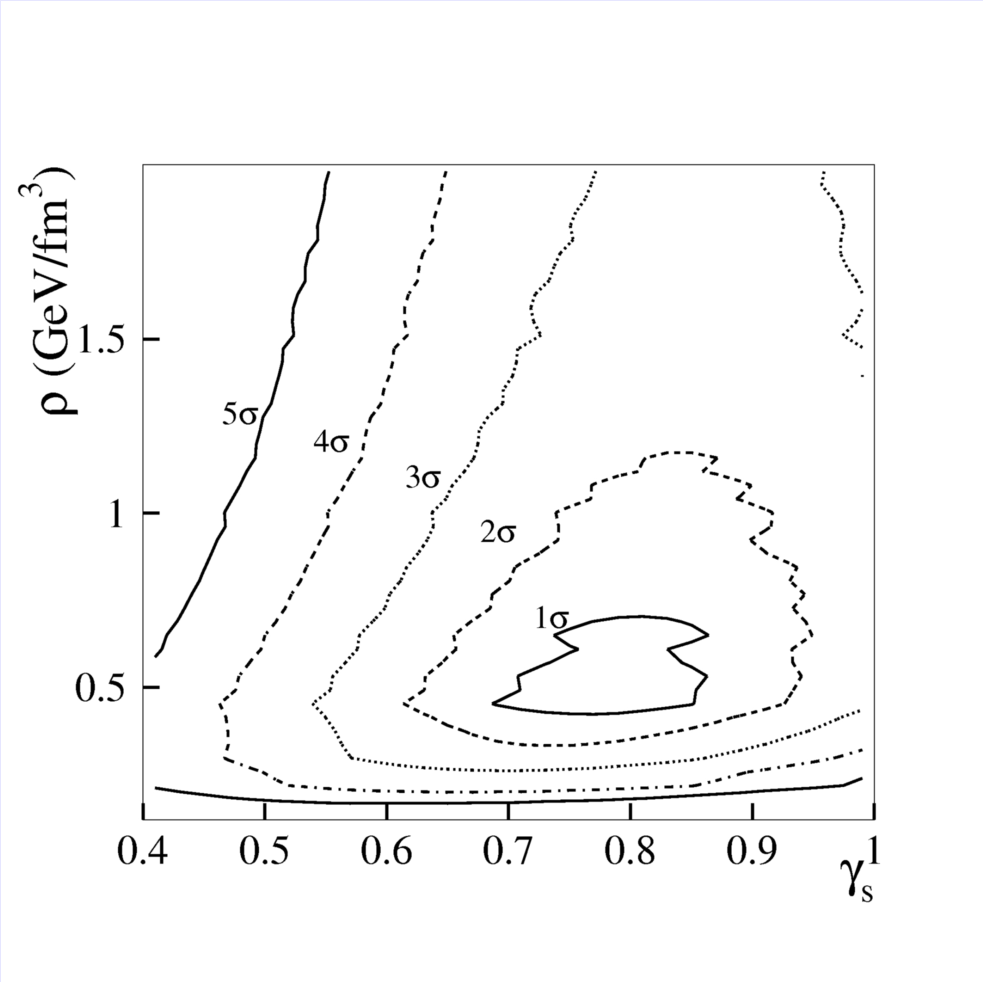
M. R. Whalley,
J. Phys.G 29 (2003) 1



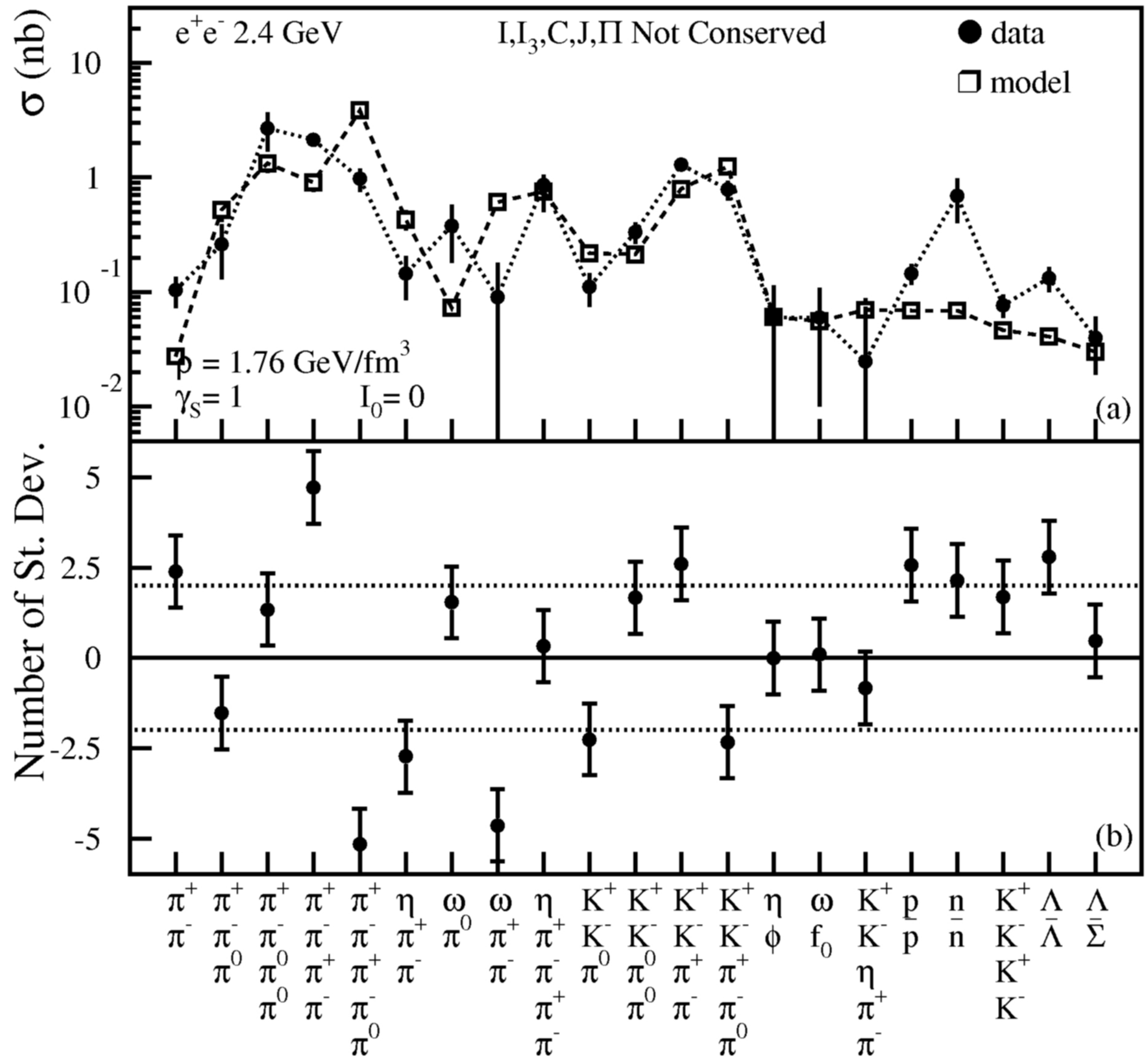
Model Parameters



χ^2 contour $\sqrt{s}$=2.4 GeV

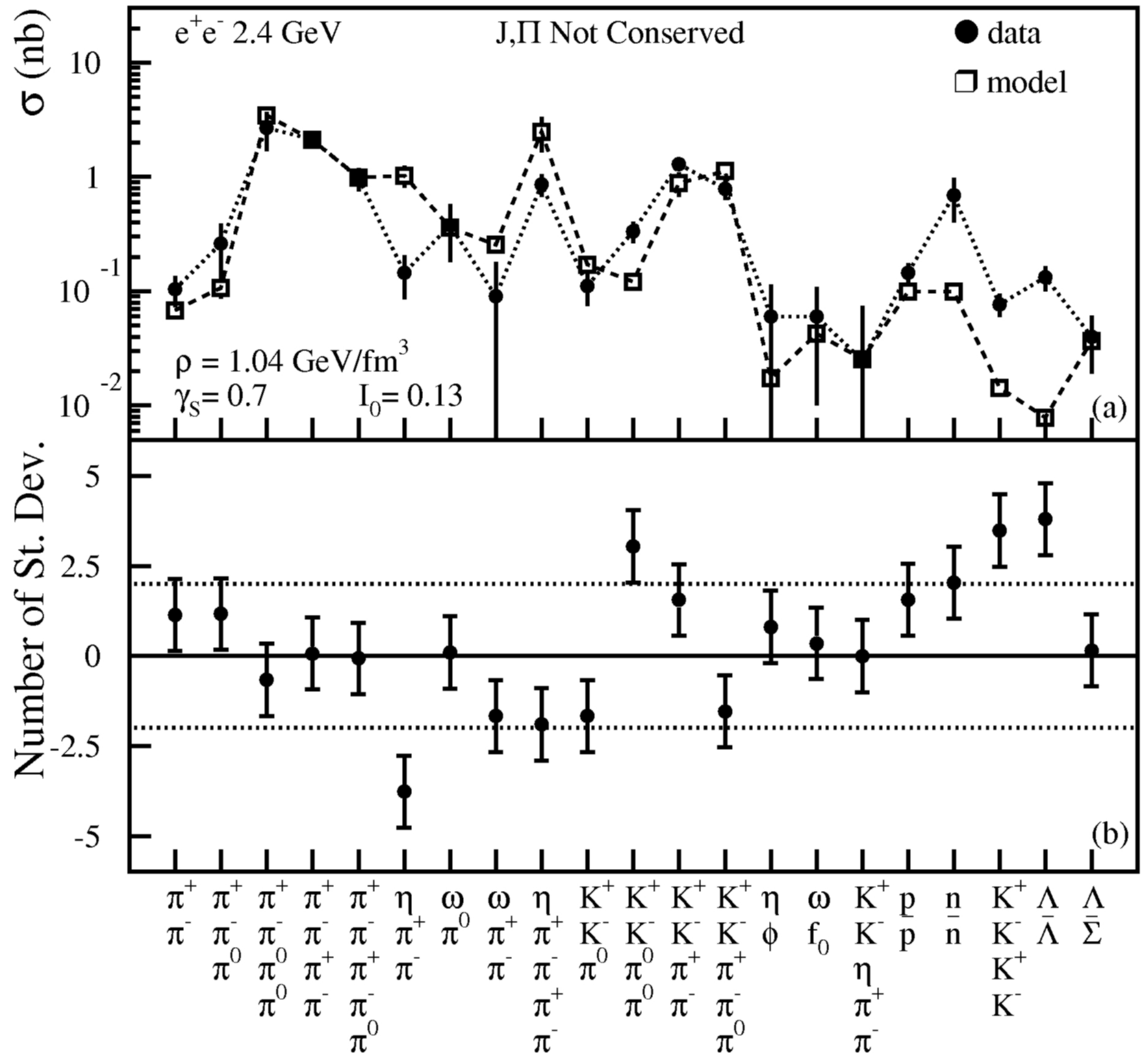


Importance of conservation laws



Q,E,P

Importance of conservation laws

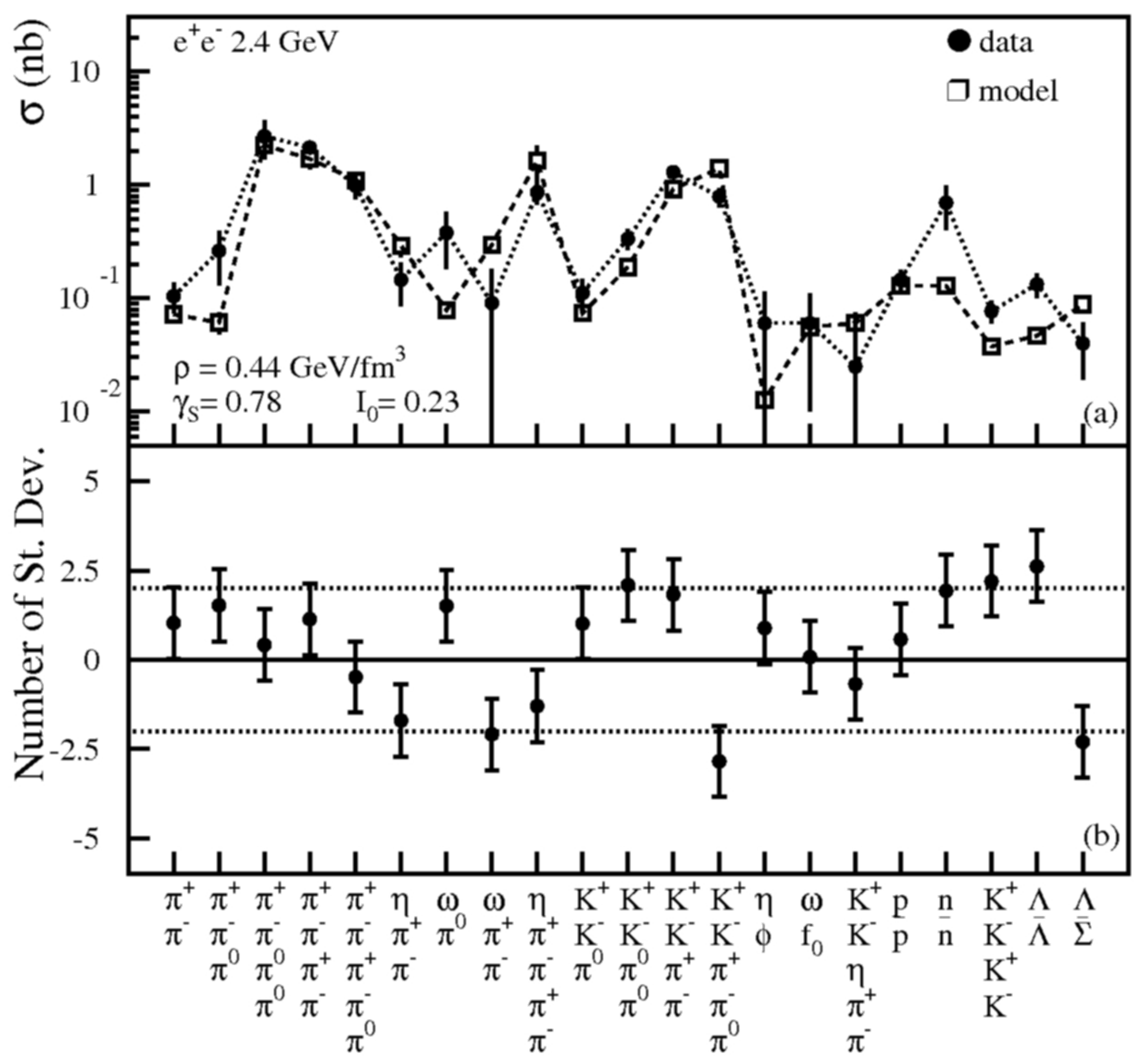


C-parity

Isospin

Q,E,P

Importance of conservation laws



Parity

J

C-parity

Isospin

Q,E,P

Conclusions

- The observed thermal features in strong interactions in high energy collisions might stem from the classically chaotic nature of QCD

- Thermal distributions are featured by quantum many-body systems which are Classically non-integrable and are owing to a property of eigenfunctions dubbed as eigenstate thermalization

- Microcanonical distributions are seen even in exclusive channels in low energy e+e+ collisions

Thanks to Giorgio Torrieri for pointing out relevant papers