Weak Interactions

Historical overview

- Weak interactions are responsible for beta radioactivity discovered by Henri Becquerel in 1896.
- Observed products of beta decay carried less energy than the decaying nucleus which suggested non-conservation of energy.
- In 1930 Pauli proposed an existence of unobserved neutral particle with a small mass to explain the continuous spectrum of beta decay and to rescue the energy conservation.
- In 1934 Enrico Fermi called the particle *neutrino*.
- In modern nomenclature the beta decay of a neutron is

$$n \to p + e^- + \overline{\nu}_e. \tag{1}$$

- In 1934 Frédéric and Irène Joliot-Curie discovered β^+ decay which occurs in nuclei where the process

$$p \to n + e^+ + \nu_e \tag{2}$$

is energetically allowed.

- To describe the neutron beta decay Enrico Fermi introduced in 1933 the four-fermion interaction which is written in a modern way as

$$\mathcal{L}^{\text{int}} = \frac{G_F}{\sqrt{2}} (\overline{\psi}_p \gamma_\mu \psi_n)(\overline{\psi}_e \gamma^\mu \psi_\nu), \tag{3}$$

where the spinor ψ_i with $i = p, n, e, \nu$ describes proton, neutron, electron and electron neutrino, respectively. G_F is the Fermi constant $G_F = 1.166 \cdot 10^{-5}$ GeV^{-2}.

- The Fermi interaction is also called the current-current interaction as it can be written in the following form

$$\mathcal{L}^{\text{int}} = \frac{G_F}{\sqrt{2}} J^\dagger_{h\,\mu} J^\mu_l, \tag{4}$$

where J^μ_h and J^μ_l are the hadron and lepton currents

$$J^\mu_h \equiv \overline{\psi}_n \gamma^\mu \psi_p, \qquad J^\mu_l \equiv \overline{\psi}_e \gamma^\mu \psi_\nu. \tag{5}$$

- In 1940 muon decays were observed

$$\mu^- \to e^- + \overline{\nu}_e + \nu_\mu, \qquad \mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu. \tag{6}$$

- In 1956 F. Reines and C. Cowan registered $\overline{\nu}_e$ through the inverse beta decay

$$\overline{\nu}_e + p \to n + e^+. \tag{7}$$

- Around 1956 there appeared the so-called $\theta - \tau$ puzzle. There were discovered two apparently different particles θ^+ and τ^+ of the same mass 494 MeV and life lifetime $1.2 \cdot 10^{-8}$ s. It was observed that θ^+ decayed as

$$\theta^+ \to \pi^+ + \pi^0, \tag{8}$$

and τ^+ as

$$\tau^+ \to \pi^+ + \pi^+ + \pi^-, \tag{9}$$

which suggested that θ^+ and τ^+ were of opposite parity.

- In 1956 T.D. Lee and C.N. Yang formulated a hypothesis that θ^+ and τ^+ is the same particle (today called as K^+) but the weak interaction does not conserve parity.

- The experiment performed by C. S. Wu et al in 1957 showed that the parity is indeed not conserved in the decay of polarized ^{60}Co
$$^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \overline{\nu}_e. \tag{10}$$

- In 1958 Feynman and Gell-Mann and Marshak and Sudarshan proposed the universal V–A theory of weak interactions where the lepton current is
$$J^\mu_l \equiv \overline{\psi}_e \gamma^\mu (\mathbb{1} + \gamma^5)\psi_\nu. \tag{11}$$
The term with γ^5 is responsible for parity non-conservation.

- In 1962 it was showed that ν_e and ν_μ are different particles.

- In 1963 Nicola Cabibbo introduced a universal hadronic current.

- 1963-1968 the model of unified electro-weak interaction was formulated by S. Glashow, A. Salam and S. Weinberg.

- In 1973 the weak neutral currents were discovered.

- In 1983 there were discovered the $W^\pm$ and Z^0 bosons which are carriers of weak interaction predicted by the model of unified electro-weak interaction.

C, P, T symmetries

Since the weak interactions violate discrete C, P, T symmetries we start with a brief discussion of the symmetries.

Parity

- One asks what happens with a physical system under the spatial inversion $\mathbf{r} \to -\mathbf{r}$. One observes what the particle's momentum, which is $\mathbf{p} \equiv m\frac{d\mathbf{r}}{d\tau}$, changes its sign but the angular momentum $\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$ remains unchanged. One checks that the electric field $\mathbf{E}$ changes its sign but the magnetic field $\mathbf{B}$ does not.

- The parity operator P, which acts on the wave function $\psi(t, \mathbf{r})$, is defined as
$$P\psi(t, \mathbf{r}) = \psi(t, -\mathbf{r}). \tag{12}$$

- If the inner product of wave functions is defined in the standard way as
$$(\psi, \phi) \equiv \int d^3r\, \psi^*(t, \mathbf{r})\, \phi(t, \mathbf{r}), \tag{13}$$
one easily shows that P is Hermitian that is $(\psi, P\phi) = (P\psi, \phi)$.

- Since $P = P^\dagger$ and $P^2 = \mathbb{1}$, the eigenvalues of P are ± 1.

- We say that a particle has positive (negative) parity if it is the eigenstate of P with the eigenvalue $+1$ (-1).

- The parity is a multiplicative quantum number – if the system is composed of two parts, which are independent from each other, its parity is a product of parities of the parts.

- If the system's Hamiltonian obeys $H(\mathbf{r}) = H(-\mathbf{r})$, the wave functions $\psi(t, \mathbf{r})$ and $\psi(t, -\mathbf{r})$ are both solutions of the Schrödinger equation. We can construct the parity plus and parity minus solutions as
$$\psi^\pm(t, \mathbf{r}) = \frac{1}{\sqrt{2}}\big[\psi(t, \mathbf{r}) \pm \psi(t, -\mathbf{r})\big]. \tag{14}$$
One sees that $P\psi^\pm(t, \mathbf{r}) = \pm\psi^\pm(t, \mathbf{r})$.

- If $H(\mathbf{r}) = H(-\mathbf{r})$, the parity operator P commutes with H and <u>the parity is conserved</u>.

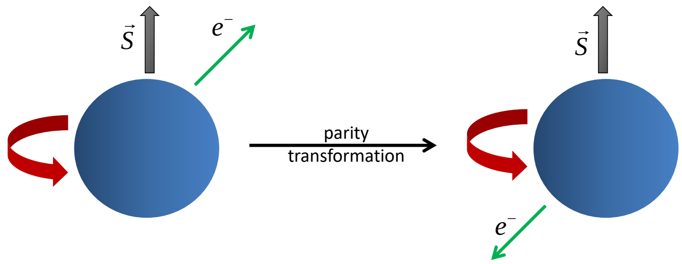

Figure 1: Scheme of the Wu's experiment

- A wave function of spherically symmetric potential is of the form

$$\psi(\mathbf{r}) = R(r) Y^{lm}(\theta, \phi) \tag{15}$$

and

$$P\psi(\mathbf{r}) = \psi(-\mathbf{r}) = R(r) Y^{lm}(\pi - \theta, \phi + \pi) = (-1)^l R(r) Y^{lm}(\theta, \phi). \tag{16}$$

The parity of a state with the orbital momentum l is $(-1)^l$.

Exercise: Prove the last equality in Eq. (16).

- Let us consider a system of two particles a and b. The wave function can be written as

$$\psi = \phi_a \phi_b \phi_{ab}, \tag{17}$$

where ϕ_i with $i = a, b$ is the wave function of particle a or b and ϕ_{ab} is the wave function of relative motion. Assuming that $P\phi_i = \epsilon_i \phi_i$ and $\phi_{ab} \sim Y^{lm}$, we have

$$P\psi = \epsilon_a \epsilon_b (-1)^l \psi. \tag{18}$$

- Photon's parity is -1.
- In case of bosons the parity of a particle and its antiparticle is the same.
- In case of fermions the parity of a particle is opposite to the parity of its antiparticle.
- Non-conservation of parity in weak interactions was demonstrated in the experiment performed by C. S. Wu et al in 1957. The experiment is schematically shown in Fig. 1. There was observed the angular distribution of electrons coming from the decays of polarized ^{60}Co

$$^{60}\text{Co} \rightarrow {}^{60}\text{Ni} + e^- + \overline{\nu}_e. \tag{19}$$

The parity transformation transforms the system as shown in Fig. 1. Therefore, the parity is violated if $\langle \mathbf{S} \cdot \mathbf{p}_e \rangle \neq 0$, where $\langle \dots \rangle$ denotes averaging over many decays of ^{60}Co, $\mathbf{S}$ is the spin of ^{60}Co and $\mathbf{p}_e$ is the electron momentum. Since the electrons were emitted mostly antiparallelly to the spin of ^{60}Co a sizable effect of parity non-conservation was observed.

Time reversal

- The time reversal operator T, which acts on the wave function $\psi(t, \mathbf{r})$, is defined as

$$T\psi(t, \mathbf{r}) = \psi(-t, \mathbf{r}). \tag{20}$$

- If the system's Hamiltonian obeys $H(t) = H(-t)$, the Schrödinger equation transforms as

$$i\frac{\partial \psi(t, \mathbf{r})}{\partial t} = H\psi(t, \mathbf{r}) \longrightarrow -i\frac{\partial \psi(-t, \mathbf{r})}{\partial t} = H\psi(-t, \mathbf{r}). \tag{21}$$

So, the wave function $\psi(-t, \mathbf{r})$ is not a solution of the Schrödinger equation of $\psi(t, \mathbf{r})$ but it is a solution of the Schrödinger of $\psi^*(t, \mathbf{r})$. Therefore,

$$T\psi(t, \mathbf{r}) = \psi^*(t, \mathbf{r}). \tag{22}$$

- Since the transformation T changes ψ into ψ^* there are no eigenfunctions of T and there is no associated quantum number.
- Until the discovery of CP violation of weak interactions in 1964, it was believed that all interactions are invariant under the time reversal. Since the CPT symmetry holds for any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian, a violation of the CP symmetry implies a violation of T symmetry.

Charge conjugation

- The transformation of charge conjugation C changes particles into antiparticles.
- A particle, like π^0 and γ, which is its own antiparticle is an eigenstate of C.
- The C parity of γ is -1 because $A^\mu(t,\mathbf{r}) \to -A^\mu(t,\mathbf{r})$.
- Since the C parity is conserved in electromagnetic interactions and π^0 decays electromagnetically as $\pi^0 \to 2\gamma$, the C parity of π^0 is $+1$.
- A positronium, which is a bound state of e^- and e^+, is an eigenstate of the charge conjugation and its C parity is $(-1)^{l+S}$.

Pure lepton processes

- There are three flavors of leptons and there is a pair lepton-neutrino of each flavor

$$\begin{pmatrix}\nu_e \\ e\end{pmatrix}, \qquad \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}, \qquad \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}. \tag{23}$$

- The leptons and neutrinos of each flavor carry a lepton charge (lepton number) equal 1. The lepton number of each flavor is conserved. The charge of antileptons and antineutrinos is obviously opposite to that of leptons and neutrinos.
- Each lepton and the corresponding neutrino are described by the Dirac spinors ψ_l and ψ_ν which satisfy the Dirac equations. The leptons are massive but the neutrinos are assumed to be massless in the Standard Model.
- The Dirac matrix γ^5 is defined as $\gamma^5 \equiv -i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma_5$ and it anticommutes with γ^μ that is $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$. One also shows that $(\gamma^5)^2 = \mathbb{1}$. We note that the matrix γ^5 is often defined with the opposite sign.
- The operators

$$P_L \equiv \frac{1}{2}(\mathbb{1} + \gamma^5), \qquad P_R \equiv \frac{1}{2}(\mathbb{1} - \gamma^5), \tag{24}$$

  are the projection operators that is they obey $P_L^2 = P_L$ and $P_R^2 = P_R$. When the matrix γ^5 is defined with the opposite sign the projectors P_L and P_R are interchanged which sometimes leads to a confusion. The operators P_L and P_R are mutually orthogonal and complementary that is $P_L P_R = P_R P_L = 0$ and $P_L + P_R = \mathbb{1}$.

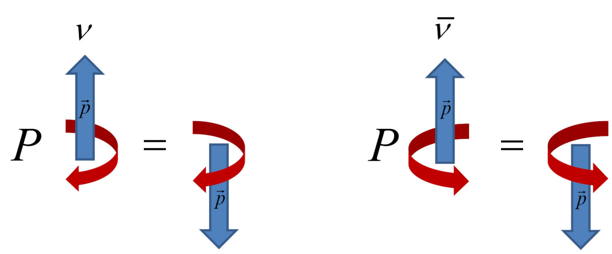

Figure 2: P transformation of a neutrino and antineutrino

- Helicity is the projection of the particle's spin onto the direction of particle's momentum. The helicity is positive or right-handed if the direction of its spin is the same as the direction of its motion. It is negative or left-handed if the directions of spin and motion are opposite.
- The projection operators P_L and P_R acting on a spinor ψ produce a state of left-handed and right-handed helicity $\psi_L = P_L\psi$ and $\psi_R = P_R\psi$.
- Leptons and antileptons can be both left-handed and right-handed. The neutrinos instead are always left-handed and antineutrinos are right-handed.
- One observes that
$$\overline{\psi}_L \equiv \psi_L^\dagger \gamma^0 = \overline{\psi} P_R, \qquad \overline{\psi}_R \equiv \psi_R^\dagger \gamma^0 = \overline{\psi} P_L, \tag{25}$$
- Since neutrinos are left-handed and antineutrinos are right-handed, the parity operator acting on a neutrino produces a non-existent right handed neutrino. Analogously, the parity operator acting on an antineutrino produces a non-existent left-handed antineutrino. Therefore, neutrinos and antineutrinos fully violate the P symmetry as depicted in Fig. 2.
- The transition lepton current is defined as
$$J^\mu(x) = \sum_{l\,=\,e,\mu,\tau} J_l^\mu(x), \tag{26}$$
where
$$J_l^\mu(x) = \overline{\psi_l}(x)\gamma^\mu(1+\gamma^5)\psi_\nu(x), \tag{27}$$
ψ_l and ψ_ν are the Dirac spinors of the lepton and corresponding neutrino, respectively.
- Keeping in mind that
$$\psi^\dagger = \overline{\psi}\gamma^0, \qquad \overline{\psi}^\dagger = \gamma^0\psi, \qquad (\gamma^0)^2 = \mathbb{1}, \qquad (\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0, \qquad (\gamma^5)^\dagger = \gamma^5, \qquad \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5, \tag{28}$$
the Hermitian conjugate current is found to be
$$\left(J_l^\mu(x)\right)^\dagger = \overline{\psi}_\nu(x)(1-\gamma^5)\gamma^\mu\psi_l(x) = \overline{\psi}_\nu(x)\gamma^\mu(1+\gamma^5)\psi_l(x). \tag{29}$$
- Since
$$(1-\gamma^5)\gamma^\mu(1+\gamma^5) = 2\gamma^\mu(1+\gamma^5), \tag{30}$$
the current J_l^μ can written as
$$J_l^\mu = \frac{1}{2}\overline{\psi_l}(1-\gamma^5)\gamma^\mu(1+\gamma^5)\psi_\nu = \frac{1}{2}\overline{\psi_l}_L\gamma^\mu\psi_{\nu L}, \tag{31}$$
where only left-handed spinors enter.
- We note that
$$\overline{\psi}\gamma^\mu\psi = \overline{\psi}_L\gamma^\mu\psi_L + \overline{\psi}_R\gamma^\mu\psi_R \tag{32}$$
and consequently, the left- and right-handed electrons contribute to the electromagnetic current.
- There are four local bilinear forms of spinors:
$$S_{ab} \equiv \overline{\psi}_a\psi_b, \qquad A_{ab} \equiv \overline{\psi}_a\gamma^5\psi_b, \qquad V_{ab}^\mu \equiv \overline{\psi}_a\gamma^\mu\psi_b, \qquad A_{ab}^\mu \equiv \overline{\psi}_a\gamma^\mu\gamma^5\psi_b. \tag{33}$$
S and A are Lorentz scalars while V^μ and A^μ are Lorentz four-vectors.
- Under the charge conjugation, the forms transform as
$$CS_{ab} = S_{ba}, \qquad CA_{ab} = A_{ba}, \qquad CV_{ab}^\mu = -V_{ba}^\mu, \qquad CA_{ab}^\mu = A_{ba}^\mu. \tag{34}$$
- Under the spatial reflection, the forms transform as
$$PS_{ab} = S_{ab}, \qquad PA_{ab} = -A_{ab}, \qquad PV_{ab}^\mu = (V^0, -\mathbf{V})_{ab}, \qquad PA_{ab}^\mu = (-A^0, \mathbf{A})_{ab}. \tag{35}$$
For this reason A is called axial scalar or pseudoscalar and A^μ axial vector or pseudovector.

- Under the time inversion, the forms transform as

$$TS_{ab} = S_{ba}, \qquad TA_{ab} = -A_{ba}, \qquad TV^{\mu}_{ab} = (V^0, -\mathbf{V})_{ba}, \qquad TA^{\mu}_{ab} = (-A^0, \mathbf{A})_{ba}. \tag{36}$$

- The Lagrangian density of the Fermi-like (four-fermion or current-current) interaction is

$$\mathcal{L}^{\rm int}(x) = \frac{G_F}{\sqrt{2}}\, J^{\dagger}_{\mu}(x) J^{\mu}(x), \tag{37}$$

  where the Fermi constant is $G_F = 1.166 \cdot 10^{-5}$ GeV^{-2}. The interaction is of zero range and for this reason it is also called 'contact' interaction.

- One checks that the Lagrangian (37) is not invariant under the spatial reflection P and under the charge conjugation C. However it is invariant under the CP combined transformation. It is also invariant under the time inversion T, as it is invariant under the CPT transformation.

- The Lagrangian (37) describes purely leptonic process like the muon decays

$$\mu^- \to e^- + \overline{\nu}_e + \nu_\mu, \qquad \mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu. \tag{38}$$

- In case of polarized muons, the distribution of orientations of electron spin and of electron momentum reveal lack of P symmetry of the process.

Hadronic processes

- There is a whole variety of hadron decays driven by weak interactions. For example, the dominant decay channels of lightest hadrons – pions – are

$$\pi^- \to \mu^- + \overline{\nu}_\mu, \qquad \pi^+ \to \mu^+ + \nu_\mu. \tag{39}$$

  Exactly the same are the dominant decay channels of the lightest strange mesons K^+ and K^-.

- The two dominant decay channels of the hyperon Λ are

$$\Lambda^0 \to p + \pi^-, \qquad \Lambda^0 \to n + \pi^0. \tag{40}$$

- Assuming that π^- can be represented by the virtual pair $\overline{p}\,n$ while π^+ by $p\,\overline{n}$, the pion decays (39) can be described using the four-fermion Lagrangian with the baryon current $J^\mu \equiv \overline{\psi}_n \gamma^\mu (1+\gamma^5)\psi_p$.

- The decays of K^+ and K^- can be analogously described using the baryon current $J^\mu \equiv \overline{\psi}_\Lambda \gamma^\mu (1+\gamma^5)\psi_p$ which assumes non-conservation of strangeness in weak interactions.

- The Lagrangian density

$$\mathcal{L}^{\rm int} = \frac{G}{\sqrt{2}}\, (\overline{\psi}_p \gamma_\mu (1+\gamma^5)\psi_n)(\overline{\psi}_\Lambda \gamma^\mu (1+\gamma^5)\psi_p), \tag{41}$$

  describes the lambda decays (40).

- In every process mentioned above the coupling constant must be properly chosen to reach an agreement with experimental results.

- In 1963 Nicola Cabibbo introduced a universal hadronic current which in modern terminology is written in terms of quarks. Let us first discuss the current for a case of two not three generations of quarks. Then, analogously to two lepton doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \tag{42}$$

  one introduces two quark doublets

$$\begin{pmatrix} u \\ d' \end{pmatrix}, \qquad \begin{pmatrix} c \\ s' \end{pmatrix}, \tag{43}$$

  where u and c are the up and charm quarks while d' and s' are 'rotated' down and strange quarks that is

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta_c\, d + \sin\theta_c\, s \\ \cos\theta_c\, s - \sin\theta_c\, d \end{pmatrix}. \tag{44}$$

  The rotation matrix is known as the Cabibbo matrix and $\theta_c \approx 13^\circ$ is the Cabibbo angle.

- The transition quark current is

$$J_q^\mu(x) = \overline{\psi_{d'}}(x)\gamma^\mu(1+\gamma^5)\psi_u(x) + \overline{\psi_{s'}}(x)\gamma^\mu(1+\gamma^5)\psi_c(x). \tag{45}$$

- In case of three generations of quarks, one introduces the 'rotated' down, strange and bottom quarks d', s', b' and each 'rotated' quark includes contributions from d, s, b quarks. The 3×3 matrix, which describes the quark mixing, was introduced by Makoto Kobayashi and Toshihide Maskawa in 1973 and is known as the Cabibbo-Kobayashi-Maskawa or CKM matrix. It is determined by three angles analogous to the Euler angles, which describe the orientation of a rigid body with respect to a fixed coordinate system, and the phase.

- The total curent with enters the Lagrangian of four-fermion interaction includes the universal lepton and quark currents.

- The Lagrangian is successful in describing a variety of weak process which involve leptons and hadrons. However, the theory is not renormalizable – it makes sense only at the lowest order of perturbative expansion while radiative correction cannot be computed. So, a better theory is required.

Weak-force carriers

- We are going to show that the four-fermion contact interaction of the Fermi Lagrangian can be treated as an approximation of a theory where a massive complex vector field mediates the interaction.

- The complex vector field V^μ of mass M generated by the current J^μ is

$$V^\mu(x) = \int d^4x'\, D^{\mu\nu}(x-x')\, J_\nu(x'), \tag{46}$$

where the propagator $D^{\mu\nu}(x)$ equals

$$D^{\mu\nu}(x) = -\int \frac{d^4p}{(2\pi)^4} \frac{e^{-ipx}}{p^2 - M^2 + i0^+}\Big(g^{\mu\nu} - \frac{p^\mu p^\nu}{M^2}\Big). \tag{47}$$

- Substituting the propagator (47) into Eq. (48) and assuming that $|p_0| \ll M$ and $|\mathbf{p}| \ll M$, the integral in Eq. (48) is computed as

$$\begin{aligned} V^\mu(x) &= -\int d^4x' \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ipx}}{p^2 - M^2 + i0^+}\Big(g^{\mu\nu} - \frac{p^\mu p^\nu}{M^2}\Big) J_\nu(x') \\ &\approx \int d^4x' \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-x')}}{M^2}\, g^{\mu\nu} J_\nu(x') \approx \frac{1}{M^2}\int d^4x' \delta^{(4)}(x-x')\, J^\mu(x') = \frac{J^\mu(x)}{M^2}. \end{aligned} \tag{48}$$

- We see that the current-current Lagrangian (37) can be written as

$$\mathcal{L}^{\rm int}(x) = \frac{G_F}{\sqrt{2}}\, J_\mu^\dagger(x) J^\mu(x) = J_\mu^\dagger(x) V^\mu(x), \tag{49}$$

when the mass of the carrier of weak interaction equals $M = \big(\frac{\sqrt{2}}{G_F}\big)^{1/2} \approx 350$ GeV.

- Since the carriers of weak interactions have to be massive, it seems that V^μ cannot be a gauge field. As we will see, the Higgs mechanism opens up such a possibility.