Perturbative processes of QCD

Interaction processes of quarks and gluons where a characteristic momentum transfer is much bigger than the scale parameter of QCD that is $Q^2 \gg \Lambda_{\rm QCD}$ can be described in terms of perturbation expansion. The problem is, however, complicated as quarks and gluons are confined in hadrons.

Exclusive and inclusive processes

One should distinguish exclusive and inclusive reactions.

- A process is called exclusive if the final state is fully specified that is we know what are particles and their momenta in the final state.
- In an inclusive process no more than one particle of the final state and its momentum are specified.
- A deep inelastic scattering of an electron on proton is a typical example of the inclusive process. Only momentum of the final state electron is specified. Therefore, the cross section of deep inelastic scattering $\frac{d\sigma}{dE'd\Omega}$ is the cross section summed over all possible processes where the final state electron has the energy E' and is scattered into the solid angle Ω.

Factorization

- Factorization is a remarkable property of a large class of inclusive hadron cross sections which allows one to factorize the cross section into a hard part that is of high momentum transfer, which is usually a parton cross section, and the soft part which is usually taken into account by means of structure functions.
- Thanks to factorization hard hadron processes can be described in terms of parton cross sections.
- Proofs of factorization theorems are difficult and technically advanced.

$\mathbf{e^+e^- \to}$ hadrons

- The simplest perturbative process is $e^+e^- \to \textit{hadrons}$ at the CM energy $\sqrt{s} \gg \Lambda_{\rm QCD}$. The process proceeds through the subprocess $e^+e^- \to q\bar{q}$ shown in Fig. 1 followed by hadronization of quark and antiquark.

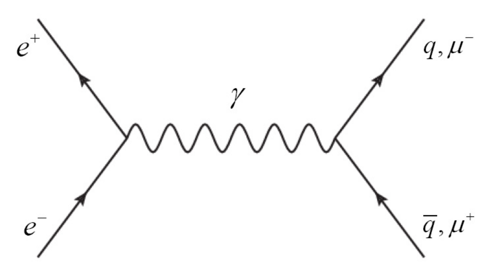

Figure 1: The diagram of the processes $e^+e^- \to q\bar{q}$ and $e^+e^- \to \mu^-\mu^+$

- When $\sqrt{s} \gg \Lambda_{\rm QCD}$ the relative momentum of quark and antiquark in the CM frame equals $\sqrt{s}$ if the quark masses are neglected. Therefore, the quark-antiquark interaction is weak and can be neglected.
- Since the processes $e^+e^- \to q\bar{q}$ and $e^+e^- \to \mu^-\mu^+$ are the described by the same diagram the ratio of cross sections equals

$$R = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^-\mu^+)} = \sum_{\rm colors} \sum_{\rm flavors} q^2 + \mathcal{O}(\alpha_s), \tag{1}$$

where q is a quark electric charge in units of e and $\alpha_s = \frac{g^2}{4\pi}$ is the strong coupling constant.

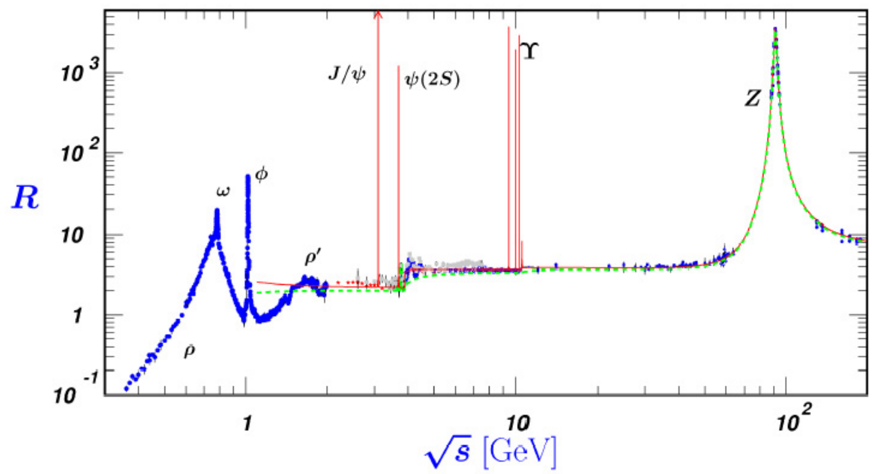

Figure 2: The experimental data on the ratio R

- If we take into account three lightest quarks u, d, s of three colors, the ratio R equals

$$R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2, \tag{2}$$

  which agrees with experimental data shown in Fig. 2 for 2 GeV $< \sqrt{s} <$ 3 GeV.

- Comparison of the theoretical value (1) with experimental data makes sense in the regions beyond the resonant production of vector mesons.

- If we take into account five lightest quarks u, d, s, c, b of three colors, the ratio R increases

$$R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9}\right) = \frac{11}{3} \approx 3.7, \tag{3}$$

  which again agrees with experimental data shown in Fig. 2 for 10 GeV $< \sqrt{s} <$ 60 GeV.

Deep Inelastic Scattering

- As explained in the Lecture II, the deep inelastic scattering is a lepton scattering on a nucleon with a momentum transfer much bigger than the nucleon mass.

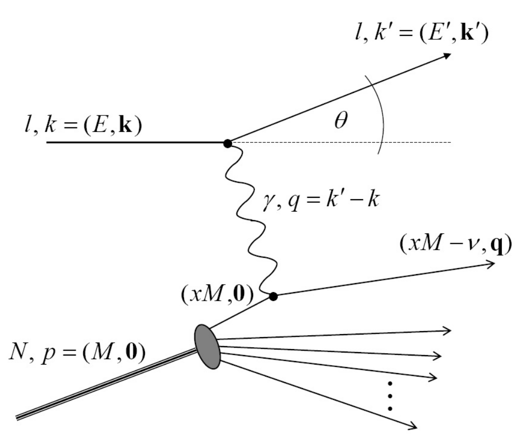

Figure 3: Deep inelastic scattering of electron or muon on a nucleon

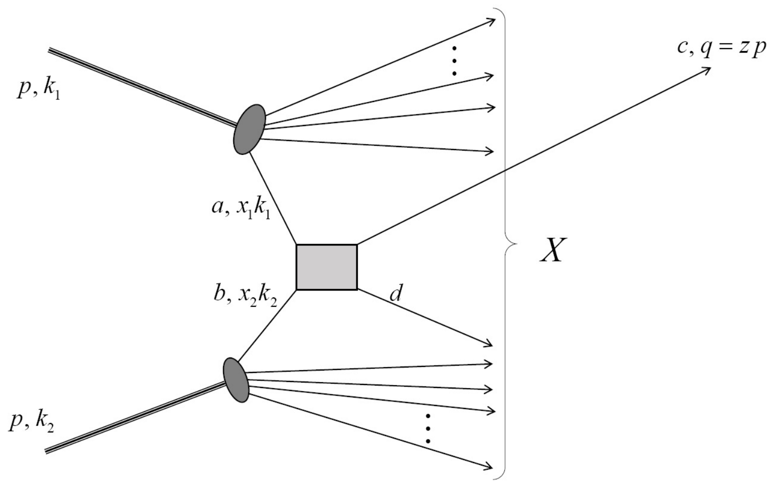

Figure 4: Schematic view of the inclusive process $pp \to \pi^+ X$

- The process of electron or muon deep inelastic scattering on a nucleon is illustrated in Fig. 3.
- At the leading order approximation the DIS cross section can be written as

$$\frac{d\sigma(k,p)}{dE'd\Omega} = \sum_f \int_0^1 dx\, F_f(x)\, \frac{d\hat{\sigma}^f(k,xp)}{dE'd\Omega}, \tag{4}$$

where $\frac{d\hat{\sigma}(k,xp)}{dE'd\Omega}$ is the lepton-quark cross section and $F_f(x)$ is the structure function which gives a probability to find a quark of a flavor f which carries a fraction x of proton momentum.

Drell-Yan processes

- A Drell-Yan process is an inclusive production lepton-antilepton pair with an invariant mass $M \gg \Lambda_{\rm QCD}$ in a hadron-hadron collision.
- As an example we consider $pp \to e^+e^-X$. A partonic subprocess is $q\bar{q} \to e^+e^-$ and it corresponds to the inverted diagram shown in Fig. 1.
- At the leading order approximation the cross section can be written as

$$\frac{d\sigma(k,p)}{dq^2} = \sum_f \sum_{\bar{f}} \int_0^1 dx \int_0^1 d\bar{x}\, F_f(x)\, \bar{F}_{\bar{f}}(\bar{x})\, \frac{d\hat{\sigma}^{f\bar{f}}(xk,\bar{x}p)}{dq^2}. \tag{5}$$

- Since the probability to find an antiquark in a proton is much smaller than the probability to find a quark, we have

$$\sigma(pp \to e^+e^-) \ll \sigma(p\bar{p} \to e^+e^-). \tag{6}$$

Inclusive production of hard hadrons in *pp* collisions

- Let us consider an inclusive process $pp \to \pi^+X$. We assume that the pion transverse momentum $p_T \gg \Lambda_{\rm QCD}$.
- The process is illustrated in Fig. 4. The rectangular box denotes the binary parton subprocess $ab \to cd$. In case of gluon-quark scattering, the box represents the diagrams shown in Fig. 5. The parton of four-momentum q hadronizes into the final state pion of momentum p.

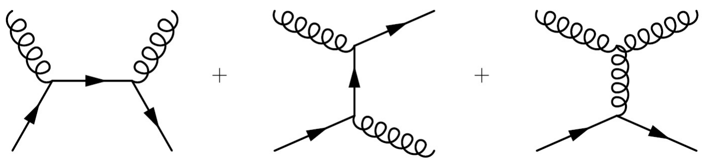

Figure 5: Diagrams representing quark-gluon scattering

- At the leading order approximation the cross section can be written as

$$\frac{d\sigma^{pp\to\pi^+X}}{d^3p} = \sum_{a,b,c} \int_0^1 dx_a \int_0^1 dx_b \int_0^1 dz\, F(x_a)\, F(x_b)\, \frac{d\hat{\sigma}^{ab\to cd}}{d^3q}\, D^{c\to\pi^+}(z), \tag{7}$$

where $D^{c\to\pi^+}(z)$ is the fragmentation function which gives a probability that a parton c of momentum zp hadronizes into pion of momentum p.

Production of jets

- A jet is a set of hadrons in a narrow cone which originate from the hadronization of a quark or gluon produced in high-energy collisions.
- In case of e^+e^- annihilation a whole accessible collision energy $\sqrt{s}$ goes into the production of two or more jets. Two and three jets occur via the parton subprocesses shown in Fig. 6.
- The cross section of two-jet production in e^+e^- annihilation at the leading order of α_s is

$$\sigma = \frac{4\pi\alpha^2}{s^2}\Big(\sum_{i=1}^{N_f} q_i^2 + \mathcal{O}(\alpha_s)\Big), \tag{8}$$

where N_f is a number of quark flavors of masses much smaller than $\sqrt{s}$ and q_i is the quark electric charge in units of e.

- A discovery of three-jet events in e^+e^- annihilation at PETRA in DESY in 1979 provided a direct evidence for the existence of gluons. A measurement of the distribution of jet relative angles confirmed that gluons are spin one particles.
- The cross section of inclusive jet production in pp collisions is

$$\sigma^{pp\to\text{jet}+X} = \sum_{a,b,c,d} \int_0^1 dx_a \int_0^1 dx_b \int d^3q\, F(x_a)\, F(x_b)\, \frac{d\hat{\sigma}^{ab\to cd}}{d^3q}. \tag{9}$$

As in case of inclusive hard hadron production there is a verity of parton subprocesses which lead to the jet production.

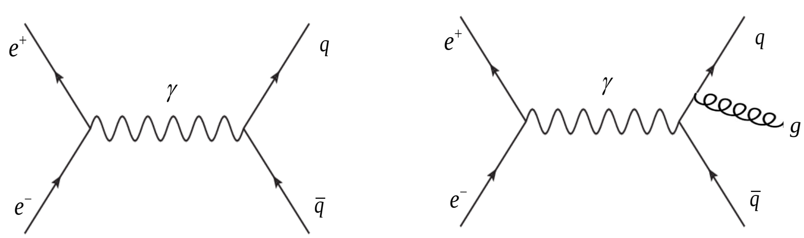

Figure 6: Parton subprocesses of e^+e^- annihilation into two (left diagram) and three (right diagram) jets

- Jets produced in relativistic heavy-ion collisions propagate across the matter which is also produced in the collisions. When the matter is in a deconfined phase, that is as a quark-gluon plasma, jet partons experience a significant energy loss which leads to the phenomenon of jet quenching observed in relativistic heavy-ion collisions

Discussion of perturbative processes

- The formulas of cross section of perturbative processes, which are given here, are written rather in the parton model then in QCD, as then the formulas are much more complicated.
- The cross sections of parton subprocesses, which enter the hadron cross sections, are determined by QCD in terms of Feynman diagrams.
- The structure and fragmentation functions, which enter the hadron cross sections, are inferred from experiment.
- The structure functions measured in one process can be used to predict another one. For example, knowing the structure functions from DIS one can predict the cross section of the Drell-Yan process.