

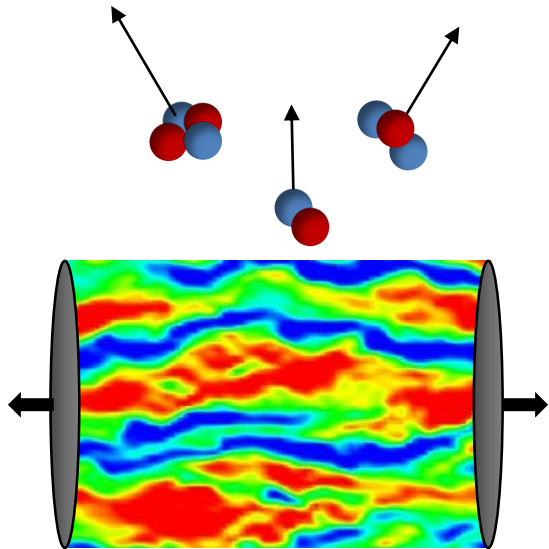
Production of light nuclei at colliders - coalescence vs. thermal model

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with **Sylwia Bazak & Patrycja Słoń**

Production of light nuclei at RHIC & LHC



baryonless matter

${}^2\text{H}$, ${}^2\bar{\text{H}}$, ${}^3\text{H}$, ${}^3\bar{\text{H}}$, ${}^3\text{He}$, ${}^3\bar{\text{He}}$, ${}^4\text{He}$, ${}^4\bar{\text{He}}$, ${}^3_{\Lambda}\text{H}$, ${}^3_{\Lambda}\bar{\text{H}}$

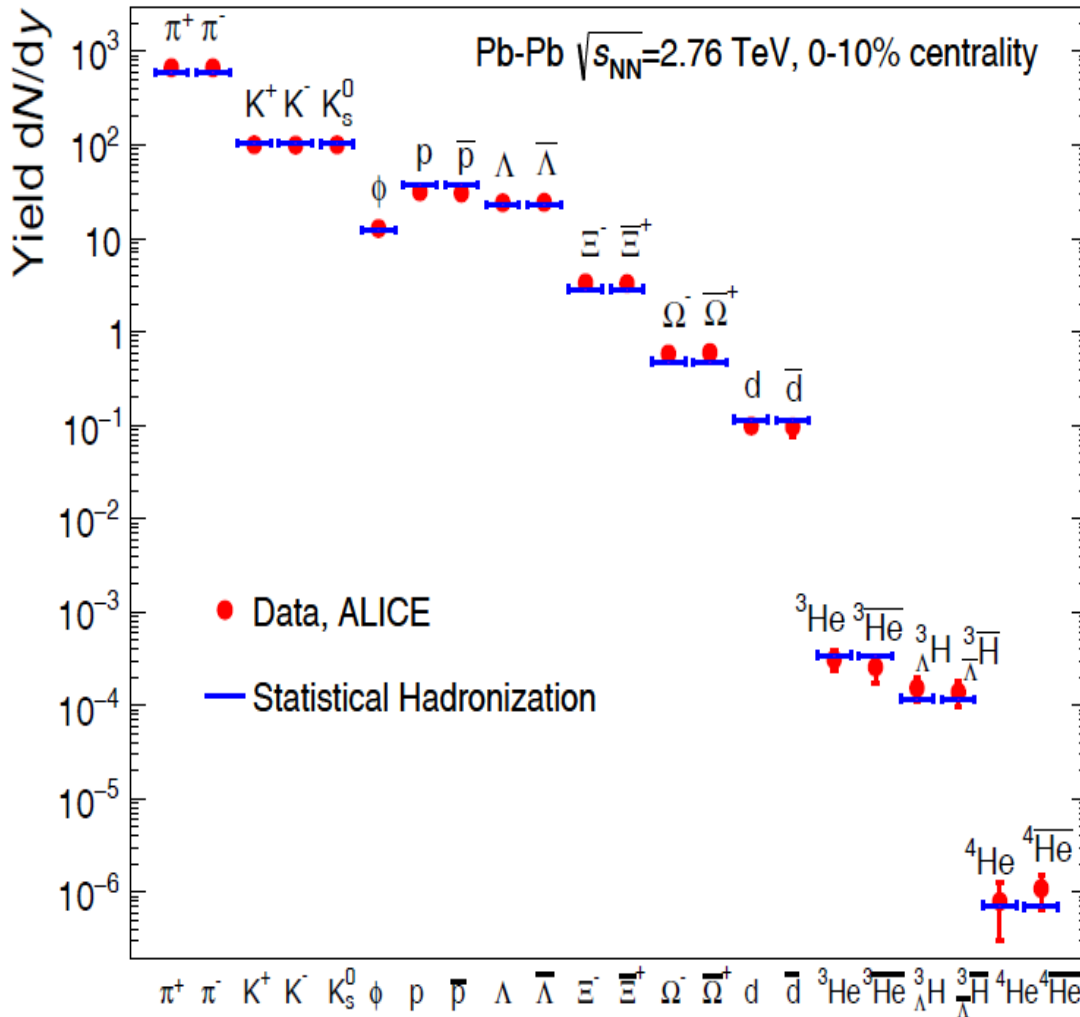
Genuine production!

Matter-antimatter symmetry!

Two approaches to production of light nuclei

- ▶ Coalescence model – final state interactions of nucleons
- ▶ Thermal model – direct production from thermalized hadron matter

Thermal model prediction



baryonless fireball

$$\text{Yield} \sim g e^{-\frac{m}{T}}$$

$$T = 156 \text{ MeV}$$

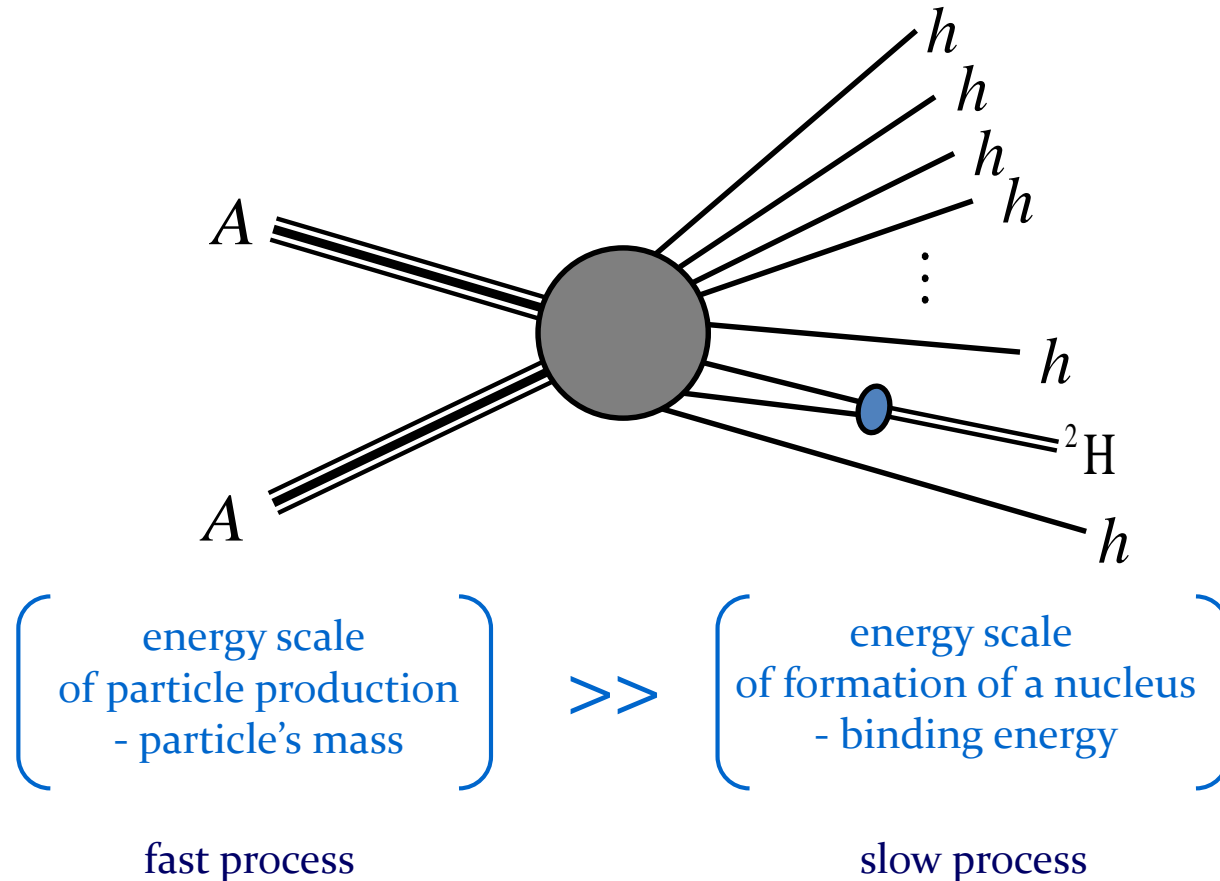
Can light nuclei exist in a fireball?

- ▶ Interparticle spacing in a hadron gas is about 1.5 fm at $T = 156$ MeV.
- ▶ Root mean square radius of a deuteron is 2.0 fm.
- ▶ Binding energy of a deuteron is $\varepsilon_B = 2.2$ MeV.
- ▶ A characteristic time of deuteron formation t is longer than 2 fm/c.
- ▶ A hadron gas at $T = 156$ MeV is essentially a classical system.

*Snowflakes in hell ?
or
Snowflakes from hell ?*



Final state interaction – conventional approach to production of light nuclei



S.T. Butler & C.A. Pearson, Phys. Rev. **129**, 836 (1963)

A. Schwarzschild & C. Zupancic, Phys. Rev. **129**, 854 (1963)

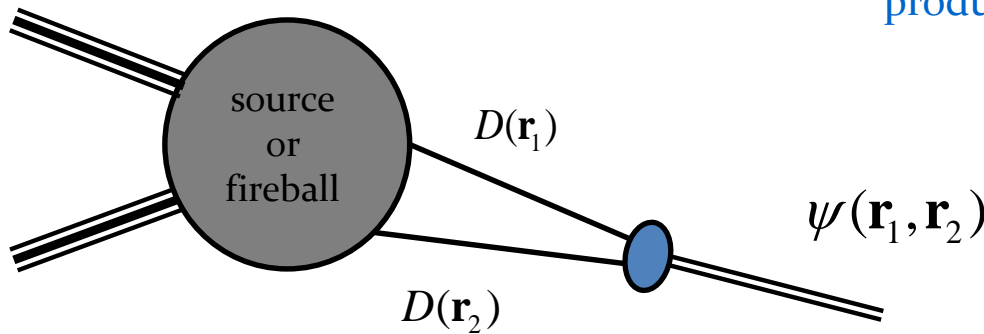
Factorization of production of nucleons and formation of a nucleus

Deuteron production cross section

$$\frac{d\sigma^D}{d^3\mathbf{P}_D} = W \frac{d\sigma^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p} \quad \frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

deuteron formation

production of np pair



spin factor

$$W = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 D(\mathbf{r}_1) D(\mathbf{r}_2) |\psi(\mathbf{r}_1, \mathbf{r}_2)|^2$$

H. Sato and K. Yazaki, Phys. Lett. B **98**, 153 (1981)

Deuteron formation rate vs. n-p correlation

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}} \varphi(\mathbf{r}) \quad \mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$W = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r} D_r(\mathbf{r}) |\varphi(\mathbf{r})|^2$$

$$D_r(\mathbf{r}) \equiv \int d^3\mathbf{R} D\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) D\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right)$$

n-p – correlation function

$$C(\mathbf{q}) = \int d^3\mathbf{r} D_r(\mathbf{r}) |\varphi_{\mathbf{q}}(\mathbf{r})|^2$$

$\varphi(\mathbf{r})$ – wave function of a bound state

$\varphi_{\mathbf{q}}(\mathbf{r})$ – wave function of a scattering state

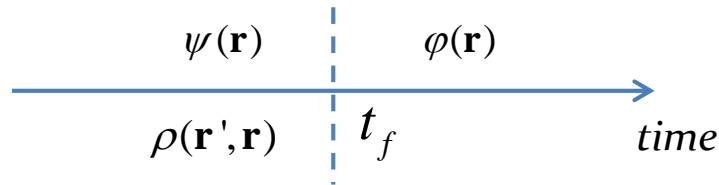
If emission time included

$$R_s \rightarrow \sqrt{R_s^2 + v^2 \tau^2}$$

St. Mrówczyński, Phys. Lett. B 277, 43 (1992)

Quantum-mechanical meaning of the formation rate formula

Sudden approximation



Transition matrix element

$$W = \left| \int d^3\mathbf{r} \psi^*(\mathbf{r}) \phi(\mathbf{r}) \right|^2 = \int d^3\mathbf{r} d^3\mathbf{r}' \phi^*(\mathbf{r}') \underbrace{\psi(\mathbf{r}') \psi^*(\mathbf{r})}_{\rho(\mathbf{r}', \mathbf{r})} \phi(\mathbf{r})$$

density matrix

$$W = \int d^3\mathbf{r} d^3\mathbf{r}' \phi^*(\mathbf{r}') \rho(\mathbf{r}', \mathbf{r}) \phi(\mathbf{r})$$

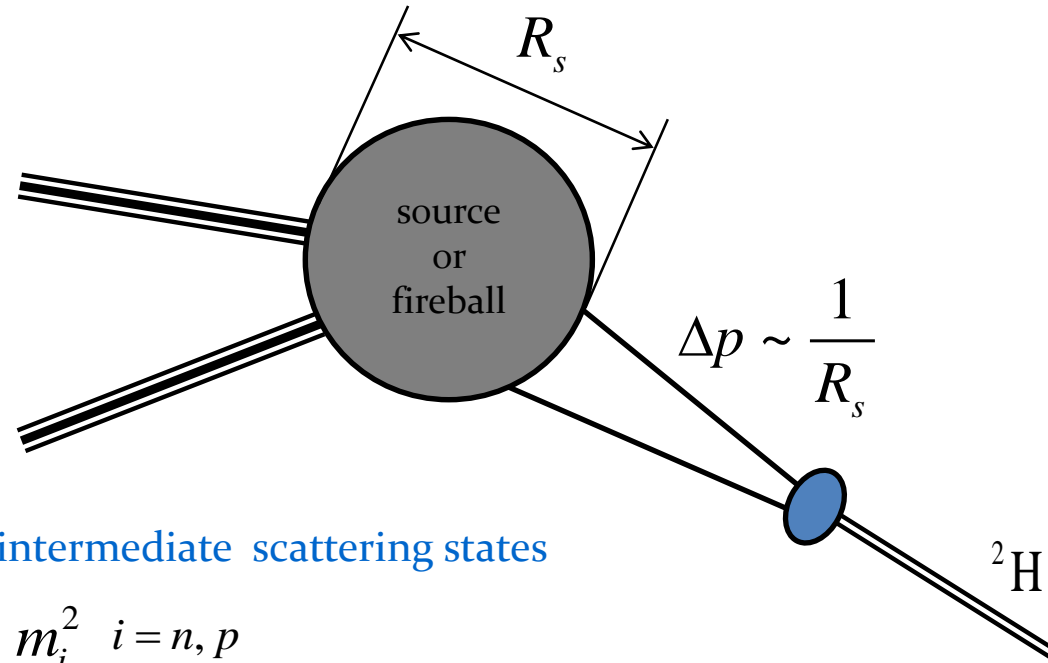
If density matrix is diagonal (random phase approximation)

$$\rho(\mathbf{r}', \mathbf{r}) = D(\mathbf{r}) \delta^{(3)}(\mathbf{r}' - \mathbf{r})$$

\Rightarrow

$$W = \int d^3\mathbf{r} D(\mathbf{r}) |\phi(\mathbf{r})|^2$$

Energy-momentum conservation



Nucleons are intermediate scattering states

$$E_i^2 - \mathbf{p}_i^2 \neq m_i^2 \quad i = n, p$$

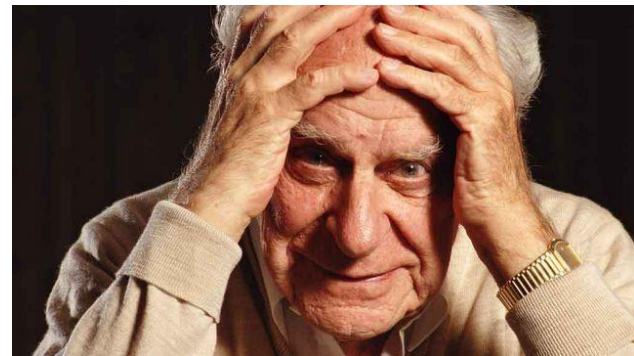
Energy-momentum conservation

$$\begin{cases} \mathbf{p}_p + \mathbf{p}_n = \mathbf{p}_D \\ E_p + E_n = E_D \end{cases}$$

St. Mrówczyński, J. Phys. G **11**, 1087 (1987)

Thermal vs. coalescence model

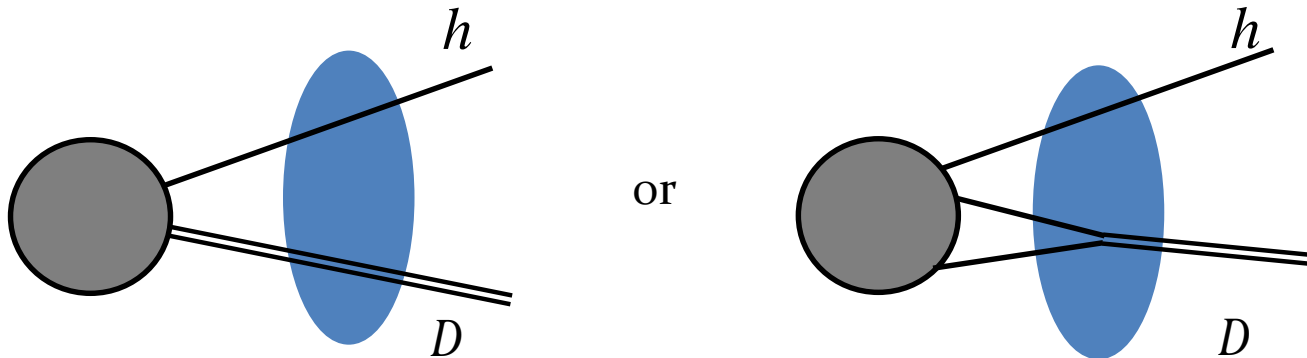
- ▶ The two models usually give quantitatively similar predictions.
- ▶ How to falsify one of the models experimentally?



Karl Popper 1902-1994

The first idea: h - D & D - D correlations

- ▶ Hadron-deuteron correlations carry information about a source of deuterons.
- ▶ A measurement of p - D & p - p correlation functions can falsify the thermal or coalescence model.

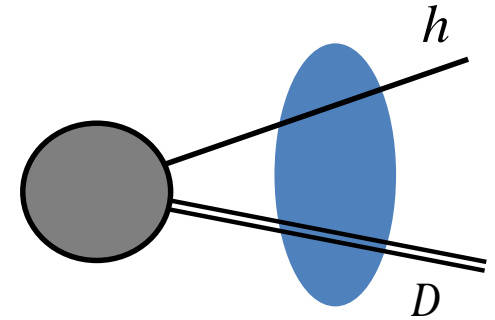


Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = R(\mathbf{p}_h, \mathbf{p}_D) \frac{dN_h}{d\mathbf{p}_h} \frac{dN_D}{d\mathbf{p}_D}$$



Theoretical formula

$$R(\mathbf{p}_h, \mathbf{p}_D) = \int d^3r_h d^3r_D D(\mathbf{r}_h) D(\mathbf{r}_D) |\psi(\mathbf{r}_h, \mathbf{r}_D)|^2$$

↑ ↑
distribution
of emission points

←
h-*D* wave function

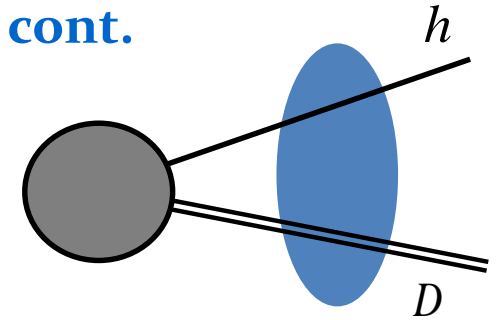
S.E. Koonin, Phys. Lett. B **70**, 43 (1977)

R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle cont.

Separation of CM and relative motion



$$\left\{ \begin{array}{l} \mathbf{R} \equiv \frac{m_D \mathbf{r}_D + m_h \mathbf{r}_h}{m_D + m_h} \\ \mathbf{r} \equiv \mathbf{r}_D - \mathbf{r}_h \end{array} \right. \quad \psi(\mathbf{r}_h, \mathbf{r}_D) = e^{i\mathbf{P}\mathbf{R}} \phi_{\mathbf{q}}(\mathbf{r})$$

$$R(\mathbf{q}) = \int d^3r D_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2$$

„Relative” source function

$$D_r(\mathbf{r}) \equiv \int d^3R D\left(\mathbf{R} - \frac{m_D}{m_D + m_h} \mathbf{r}\right) D\left(\mathbf{R} + \frac{m_h}{m_D + m_h} \mathbf{r}\right) = \left(\frac{1}{4\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_s^2}\right)$$

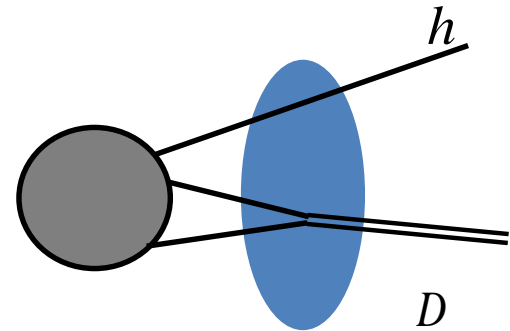
$$D(\mathbf{r}) = \left(\frac{1}{2\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right)$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = R(\mathbf{p}_h, \mathbf{p}_D) W_D \frac{dN_h}{d\mathbf{p}_h} \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$



Theoretical formula

$$R(\mathbf{p}_h, \mathbf{p}_D) W_D = \int d^3 r_h d^3 r_n d^3 r_p D(\mathbf{r}_h) D(\mathbf{r}_n) D(\mathbf{r}_p) |\psi_{hD}(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p)|^2$$

Deuteron formation rate

$$\frac{dN_D}{d\mathbf{p}_D} = W_D \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p} \quad \frac{1}{2} \mathbf{p}_D = \mathbf{p}_n = \mathbf{p}_p$$

$$W_D = \frac{3}{4} (2\pi)^3 \int d^3 \mathbf{r}_n d^3 \mathbf{r}_p D(\mathbf{r}_n) D(\mathbf{r}_p) |\psi_D(\mathbf{r}_n, \mathbf{r}_p)|^2 = \frac{3}{4} (2\pi)^3 \int d^3 r_{np} D_r(\mathbf{r}_{np}) |\phi_D(\mathbf{r}_{np})|^2$$

spin factor

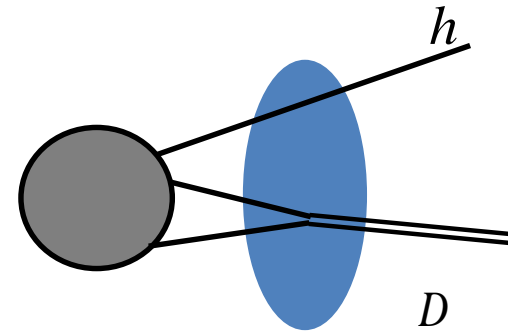
$$\psi_D(\mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_D(\mathbf{r}_{np})$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton cont

Separation of CM and relative motion

$$\left\{ \begin{array}{l} \mathbf{R} \equiv \frac{m_p \mathbf{r}_p + m_n \mathbf{r}_n + m_h \mathbf{r}_h}{m_p + m_n + m_h} \\ \mathbf{r}_{np} \equiv \mathbf{r}_p - \mathbf{r}_n \\ \mathbf{r} \equiv \mathbf{r}_h - \frac{m_p \mathbf{r}_p + m_n \mathbf{r}_n}{m_p + m_n} \end{array} \right.$$



$$\psi(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_{\mathbf{q}}(\mathbf{r}) \varphi_D(\mathbf{r}_{np})$$

$$R(\mathbf{q}) = \frac{1}{W_D} \int d^3 R d^3 r_{np} d^3 r D(\mathbf{r}_h) D(\mathbf{r}_n) D(\mathbf{r}_p) |\phi_{\mathbf{q}}(\mathbf{r})|^2 |\varphi_D(\mathbf{r}_{np})|^2$$

For Gaussian source

$$R(\mathbf{q}) = \int d^3 r D_{3r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

$$D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2} \right)$$

Thermal vs. coalescence model

Thermal model

$$R(\mathbf{q}) = \int d^3 r D_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

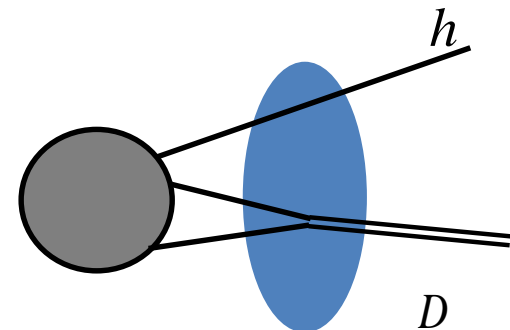
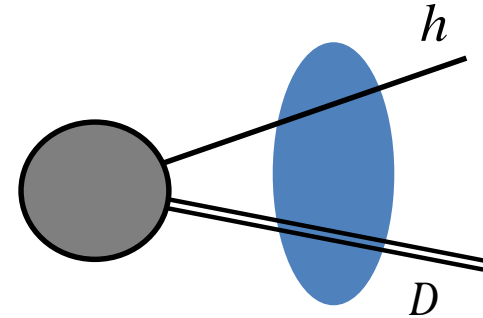


$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2} \right)$$

$$D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2} \right)$$

Coalescence model

$$R(\mathbf{q}) = \int d^3 r D_{3r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$



h -D correlation function

The wave function in scattering asymptotic state

$$\phi_{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{q}\mathbf{r}} + f(\mathbf{q}) \frac{e^{iqr}}{r}$$

The s -wave amplitude

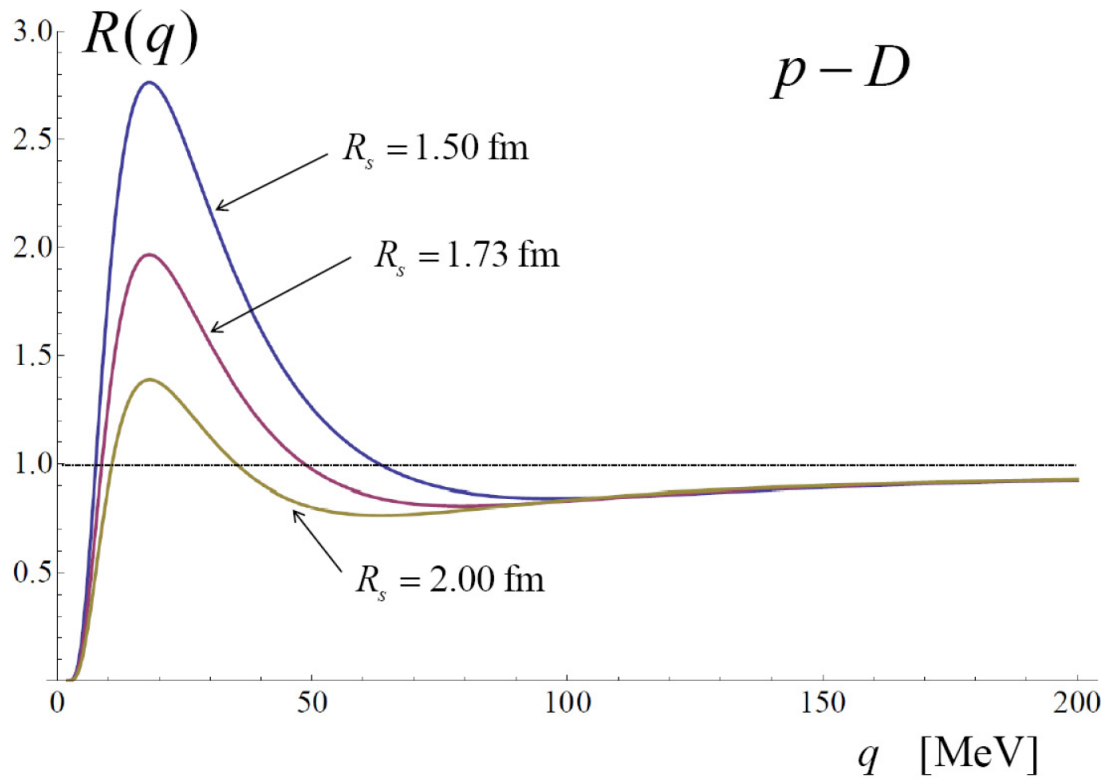
$$f(\mathbf{q}) = -\frac{a}{1 - iqa} \quad a - \text{scattering length}$$

Coulomb interaction via Gamow factor

$$G(q) = \pm \frac{2\pi}{a_B q} \frac{1}{\exp\left(\pm \frac{2\pi}{a_B q}\right) - 1} \quad a_B = \frac{1}{\mu\alpha} - \text{Bohr radius}$$

R. Lednicky and V.L. Lyuboshitz, *Yad. Fiz.* **35**, 1316 (1982)

p-D correlation functions



$$R(q) = \frac{1}{3} R_{1/2}(q) + \frac{2}{3} R_{3/2}(q)$$

$$a_{1/2} = 4.0 \text{ fm}$$

$$a_{3/2} = 11.0 \text{ fm}$$

$$2.00 = \sqrt{\frac{4}{3}} 1.73 = \frac{4}{3} 1.50$$

R_s from *p-D* correlation function vs. *R_s* from *p-p* correlation function

Deuteron-deuteron correlation function

Direct production

$$R(\mathbf{q}) = \int d^3 r D_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$



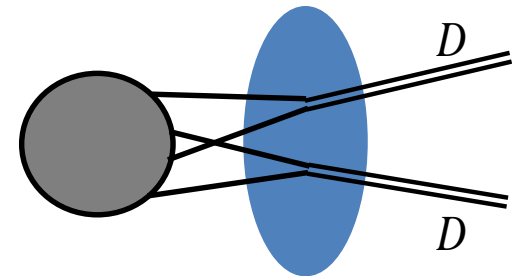
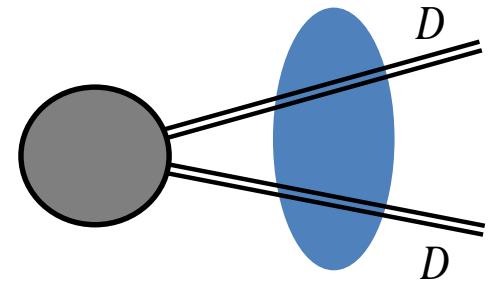
$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2} \right)$$

$$D_{4r}(\mathbf{r}) = \left(\frac{1}{2\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R^2} \right)$$

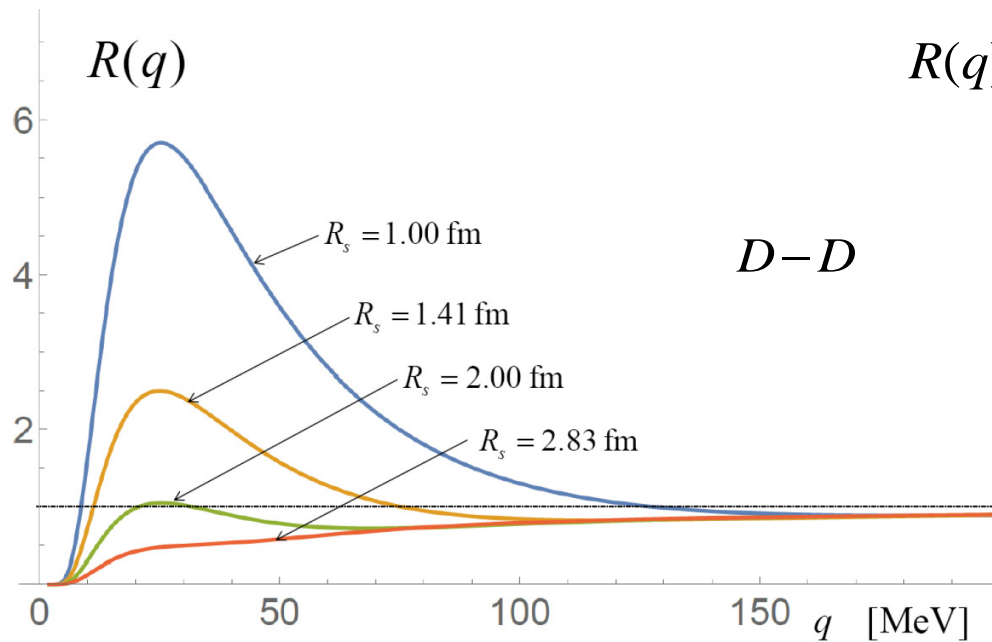
Final state interaction



$$R(\mathbf{q}) = \int d^3 r D_{4r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$



Deuteron-deuteron correlation function



$$R(q) = \frac{1}{9} R_0(q) + \frac{3}{9} R_1(q) + \frac{5}{9} R_2(q)$$

$$a_0 = (10.2 + 0.2i) \text{ fm}$$


$$a_2 = 7.5 \text{ fm}$$

$$2.83 = \sqrt{2} \cdot 2.00 = (\sqrt{2})^2 \cdot 1.41 = (\sqrt{2})^3 \cdot 1.00$$

R_s from $D-D$ correlation function vs. R_s from $p-p$ & $p-D$ correlation function

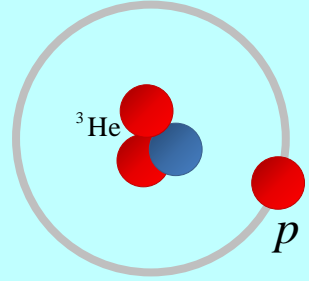
The second idea: ${}^4\text{He}$ vs. ${}^4\text{Li}$

${}^4\text{He}$



$r_{\text{RMS}} = 1.68 \text{ fm}$
 $\varepsilon_B = 28.3 \text{ MeV}$
 $m = 3727.4 \text{ MeV}$
 $s = 0$

${}^4\text{Li}$



${}^4\text{Li} \rightarrow {}^3\text{He} + p$
 $\Gamma = 6 \text{ MeV}$
 $m = m_{{}^3\text{He}} + m_p + 4.1 \text{ MeV}$
 $m = 3749.7 \text{ MeV}$
 $s = 2$

▶ Thermal model $\frac{\text{Yield}({}^4\text{Li})}{\text{Yield}({}^4\text{He})} = \frac{2S_{\text{Li}} + 1}{2S_{\text{He}} + 1} = 5$

▶ Coalescence model $\frac{\text{Yield}({}^4\text{Li})}{\text{Yield}({}^4\text{He})} = \frac{W_{\text{Li}}}{W_{\text{He}}}$

S. Bazak & St. Mrówczyński, Mod. Phys. Lett. A **33**, 1850142 (2018)

S. Bazak & St. Mrówczyński, Eur. Phys. J. A **56**, 193 (2020)

Formation rates of ${}^4\text{He}$ & ${}^4\text{Li}$

$$W = g_S g_I (2\pi)^9 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \int d^3\mathbf{r}_3 d^3\mathbf{r}_4 D(\mathbf{r}_1) D(\mathbf{r}_2) D(\mathbf{r}_3) D(\mathbf{r}_4) |\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)|^2$$

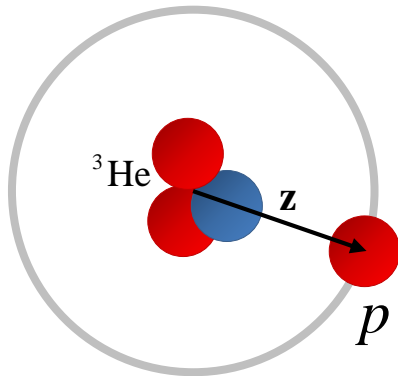
▶ ${}^4\text{He}$



$$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

$$|\psi_{\text{He}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)|^2 \sim \exp\left[-\alpha(\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2)\right]$$

▶ ${}^4\text{Li}$



J. C. Bergstrom, Nucl. Phys. A **327**, 458 (1979)

$$\mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3)$$

$$|\psi_{\text{Li}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)|^2 \sim \exp\left[-\beta(\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{23}^2)\right] \mathbf{z}^4 \exp(-\gamma \mathbf{z}^2) |Y_{lm}(\Omega_{\mathbf{z}})|^2$$

Formation rates of ${}^4\text{He}$ & ${}^4\text{Li}$

$$W = g_S g_I (2\pi)^9 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \int d^3\mathbf{r}_3 d^3\mathbf{r}_4 D(\mathbf{r}_1) D(\mathbf{r}_2) D(\mathbf{r}_3) D(\mathbf{r}_4) |\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)|^2$$

Source function

$$D(\mathbf{r}_i) = \frac{1}{(2\pi R_s^2)^{3/2}} \exp\left(-\frac{\mathbf{r}_i^2}{2R_s^2}\right) \quad i = 1, 2, 3, 4$$

If emission time included

$$R_s \rightarrow \sqrt{R_s^2 + v^2 \tau^2}$$

Jacobi variables

$$\left\{ \begin{array}{l} \mathbf{R} \equiv \frac{1}{4}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4) \\ \mathbf{x} \equiv \mathbf{r}_2 - \mathbf{r}_1 \\ \mathbf{y} \equiv \mathbf{r}_3 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2) \\ \mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) \end{array} \right.$$

$$\blacktriangleright \quad \mathbf{r}_1^2 + \mathbf{r}_2^2 + \mathbf{r}_3^2 + \mathbf{r}_4^2 = 4\mathbf{R}^2 + \frac{1}{2}\mathbf{x}^2 + \frac{2}{3}\mathbf{y}^2 + \frac{3}{4}\mathbf{z}^2$$

$$\blacktriangleright \quad \mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2 = 2\mathbf{x}^2 + \frac{8}{3}\mathbf{y}^2 + 3\mathbf{z}^2$$

$$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

Fully analytic calculations
are possible!

Formation rates of ${}^4\text{He}$ & ${}^4\text{Li}$

$$\triangleright W_{\text{He}} = \frac{\pi^{9/2}}{2^{9/2}} \frac{1}{\left(R_s^2 + R_\alpha^2\right)^{9/2}}$$

$$\triangleright W_{\text{Li}} = \frac{3\pi^{9/2}}{2^{11/2}} \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \frac{R_s^4}{\left(R_s^2 + \frac{1}{2}R_c^2\right)^3 \left(R_s^2 + \frac{4}{7}R_{\text{Li}}^2 - \frac{3}{7}R_c^2\right)^{7/2}} \begin{pmatrix} l=1 \\ l=2 \end{pmatrix}$$

Since ${}^4\text{Li}$ is $J^P = 2^-$ then $l=1$.

R_s – root mean square radius of the source

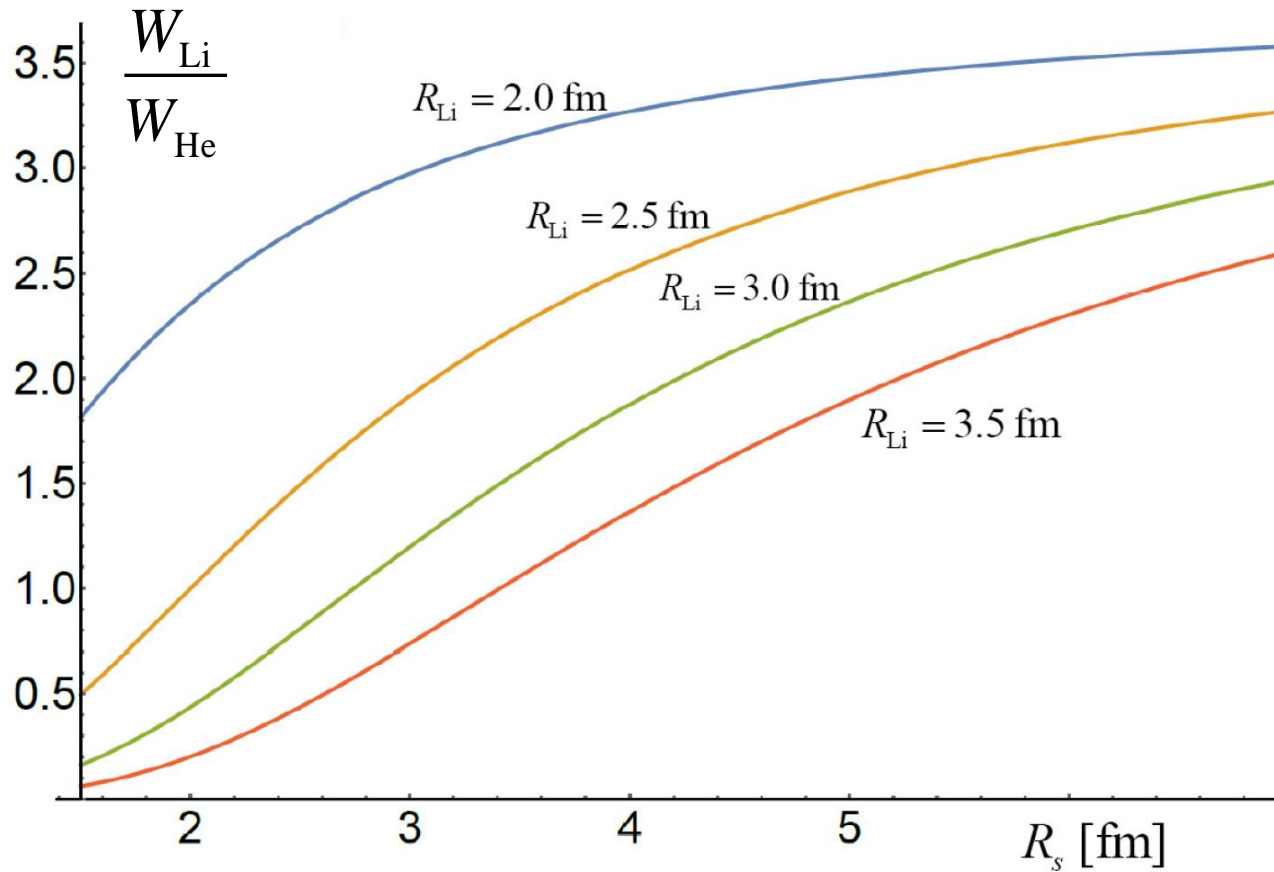
R_α – root mean square radius of ${}^4\text{He}$

R_{Li} – root mean square radius of ${}^4\text{Li}$

R_c – root mean square radius of ${}^3\text{He}$ cluster in ${}^4\text{Li}$

Ratio of yields of ${}^4\text{Li}$ to ${}^4\text{He}$

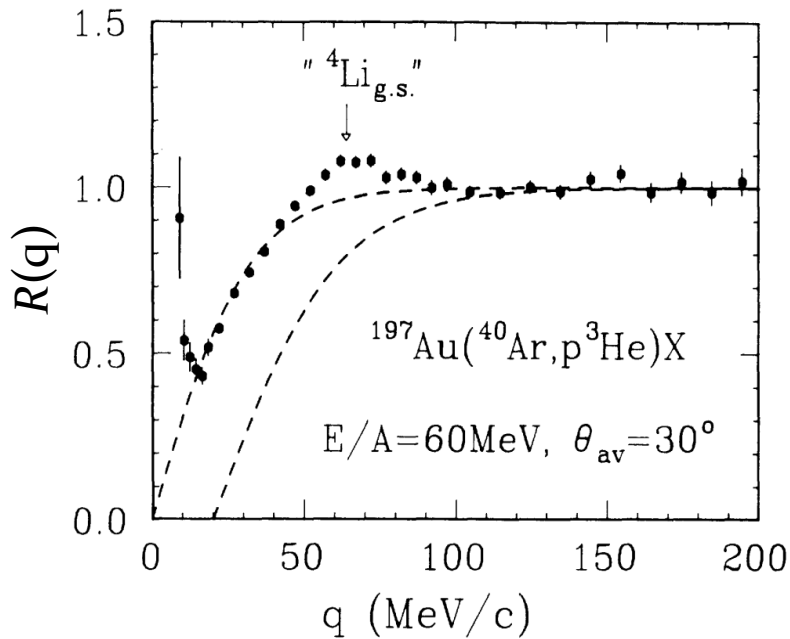
In the thermal model the ratio equals 5.



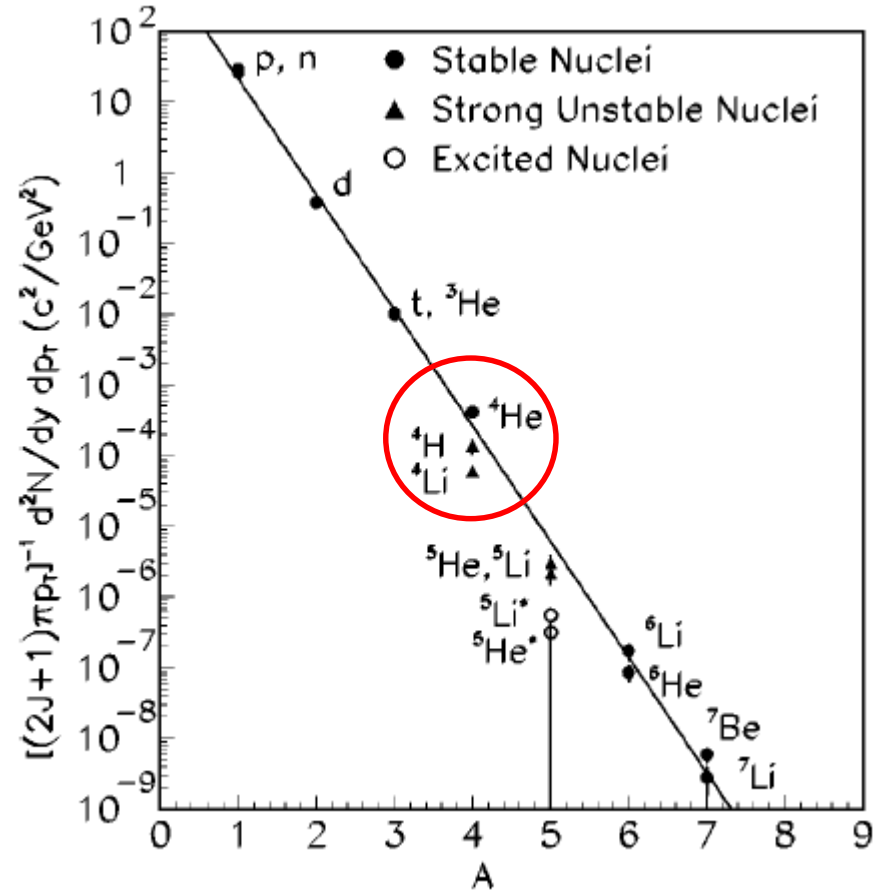
How to observe ${}^4\text{Li}$?

Measurement of the correlation function of ${}^3\text{He}$ - p is needed

${}^{197}\text{Au} + {}^{197}\text{Pt}$ @ AGS

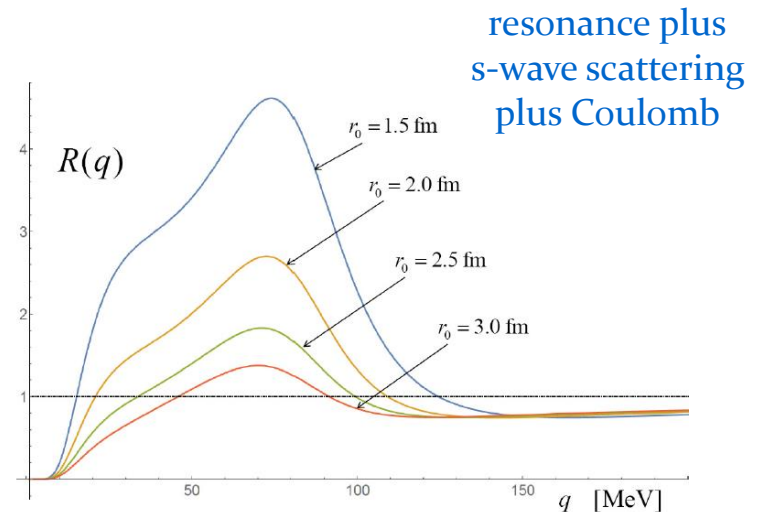
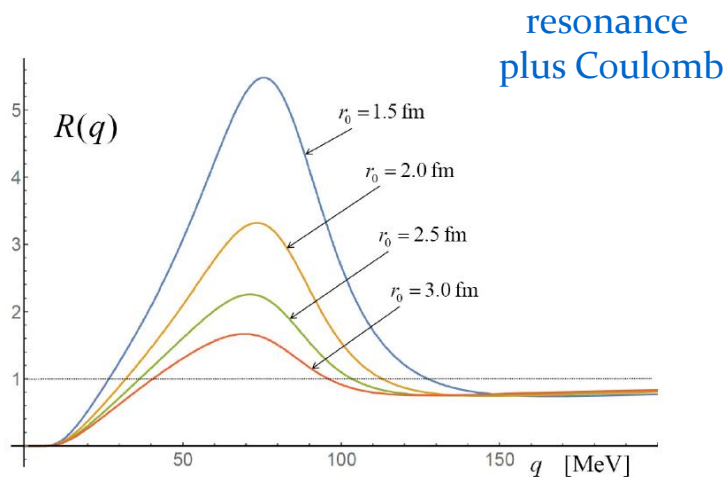
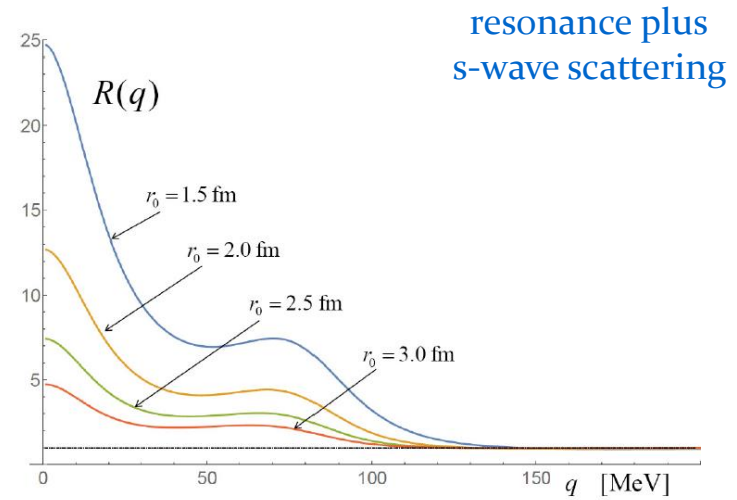
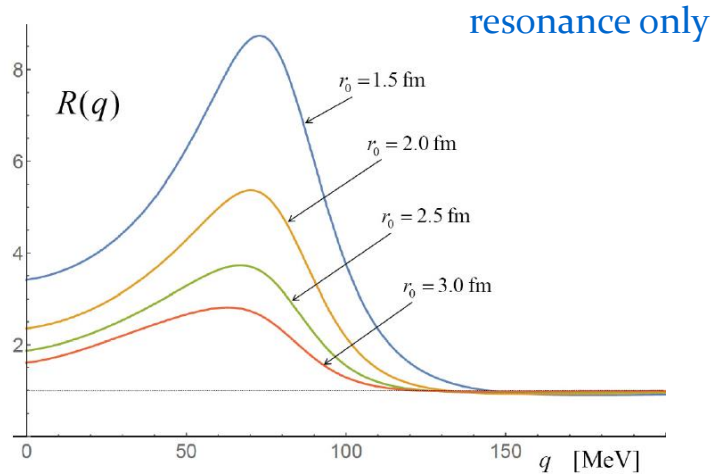


J. Pochodzala et al. Phys. Rev. C **35**, 1695 (1987)



T. A. Armstrong et al. Phys. Rev. C **65**, 014906 (2001)

Correlation function p - ^3He



How to measure yield of ${}^4\text{Li}$

$$\frac{dN_{\text{Li}}}{d\mathbf{p}} = S_R \frac{dN_p}{d\mathbf{p}} \frac{dN_{{}^3\text{He}}}{d\mathbf{p}}$$

\mathbf{p} - momentum per nucleon

$$S_R \equiv \int d^3q R_R(\mathbf{q})$$

correlation function where only the ${}^4\text{Li}$ resonance contributes

Conclusions

p - D & D - D correlations

- ▶ Hadron-deuteron and deuteron-deuteron correlations carry information about source of deuterons.
- ▶ Measurement of p - p , p - D & D - D correlation functions can tell us whether deuterons are directly emitted from a fireball like protons or deuterons are formed due to final state interactions.
- ▶ p - D and D - D correlation functions show a sufficient sensitivity to a size of particle source to falsify the thermal or coalescence model.

${}^4\text{He}$ vs. ${}^4\text{Li}$

- ▶ The thermal and coalescence models give different predictions on the ratio of yields of ${}^4\text{Li}$ to ${}^4\text{He}$.
- ▶ In the thermal model the ratio of yields is independent of collision centrality.
- ▶ In the coalescence model the ratio is maximal for central collisions and rapidly decreases when one goes to peripheral collisions.
- ▶ Since ${}^4\text{Li}$ can be observed through the correlation function of ${}^3\text{He}$ - p , the correlation needs to be measured.