Production of Light Nuclei in Relativistic Heavy-Ion Collisions

- how to falsify thermal model

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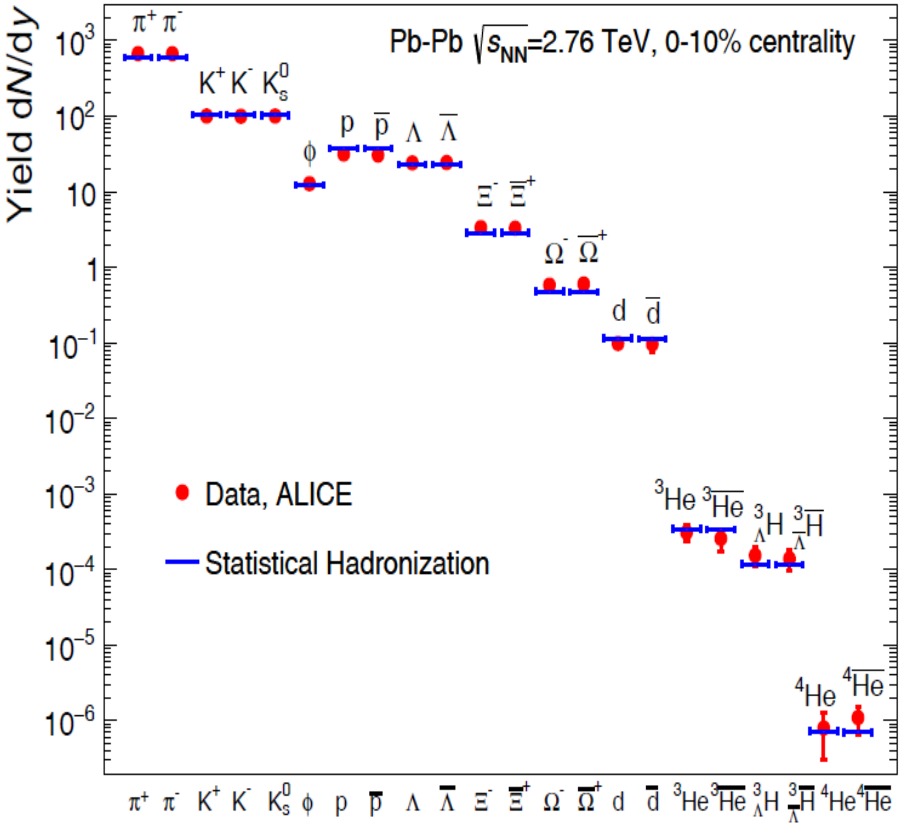
Background

- Production of ^2H, $^2\overline{H}$, ^3H, $^3\overline{H}$, ^3He, $^3\overline{He}$, ^4He, $^4\overline{He}$, $^3_\Lambda H$, $^3_{\overline{\Lambda}}\overline{H}$ is observed in midrpidity at RHIC & LHC.

- Thermal model properly describes yields of light nuclei.



baryonless fireball

$$\text{Yield} \sim g\, e^{-\frac{m}{T}}$$

$$T = 156 \text{ MeV}$$

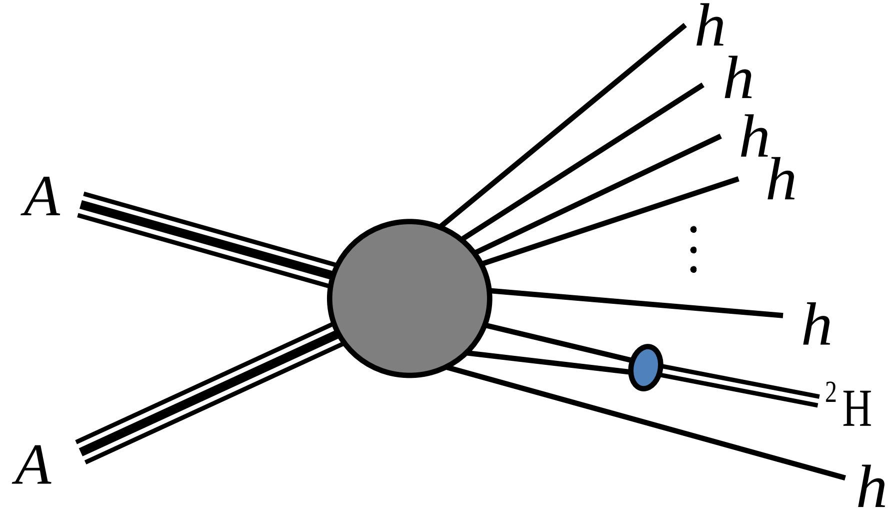
A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature **561**, 321 (2018)

Can light nuclei exist in a fireball?

- Interparticle spacing in a hadron gas is about 1.5 fm at T = 156 MeV.
- Root mean square radius of a deuteron is 2.0 fm.
- Binding energy of a deuteron is 2.2 MeV.
- A hadron gas at T = 156 MeV is essentially a classical system.

- Snowflakes in hell ?
- No, snowflakes from hell.

Final state interaction – conventional approach to production of light nuclei



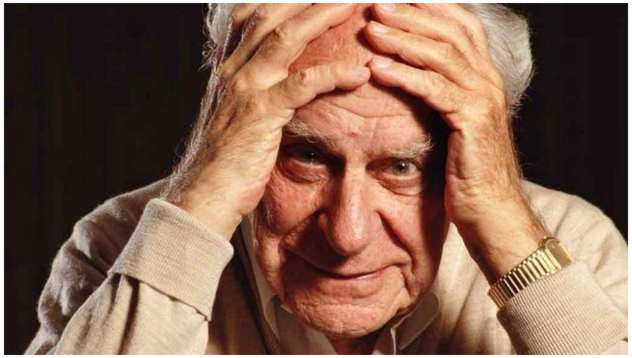
energy scale of particle production - particle's mass	>>	energy scale of formation of a nucleus - binding energy
fast process		slow process

S.T. Butler & C.A. Pearson, Phys. Rev. **129**, 836 (1963)
A. Schwarzschild & C. Zupancic, Phys. Rev. **129**, 854 (1963)
H. Sato and K. Yazaki, Phys. Lett. B **98** , 153 (1981)

Thermal vs. coalescence model

- The two models usually give quantitatively similar predictions.

- How to falsify one of the models experimentally?

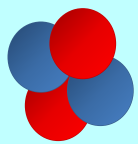


Karl Popper 1902-1994

St. Mrówczyński, Acta Phys. Pol. B **48**, 707 (2017)

The first idea: ^{4}He vs. ^{4}Li

$^{4}\mathrm{He}$



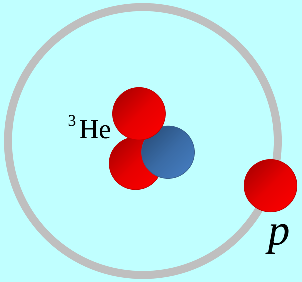
$r_{\mathrm{RMS}} = 1.68\ \mathrm{fm}$

$\varepsilon_B = 28.3\ \mathrm{MeV}$

$m = 3727.4\ \mathrm{MeV}$

$s = 0$

$^{4}\mathrm{Li}$



$^{4}\mathrm{Li} \rightarrow {}^{3}\mathrm{He} + p$

$\Gamma = 6\ \mathrm{MeV}$

$m = m_{^3\mathrm{He}} + m_p + 4.1\ \mathrm{MeV}$

$m = 3749.7\ \mathrm{MeV}$

$s = 2$

- Thermal model $$\frac{\mathrm{Yield}(^4\mathrm{Li})}{\mathrm{Yield}(^4\mathrm{He})} = \frac{2S_{\mathrm{Li}}+1}{2S_{\mathrm{He}}+1} = 5$$

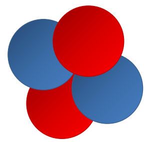
- Coalescence model $$\frac{\mathrm{Yield}(^4\mathrm{Li})}{\mathrm{Yield}(^4\mathrm{He})} = \frac{W_{\mathrm{Li}}}{W_{\mathrm{He}}}$$

S. Bazak & St. Mrówczyński, Mod. Phys. Letters A **33**, 1850142 (2018)

Formation rates of ^{4}He & ^{4}Li

$$W = g_S g_I (2\pi)^9 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \int d^3\mathbf{r}_3 d^3\mathbf{r}_4 \, D(\mathbf{r}_1)D(\mathbf{r}_2)D(\mathbf{r}_3)D(\mathbf{r}_4) \left|\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2$$

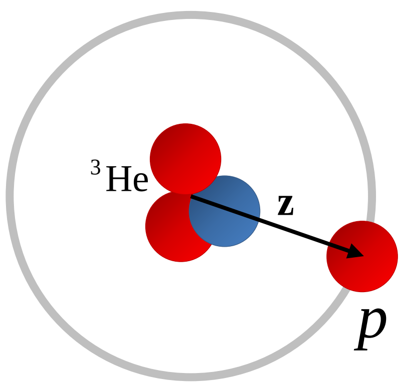
- ^{4}He



$$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

$$\left|\psi_{\mathrm{He}}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2 \sim \exp\left[-\alpha(\mathbf{r}_{12}^2+\mathbf{r}_{13}^2+\mathbf{r}_{14}^2+\mathbf{r}_{23}^2+\mathbf{r}_{24}^2+\mathbf{r}_{34}^2)\right]$$

- ^{4}Li



J. C. Bergstrom, Nucl. Phys. A **327**, 458 (1979)

$$\mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)$$

$$\left|\psi_{\mathrm{Li}}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2 \sim \exp\left[-\beta(\mathbf{r}_{12}^2+\mathbf{r}_{13}^2+\mathbf{r}_{23}^2)\right]\mathbf{z}^4 \exp\left(-\gamma \mathbf{z}^2\right)\left|Y_{lm}(\Omega_{\mathbf{z}})\right|^2$$

Formation rates of ^{4}He & ^{4}Li

$$W = g_S g_I (2\pi)^9 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \int d^3\mathbf{r}_3 d^3\mathbf{r}_4 \, D(\mathbf{r}_1)D(\mathbf{r}_2)D(\mathbf{r}_3)D(\mathbf{r}_4)\left|\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2$$

Source function

$$D(\mathbf{r}_i) = \frac{1}{(2\pi R_s^2)^{3/2}} \exp\left(-\frac{\mathbf{r}_i^2}{2R_s^2}\right) \qquad i = 1,2,3,4$$

If emission time included

$$R_s \to \sqrt{R_s^2 + v^2\tau^2}$$

Jacobi variables

$$\begin{cases} \mathbf{R} \equiv \frac{1}{4}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4) \\ \mathbf{x} \equiv \mathbf{r}_2 - \mathbf{r}_1 \\ \mathbf{y} \equiv \mathbf{r}_3 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2) \\ \mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) \end{cases}$$

- $$\mathbf{r}_1^2 + \mathbf{r}_2^2 + \mathbf{r}_3^2 + \mathbf{r}_4^2 = 4\mathbf{R}^2 + \frac{1}{2}\mathbf{x}^2 + \frac{2}{3}\mathbf{y}^2 + \frac{3}{4}\mathbf{z}^2$$
- $$\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2 = 2\mathbf{x}^2 + \frac{8}{3}\mathbf{y}^2 + 3\mathbf{z}^2$$

  $$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

Fully analytic calculations are possible!

Formation rates of ^{4}He & ^{4}Li

$$W_{\mathrm{He}} = \frac{\pi^{9/2}}{2^{9/2}} \frac{1}{\left(R_s^2 + R_\alpha^2\right)^{9/2}}$$

$$W_{\mathrm{Li}} = \frac{3\pi^{9/2}}{2^{11/2}} \begin{pmatrix} \frac{5}{2} \\ 1 \end{pmatrix} \frac{R_s^4}{\left(R_s^2 + \frac{1}{2} R_c^2\right)^3 \left(R_s^2 + \frac{4}{7} R_{\mathrm{Li}}^2 - \frac{3}{7} R_c^2\right)^{7/2}} \qquad \begin{pmatrix} l = 1 \\ l = 2 \end{pmatrix}$$

R_s – root mean square radius of the source

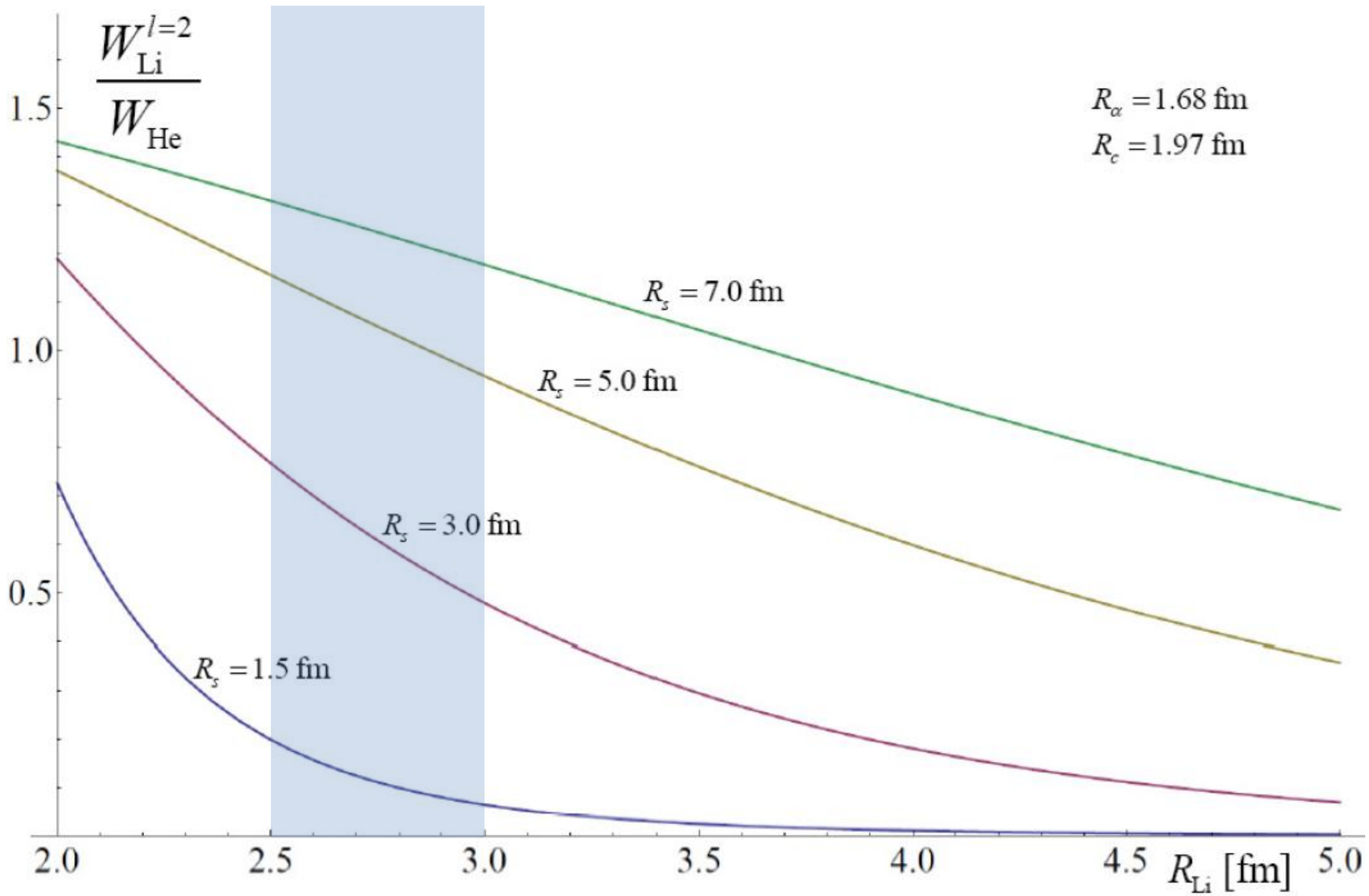
R_α – root mean square radius of ^{4}He

R_{Li} – root mean square radius of ^{4}Li

R_c – root mean square radius of ^{3}He cluster in ^{4}Li

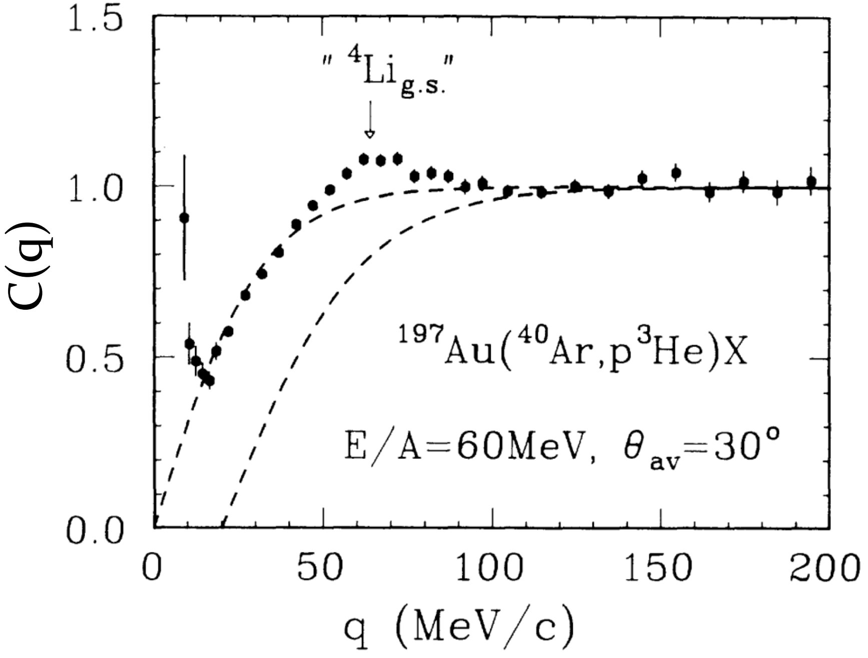
Ratio of yields of ^{4}Li to ^{4}He

In the thermal model the ratio equals 5.



How to observe ^{4}Li?

Measurement of the correlation function of ^{3}He-*p* is needed



J. Pochodzala et al. Phys. Rev. C **35**, 1695 (1987)

The second idea: *h*-*D* correlations

- Hadron-deuteron correlations carry information about a source of deuterons.

- A measurement of K^- -*D* or *p*-*D* correlation functions can falsify the thermal or coalescence model.

P. Słoń & St. Mrówczyński, arXiv:1904.08320 [nucl-th]

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle

Experimental definition

$$\frac{dN_{\pi D}}{d\mathbf{p}_\pi d\mathbf{p}_D} = R(\mathbf{p}_\pi, \mathbf{p}_D)\,\frac{dN_\pi}{d\mathbf{p}_\pi}\,\frac{dN_D}{d\mathbf{p}_D}$$

pion a generic hadron

Theoretical formula

$$R(\mathbf{p}_\pi, \mathbf{p}_D) = \int d^3r_\pi\, d^3r_D\, D(\mathbf{r}_\pi)\, D(\mathbf{r}_D) \left|\psi(\mathbf{r}_\pi, \mathbf{r}_D)\right|^2$$

distribution of emission points

π-*D* wave function

S.E. Koonin, Phys. Lett. B **70**, 43 (1977)
R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle cont.

Center-of-mass variables

$$\begin{cases} \mathbf{R} = \dfrac{m_\pi \mathbf{r}_\pi + m_D \mathbf{r}_D}{m_\pi + m_D} \\ \mathbf{r}_{\pi D} = \mathbf{r}_\pi - \mathbf{r}_D \end{cases} \qquad \begin{cases} \mathbf{P} = (\mathbf{p}_1 + \mathbf{p}_2) \\ \mathbf{q} = \dfrac{m_\pi \mathbf{p}_D - m_D \mathbf{p}_\pi}{m_\pi + m_D} \end{cases} \qquad \psi(\mathbf{r}_\pi, \mathbf{r}_D) = e^{i\mathbf{PR}} \phi_{\mathbf{q}}(\mathbf{r}_{\pi D})$$

$$R(\mathbf{q}) = \int d^3 r_{\pi D} D_r(\mathbf{r}_{\pi D}) \left| \phi_{\mathbf{q}}(\mathbf{r}_{\pi D}) \right|^2$$

`Relative' source

$$D_r(\mathbf{r}_{\pi D}) \equiv \int d^3\mathbf{R} \; D\left(\mathbf{R} - \frac{m_\pi}{m_\pi + m_D} \mathbf{r}_{\pi D} \right) D\left(\mathbf{R} + \frac{m_D}{m_\pi + m_D} \mathbf{r}_{\pi D} \right)$$

$$D(\mathbf{r}) = \left(\frac{1}{2\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R^2} \right) \quad \Rightarrow \quad D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2} \right)$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{\pi D}}{d\mathbf{p}_\pi d\mathbf{p}_D} = R(\mathbf{p}_\pi, \mathbf{p}_D)\, A \frac{dN_\pi}{d\mathbf{p}_\pi} \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$

$$\frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

Deuteron formation rate

$$\frac{dN_D}{d\mathbf{p}_D} = A \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$

$$A = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}_n d^3\mathbf{r}_p\, D(\mathbf{r}_n) D(\mathbf{r}_p) \left|\psi_D(\mathbf{r}_n, \mathbf{r}_p)\right|^2 = \frac{3}{4}(2\pi)^3 \int d^3 r_{np}\, D_r(\mathbf{r}_{np}) \left|\phi_D(\mathbf{r}_{np})\right|^2$$

spin factor

$$\psi_D(\mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{PR}} \phi_D(\mathbf{r}_{np})$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton cont.

Theoretical formula

$$R(\mathbf{p}_\pi,\mathbf{p}_D)=\frac{1}{A}\int d^3r_\pi\, d^3r_n\, d^3r_p\, D(\mathbf{r}_\pi)\,D(\mathbf{r}_n)\, D(\mathbf{r}_p)\left|\psi_{\pi D}(\mathbf{r}_\pi,\mathbf{r}_n,\mathbf{r}_p)\right|^2$$

Center-of-mass (Jacobi) variables

$$\begin{cases}\mathbf{R}=\dfrac{m_\pi\mathbf{r}_\pi+m_n\mathbf{r}_n+m_p\mathbf{r}_p}{m_\pi+m_n+m_p}\\ \mathbf{r}_{np}=\mathbf{r}_n-\mathbf{r}_p\\ \mathbf{r}_{\pi D}=\mathbf{r}_\pi-\dfrac{m_n\mathbf{r}_n+m_p\mathbf{r}_p}{m_n+m_p}\end{cases}$$

$$\psi_{\pi D}(\mathbf{r}_\pi,\mathbf{r}_n,\mathbf{r}_p)=e^{i\mathbf{PR}}\,\phi_{\mathbf{q}}(\mathbf{r}_{\pi D})\,\phi_D(\mathbf{r}_{np})$$

$$D_{3r}(\mathbf{r}_{\pi D})\,D_r(\mathbf{r}_{np})\equiv\int d^3\mathbf{R}\; D(\mathbf{r}_\pi)\,D(\mathbf{r}_n)\,D(\mathbf{r}_p)$$

$$D(\mathbf{r})=\left(\frac{1}{2\pi R^2}\right)^{3/2}\exp\left(-\frac{\mathbf{r}^2}{2R^2}\right)$$

Deuteron formation rate cancels out!

$$R(\mathbf{q})=\int d^3r_{\pi D}\,D_{3r}(\mathbf{r}_{\pi D})\left|\phi_{\mathbf{q}}(\mathbf{r}_{\pi D})\right|^2$$

$$D_r(\mathbf{r})=\left(\frac{1}{4\pi R^2}\right)^{3/2}\exp\left(-\frac{\mathbf{r}^2}{4R^2}\right)$$

$$D_{3r}(\mathbf{r})=\left(\frac{1}{3\pi R^2}\right)^{3/2}\exp\left(-\frac{\mathbf{r}^2}{3R^2}\right)$$

Thermal vs. coalescence model

- Thermal model

$$R(\mathbf{q}) = \int d^3 r_{\pi D} D_r(\mathbf{r}_{\pi D}) \left| \phi_{\mathbf{q}}(\mathbf{r}_{\pi D}) \right|^2$$

$$D(\mathbf{r}) = \left(\frac{1}{2\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R^2} \right)$$

$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2} \right)$$

$$D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2} \right)$$

- Coalescence model

$$R(\mathbf{q}) = \int d^3 r_{\pi D} D_{3r}(\mathbf{r}_{\pi D}) \left| \phi_{\mathbf{q}}(\mathbf{r}_{\pi D}) \right|^2$$

Computation of correlation function

$$R(\mathbf{q}) = \int d^3 r_{\pi D} D_r(\mathbf{r}_{\pi D}) \left|\phi_{\mathbf{q}}(\mathbf{r}_{\pi D})\right|^2$$

Source function

$$D_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2}\right) \quad \text{or} \quad D_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2}\right)$$

Wave function

$$\phi_{\mathbf{q}}(\mathbf{r}) = e^{i\mathbf{qr}} + f(q)\frac{e^{iqr}}{r}$$

Coulomb effect

$$R(\mathbf{q}) \rightarrow G(\mathbf{q})R(\mathbf{q})$$

S-wave amplitude

$$f(q) = -\frac{a}{1+iqa}$$

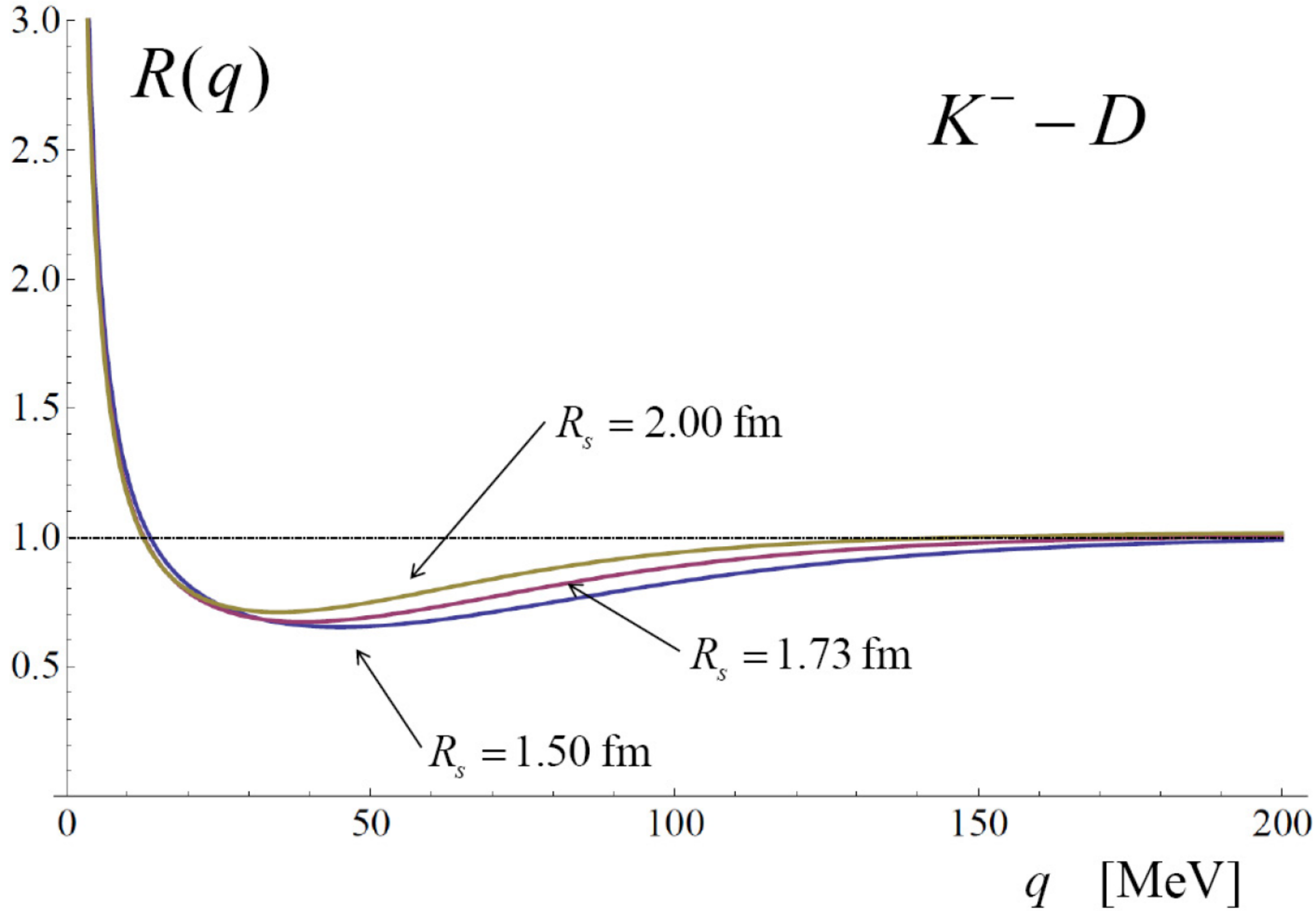
a - scattering length

Gamov factor

$$G(\mathbf{q}) = \pm\frac{2\pi}{a_B q}\frac{1}{\exp\left(\frac{2\pi}{a_B q}\right)-1}$$

a_B - Bohr radius

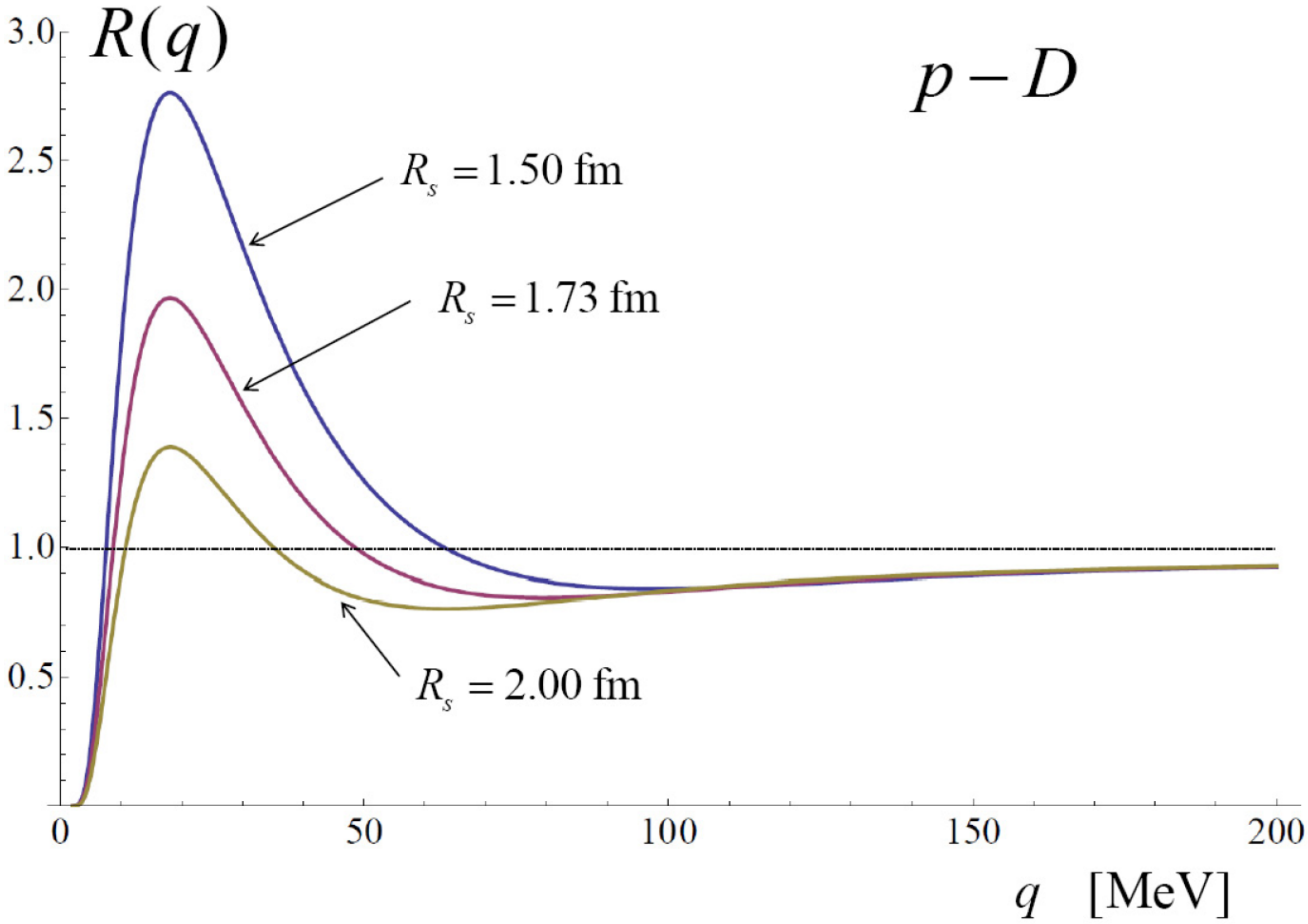
K^--D correlation functions



$a = (1.46 - 1.08i)\ \text{fm}$

$$2.00 = \sqrt{\frac{4}{3}}1.73 = \frac{4}{3}1.50$$

p-*D* correlation functions



$a_{1/2} = 4.0$ fm

$a_{3/2} = 11.0$ fm

$$R(q) = \frac{1}{3} R_{1/2}(q) + \frac{2}{3} R_{3/2}(q)$$

$$2.00 = \sqrt{\frac{4}{3}}\, 1.73 = \frac{4}{3}\, 1.50$$

Conclusions

^{4}He vs. ^{4}Li

- The thermal and coalescence models give different predictions on the ratio of yields of ^{4}Li to ^{4}He.
- In the thermal model the ratio of yields is independent of collision centrality.
- In the coalescence model the ratio is maximal for central collisions and rapidly decreases when one goes to peripheral collisions.
- Since ^{4}Li can be observed through the correlation function of ^{3}He-p, the correlation needs to be computed.

***h-D* correlations**

- Hadron-deuteron correlations carry information about source of deuterons.
- Measurement of *h-D* correlation function can tell us whether deuterons are directly emitted from a fireball or deuterons are formed due to final state interactions.
- K^- -*D* & *p-D* correlation functions show a sufficient sensitivity to a size of particle source to falsify the thermal or coalescence model.