Heavy Quarks in Turbulent QGP

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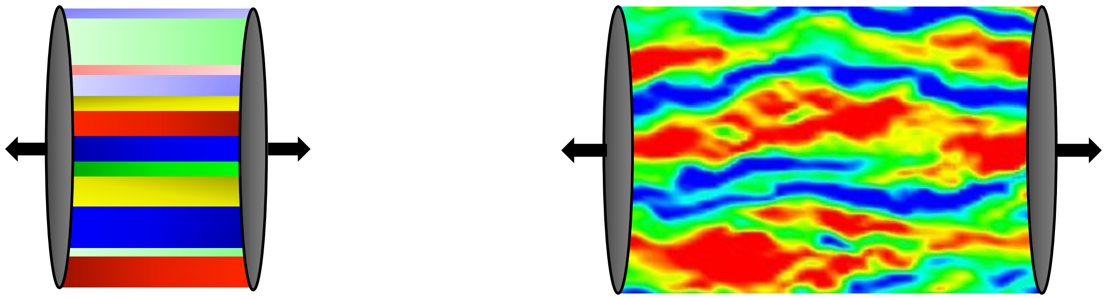
Turbulent QGP - quark-gluon plasma populated with strong classical chromodynamic fields

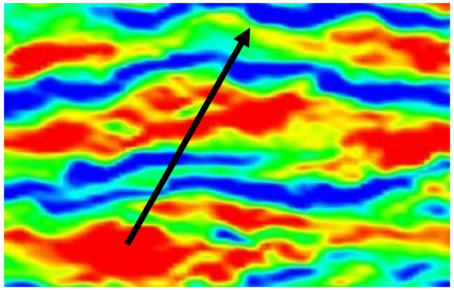
Motivation

- We consider the earliest stages of relativistic heavy-ion collisions.

- According to CGC, color charges confined in the colliding nuclei generate strong chromodynamic fields.



- How heavy quarks propagate through the turbulent QGP?

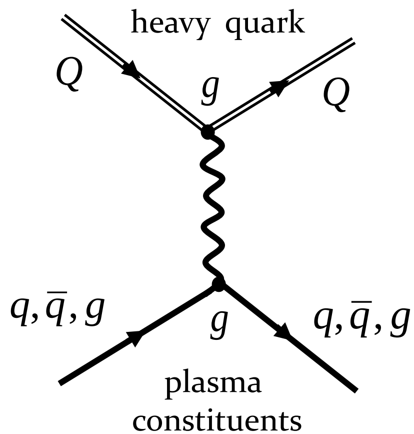


$$\frac{dE}{dx}, \ \hat{q} \ \ ?$$

Parametric Estimates

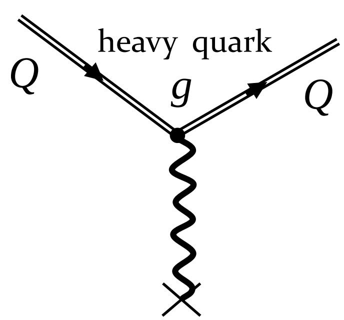
$$\boxed{\frac{dE}{dx},\ \hat{q} \quad \propto \quad |M|^2}$$

- equilibrium plasma



$$\left\{ \begin{array}{l} |M|^2 \propto g^4 \\ \dfrac{dE}{dx} \quad \propto \quad g^4 T^2 \\ \hat{q} \quad \propto \quad g^4 T^3 \end{array} \right.$$

- turbulent plasma



$$\left\{ \begin{array}{l} |M|^2 \propto g^2 \\ \varepsilon_{\text{plasma}} = \varepsilon_{\text{field}} \propto T^4 \\ \dfrac{dE}{dx} \quad \propto \quad g^2 T^2 \\ \hat{q} \quad \propto \quad g^2 T^3 \end{array} \right.$$

Fokker-Planck Equation

- Transport of heavy quarks is usually described in terms of Fokker-Planck equation.

$$\underbrace{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)}_{\text{drift}} n(t,\mathbf{r},\mathbf{p}) = \underbrace{\left(\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j + \nabla_p^i Y^i(\mathbf{v})\right)}_{\text{collisions}} n(t,\mathbf{r},\mathbf{p})$$

$n(t,\mathbf{r},\mathbf{p})$ - distribution function of heavy quarks $\qquad \mathbf{v}\equiv\frac{\mathbf{p}}{E_{\mathbf{p}}}, \quad \nabla_p^i \equiv \frac{\partial}{\partial p_i}$

$$\begin{cases} \dfrac{dE}{dx} = -\dfrac{v^i}{v} Y^i(\mathbf{v}) & \text{collisional energy loss} \\[2ex] \hat{q} = \dfrac{2}{v}\left(\delta^{ij} - \dfrac{v^i v^j}{v^2}\right) X^{ji}(\mathbf{v}) & \text{momentum broadening} \end{cases}$$

$$n(t,\mathbf{r},\mathbf{p}) = n_{\text{eq}}(\mathbf{p}) \sim e^{-\frac{E_{\mathbf{p}}}{T}}$$

solves FK equation

$$\Leftrightarrow$$

$$Y^j(\mathbf{v}) = \frac{v^i}{T} X^{ij}(\mathbf{v})$$

Origin of Fokker-Planck Equation

Boltzmann equation

Diffusive approximation

$$p >> \Delta p$$

p – momentum of a heavy-quark

Δp – momentum transfer in a single collision

Fokker-Planck equation

- How to obtain a Fokker-Planck equation for turbulent QGP?

  Apply the *quasilinear* method known in plasma physics.

Derivation of Fokker-Planck Equation

The dynamics is assumed to be dominated by strong classical fields.

Vlasov equation

$$\boxed{p_\mu D^\mu Q(t,\mathbf{r},\mathbf{p}) - \frac{g}{2} p^\mu \{F_{\mu\nu}(t,\mathbf{r}), \partial_p^\nu Q(t,\mathbf{r},\mathbf{p})\} = 0}$$

free streaming

mean-field force

$Q(t,\mathbf{r},\mathbf{p})$ - exact distribution function of heavy quarks which is the $N_c \times N_c$ matrix

$$D^\mu \equiv \partial^\mu - ig[A^\mu, \ldots.], \qquad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

Derivation of Fokker-Planck Equation

Regular and fluctuating quantities

fluctuating part

$$Q(t,\mathbf{r},\mathbf{p}) = \langle Q(t,\mathbf{r},\mathbf{p}) \rangle + \delta Q\,(t,\mathbf{r},\mathbf{p})$$

regular colorless part

$$\langle Q(t,\mathbf{r},\mathbf{p}) \rangle = n(t,\mathbf{r},\mathbf{p}) I$$

$n(t,\mathbf{r},\mathbf{p})$ - averaged distribution function

- $$|n| >> |\delta Q|, \quad |\nabla_p n| >> |\nabla_p \delta Q|$$
- $$\left|\frac{\partial n}{\partial t}\right| << \left|\frac{\partial \delta Q}{\partial t}\right|, \qquad |\nabla n| << |\nabla \delta Q|$$
- $$\langle \mathbf{E} \rangle = 0, \quad \langle \mathbf{B} \rangle = 0, \quad \mathbf{E}, \mathbf{B}, A^\mu \sim \delta Q$$

Derivation of Fokker-Planck Equation

$$Q(t,\mathbf{r},\mathbf{p}) = n(t,\mathbf{r},\mathbf{p})I + \delta Q(t,\mathbf{r},\mathbf{p})$$

Vlasov equation

$$\left(D^0 + \mathbf{v}\cdot\mathbf{D}\right)Q - g\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right)\cdot\nabla_p Q = 0$$

ensemble averaging $\Big\Downarrow \; \mathrm{Tr}\langle \ldots \rangle$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)n(t,\mathbf{r},\mathbf{p}) = \overbrace{\frac{g}{N_c}\mathrm{Tr}\left\langle\left(\mathbf{E}(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{r})\right)\cdot\nabla_p \delta Q(t,\mathbf{r},\mathbf{p})\right\rangle}^{\text{collision term}}$$

Fluctuations provide a collision term.

Derivation of Fokker-Planck Equation

How to compute the collision term?

$$C \equiv \frac{g}{N_c} \mathrm{Tr}\left\langle \left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)\cdot\nabla_p \delta Q\right\rangle = ?$$

$$Q(t,\mathbf{r},\mathbf{p}) = n(t,\mathbf{r},\mathbf{p})I + \delta Q\,(t,\mathbf{r},\mathbf{p})$$

Vlasov equation

$$\left(D^0 + \mathbf{v}\cdot\mathbf{D}\right)Q - g\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)\cdot\nabla_p Q = 0$$

$$n \gg |\delta Q|$$
$$|\nabla_p n| \gg |\nabla_p \delta Q|$$

linearization

$$\left|\frac{\partial n}{\partial t}\right| \ll \left|\frac{\partial \delta Q}{\partial t}\right|$$
$$|\nabla n| \ll |\nabla \delta Q|$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\delta Q = g\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)\nabla_p n$$

Derivation of Fokker-Planck Equation

Solution of the linearized transport equation

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\delta Q(t,\mathbf{r},\mathbf{p})=g\big(\mathbf{E}(t,\mathbf{r})+\mathbf{v}\times\mathbf{B}(t,\mathbf{r})\big)\nabla_p n(\mathbf{p})$$

$$\delta Q(t,\mathbf{r},\mathbf{p})=g\int_0^t dt'\big(\mathbf{E}(t',\mathbf{r}-\mathbf{v}(t-t'))+\mathbf{v}\times\mathbf{B}(t',\mathbf{r}-\mathbf{v}(t-t'))\big)\nabla_p n(\mathbf{p})$$
$$+\,\delta Q_0(\mathbf{r}-\mathbf{v}t,\mathbf{p})$$

initial value

$C\equiv\frac{g}{N_c}\mathrm{Tr}\left\langle\big(\mathbf{E}+\mathbf{v}\times\mathbf{B}\big)\cdot\nabla_p\delta Q\right\rangle$ expressed by $\left\langle E^iE^j\right\rangle,\left\langle B^iE^j\right\rangle,\left\langle B^iB^j\right\rangle$

Collision term is given by the field correlators

Field correlators in Equilibrium QGP

space-time translational invariance

flucuation spectrum

$$\boxed{\left\langle E_a^i(t,\mathbf{r}) E_b^j(t',\mathbf{r}')\right\rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3}\, e^{-i(\omega(t-t')-\mathbf{k}(\mathbf{r}-\mathbf{r}'))}} \overbrace{\left\langle E_a^i E_b^j \right\rangle_{\omega,\mathbf{k}}}^{\text{flucuation spectrum}}$$

- $$\left\langle E_a^i E_b^j \right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab} \frac{\omega^4}{e^{\beta|\omega|}-1} \left[\frac{k^i k^j}{\mathbf{k}^2} \frac{\mathrm{Im}\,\varepsilon_L(\omega,\mathbf{k})}{|\omega^2 \varepsilon_L(\omega,\mathbf{k})|^2} + \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right) \frac{\mathrm{Im}\,\varepsilon_T(\omega,\mathbf{k})}{|\omega^2 \varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2|^2} \right]$$

- $$\left\langle B_a^i B_b^j \right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab} \frac{\omega^2 \mathbf{k}^2}{e^{\beta|\omega|}-1} \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right) \frac{\mathrm{Im}\,\varepsilon_T(\omega,\mathbf{k})}{|\omega^2 \varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2|^2}$$

- $$\left\langle B_a^i E_b^j \right\rangle_{\omega,\mathbf{k}} = \left\langle E_a^j B_b^i \right\rangle_{\omega,\mathbf{k}} = 2\delta^{ab} \frac{\omega^3}{e^{\beta|\omega|}-1} \varepsilon^{imj} k^m \frac{\mathrm{Im}\,\varepsilon_T(\omega,\mathbf{k})}{|\omega^2 \varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2|^2}$$

$\varepsilon_{L,T}(\omega,\mathbf{k})$ - chromodielectric functions

St. Mrówczyński, Physical Review D **77**, 105022 (2008)

Fokker-Planck Equation of Equilibrium QGP

$$\frac{g}{N_c}\mathrm{Tr}\left[\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)\cdot\nabla_p\delta Q\right]=[\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j+\nabla_p^i Y^i(\mathbf{v})]n(\mathbf{p})$$

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)n(t,\mathbf{r},\mathbf{p})=\left(\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j+\nabla_p^i Y^i(\mathbf{v})\right)n(t,\mathbf{r},\mathbf{p})$$

Isotropy

$$X^{ij}(\mathbf{v})\equiv X_L(v)\frac{v^i v^j}{\mathbf{v}^2}+X_T(v)\left(\delta^{ij}-\frac{v^i v^j}{\mathbf{v}^2}\right),\qquad Y^j(\mathbf{v})=\frac{v^i}{T}X^{ij}(\mathbf{v})=\frac{v^i}{T}X_L(v)$$

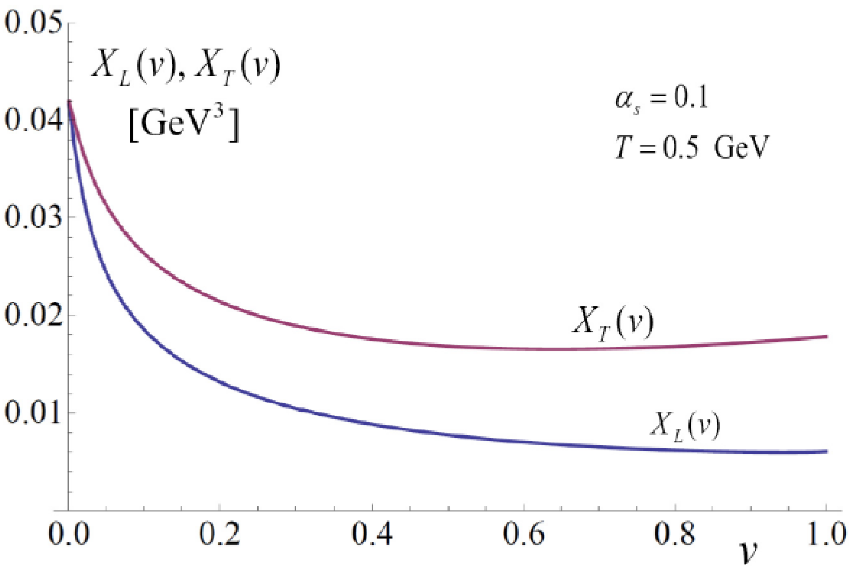
$$v<<1,\quad g<<1$$

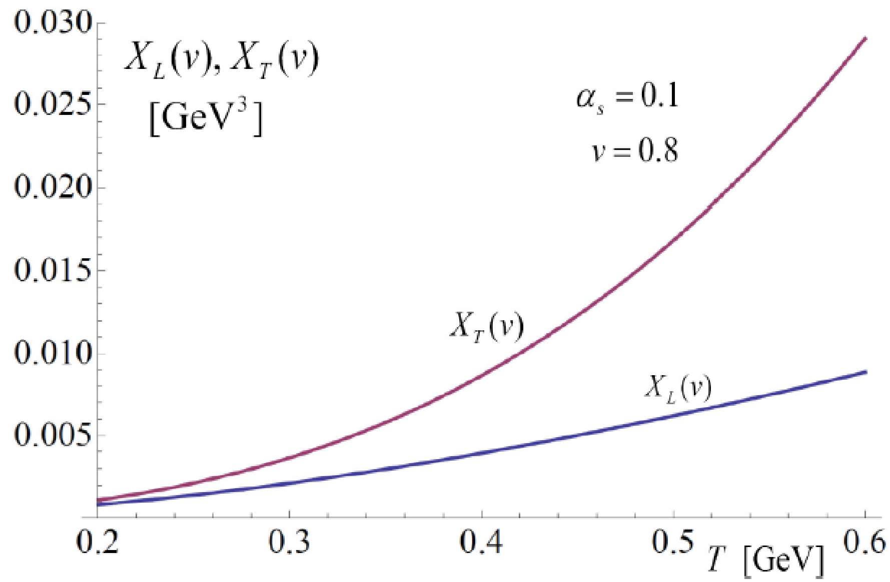
$$X_L(v)=X_T(v)\approx\frac{g^2C_F}{12\pi}m_D^2 T\log\left(\frac{T}{m_D}\right)$$

$$C_F\equiv\frac{N_c^2-1}{2N_c}$$

G. D. Moore and D. Teaney, Physical Review D **71**, 064904 (2005)

Fokker-Planck Equation of Equilibrium QGP





Quantitative agreement with $X_L(v)$ & $X_T(v)$ obtained from the Boltzmann collision term by means of the diffusive approximation.

B. Svetitsky, Physical Review D **37**, 2484 (1988)

Modeling of Isotropic, Homogenous & Stationary Turbulent QGP

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle,\quad \left\langle E_a^i(t,\mathbf{r})B_b^j(t',\mathbf{r}')\right\rangle,\quad \left\langle B_a^i(t,\mathbf{r})B_b^j(t',\mathbf{r}')\right\rangle \quad ?$$

‘Gaussian *E* & *B*’

colorlessness

isotropy

$$\begin{cases} \left\langle E_a^i(t,\mathbf{r})E_b^j(0,\mathbf{0})\right\rangle = \delta^{ab}\delta^{ij} M_E \exp\left(-\dfrac{t^2}{\tau^2}-\dfrac{\mathbf{r}^2}{\sigma^2}\right) \\ \left\langle E_a^i(t,\mathbf{r})B_b^j(0,\mathbf{0})\right\rangle = 0 \\ \left\langle B_a^i(t,\mathbf{r})B_b^j(0,\mathbf{0})\right\rangle = \delta^{ab}\delta^{ij} M_B \exp\left(-\dfrac{t^2}{\tau^2}-\dfrac{\mathbf{r}^2}{\sigma^2}\right) \end{cases}$$

$$X_L(v) = \sqrt{\frac{\pi}{2}}g^2 C_F \frac{M_E \tau\sigma}{\sqrt{\sigma^2+v^2\tau^2}}, \qquad X_T(v) = \sqrt{\frac{\pi}{2}}g^2 C_F \frac{\left(M_E+v^2 M_B\right)\tau\sigma}{\sqrt{\sigma^2+v^2\tau^2}}$$

M. Asakawa, S. A. Bass and B. Müller, Prog. Theor. Phys. **116**, 725 (2006)

Modeling of Isotropic, Homogenous & Stationary Turbulent QGP

'Gaussian A'

$$\boxed{\left\langle A_a^i(t,\mathbf{r}) A_b^j(0,\mathbf{0}) \right\rangle = \delta^{ab}\delta^{ij} M_A \exp\left(-\frac{t^2}{\tau^2} - \frac{\mathbf{r}^2}{\sigma^2} \right)}$$

Gauge condition

$$A_a^0(t,\mathbf{r}) = 0, \quad \nabla \cdot \mathbf{A}_a(t,\mathbf{r}) = 0$$

$$\mathbf{E}_a(t,\mathbf{r}) = -\dot{\mathbf{A}}_a(t,\mathbf{r}),$$
$$\mathbf{B}_a(t,\mathbf{r}) = \nabla \times \mathbf{A}_a(t,\mathbf{r})$$

$$\Downarrow$$

$$\left\langle E_a^i(t,\mathbf{r}) E_b^j(0,\mathbf{0}) \right\rangle, \quad \left\langle E_a^i(t,\mathbf{r}) B_b^j(0,\mathbf{0}) \right\rangle, \quad \left\langle B_a^i(t,\mathbf{r}) B_b^j(0,\mathbf{0}) \right\rangle$$

$$\begin{cases} X_L(v) = \sqrt{\dfrac{\pi}{2}} g^2 C_F \dfrac{M_A \tau \sigma v^2}{(\sigma^2 + v^2\tau^2)^{3/2}} \\[2ex] X_T(v) = \sqrt{\dfrac{\pi}{8}} g^2 C_F \dfrac{M_A \tau v^2}{\sigma} \left[\dfrac{3}{(\sigma^2 + v^2\tau^2)^{1/2}} - \dfrac{\sigma^2 + 3v^2\tau^2}{(\sigma^2 + v^2\tau^2)^{3/2}} \right] \end{cases}$$

Modeling of Isotropic, Homogenous & Stationary Turbulent QGP

'Stationary A'

$$\boxed{\left\langle A_a^i A_b^j \right\rangle_{\omega,\mathbf{k}} = \delta^{ab}\,\delta^{ij}\,\frac{2\pi\delta(\omega)}{\mathbf{k}^2+\mu^2}\,\Theta\left(|\mathbf{k}| - k_{\max}\right)M}$$

Gauge condition

$$A_a^0(t,\mathbf{r}) = 0, \quad \nabla\cdot\mathbf{A}_a(t,\mathbf{r}) = 0$$

$$\Downarrow$$

$$\mathbf{E}_a(t,\mathbf{r}) = -\dot{\mathbf{A}}_a(t,\mathbf{r}),$$

$$\mathbf{B}_a(t,\mathbf{r}) = \nabla\times\mathbf{A}_a(t,\mathbf{r})$$

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(0,\mathbf{0})\right\rangle = 0, \quad \left\langle E_a^i(t,\mathbf{r})B_b^j(0,\mathbf{0})\right\rangle = 0, \quad \left\langle B_a^i(t,\mathbf{r})B_b^j(0,\mathbf{0})\right\rangle \neq 0$$

$$\left\{\begin{array}{l} X_L(v) = 0 \\ X_T(v) \approx \dfrac{g^2 C_F}{16\pi} M k_{\max}^2 v \\ \quad\quad k_{\max} >> \mu \end{array}\right.$$

Model Parameters

Turbulent plasma *vs.* equilibrium plasma at the same energy density

- Energy density of weakly coupled equilibrium QGP

$$\varepsilon_{\text{QGP}} = \frac{\pi^2}{60}\left(4(N_c^2-1)+7N_f N_c\right)T^4$$

- Energy density accumulated in the fields

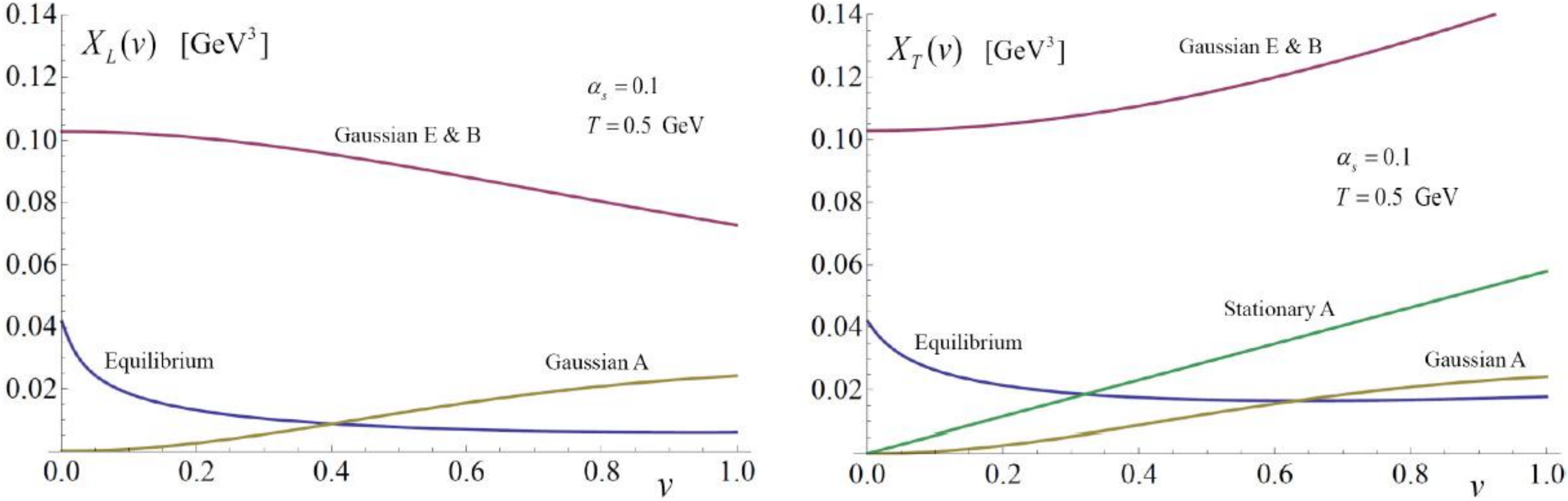
$$\varepsilon_{\text{field}} = \frac{1}{2}\left(\left\langle E_a^i(t,\mathbf{r})E_a^i(t,\mathbf{r})\right\rangle + \left\langle B_a^i(t,\mathbf{r})B_a^i(t,\mathbf{r})\right\rangle\right)$$

$$\varepsilon_{\text{QGP}} = \varepsilon_{\text{field}} = (N_c^2-1)\times\begin{cases}\frac{3}{2}\left(M_E+M_B\right) & \text{`Gaussian } E \ \& \ B\text{'}\\ \frac{3}{2}\left(\frac{1}{\tau^2}+\frac{2}{\sigma^2}\right)M_A & \text{`Gaussian } A\text{'}\\ \frac{M k_{\max}^3}{6\pi^2} & \text{`stationary } A\text{'}\end{cases}$$

$$M_E = M_B, \quad M_A, \quad \tau = \sigma = \frac{1}{m_D}, \quad k_{\max} = 5m_D$$

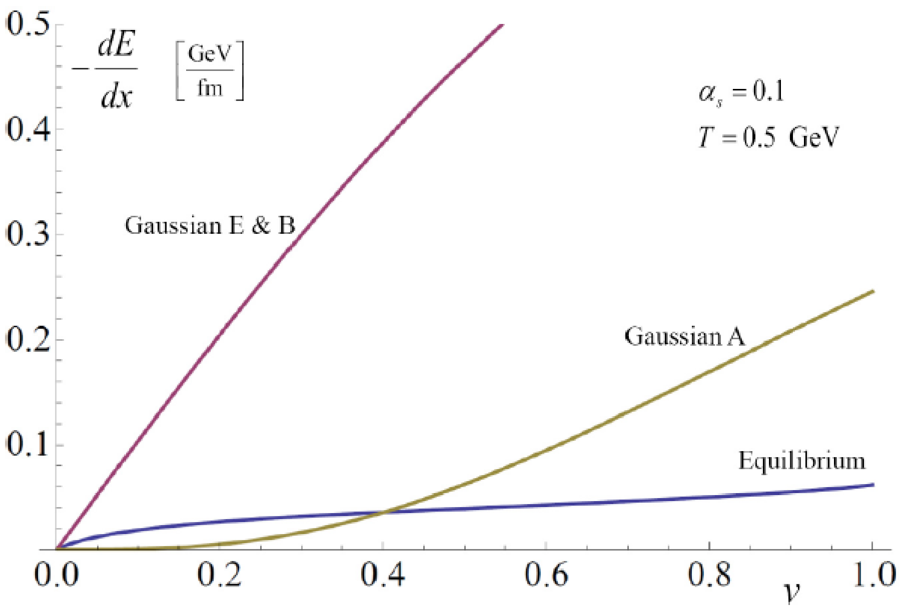
$$g, T$$

Turbulent *vs.* Equilibrium Plasmas



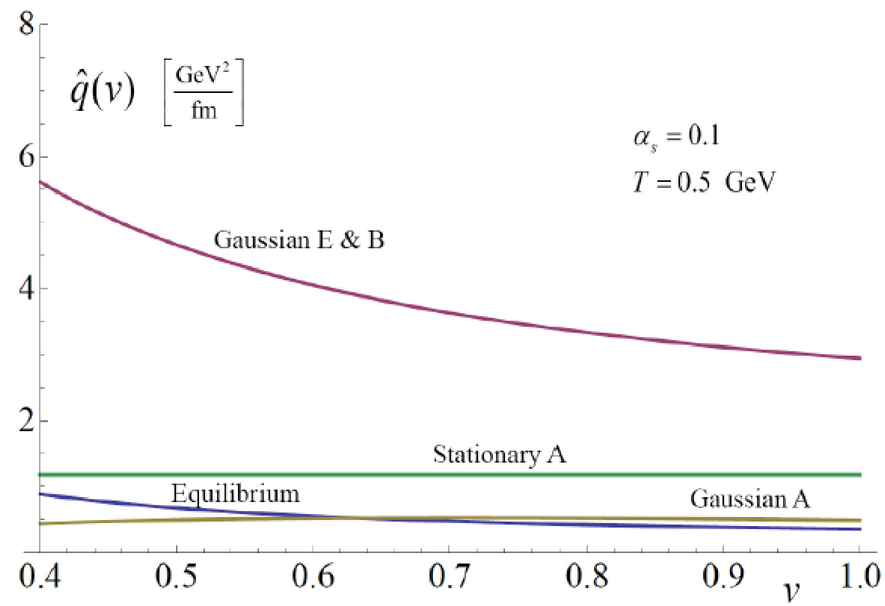
Turbulent *vs.* Equilibrium Plasmas

Collisional energy loss



$$\frac{dE}{dx} = -\frac{v}{T} X_L(v)$$

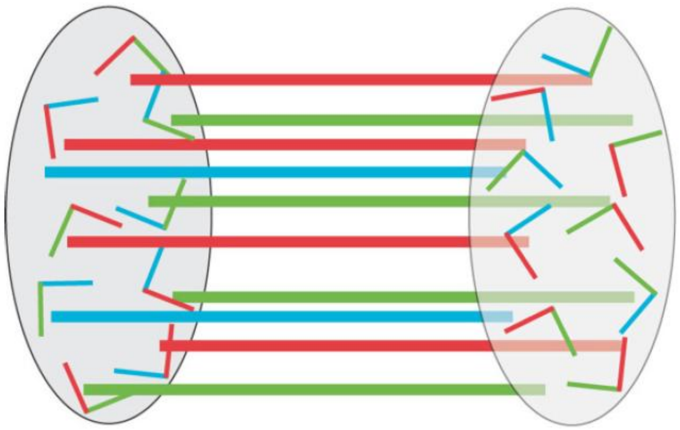
Momentum broadening



$$\hat{q} = \frac{4}{v} X_T(v)$$

Glasma

The earliest stage of relativistic heavy-ion collisions



E & B fields along the axis z

$$A_a^\mu(t,\mathbf{r}) = \left(A_a^0(t,z), A_a^x(x,y), A_a^y(x,y), A_a^z(t,z)\right)$$

Boost-invariant correlation functions

- $$\left\langle E_a^z(t_1,z_1)E_b^z(t_2,z_2)\right\rangle = \delta^{ab}\Theta(t_1^2-z_1^2)\,\Theta(t_2^2-z_2^2)\tilde{M}_E \exp\left(-\frac{(\tau_1-\tau_2)^2}{2\sigma_\tau^2}-\frac{(\eta_1-\eta_2)^2}{2\sigma_\eta^2}\right)$$
- $$\left\langle B_a^z(x_1,y_1)B_b^z(x_1,y_1)\right\rangle = \delta^{ab}\tilde{M}_B \exp\left(-\frac{x^2+x^2}{2\sigma_T^2}\right)$$

$$\tau_i \equiv \sqrt{t_i^2-z_i^2}, \qquad \eta_i \equiv \frac{1}{2}\log\left(\frac{t_i+z_i}{t_i-z_i}\right), \qquad i=1,2$$

G.Chen, R.J. Fries, J.I. Kapusta and Y. Li, Physical Review D **92**, 064912 (2015)

Glasma

$$\boxed{X^{ij}(\mathbf{v}) = \sqrt{\frac{\pi}{2}} g^2 C_F \left(\tilde{M}_E n^i n^j - \frac{V^{ij}}{v_T} \tilde{M}_B \sigma_T \right)}$$

$$n^i \equiv (0,0,1), \qquad V^{ij} \equiv \varepsilon^{ikl} v^k n^l \varepsilon^{jmn} v^m n^n = \begin{pmatrix} v_y^2 & -v_x v_y & 0 \\ -v_x v_y & v_x^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

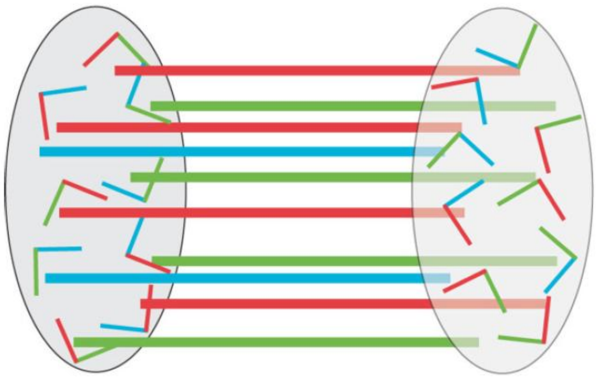
$$\begin{cases} -\dfrac{dE}{dx} = \dfrac{v^i v^j}{vT} X^{ji}(\mathbf{v}) & \text{collisional energy loss} \\[2ex] \hat{q} = \dfrac{2}{v} \left(\delta^{ij} - \dfrac{v^i v^j}{v^2} \right) X^{ji}(\mathbf{v}) & \text{momentum broadening} \end{cases}$$

Model Parameters

Density of energy released in a central collision

$$\boxed{\varepsilon_{\mathrm{coll}} = \frac{c_{\mathrm{inel}} A \sqrt{s}}{\pi R_A^2 l}}$$

$$c_{\mathrm{inel}} = 0.5, \quad A = 200, \quad \sqrt{s} = 5\,\mathrm{TeV}$$



$$R_A = 7 \text{ fm}$$

$$l = 1\,\mathrm{fm}$$

$$\varepsilon_{\mathrm{coll}} = \varepsilon_{\mathrm{field}}$$

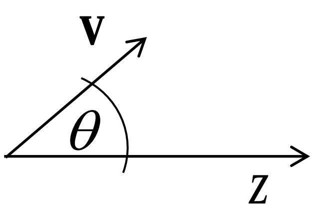
$$\Downarrow$$

$$(T = 1.2 \;\; \mathrm{GeV})$$

$$\widetilde{M}_E = \widetilde{M}_B, \qquad \sigma_T = Q_s^{-1} = 0.5 \quad [\mathrm{GeV}^{-1}]$$

Energy-loss and Momentum Broadening in the Glasma

$$\begin{cases} -\dfrac{dE}{dx} = 14\cos^2\theta \quad \left[\dfrac{\text{GeV}}{\text{fm}}\right] \\[2ex] \hat{q} = 33\left(\sin^2\theta + \sin\theta\right) \quad \left[\dfrac{\text{GeV}^2}{\text{fm}}\right] \end{cases}$$



$$v = 1$$

Typical values inferred from experimental data on jet quenching

$$\begin{cases} -\dfrac{dE}{dx} = 1.0 - 3.0 \quad \left[\dfrac{\text{GeV}}{\text{fm}}\right] \\[2ex] \hat{q} = 1.5 - 7.0 \quad \left[\dfrac{\text{GeV}^2}{\text{fm}}\right] \end{cases}$$

F. Prino & R. Rapp, Journal of Physics G **43**, 093002 (2016)

Summary & Conclusions

- The Fokker-Planck equation of heavy quarks interacting with classical chromodynamic fields rather than with plasma constituents is derived.

- The known case of equilibrium plasma is reproduced.

- The turbulent plasma with the energy dominated by chromodynamic classical fields interacts strongly with heavy quarks.

- In spite of its short lifetime the glasma can provide a significant contribution to the collisional and radiative energy loss of heavy quarks.

more details in: St. Mrówczynski, arXiv:1706.03127