Does it matter that QGP is initially unstable?

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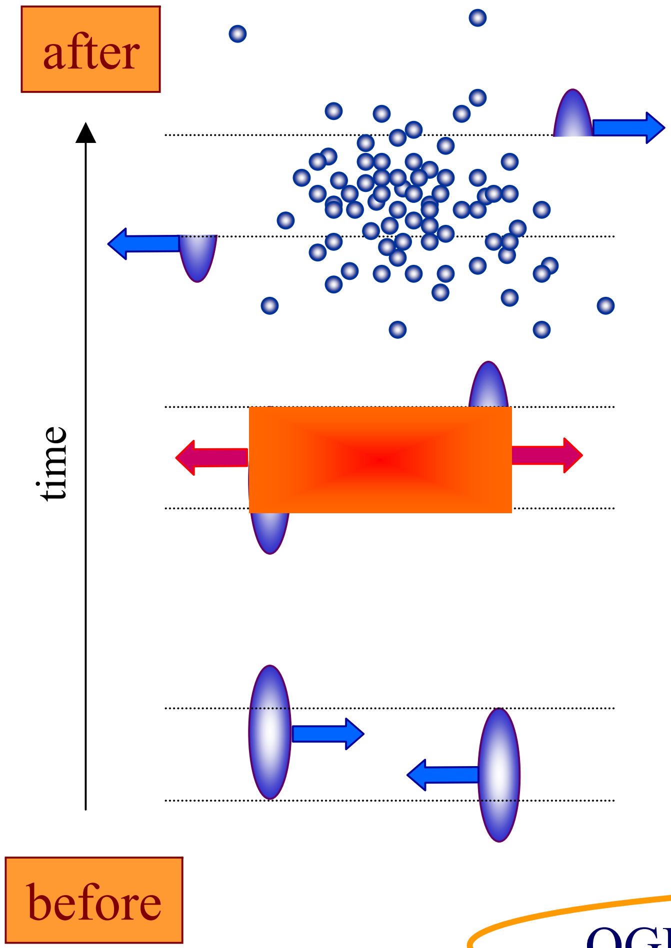
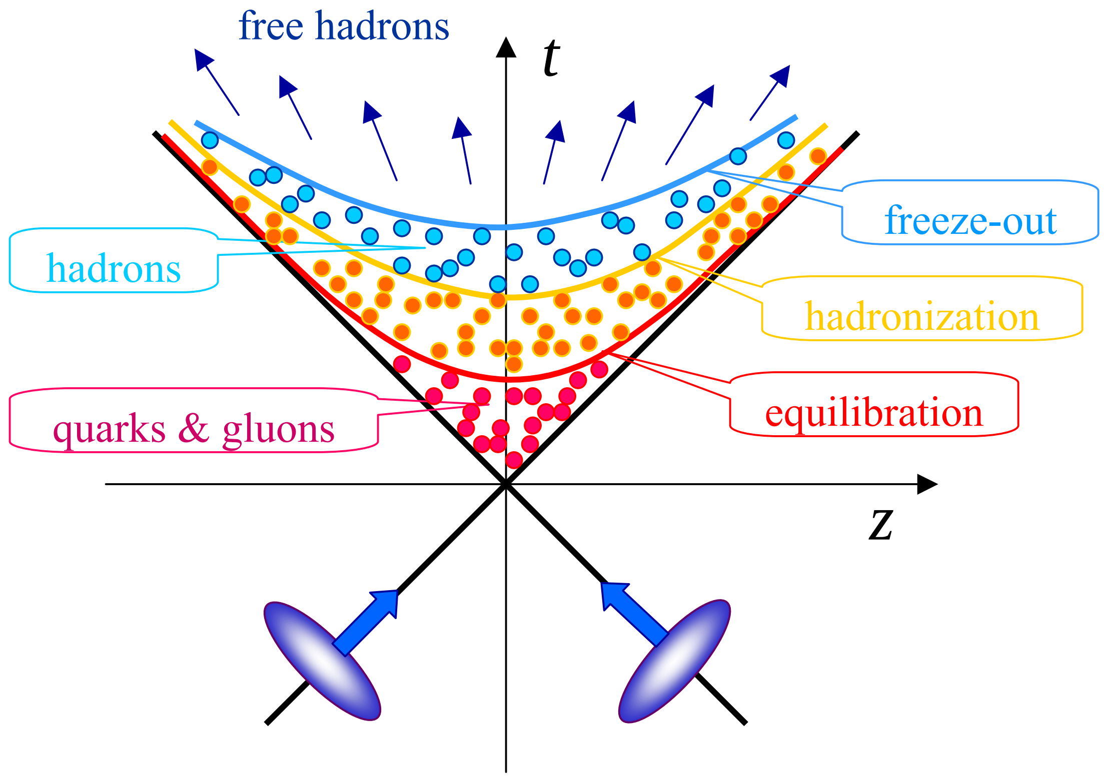
- Unstable quark-gluon plasma
- Role of instabilities in equilibration of QGP
- Energy loss in unstable QGP

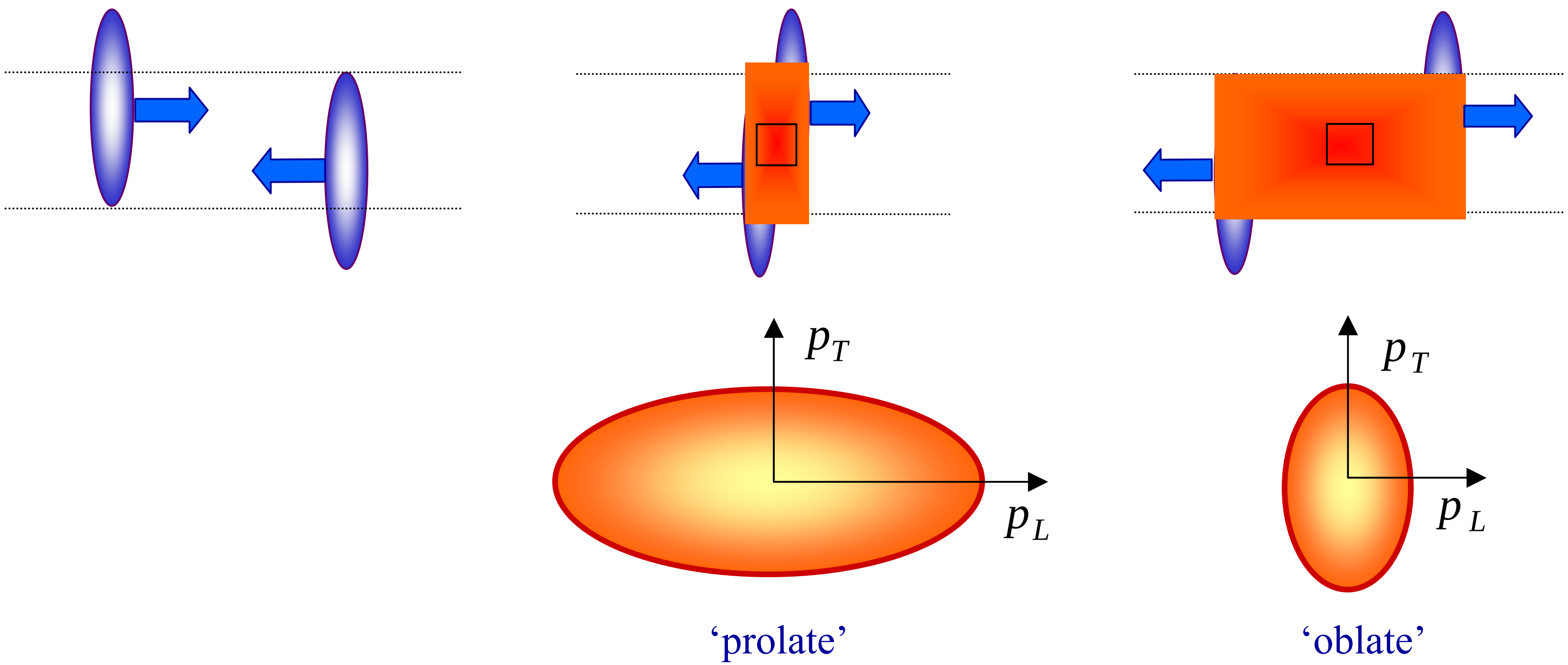
Scenario of relativistic heavy-ion collisions





QGP is out of equilibrium at the collision early stage

Anisotropic QGP



Anisotropic QGP is unstable due to magnetic plasma modes

Instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

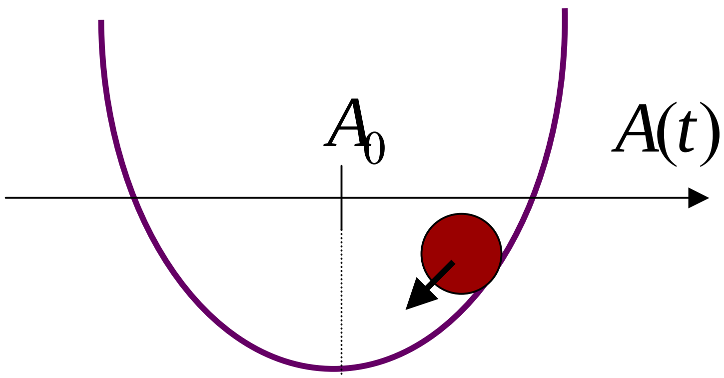
fluctuation

Instability

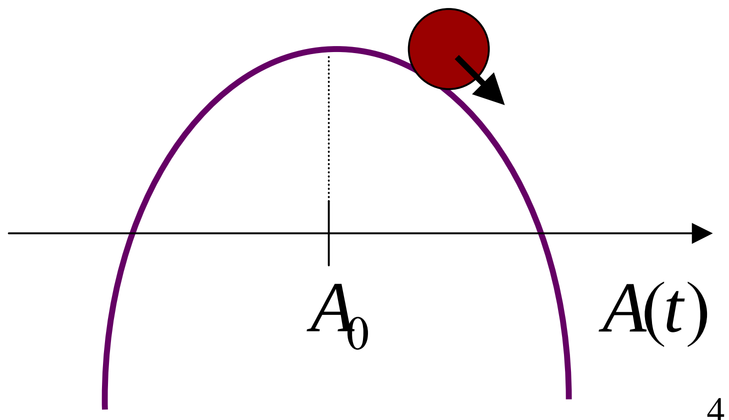
$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma > 0$$

stable configuration



unstable configuration

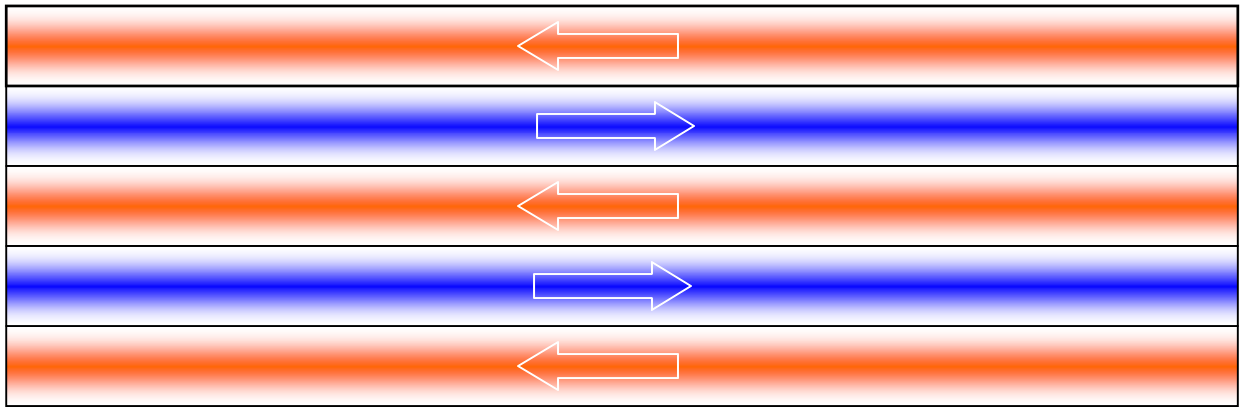


Seeds of instability

$\left\langle j_a^{\mu}(x) \right\rangle = 0$ **but current fluctuations are finite**

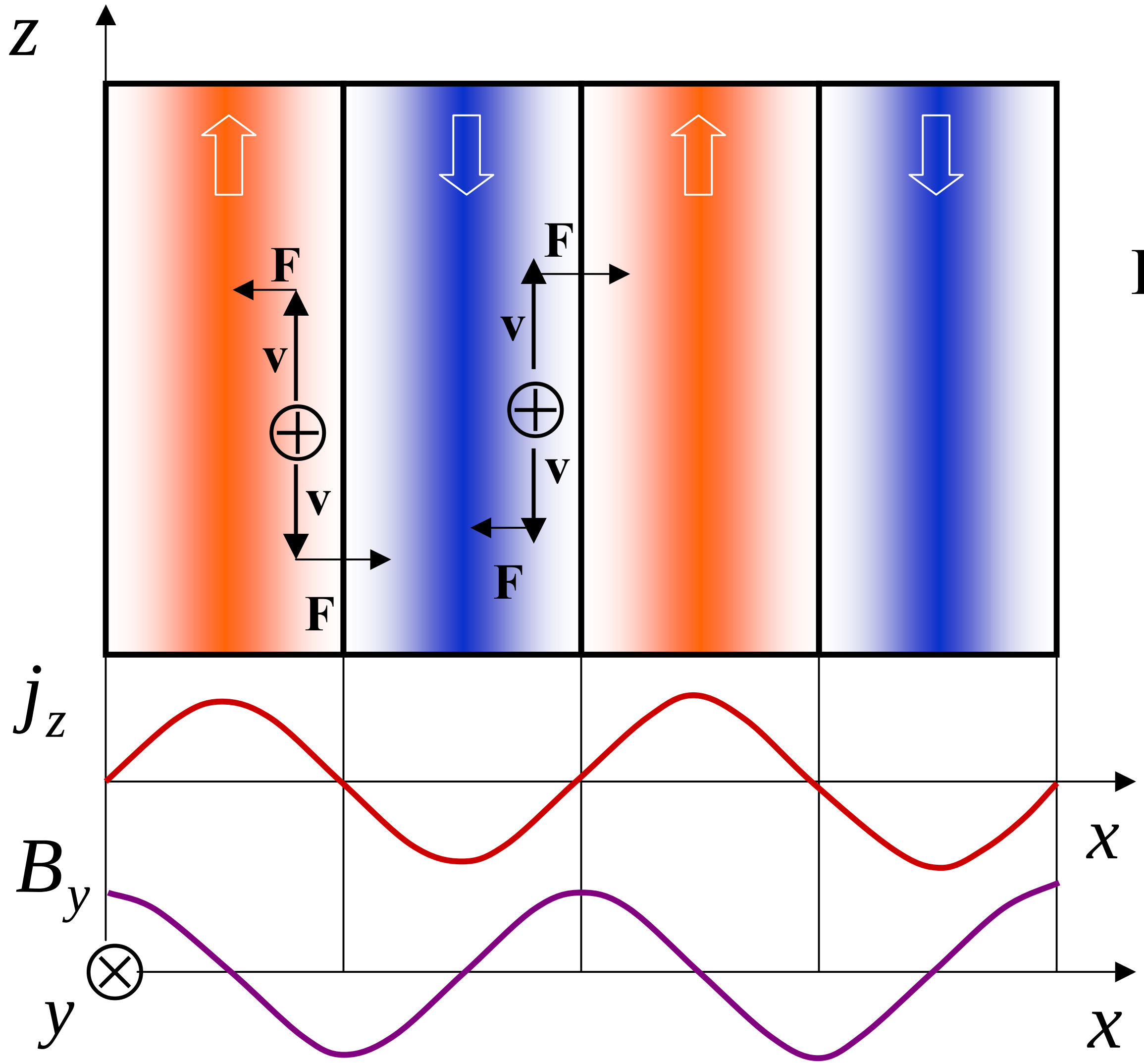
$$\left\langle j_a^{\mu}(x_1)\, j_b^{\nu}(x_2) \right\rangle = \frac{1}{2}\delta^{ab} \int \frac{d^3p}{(2\pi)^3} \frac{p^{\mu} p^{\nu}}{E_p^2} f(\mathbf{p}) \delta^{(3)}(\mathbf{x} - \mathbf{v}t) \neq 0$$

$$x_1 = (t_1, \mathbf{x}_1), \quad x_2 = (t_2, \mathbf{x}_2), \quad x = (t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2)$$



Direction of the momentum surplus

Mechanism of filamentation



Lorentz force

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

Ampere's law

$$\nabla \times \mathbf{B} = \mathbf{j}$$

Growth of instabilities – 1+1 numerical simulations

SU(2) Hard Loop Dynamics

1+1 dimensions

$$A_a^\mu = A_a^\mu(t,z)$$

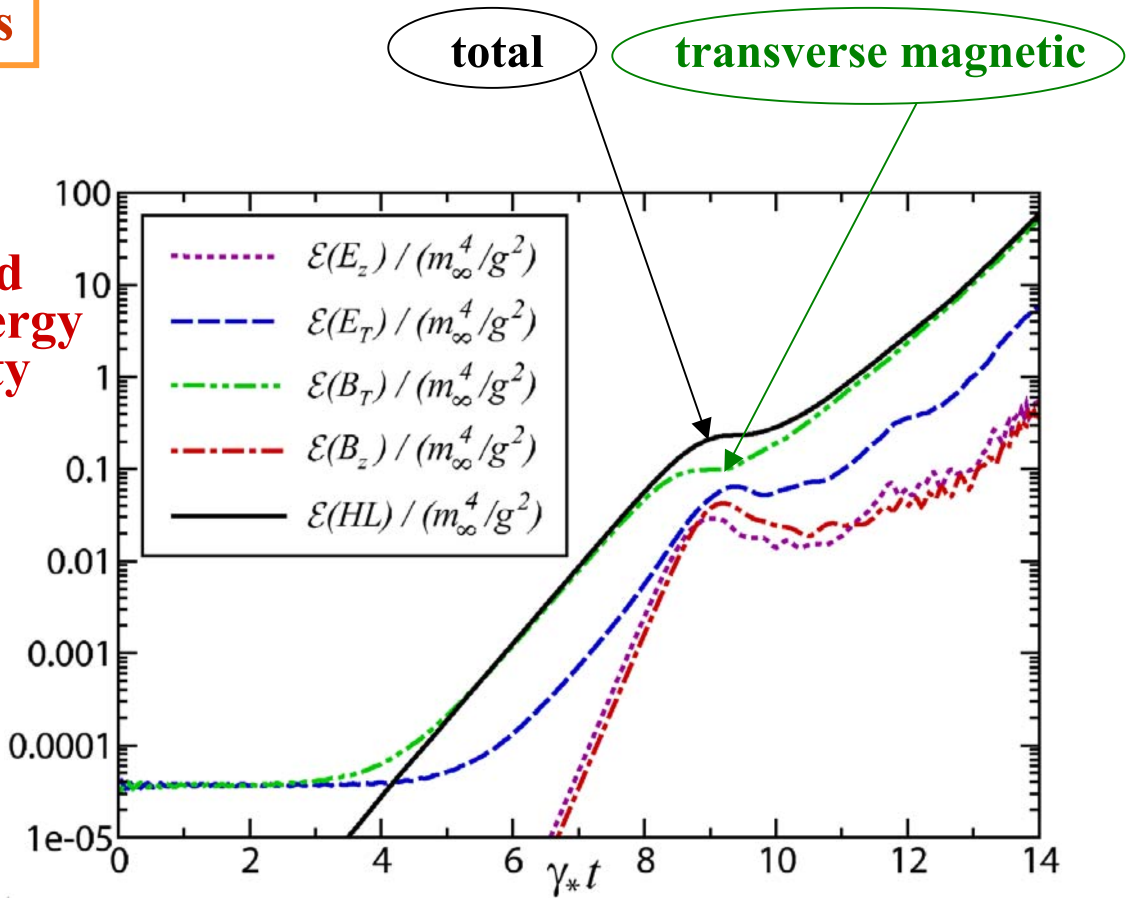
Anisotropic particle's momentum distribution

$$f(\mathbf{p}) = f_{\text{iso}}(|\mathbf{p}| + \zeta p_z)$$

$$m_D^2 = -\frac{\alpha_s}{\pi}\int_0^\infty dp\, p^2 \frac{df_{\text{iso}}(p)}{dp}$$

$$(m_D, \zeta)$$

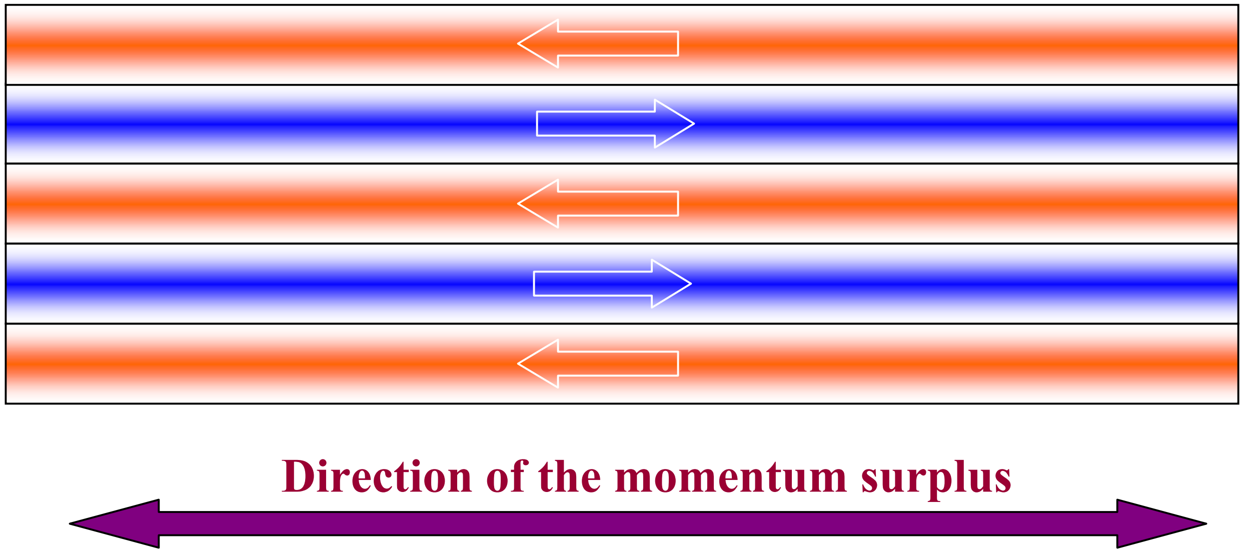
Scaled field energy density

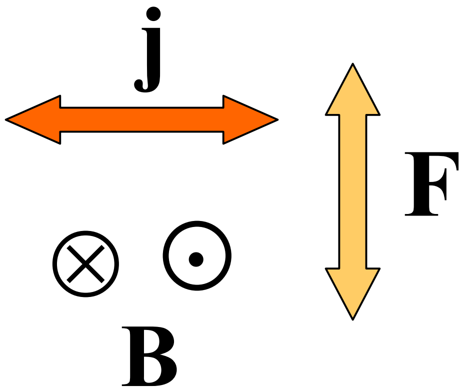


Strong anisotropy $\zeta = 10$ γ_* - maximal growth rate

A. Rebhan, P. Romatschke & M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005)

Mechanism of isotropization





$$\Delta \mathbf{p} = \int dt\, \mathbf{F}$$

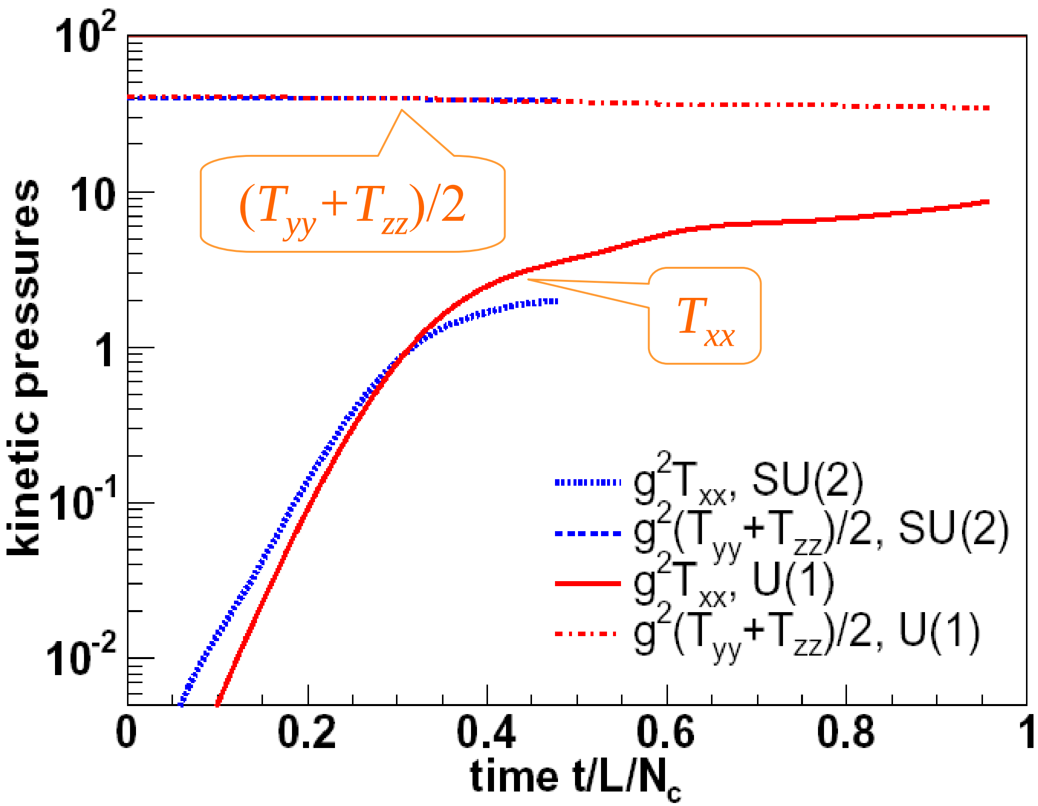
Isotropization – numerical simulation

Classical system of colored particles & fields

$$T_{ij} = \int \frac{d^3 p}{(2\pi)^3} \frac{p_i p_j}{E} f(\mathbf{p})$$

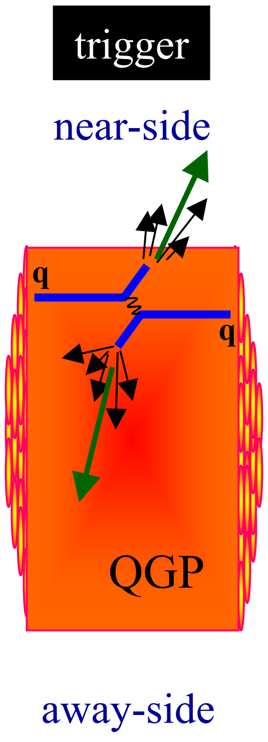
Isotropy:

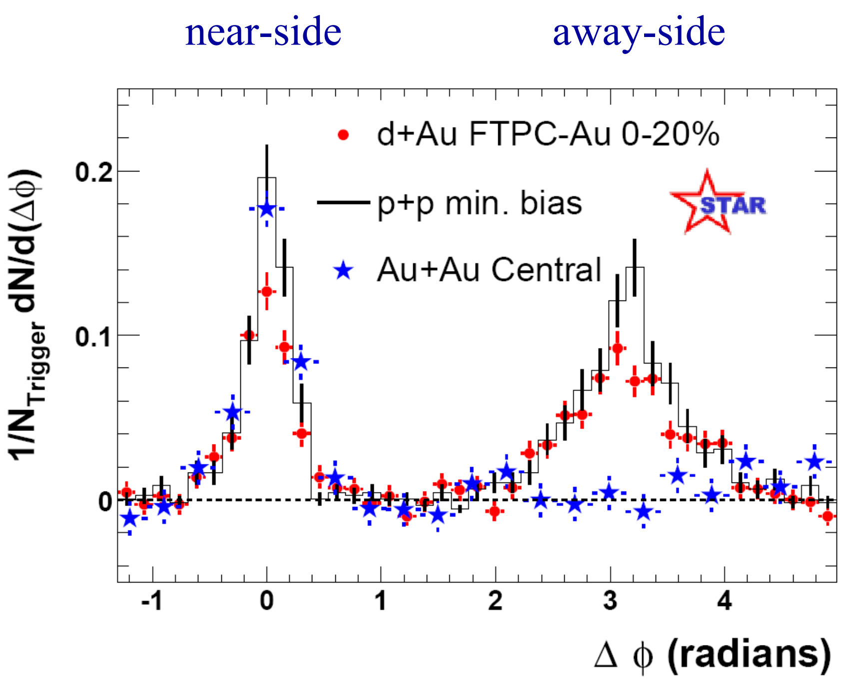
$$T_{xx} = (T_{yy} + T_{zz})/2$$



A. Dumitru & Y. Nara, Phys. Lett. **B621**, 89 (2005).

Jet quenching





Away-side jet is suppressed in central collisions

J. Adams et al. [STAR Collaboration], Nucl. Phys. **A757**, 102 (2005)

Energy loss in unstable QGP

- Is dE/dx in unstable QGP sizeable?

Yes!

- How to compute dE/dx in unstable QGP?

Solve initial value problem!

M. Carrington, K. Deja and St. Mrówczyński,
Acta Physica Polonica B - Proceedings Supplement **5**, 047 (2012); *ibid* **5**, 343 (2012)

A test parton in QGP

Wong's equation of motion (Hard Loop Approximation)

$$\begin{cases} \dfrac{dx^{\mu}(\tau)}{d\tau} = u^{\mu}(\tau) \\ \dfrac{dp^{\mu}(\tau)}{d\tau} = gQ_a(\tau)\, F_a^{\mu\nu}(x(\tau))\, u_{\nu}(\tau) \\ \dfrac{dQ_a(\tau)}{d\tau} = -gf^{abc}\, p_{\mu}(\tau)\, A_b^{\mu}(x(\tau))\, Q_c(\tau) \end{cases}$$

Simplifications

Gauge condition: $p_{\mu}(\tau)\, A_b^{\mu}\big(x(\tau)\big) = 0 \quad \Rightarrow \quad Q_a(\tau) = \text{const}$

Parton travels with constant velocity: $u^{\mu} = (\gamma, \gamma\, \mathbf{v}) = \text{const}$

Parton's energy loss

$$\frac{dE(t)}{dt} = gQ_a \mathbf{E}_a(t,\mathbf{r}(t)) \cdot \mathbf{v}$$

induced & spontaneously generated chromoelectric field

parton's current: $\mathbf{j}_a(t,\mathbf{r}) = gQ_a \mathbf{v} \delta^{(3)}(\mathbf{r} - \mathbf{v}t)$

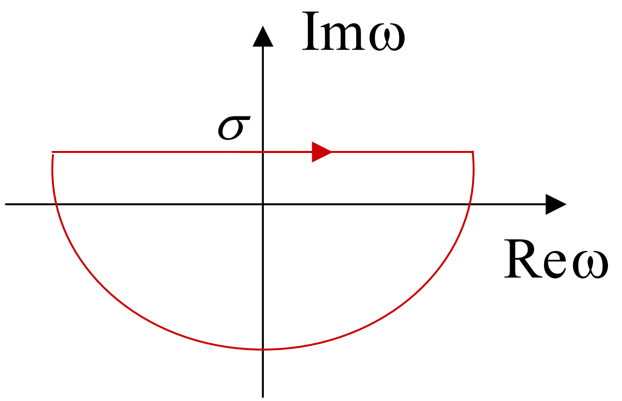
$$\frac{dE(t)}{dt} = \int d^3r \, \mathbf{E}_a(t,\mathbf{r}) \cdot \mathbf{j}_a(t,\mathbf{r})$$

Initial value problem

One-sided Fourier transformation

$$\begin{cases} f(\omega,\mathbf{k}) = \int_0^\infty dt \int d^3r\, e^{i(\omega t - \mathbf{kr})} f(t,\mathbf{r}) \\ f(t,\mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega t - \mathbf{kr})} f(\omega,\mathbf{k}) \end{cases}$$

$$0 < \sigma \in R$$



$$\mathbf{j}_a(t,\mathbf{r}) = gQ_a \mathbf{v}\,\delta^{(3)}(\mathbf{r} - \mathbf{v}t) \quad \Rightarrow \quad \mathbf{j}_a(\omega,\mathbf{k}) = \frac{igQ_a\mathbf{v}}{\omega - \mathbf{k}\cdot\mathbf{v}}$$

$$\frac{dE(t)}{dt} = gQ_a \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega - \mathbf{kv})t} \mathbf{E}_a(\omega,\mathbf{k}) \cdot \mathbf{v}$$

Induced Electric Field

Linearized Yang-Mills (Maxwell) equations (Hard Loop Approximation)

$$i\mathbf{k}\cdot\mathbf{D}(\omega,\mathbf{k})=\rho(\omega,\mathbf{k}),\qquad i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k})=0,$$
$$i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k})=i\omega\mathbf{B}(\omega,\mathbf{k})+\mathbf{B}_0(\mathbf{k}),$$
$$i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k})=\mathbf{j}(\omega,\mathbf{k})-i\omega\mathbf{E}(\omega,\mathbf{k})-\mathbf{D}_0(\mathbf{k})$$

$$D^i(\omega,\mathbf{k})=\varepsilon^{ij}(\omega,\mathbf{k})\,E^j(\omega,\mathbf{k})$$

Chromodielectric tensor

$$\varepsilon^{ij}(\omega,\mathbf{k})=\delta^{ij}+\frac{g^2}{2\omega}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{kv}+i0^+}\frac{\partial f(\mathbf{p})}{\partial p^l}\Big[\Big(1-\frac{\mathbf{kv}}{\omega}\Big)\delta^{lj}+\frac{k^l v^j}{\omega}\Big]$$

dynamical information

$$E^i(\omega,\mathbf{k})=-i(\Sigma^{-1})^{ij}(\omega,\mathbf{k})[\omega\mathbf{j}(\omega,\mathbf{k})+\mathbf{k}\times\mathbf{B}_0(\mathbf{k})-\omega\mathbf{D}_0(\mathbf{k})]^j$$

$$\Sigma^{ij}(\omega,\mathbf{k})\equiv-\mathbf{k}^2\delta^{ij}+k^ik^j+\omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

Energy-Loss formula

$$\frac{dE(t)}{dt} = gQ_a v^i \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\varpi)t}$$
$$\times (\Sigma^{-1})^{ij}(\omega,\mathbf{k}) \left[\frac{igQ_a \omega \mathbf{v}}{\omega - \varpi} + \mathbf{k} \times \mathbf{B}_0(\mathbf{k}) - \omega \mathbf{D}_0(\mathbf{k}) \right]^j$$

$$\varpi \equiv \mathbf{k} \cdot \mathbf{v}$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega,\mathbf{k})$$

Dispersion equation

$$\det[\Sigma(\omega,\mathbf{k})] = 0$$

Collective mode, qusiparticle excitation: $\omega(\mathbf{k})$

Initial values of the fields

Maxwell equations & $\mathbf{j}_a(t,\mathbf{r}) = gQ_a \mathbf{v}\delta^{(3)}(\mathbf{r}-\mathbf{v}t)$

Initial values:

$$D_0^i(\mathbf{k}) = -igQ_a \overline{\omega}\varepsilon^{ij}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{jk}(\overline{\omega},\mathbf{k})v^k$$

$$B_0^i(\mathbf{k}) = -igQ_a \varepsilon^{ijk}k^j(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k})v^l$$

When the test parton enters the plasma at $t = 0$, the instabilities are initiated.

Energy-Loss formula

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t} (\Sigma^{-1})^{ij}(\omega,\mathbf{k})$$

$$\times \left[\frac{\omega\delta^{jl}}{\omega-\overline{\omega}} - (k^j k^k - \mathbf{k}^2\delta^{jk})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k}) + \omega\overline{\omega}\varepsilon^{jk}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k}) \right]$$

$\mathbf{j}(\omega,\mathbf{k})$ $\qquad$ $\mathbf{B}_0(\mathbf{k})$ $\qquad$ $\mathbf{D}_0(\mathbf{k})$

Averaging over parton's colors: $\int dQ\, Q_a\, Q_b = C_2 \delta^{ab}, \qquad C_2 \equiv \begin{cases} \frac{1}{2} & \text{for quark} \\ N_c & \text{for gluon} \end{cases}$

$$C_R = \begin{cases} C_2 \frac{N_c^2-1}{N_c} = \frac{N_c^2-1}{2N_c} & \text{for quark (R = F)} \\ C_2 = N_c & \text{for gluon (R = G)} \end{cases}$$

Stable isotropic plasma

- $$\varepsilon^{ij}(\omega,\mathbf{k}) = \varepsilon_L(\omega,\mathbf{k})\frac{k^i k^j}{\mathbf{k}^2} + \varepsilon_T(\omega,\mathbf{k})\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

- $$(\Sigma^{-1})^{ij}(\omega,\mathbf{k}) = \frac{1}{\omega^2\varepsilon_L(\omega,\mathbf{k})}\frac{k^i k^j}{\mathbf{k}^2} + \frac{1}{\omega^2\varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi i}\int\frac{d^3k}{(2\pi)^3}e^{-i(\omega-\varpi)t}(\Sigma^{-1})^{ij}(\omega,\mathbf{k})$$
$$\times\left[\frac{\omega\,\delta^{jl}}{\omega-\varpi} - (k^j k^k - \mathbf{k}^2\delta^{jk})(\Sigma^{-1})^{kl}(\varpi,\mathbf{k}) + \omega\varpi\,\varepsilon^{jk}(\varpi,\mathbf{k})(\Sigma^{-1})^{kl}(\varpi,\mathbf{k})\right]$$

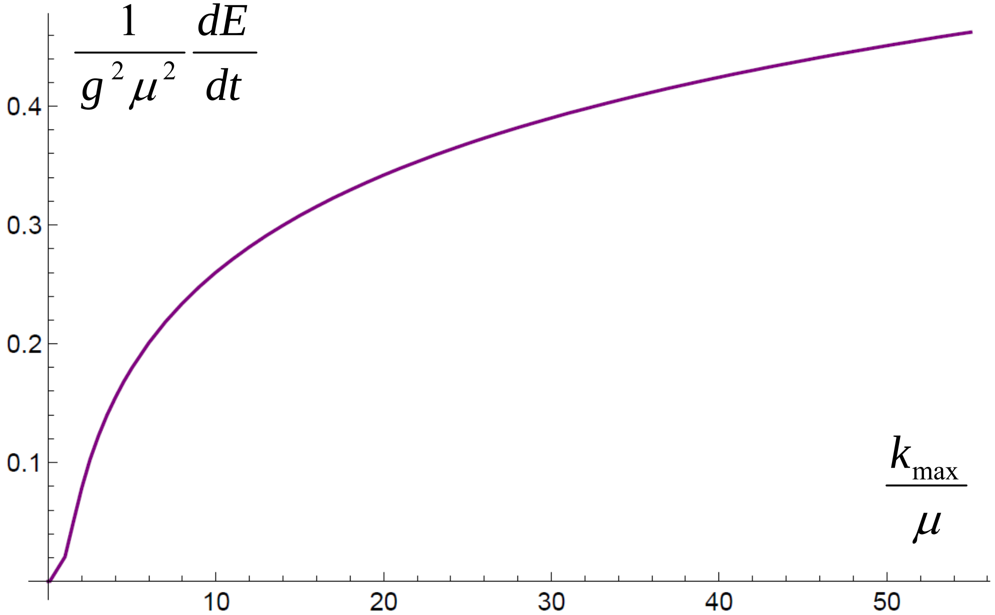
The only stationary contribution: $\omega = \varpi \equiv \mathbf{k}\cdot\mathbf{v}$

$$\frac{dE(t)}{dt} = ig^2 C_R\int\frac{d^3k}{(2\pi)^3}\frac{\varpi}{\mathbf{k}^2}\left[\frac{1}{\varepsilon_L(\varpi,\mathbf{k})} + \frac{\mathbf{k}^2\mathbf{v}^2 - \varpi^2}{\varpi^2\varepsilon_T(\varpi,\mathbf{k}) - \mathbf{k}^2}\right]$$

equivalent to the standard result by Braaten & Thoma

Energy loss in equilibrium QGP

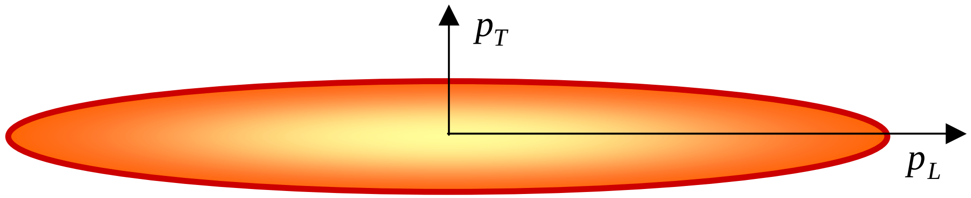
$$\frac{dE(t)}{dt} = ig^2 C_R \int \frac{d^3k}{(2\pi)^3} \frac{\overline{\omega}}{\mathbf{k}^2} \left[\frac{1}{\varepsilon_L(\overline{\omega},\mathbf{k})} + \frac{\mathbf{k}^2\mathbf{v}^2 - \overline{\omega}^2}{\overline{\omega}^2 \varepsilon_T(\overline{\omega},\mathbf{k}) - \mathbf{k}^2} \right]$$



Debye mass

$$\mu^2 \equiv g^2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

Extremely prolate system



$$f(\mathbf{p}) \sim \delta(p_x)\,\delta(p_y)$$

Collective modes

$$\det[\Sigma^{ij}(\omega,\mathbf{k})] = 0$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

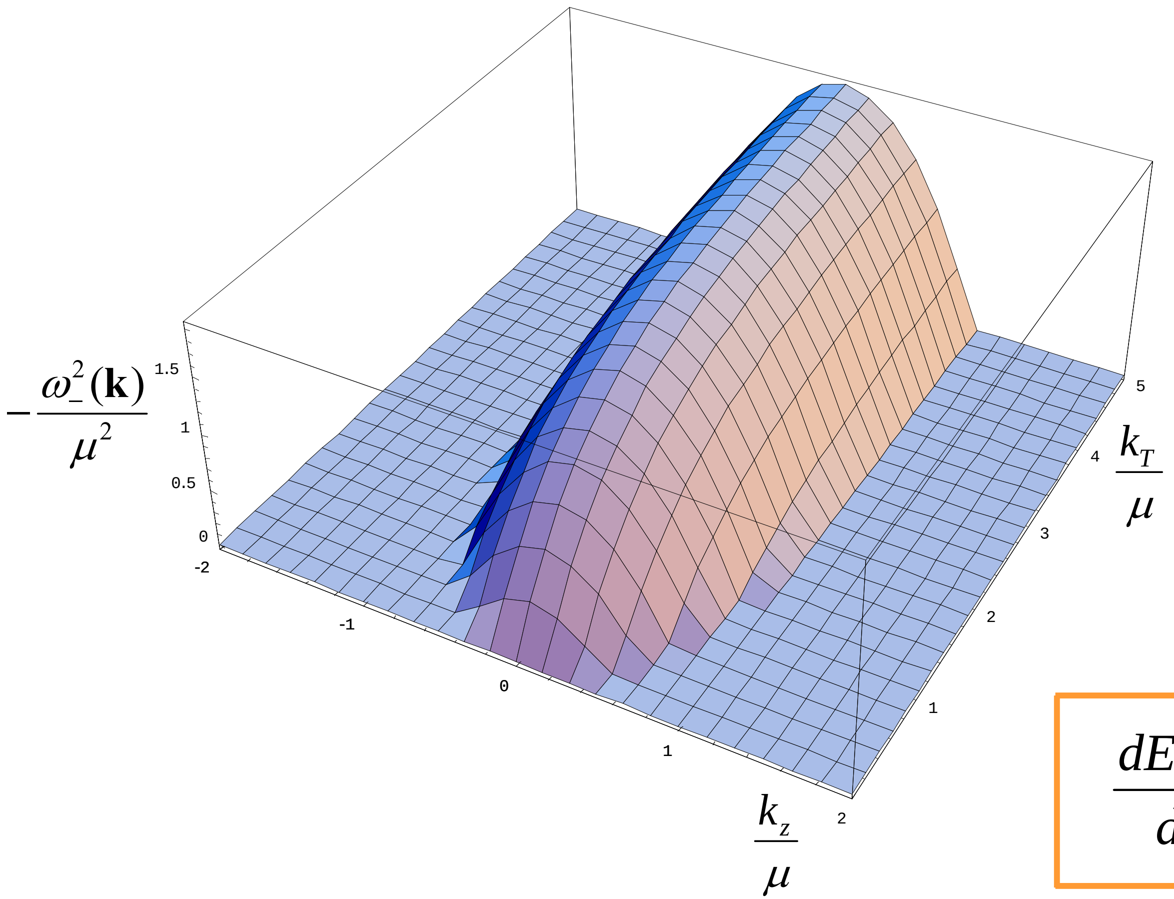
$$\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{kv}+i0^+}\frac{\partial f(\mathbf{p})}{\partial p^l}\Big[\Big(1-\frac{\mathbf{kv}}{\omega}\Big)\delta^{lj} + \frac{k^l v^j}{\omega}\Big]$$

Spectrum of collective modes

$$\begin{cases} \omega_1(\mathbf{k}) = \mu^2 + \mathbf{k}^2 \\ \omega_2(\mathbf{k}) = \mu^2 + (\mathbf{k}\cdot\mathbf{n})^2 \\ \omega_\pm(\mathbf{k}) = \frac{1}{2}\Big(\mathbf{k}^2 + (\mathbf{k}\cdot\mathbf{n})^2 \pm \sqrt{\mathbf{k}^4 + (\mathbf{k}\cdot\mathbf{n})^4 + 4\mu^2\mathbf{k}^2 - 4\mu^2(\mathbf{k}\cdot\mathbf{n})^2 - 2\mathbf{k}^2(\mathbf{k}\cdot\mathbf{n})^2}\Big) \end{cases}$$

$$\mathbf{n} \equiv (0,0,1)$$

Unstable chromomagnetic mode



$$\text{Im}\,\omega > 0$$

$$\frac{dE(t)}{dt} \sim \int \frac{d^3k}{(2\pi)^3} e^{\text{Im}\,\omega t} \ldots$$

Energy loss in extremely prolate system

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t} (\Sigma^{-1})^{ij}(\omega,\mathbf{k})$$
$$\times \left[\frac{\omega\,\delta^{jl}}{\omega-\overline{\omega}} - (k^j k^k - \mathbf{k}^2 \delta^{jk})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k}) + \omega\overline{\omega}\varepsilon^{jk}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k}) \right]$$

- Inversion of matrix Σ depending on **k** and **n**

$$\Sigma \;= aA + bB + cC + dD$$

$$\Sigma^{-1} = \alpha A + \beta B + \gamma C + \delta D$$

basis of matrices

$$A^{ij} = \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}, \quad B^{ij} = \frac{k^i k^j}{\mathbf{k}^2},$$
$$C^{ij} = \frac{n_T^i n_T^j}{\mathbf{n}_T^2}, \quad D^{ij} = n_T^i k^j + k^i n_T^j$$

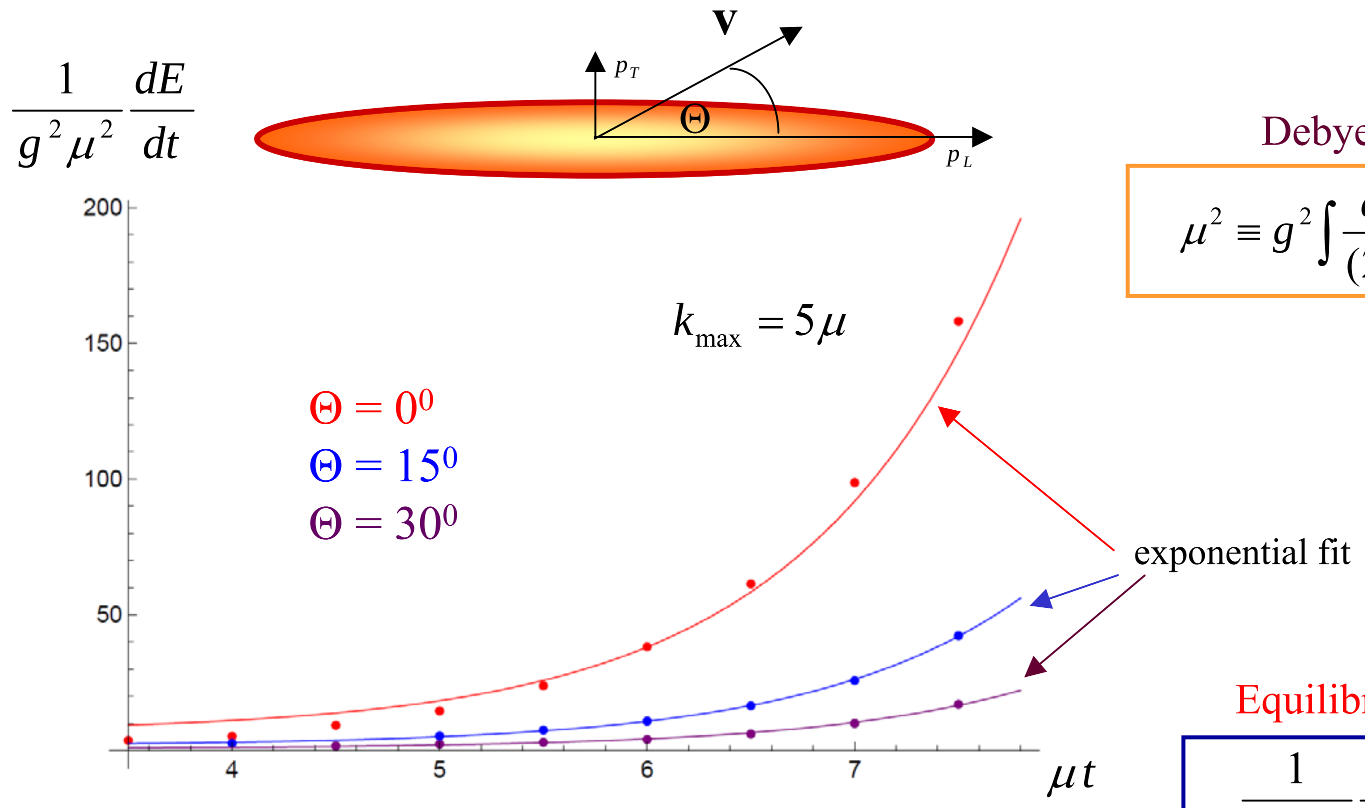
$$n_T^i \equiv \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) n^j$$

$$\mathbf{n}_T \perp \mathbf{k}$$

$$\Sigma\Sigma^{-1} = \mathbf{1} \quad \Rightarrow \quad \alpha, \beta, \gamma, \delta$$

P. Romatschke & M. Strickland, Physical Review D **68**, 036004 (2003)

Energy loss in extremely prolate system cont.



Debye mass

$$\mu^2 \equiv g^2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

Equilibrium value

$$\frac{1}{g^2\mu^2}\frac{dE}{dt} \approx 0.2$$

Conclusion

For various characteristics it matters that QGP is initially unstable