Production of light nuclei in high-energy collisions

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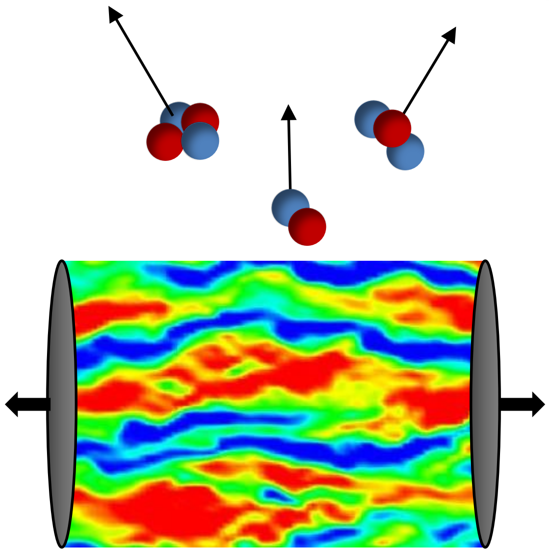
Production of light nuclei at RHIC & LHC



baryonless matter

$^{2}H,\ ^{2}\overline{H},\ ^{3}H,\ ^{3}\overline{H},\ ^{3}He,\ ^{3}\overline{He},\ ^{4}He,\ ^{4}\overline{He},\ ^{3}_{\Lambda}H,\ ^{3}_{\bar{\Lambda}}\overline{H}$

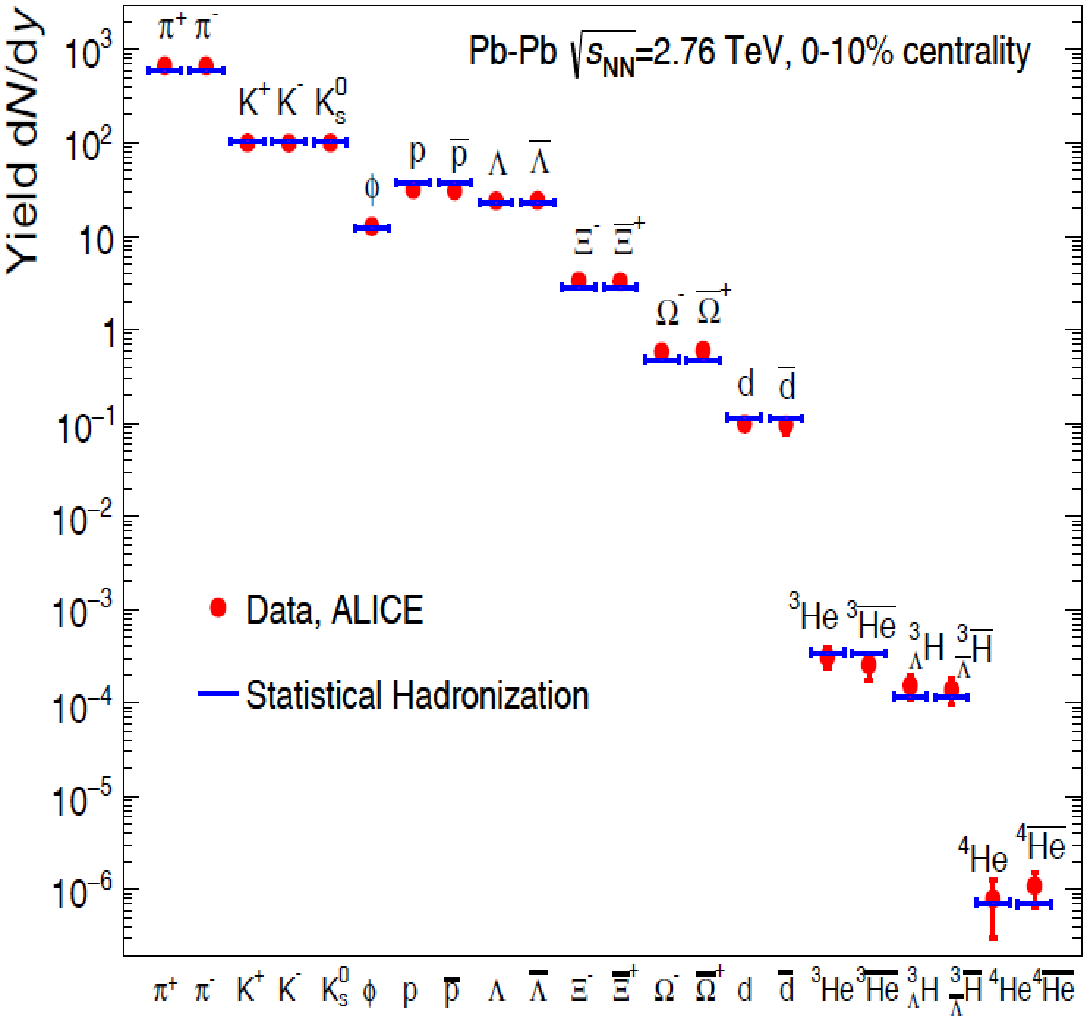
Genuine production!

Matter-antimatter symmetry!

<u>Two approaches to production of light nuclei</u>

- Coalescence model – final state interactions of nucleons
- Thermal model – direct production from thermalized hadron matter

Thermal model prediction



baryonless fireball

$$\text{Yield} \sim g\, e^{-\frac{m}{T}}$$

$$T = 156\ \text{MeV}$$

A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature **561**, 321 (2018)

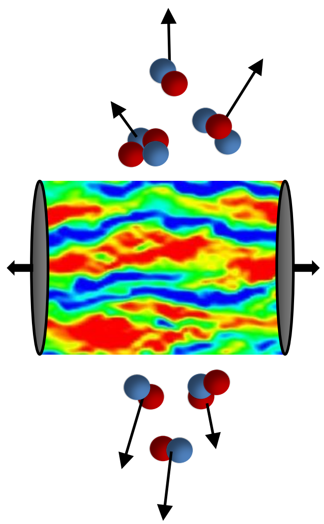
Can light nuclei exist in a fireball?

- Interparticle spacing in a hadron gas is about 1.5 fm at T = 156 MeV.
- Root mean square radius of a deuteron is 2.0 fm.
- Binding energy of a deuteron is ε_B = 2.2 MeV.
- A characteristic time of deuteron formation t is longer than 2 fm/c.
- A hadron gas at T = 156 MeV is essentially a classical system.

Snowflakes in hell ?
or
Snowflakes from hell ?

Two very different cases of producing light nuclei

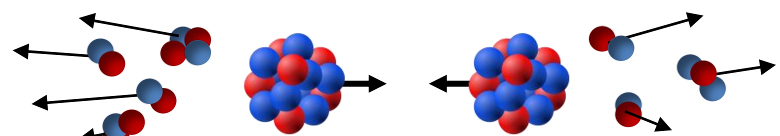
Genuine production



$y \approx 0$

hard process

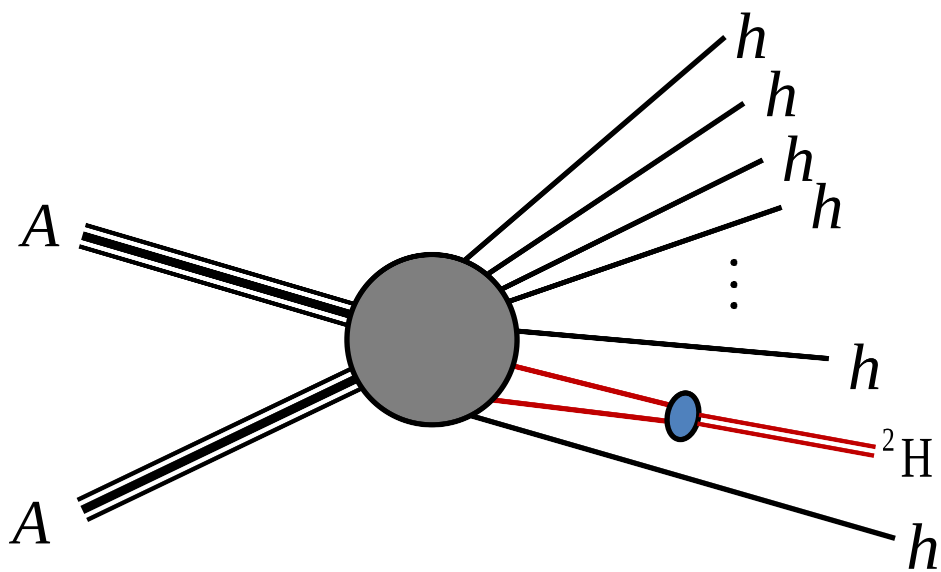
Shattering of incoming nuclei



$y \approx y_t$ $y \approx y_p$

soft process

Final state interaction



energy scale
of particle production
- deuteron mass

>>

energy scale
of formation of a nucleus
- binding energy

fast process

slow process

S.T. Butler & C.A. Pearson, Phys. Rev. **129**, 836 (1963)
A. Schwarzschild & C. Zupancic, Phys. Rev. **129**, 854 (1963)

Factorization of production of nucleons and formation of a deuteron

Deuteron yield

isospin factor

$$\frac{dN^D}{d^3\mathbf{P}_D} = \frac{1}{2} A_D \frac{dN^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p}$$

$$\frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

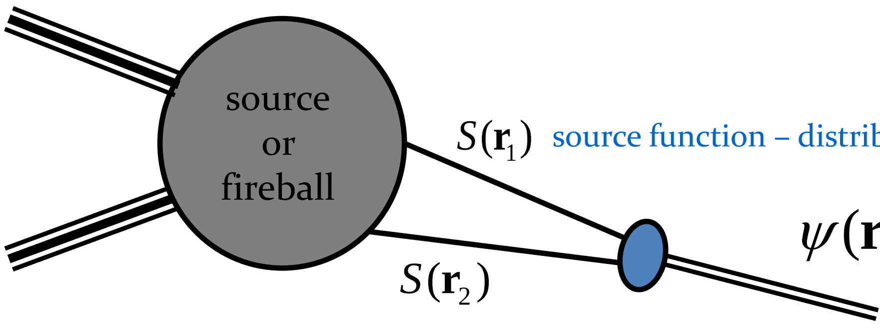
deuteron formation rate

yield of *np* pairs

$$\frac{1}{2}\frac{dN^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p} \approx \frac{dN^{pp}}{d^3\mathbf{p}_p d^3\mathbf{p}_p} \approx \left(\frac{dN^p}{d^3\mathbf{p}_p}\right)^2$$

$$\boxed{\frac{dN^D}{d^3\mathbf{P}_D} = A_D \left(\frac{dN^p}{d^3\mathbf{p}_p}\right)^2}$$

Deuteron formation rate



$S(\mathbf{r}_1)$ source function – distribution of emission points

$\psi(\mathbf{r}_1,\mathbf{r}_2)$ deuteron wave fuction

spin factor

$$A_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2\, S(\mathbf{r}_1) S(\mathbf{r}_2) \left|\psi(\mathbf{r}_1,\mathbf{r}_2)\right|^2$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \qquad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$\psi(\mathbf{r}_1,\mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}} \varphi_D(\mathbf{r})$$

$$\boxed{A_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}\, S_r(\mathbf{r}) \left|\varphi_D(\mathbf{r})\right|^2}$$

$$S_r(\mathbf{r}) \equiv \int d^3\mathbf{R}\, S\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) S\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right)$$

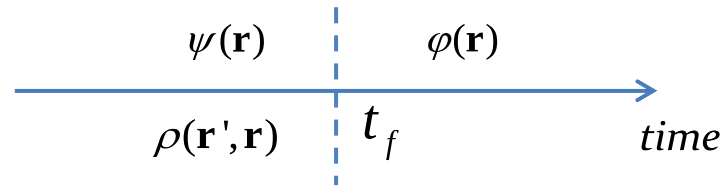
distribution of relative distance of n and p

H. Sato and K. Yazaki, Phys. Lett. B **98**, 153 (1981)

Quantum-mechanical meaning of the formation rate formula

Sudden approximation

$$E\Delta t << 1$$



Transition matrix element

$$M = \left|\int d^3\mathbf{r}\,\psi^*(\mathbf{r})\,\varphi(\mathbf{r})\right|^2 = \int d^3\mathbf{r}\,d^3\mathbf{r}'\,\varphi^*(\mathbf{r}')\underbrace{\psi(\mathbf{r}')\psi^*(\mathbf{r})}_{\rho(\mathbf{r}',\mathbf{r})}\varphi(\mathbf{r})$$

density matrix

$$M = \int d^3\mathbf{r}\,d^3\mathbf{r}'\,\varphi^*(\mathbf{r}')\,\rho(\mathbf{r}',\mathbf{r})\,\varphi(\mathbf{r})$$

If density matrix is diagonal

$$\rho(\mathbf{r}',\mathbf{r}) = S(\mathbf{r})\,\delta^{(3)}(\mathbf{r}'-\mathbf{r}) \qquad \Rightarrow \qquad \boxed{M = \int d^3\mathbf{r}\,S(\mathbf{r})\left|\varphi(\mathbf{r})\right|^2}$$

Diagonal density matrix

$$\langle\psi|\hat{A}|\psi\rangle = \sum_{i,j} c_i^* c_j \langle\alpha_i|\hat{A}|\alpha_j\rangle = \sum_{i,j} \rho_{ji} A_{ij}$$

$$|\psi\rangle = \sum_i c_i |\alpha_i\rangle$$

$$\rho_{ji} \equiv c_i^* c_j \qquad A_{ij} \equiv \langle\alpha_i|\hat{A}|\alpha_j\rangle$$

density matrix

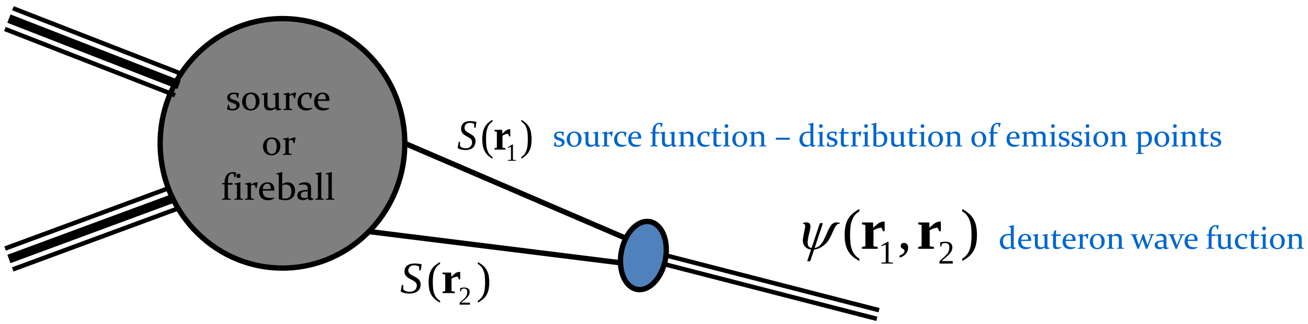
$\overline{\ldots\ldots}$ – averaging over time or events

$$\overline{\langle\psi|\hat{A}|\psi\rangle} = \sum_{i,j} \overline{c_i^* c_j} \langle\alpha_i|\hat{A}|\alpha_j\rangle = \sum_i |c_i|^2 A_{ii}$$

$$\overline{\rho_{ji}} = \overline{c_i^* c_j} = \delta^{ij} |c_i|^2$$ random phase approximation

diagonal density matrix

Deuteron formation rate



spin factor

$$A_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2\, S(\mathbf{r}_1) S(\mathbf{r}_2) \left|\psi(\mathbf{r}_1,\mathbf{r}_2)\right|^2$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \qquad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

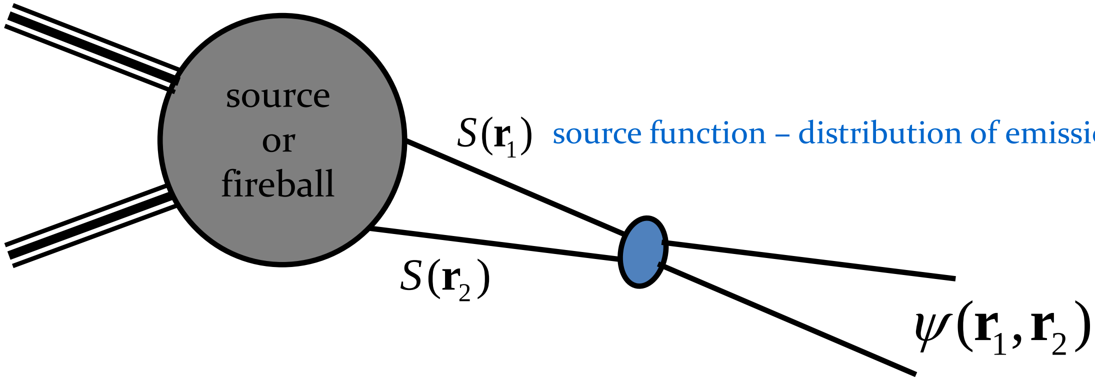
$$\psi(\mathbf{r}_1,\mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}} \varphi_D(\mathbf{r})$$

$$\boxed{A_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}\, S_r(\mathbf{r}) \left|\varphi_D(\mathbf{r})\right|^2}$$

$$S_r(\mathbf{r}) \equiv \int d^3\mathbf{R}\, S\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) S\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right)$$ distribution of relative distance of n and p

H. Sato and K. Yazaki, Phys. Lett. B **98**, 153 (1981)

n-p correlation function



$S(\mathbf{r}_1)$ source function – distribution of emission points

$\psi(\mathbf{r}_1,\mathbf{r}_2)$ wave fuction of n-p in scattering state

$$C(\mathbf{q}) = \int d^3\mathbf{r}_1 d^3\mathbf{r}_2\, S(\mathbf{r}_1)S(\mathbf{r}_2)\left|\psi_{\mathbf{q}}(\mathbf{r}_1,\mathbf{r}_2)\right|^2$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2), \qquad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

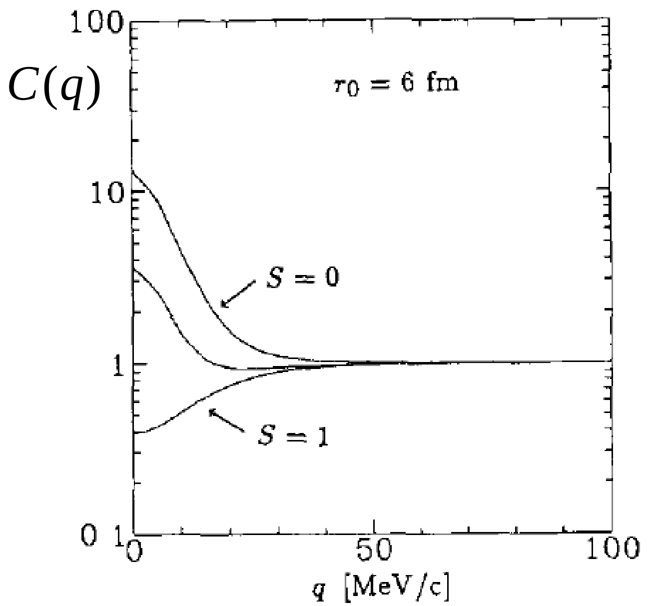
$$\psi_{\mathbf{q}}(\mathbf{r}_1,\mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}}\varphi_{\mathbf{q}}(\mathbf{r})$$

$$\boxed{C(\mathbf{q}) = \int d^3\mathbf{r}\, S_r(\mathbf{r})\left|\varphi_{\mathbf{q}}(\mathbf{r})\right|^2}$$

$$S_r(\mathbf{r}) \equiv \int d^3\mathbf{R}\, S\left(\mathbf{R}-\frac{1}{2}\mathbf{r}\right)S\left(\mathbf{R}+\frac{1}{2}\mathbf{r}\right)$$

S.E. Koonin, Phys. Lett. B **70**, 43 (1977)

n-p correlation function



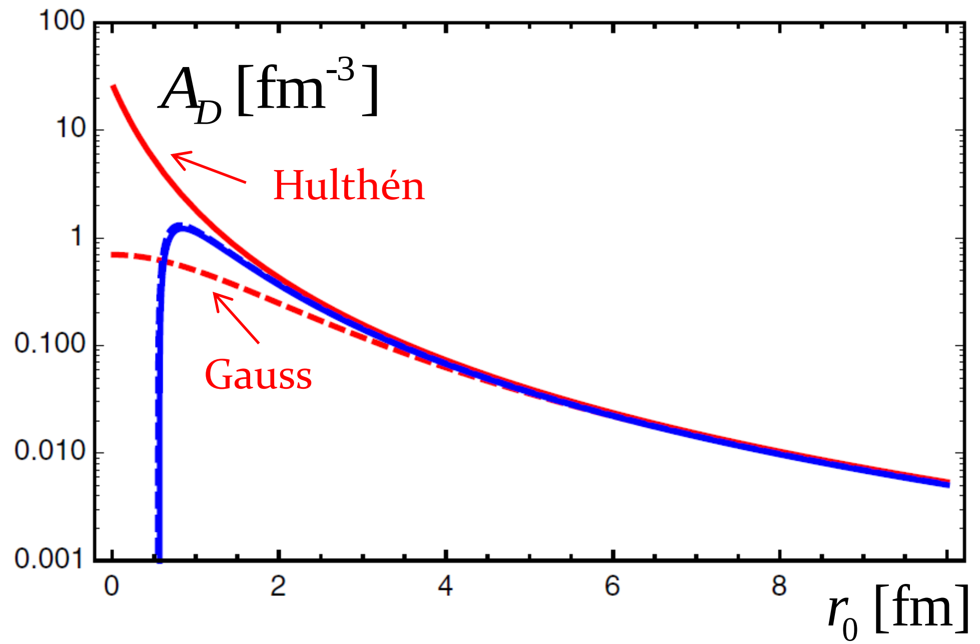
$$S(\mathbf{r}) = \left(\frac{1}{2\pi r_0^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2r_0^2}\right)$$

Lednicky-Lyuboshitz formula

St. Mrówczyński, Phys. Lett. B **277**, 43 (1992)

Sum rule due to completeness of quantum states

$$\int d^3\mathbf{q}\big(C_1(\mathbf{q}) - C_0(\mathbf{q})\big) = -A_D$$



R. Maj & St. Mrówczyński, Phys. Rev. C **101**, 014901 (2020)
R. Maj & St. Mrówczyński, Phys. Rev. C **71**, 044905 (2005)
St. Mrówczyński, Phys. Lett. B **345**, 393 (1995)

Emission time

Instantaneous emission

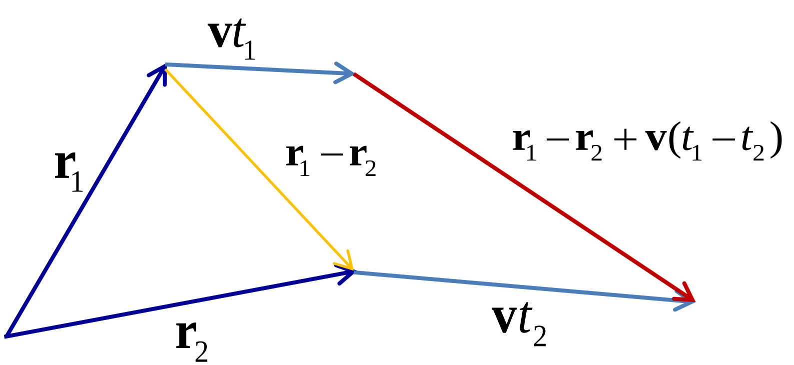
$$A_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2\, S(\mathbf{r}_1) S(\mathbf{r}_2) \left|\psi(\mathbf{r}_1, \mathbf{r}_2)\right|^2$$

Emission extended in time

$$A_D = \frac{3}{4}(2\pi)^3 \int dt_1 d^3\mathbf{r}_1 dt_2 d^3\mathbf{r}_2\, S(t_1, \mathbf{r}_1) S(t_2, \mathbf{r}_2) \left|\psi(\mathbf{r}_1 + \mathbf{v}t_1, \mathbf{r}_2 + \mathbf{v}t_2)\right|^2$$

$$\int dt\, d^3\mathbf{r}\, S(t, \mathbf{r}) = 1$$

$$\mathbf{v} = \frac{\mathbf{P}_D}{E_D}$$



S.E. Koonin, Phys. Lett. B **70**, 43 (1977)

Emission time cont.

$$A_D = \frac{3}{4}(2\pi)^3 \int dt_1 d^3\mathbf{r}_1 dt_2 d^3\mathbf{r}_2 \, S(t_1, \mathbf{r}_1 - \mathbf{v}t_1) S(t_2, \mathbf{r}_2 - \mathbf{v}t_2) \left|\psi(\mathbf{r}_1, \mathbf{r}_2)\right|^2$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \qquad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$T \equiv \frac{1}{2}(t_1 + t_2), \qquad t \equiv t_1 - t_2$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}} \varphi(\mathbf{r})$$

$$\boxed{A_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r} \, S_r(\mathbf{r}) \left|\varphi(\mathbf{r})\right|^2}$$

$$S_r(\mathbf{r}) \equiv \int dt \;\, S_r\left(t, \mathbf{r} - \mathbf{v}t\right)$$

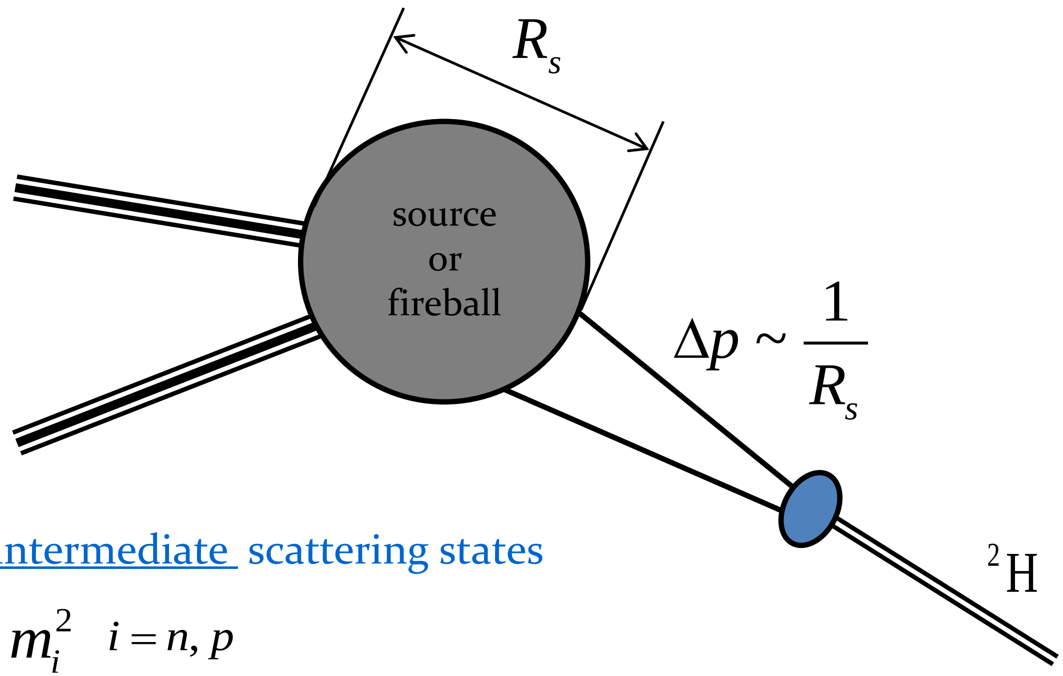
$$S_r(t, \mathbf{r}) \equiv \int dT \, d^3\mathbf{R} \, S\left(T - \frac{1}{2}t, \mathbf{R} - \frac{1}{2}\mathbf{r}\right) S\left(T + \frac{1}{2}t, \mathbf{R} + \frac{1}{2}\mathbf{r}\right)$$

$$S(t, \mathbf{r}) = \left(\frac{1}{2\pi\tau^2}\right)^{1/2} \left(\frac{1}{2\pi R_s^2}\right)^{3/2} \exp\left(-\frac{t^2}{2\tau^2}\right) \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right)$$

$$S_r(\mathbf{r}) = \left(\frac{1}{2\pi(R_s^2 + v^2\tau^2)}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2(R_s^2 + v^2\tau^2)}\right)$$

$$\boxed{R_s \to \sqrt{R_s^2 + v^2\tau^2}}$$

Energy-momentum conservation



Nucleons are <u>intermediate</u> scattering states

$$E_i^2 - \mathbf{p}_i^2 \neq m_i^2 \quad i = n, p$$

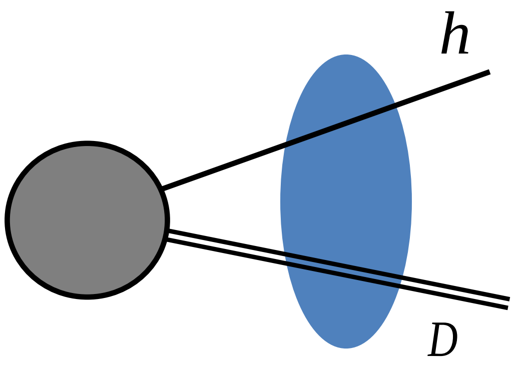
Energy-momentum conservation

$$\begin{cases} \mathbf{p}_p + \mathbf{p}_n = \mathbf{p}_D \\ E_p + E_n = E_D \end{cases}$$

St. Mrówczyński, J. Phys. G **11**, 1087 (1987)

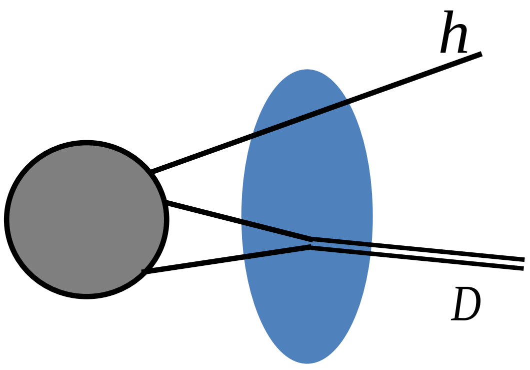
Hadron-deuteron correlations

Hadron-deuteron correlations carry information about a mechanism of deuteron production.



direct production

or

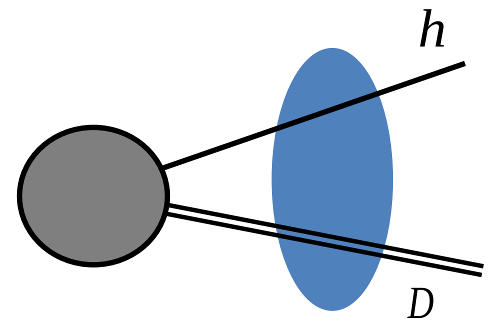


final state interaction

St. Mrówczyński & P. Słoń, Acta Phys. Pol. B **51**, 1739 (2020)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle



Experimental definition

$$\boxed{\frac{dN_{hD}}{d\mathbf{p}_h\, d\mathbf{p}_D} = C(\mathbf{p}_h, \mathbf{p}_D)\, \frac{dN_h}{d\mathbf{p}_h}\, \frac{dN_D}{d\mathbf{p}_D}}$$

Theoretical formula

$$\boxed{C(\mathbf{p}_h, \mathbf{p}_D) = \int d^3 r_h\, d^3 r_D\, S(\mathbf{r}_h)\, S(\mathbf{r}_D) \left|\psi(\mathbf{r}_h, \mathbf{r}_D)\right|^2}$$

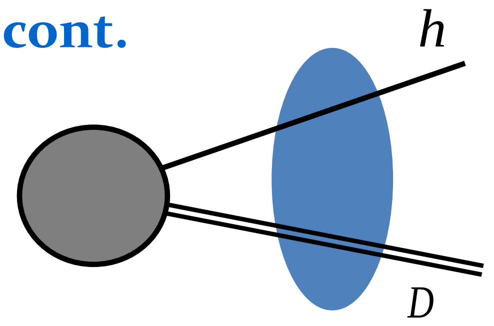
distribution
of emission points

h-D wave function

S.E. Koonin, Phys. Lett. B **70**, 43 (1977)
R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle cont.



Separation of CM and relative motion

$$\begin{cases} \mathbf{R} \equiv \dfrac{m_D \mathbf{r}_D + m_h \mathbf{r}_h}{m_D + m_h} \\ \mathbf{r} \equiv \mathbf{r}_D - \mathbf{r}_h \end{cases}$$

$$\psi(\mathbf{r}_h, \mathbf{r}_D) = e^{i\mathbf{PR}} \phi_{\mathbf{q}}(\mathbf{r})$$

$$\boxed{C(\mathbf{q}) = \int d^3r \; S_r(\mathbf{r}) \left| \phi_{\mathbf{q}}(\mathbf{r}) \right|^2}$$

„Relative" source function

$$S_r(\mathbf{r}) \equiv \int d^3R \; S\left(\mathbf{R} - \frac{m_D}{m_D + m_h}\mathbf{r}\right) S\left(\mathbf{R} + \frac{m_h}{m_D + m_h}\mathbf{r}\right) = \left(\frac{1}{4\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_s^2}\right)$$

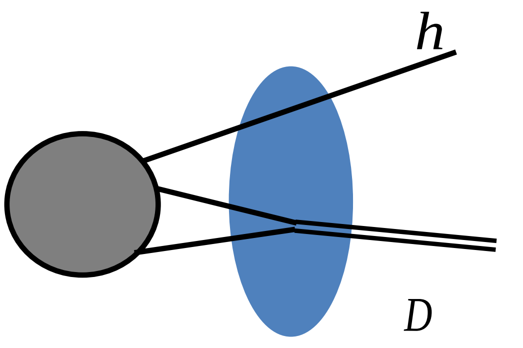
$$S(\mathbf{r}) = \left(\frac{1}{2\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right)$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h\, d\mathbf{p}_D} = C(\mathbf{p}_h, \mathbf{p}_D)\, A_D \frac{dN_h}{d\mathbf{p}_h} \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$



Theoretical formula

$$C(\mathbf{p}_h, \mathbf{p}_D)\, A_D = \int d^3 r_h\, d^3 r_n\, d^3 r_p\, S(\mathbf{r}_h) S(\mathbf{r}_n)\; S(\mathbf{r}_p) \left|\psi_{hD}(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p)\right|^2$$

Deuteron formation rate $\quad \frac{dN_D}{d\mathbf{p}_D} = A_D \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p} \qquad \frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$

$$A_D = \frac{3}{8}(2\pi)^3 \int d^3\mathbf{r}_n d^3\mathbf{r}_p\, S(\mathbf{r}_n) S(\mathbf{r}_p) \left|\psi_D(\mathbf{r}_n, \mathbf{r}_p)\right|^2 = \frac{3}{8}(2\pi)^3 \int d^3 r_{np}\, S_r(\mathbf{r}_{np}) \left|\phi_D(\mathbf{r}_{np})\right|^2$$

spin-isopsin factor

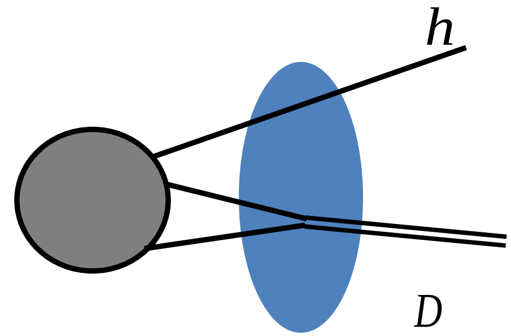
$$\psi_D(\mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{PR}} \phi_D(\mathbf{r}_{np})$$

St. Mrówczyński & P. Słoń, Acta Phys. Pol. B **51**, 1739 (2020)

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton cont

Separation of CM and relative motion



$$\begin{cases} \mathbf{R} \equiv \dfrac{m_p \mathbf{r}_p + m_n \mathbf{r}_n + m_h \mathbf{r}_h}{m_p + m_n + m_h} \\ \mathbf{r}_{np} \equiv \mathbf{r}_p - \mathbf{r}_n \\ \mathbf{r} \equiv \mathbf{r}_h - \dfrac{m_p \mathbf{r}_p + m_n \mathbf{r}_n}{m_p + m_n} \end{cases}$$

$$\psi(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_{\mathbf{q}}(\mathbf{r}) \varphi_D(\mathbf{r}_{np})$$

$$C(\mathbf{q}) = \frac{1}{A_D} \int d^3R \, d^3r_{np} \, d^3r \, S(\mathbf{r}_h) S(\mathbf{r}_n) S(\mathbf{r}_p) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2 \left|\varphi_D(\mathbf{r}_{np})\right|^2$$

For Gaussian source

$$\boxed{C(\mathbf{q}) = \int d^3r \, S_{3r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2}$$

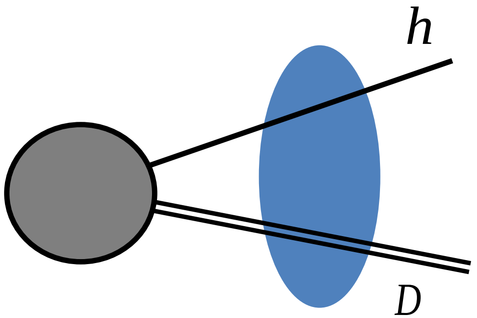
$$S_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2}\right)$$

For a non-Gaussian source, A_D remains in the correlation function!

Direct vs. final state interaction

Direct production

$$C(\mathbf{q}) = \int d^3r S_r(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



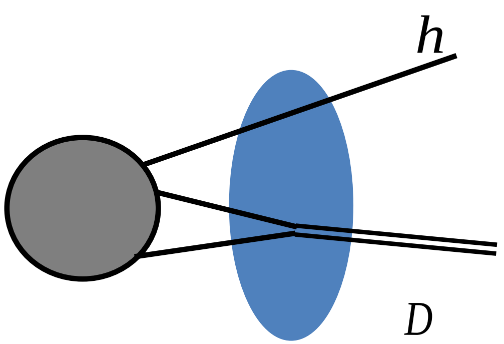
$$S_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2}\right)$$

$$\sqrt{\frac{4}{3}} \approx 1.15$$

$$S_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2}\right)$$

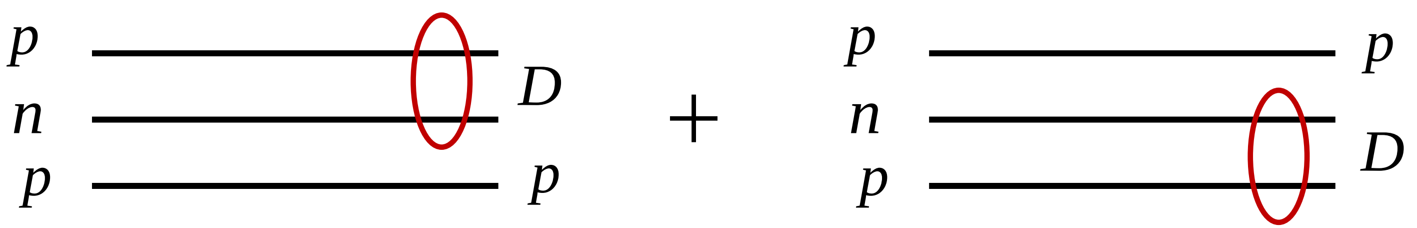
Final state interaction

$$C(\mathbf{q}) = \int d^3r S_{3r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



p-*D* correlation function

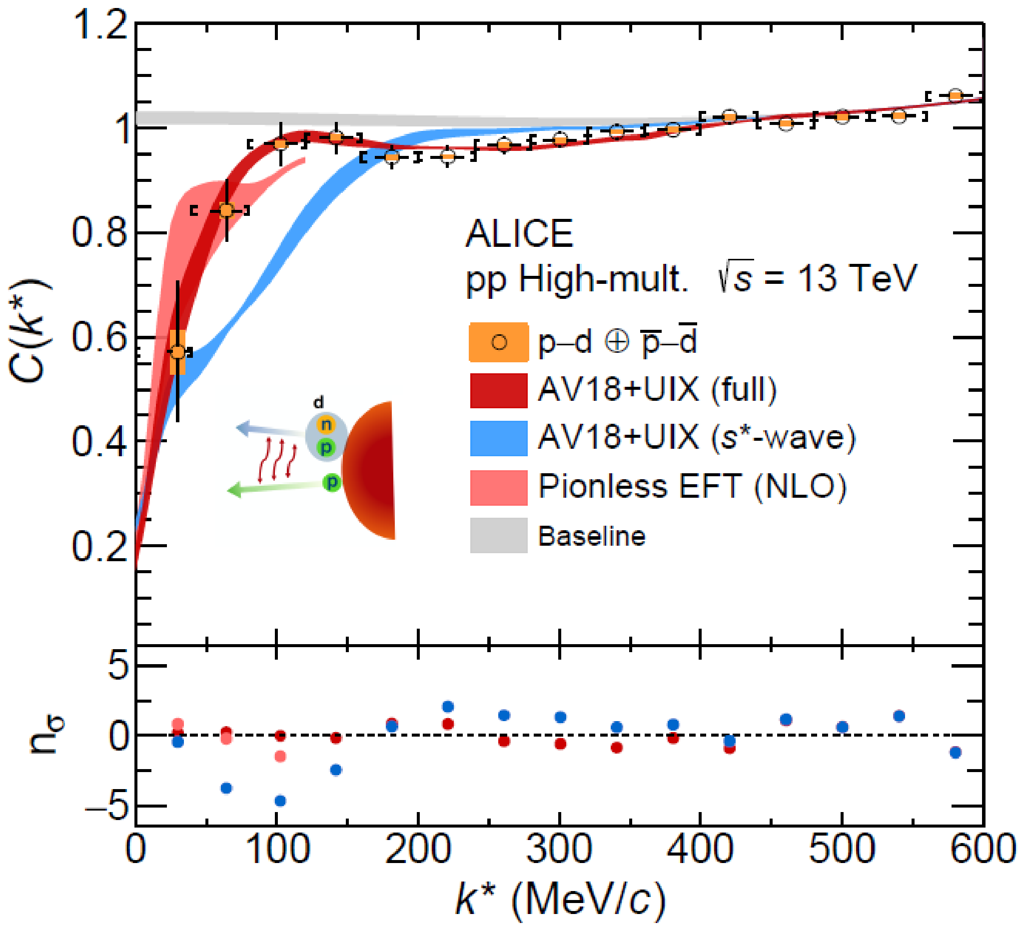
$$\psi^{\mathbf{q}}_{pD}(\mathbf{r}_n, \mathbf{r}_{p_1}, \mathbf{r}_{p_2})$$



Full three-body calculations

$$C(\mathbf{q}) = \frac{1}{A_D} \int d^3r_n \, d^3r_{p_1} \, d^3r_{p_2} \, S(\mathbf{r}_n) S(\mathbf{r}_{p_1}) S(\mathbf{r}_{p_2}) \left| \psi^{\mathbf{q}}_{pD}(\mathbf{r}_n, \mathbf{r}_{p_1}, \mathbf{r}_{p_2}) \right|^2$$

p-D correlation function



$$R_s = 1.43 \pm 0.16 \text{ fm}$$

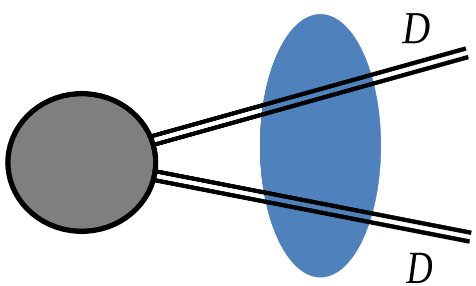
ALICE arXiv:2308.16120

M. Viviani et al, Phys. Rev. C **108**, 064002 (2023)

Deuteron-deuteron correlation function

Direct production

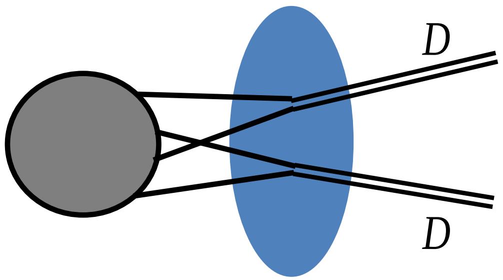
$$C(\mathbf{q}) = \int d^3r\, S_r(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$



$$S_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2}\right)$$

$\sqrt{2} \approx 1.41$

$$S_{4r}(\mathbf{r}) = \left(\frac{1}{2\pi R^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R^2}\right)$$



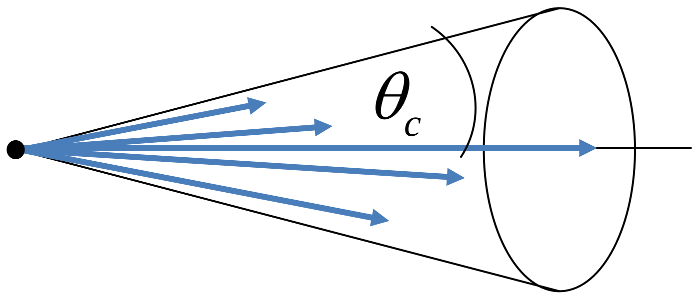
Final state interaction
& factorizatiom

$$C(\mathbf{q}) = \int d^3r\, S_{4r}(\mathbf{r}) \left|\phi_{\mathbf{q}}(\mathbf{r})\right|^2$$

St. Mrówczyński & P. Słoń, Phys. Rev. C **104**, 024909 (2021)

Jet-associated deuteron production

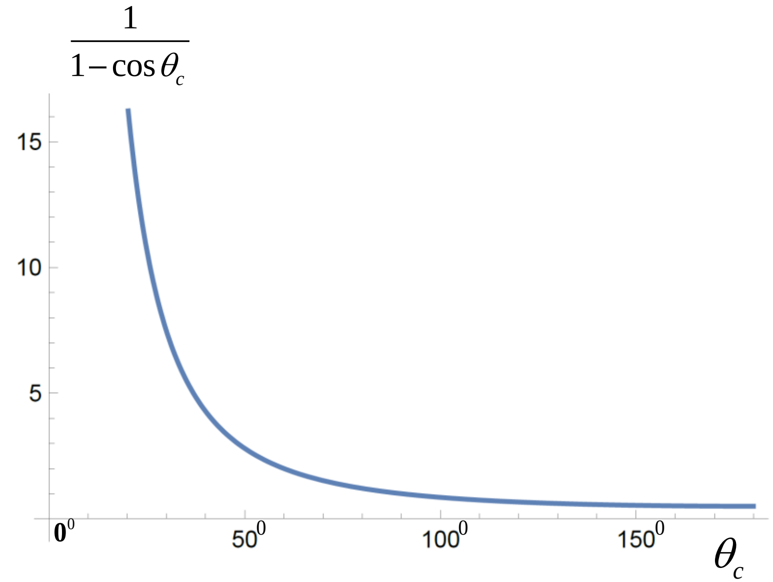
ALICE Collaboration, Phys. Lett. B **819**, 136440 (2021)
ALICE Collaboration, Phys. Rev. Lett. **131**, 042301 (2023), Erratum, *ibid*. **132**, 109901(E) (2024)



$$\frac{dN_D}{d^3\mathbf{P}_D} = A_D \boxed{\left(\frac{dN^p}{d^3\mathbf{p}_p}\right)^2} \qquad \mathbf{P}_D = 2\mathbf{p}_p$$

$$\frac{dN_p}{d^3\mathbf{p}} = N_p \frac{e^{-\alpha p}}{\pi\alpha^3} \frac{\Theta(\cos\theta - \cos\theta_c)}{1-\cos\theta_c}$$

$$\int d^3\mathbf{p} \frac{dN_p}{d^3\mathbf{p}} = N_p$$



Deuteron yield

$$N_D = \int d^3\mathbf{p} \frac{dN_D}{d^3\mathbf{p}} = N_p^2 A_D \frac{2}{\pi\alpha^3} \frac{1}{1-\cos\theta_c}$$

St. Mrówczyński, arXiv: 2312.17695

Jet-associated deuteron production

$$\frac{dN_D}{d^3\mathbf{P}_D} = A_D \left(\frac{dN^p}{d^3\mathbf{p}_p} \right)^2 \qquad \mathbf{P}_D = 2\mathbf{p}_p$$

$$E_D \frac{dN_D}{d^3\mathbf{P}_D} = B_2 \left(E_p \frac{dN^p}{d^3\mathbf{p}_p} \right)^2 \qquad E_D = 2E_p$$

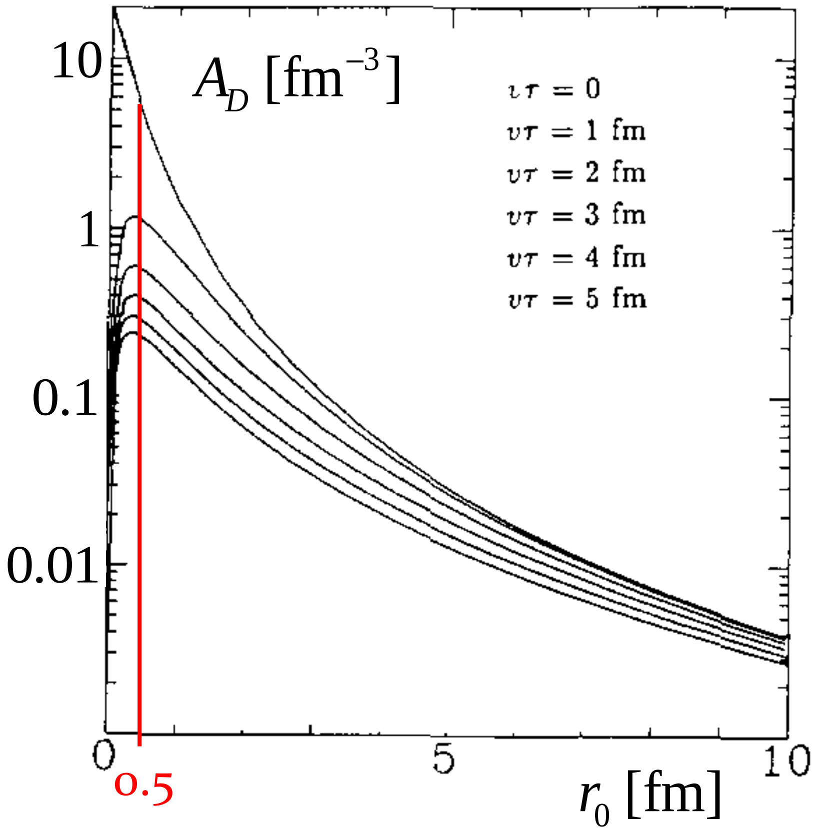
$$B_2 \approx 0.13 \pm 0.7 \text{ GeV}^2$$

$$A_D = \frac{1}{2} B_2 m$$

$$A_D \approx 8 \pm 4 \text{ fm}^{-3}$$

Hulthén wave function

$$\phi_D(r) = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}} \frac{\exp(-\alpha r) - \exp(-\beta r)}{r}$$



St. Mrówczyński, Phys. Lett. B **277**, 43 (1992)

ALICE Collaboration, Phys. Rev. Lett. 131, 042301 (2023)

Jet-associated deuteron production

$$A_D = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}\, S_r(\mathbf{r}) |\varphi(\mathbf{r})|^2 \approx \frac{3}{4}(2\pi)^3 |\varphi(r=0)|^2 \int d^3\mathbf{r}\, S_r(\mathbf{r})$$

$$r_0 << r_D$$

$$A_D = \frac{3}{4}(2\pi)^3 |\varphi(r=0)|^2 = 3\pi^2 \alpha\beta(\alpha+\beta) \approx 20.2 \text{ fm}^{-2}$$

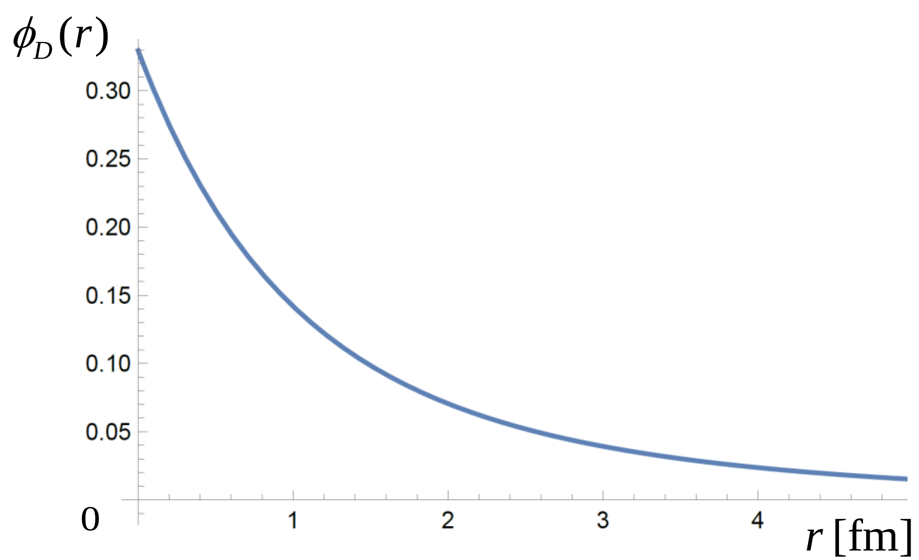
Exp: $A_D \approx 8 \pm 4 \text{ fm}^{-3}$

$r_0 < 0.5$ fm

Hulthén wave function

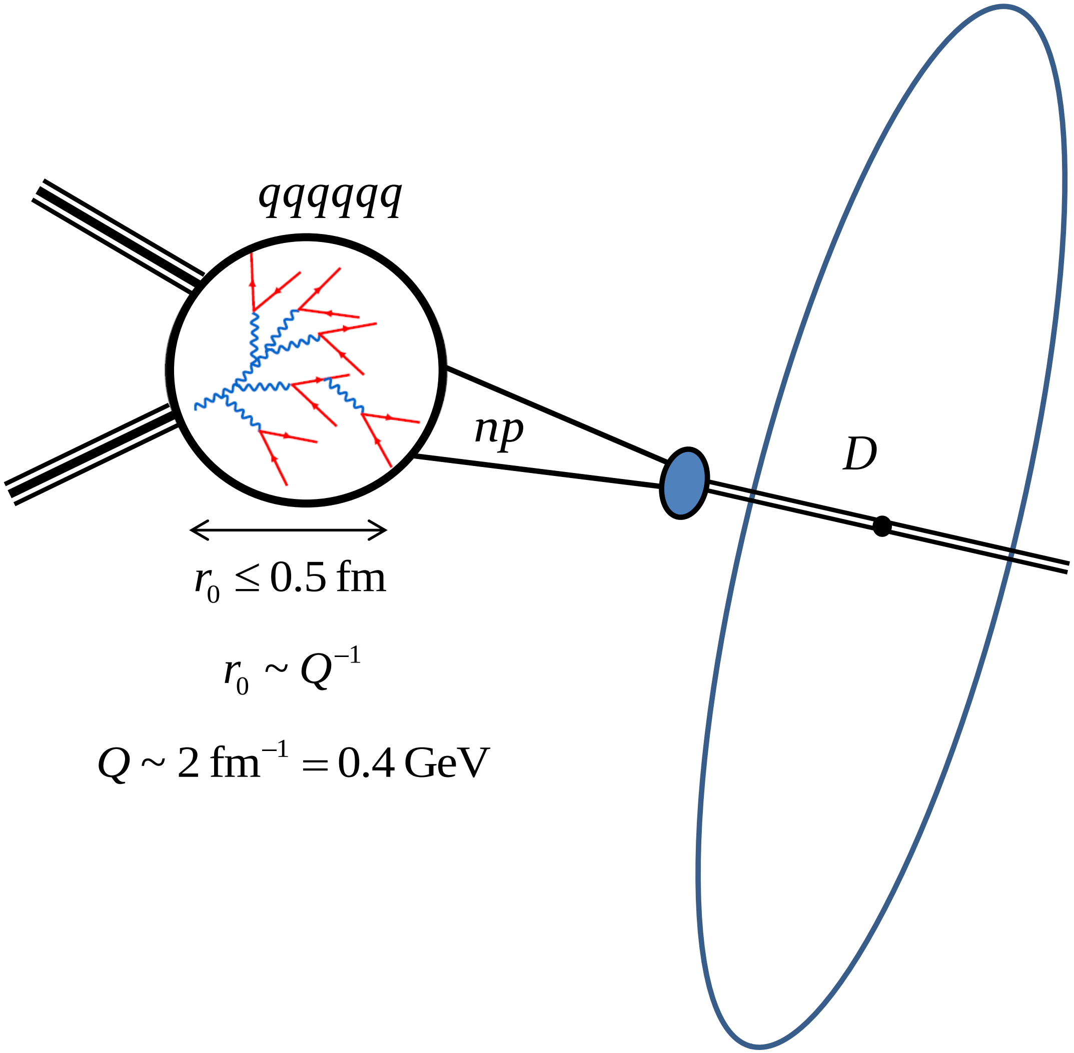
$$\phi_D(r) = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}} \frac{\exp(-\alpha r) - \exp(-\beta r)}{r}$$

$$\alpha = 0.23 \text{ fm}^{-1}, \quad \beta = 1.61 \text{ fm}^{-1}$$



St. Mrówczyński, arXiv: 2312.17695

Jet-associated deuteron production



$$r_0 \leq 0.5\ \mathrm{fm}$$

$$r_0 \sim Q^{-1}$$

$$Q \sim 2\ \mathrm{fm}^{-1} = 0.4\ \mathrm{GeV}$$

St. Mrówczyński, arXiv: 2312.17695

Big bound states from small sources

positronium

$$\pi^0 \to \gamma\,(e^+e^-)$$

L. G. Afanasev et al., Phys. Lett. B **236**, 116 (1990)

pionium

$$p\mathrm{Be} \to X\ (\pi^+\pi^-)$$

DIRAC Collaboration, Phys. Rev. Lett**.** **122**, 082003 (2019)

π μ atom

$$K_L^0 \to (\pi^\pm \mu^\mp)\,\nu_\mu$$

$$\frac{r_B}{r_0} \sim 10^5$$

S. H. Aronson et al., Phys. Rev. D **33**, 3180 (1986)