Energy loss in Unstable Quark-Gluon Plasma

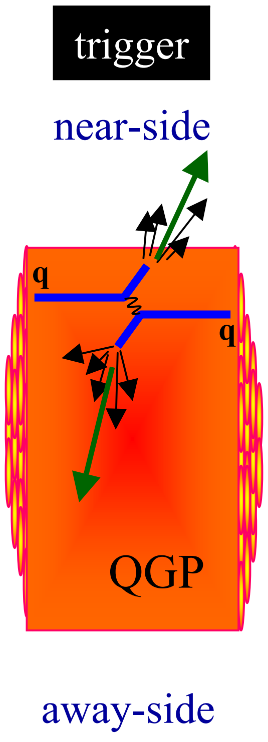
Stanisław Mrówczyński

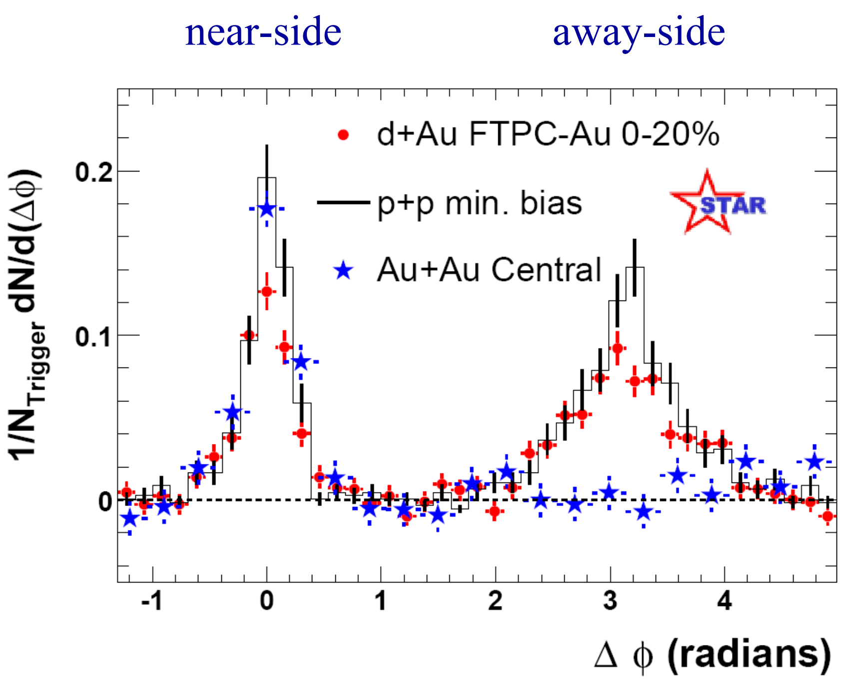
Jan Kochanowski University, Kielce, Poland
& National Centre of Nuclear Reserach, Warsaw, Poland

in collaboration with

Margaret Carrington & Katarzyna Deja

Motivation - jet quenching

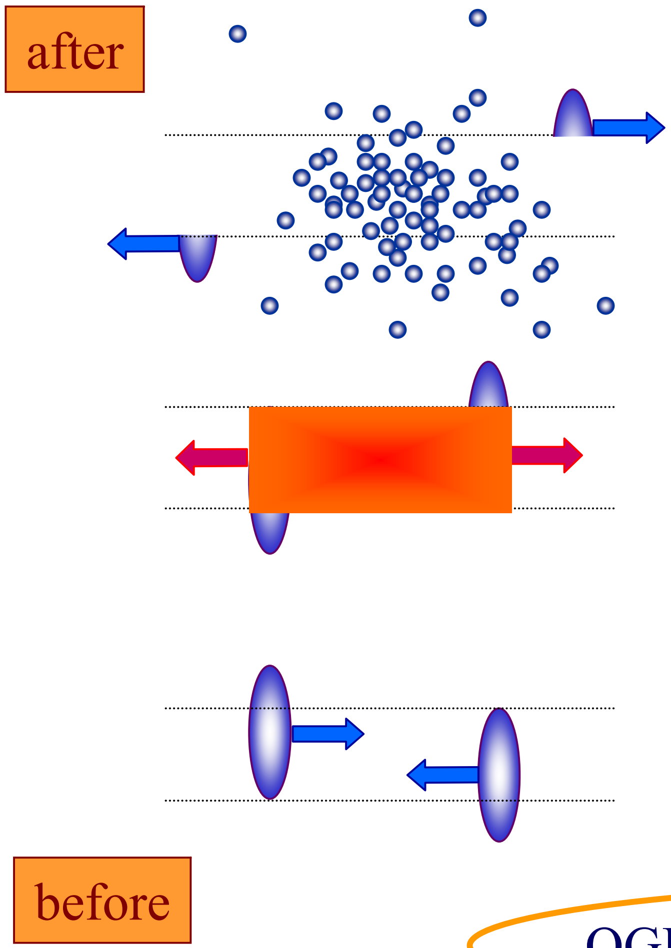


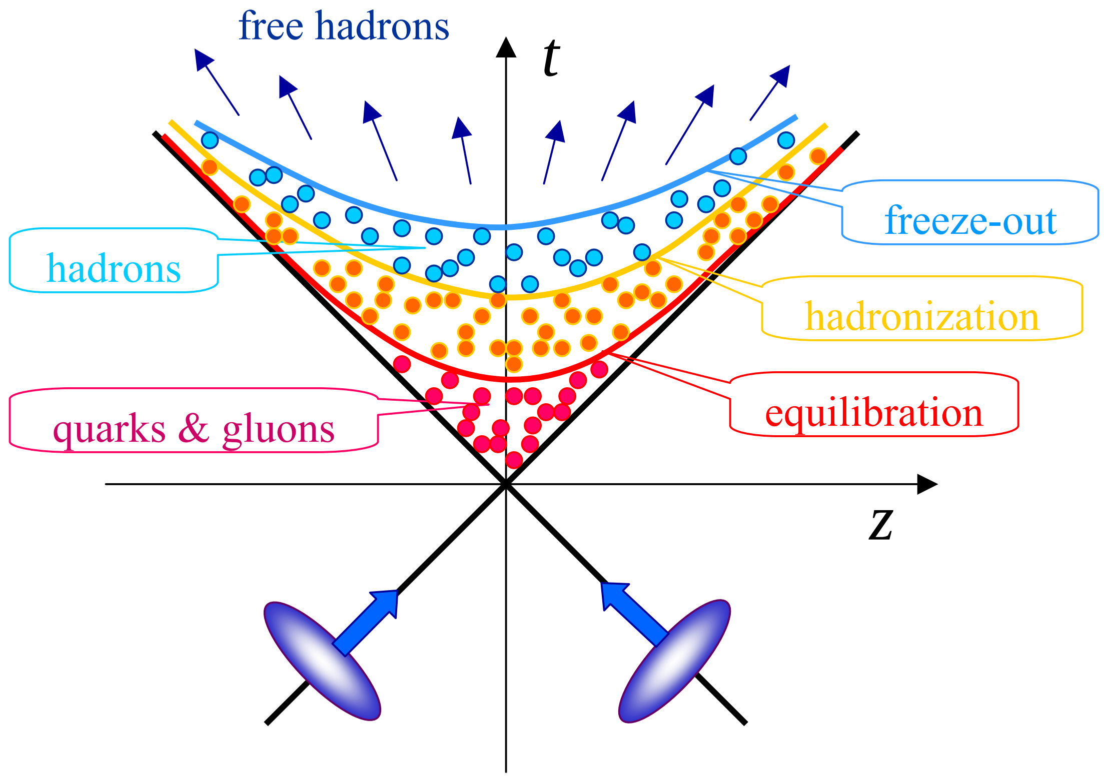


Away-side jet is suppressed
in central collisions

J. Adams et al. [STAR Collaboration], Nucl. Phys. **A757**, 102 (2005)

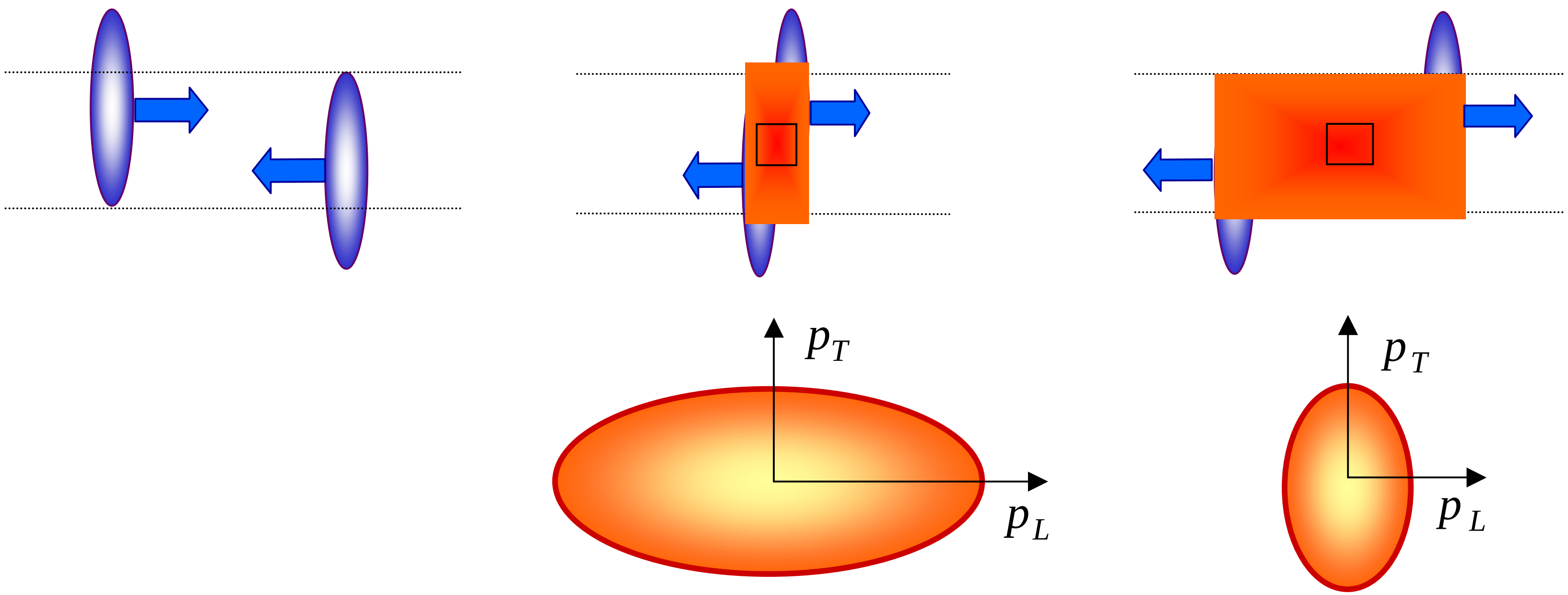
Scenario of relativistic heavy-ion collisions





QGP is out of equilibrium at the collision early stage

Anisotropic QGP



‘prolate’ ‘oblate’

Anisotropic QGP is unstable due to magnetic plasma modes

Instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

fluctuation

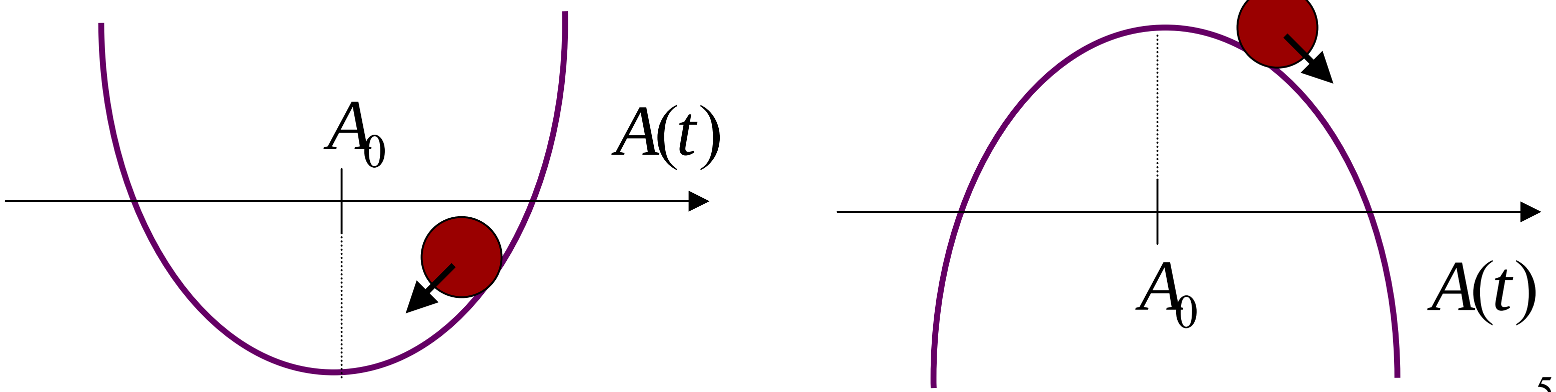
Instability

$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma > 0$$

stable configuration

unstable configuration

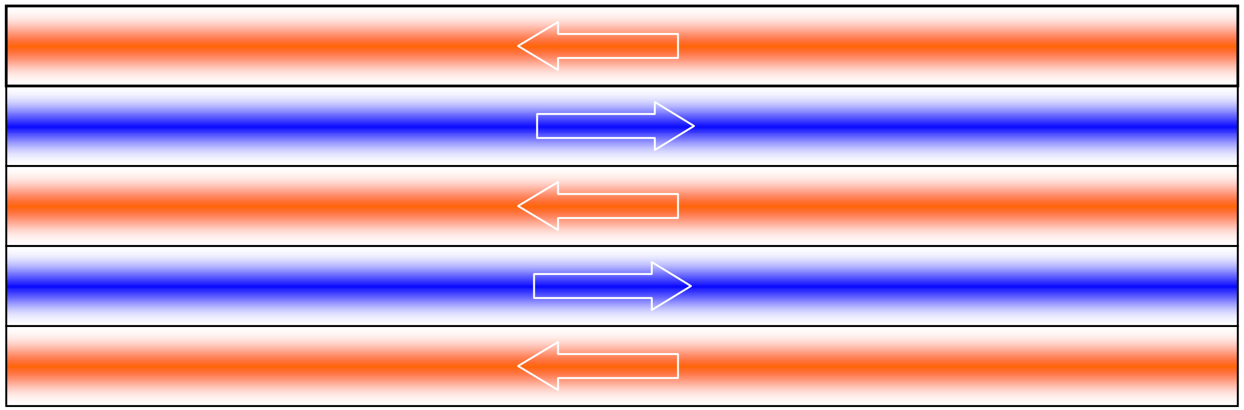


Seeds of instability

$\left\langle j_a^{\mu}(x)\right\rangle = 0$ **but current fluctuations are finite**

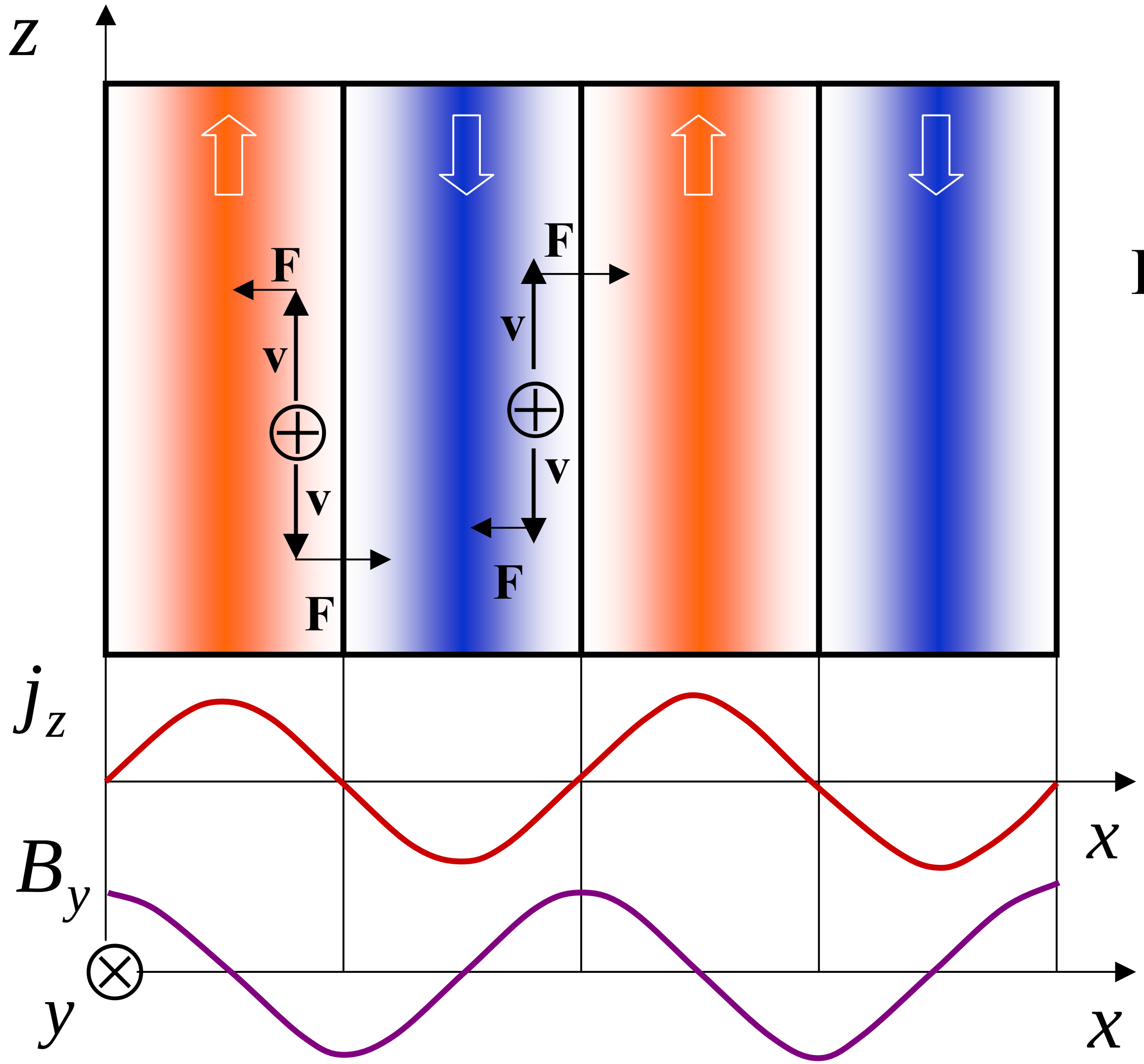
$$\boxed{\left\langle j_a^{\mu}(x_1)\, j_b^{\nu}(x_2)\right\rangle = \frac{1}{2}\delta^{ab}\int \frac{d^3p}{(2\pi)^3}\,\frac{p^{\mu}p^{\nu}}{E_p^2}\, f(\mathbf{p})\delta^{(3)}(\mathbf{x}-\mathbf{v}t) \neq 0}$$

$$x_1 = (t_1, \mathbf{x}_1), \quad x_2 = (t_2, \mathbf{x}_2), \quad x = (t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2)$$



Direction of the momentum surplus

Mechanism of filamentation



Lorentz force

$$\mathbf{F} = q\,\mathbf{v}\times\mathbf{B}$$

Ampere's law

$$\nabla\times\mathbf{B} = \mathbf{j}$$

Energy loss in unstable QGP

- Is *dE/dx* in unstable QGP sizeable?

  Yes!

- How to compute *dE/dx* in unstable QGP?

  Solve initial value problem!

A test parton in QGP

Wong's equation of motion (Hard Loop Approximation)

$$\begin{cases} \dfrac{dx^{\mu}(\tau)}{d\tau} = u^{\mu}(\tau) \\ \dfrac{dp^{\mu}(\tau)}{d\tau} = gQ_a(\tau)\, F_a^{\mu\nu}(x(\tau))\, u_\nu(\tau) \\ \dfrac{dQ_a(\tau)}{d\tau} = -gf^{abc}\, p_\mu(\tau)\, A_b^{\mu}(x(\tau))\, Q_c(\tau) \end{cases}$$

Simplifications

Gauge condition: $p_\mu(\tau)\, A_b^{\mu}\big(x(\tau)\big) = 0 \quad \Rightarrow \quad Q_a(\tau) = \text{const}$

Parton travels with constant velocity: $u^{\mu} = (\gamma, \gamma\, \mathbf{v}) = \text{const}$

Parton's energy loss

$$\boxed{\frac{dE(t)}{dt} = gQ_a \, \mathbf{E}_a(t, \mathbf{r}(t)) \cdot \mathbf{v}}$$

induced & spontaneously generated chromoelectric field

parton's current: $\mathbf{j}_a(t,\mathbf{r}) = gQ_a \mathbf{v}\delta^{(3)}(\mathbf{r} - \mathbf{v}t)$

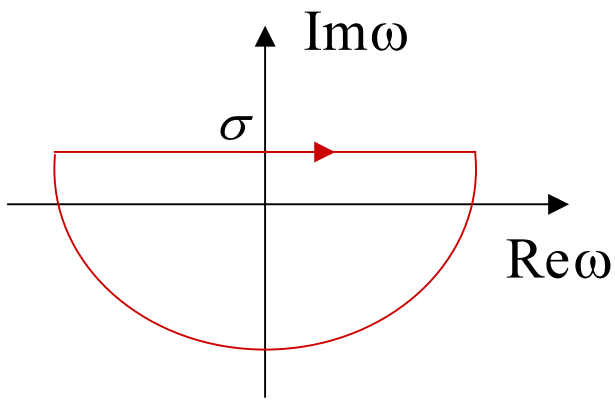
$$\boxed{\frac{dE(t)}{dt} = \int d^3r \, \mathbf{E}_a(t,\mathbf{r}) \cdot \mathbf{j}_a(t,\mathbf{r})}$$

Initial value problem

One-sided Fourier transformation

$$\begin{cases} f(\omega,\mathbf{k}) = \int_0^\infty dt \int d^3r \, e^{i(\omega t - \mathbf{kr})} f(t,\mathbf{r}) \\ f(t,\mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega t - \mathbf{kr})} f(\omega,\mathbf{k}) \end{cases}$$

$$0 < \sigma \in R$$



$$\mathbf{j}_a(t,\mathbf{r}) = gQ_a \mathbf{v} \delta^{(3)}(\mathbf{r} - \mathbf{v}t) \quad \Rightarrow \quad \mathbf{j}_a(\omega,\mathbf{k}) = \frac{igQ_a \mathbf{v}}{\omega - \mathbf{k}\cdot\mathbf{v}}$$

$$\frac{dE(t)}{dt} = gQ_a \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega - \mathbf{kv})t} \mathbf{E}_a(\omega,\mathbf{k}) \cdot \mathbf{v}$$

Induced Electric Field

Linearized Yang-Mills (Maxwell) equations (Hard Loop Approximation)

$$\boxed{\begin{gathered} i\mathbf{k}\cdot\mathbf{D}(\omega,\mathbf{k}) = \rho(\omega,\mathbf{k}), \qquad i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0, \\ i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = i\omega\mathbf{B}(\omega,\mathbf{k}) + \mathbf{B}_0(\mathbf{k}), \\ i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = \mathbf{j}(\omega,\mathbf{k}) - i\omega\mathbf{E}(\omega,\mathbf{k}) - \mathbf{D}_0(\mathbf{k}) \end{gathered}}$$

$$D^i(\omega,\mathbf{k}) = \varepsilon^{ij}(\omega,\mathbf{k})\,E^j(\omega,\mathbf{k})$$

Chromodielectric tensor

$$\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega-\mathbf{kv}+i0^+}\frac{\partial f(\mathbf{p})}{\partial p^l}\Big[\Big(1-\frac{\mathbf{kv}}{\omega}\Big)\delta^{lj} + \frac{k^l v^j}{\omega}\Big]$$

dynamical information

$$\boxed{E^i(\omega,\mathbf{k}) = -i(\Sigma^{-1})^{ij}(\omega,\mathbf{k})[\omega\mathbf{j}(\omega,\mathbf{k}) + \mathbf{k}\times\mathbf{B}_0(\mathbf{k}) - \omega\mathbf{D}_0(\mathbf{k})]^j}$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

Energy-Loss formula

$$\frac{dE(t)}{dt} = gQ_a v^i \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\varpi)t}$$
$$\times (\Sigma^{-1})^{ij}(\omega,\mathbf{k}) \left[\frac{igQ_a\omega\,\mathbf{v}}{\omega-\varpi} + \mathbf{k}\times\mathbf{B}_0(\mathbf{k}) - \omega\mathbf{D}_0(\mathbf{k}) \right]^j$$

$$\varpi \equiv \mathbf{k}\cdot\mathbf{v}$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega,\mathbf{k})$$

Dispersion equation

$$\boxed{\det[\Sigma(\omega,\mathbf{k})] = 0}$$

Collective mode, qusiparticle excitation: $\omega(\mathbf{k})$

Initial values of the fields

Maxwell equations & $\mathbf{j}_a(t,\mathbf{r}) = gQ_a\mathbf{v}\delta^{(3)}(\mathbf{r}-\mathbf{v}t)$

Initial values:

$$D_0^i(\mathbf{k}) = -igQ_a\overline{\omega}\varepsilon^{ij}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{jk}(\overline{\omega},\mathbf{k})v^k$$

$$B_0^i(\mathbf{k}) = -igQ_a\varepsilon^{ijk}k^j(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k})v^l$$

When the test parton enters the plasma at $t = 0$, the instabilities are initiated.

Energy-Loss formula

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\varpi)t} (\Sigma^{-1})^{ij}(\omega,\mathbf{k}) \times \left[\underbrace{\frac{\omega\,\delta^{jl}}{\omega-\varpi}}_{\mathbf{j}(\omega,\mathbf{k})} - \underbrace{(k^j k^k - \mathbf{k}^2\delta^{jk})(\Sigma^{-1})^{kl}(\varpi,\mathbf{k})}_{\mathbf{B}_0(\mathbf{k})} + \underbrace{\omega\varpi\,\varepsilon^{jk}(\varpi,\mathbf{k})(\Sigma^{-1})^{kl}(\varpi,\mathbf{k})}_{\mathbf{D}_0(\mathbf{k})} \right]$$

Averaging over parton's colors: $\int dQ\, Q_a\, Q_b = C_2 \delta^{ab}, \quad C_2 \equiv \begin{cases} \frac{1}{2} & \text{for} \quad \text{quark} \\ N_c & \text{for} \quad \text{gluon} \end{cases}$

$$C_R = \begin{cases} C_2 \dfrac{N_c^2 - 1}{N_c} = \dfrac{N_c^2 - 1}{2N_c} & \text{for} \quad \text{quark} \quad (\mathrm{R} = \mathrm{F}) \\ C_2 = N_c & \text{for} \quad \text{gluon} \quad (\mathrm{R} = \mathrm{G}) \end{cases}$$

Stable isotropic plasma

- $$\varepsilon^{ij}(\omega,\mathbf{k}) = \varepsilon_L(\omega,\mathbf{k})\frac{k^i k^j}{\mathbf{k}^2} + \varepsilon_T(\omega,\mathbf{k})\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

- $$(\Sigma^{-1})^{ij}(\omega,\mathbf{k}) = \frac{1}{\omega^2\varepsilon_L(\omega,\mathbf{k})}\frac{k^i k^j}{\mathbf{k}^2} + \frac{1}{\omega^2\varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i}\int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\bar{\omega})t}(\Sigma^{-1})^{ij}(\omega,\mathbf{k})$$
$$\times\left[\frac{\omega\,\delta^{jl}}{\omega-\bar{\omega}} - (k^j k^k - \mathbf{k}^2\delta^{jk})(\Sigma^{-1})^{kl}(\bar{\omega},\mathbf{k}) + \omega\bar{\omega}\,\varepsilon^{jk}(\bar{\omega},\mathbf{k})(\Sigma^{-1})^{kl}(\bar{\omega},\mathbf{k})\right]$$

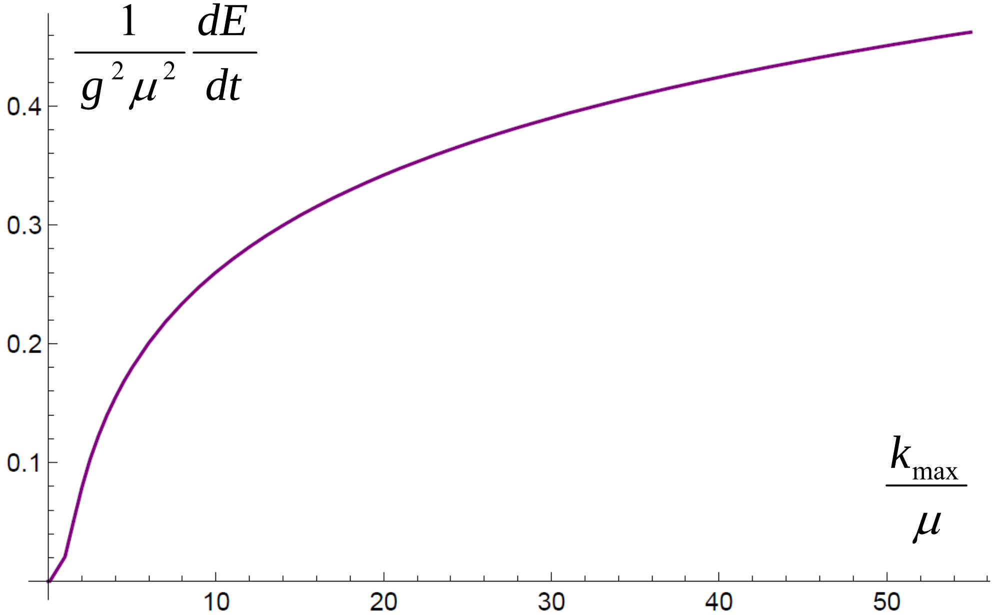
The only stationary contribution: $\omega = \bar{\omega} \equiv \mathbf{k}\cdot\mathbf{v}$

$$\boxed{\frac{dE(t)}{dt} = ig^2 C_R \int \frac{d^3k}{(2\pi)^3}\frac{\bar{\omega}}{\mathbf{k}^2}\left[\frac{1}{\varepsilon_L(\bar{\omega},\mathbf{k})} + \frac{\mathbf{k}^2\mathbf{v}^2 - \bar{\omega}^2}{\bar{\omega}^2\varepsilon_T(\bar{\omega},\mathbf{k}) - \mathbf{k}^2}\right]}$$

equivalent to the standard result by Braaten & Thoma

Energy loss in equilibrium QGP

$$\frac{dE(t)}{dt} = ig^2 C_R \int \frac{d^3k}{(2\pi)^3} \frac{\overline{\omega}}{\mathbf{k}^2} \left[\frac{1}{\varepsilon_L(\overline{\omega},\mathbf{k})} + \frac{\mathbf{k}^2\mathbf{v}^2 - \overline{\omega}^2}{\overline{\omega}^2 \varepsilon_T(\overline{\omega},\mathbf{k}) - \mathbf{k}^2} \right]$$



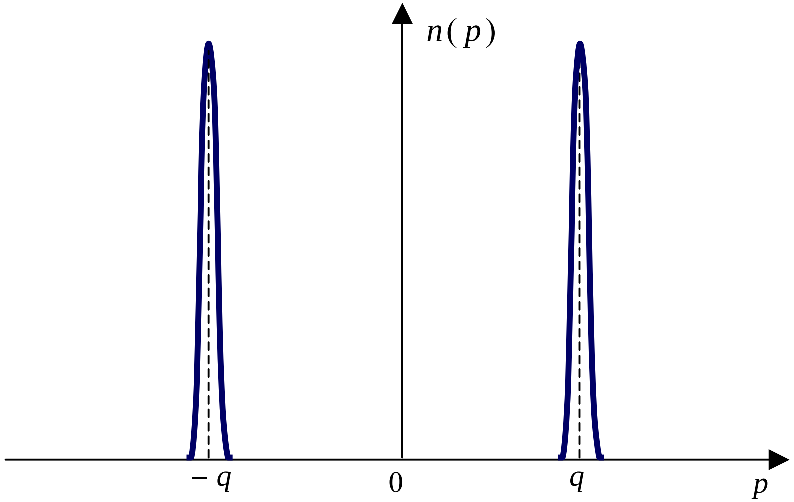
Debye mass

$$\mu^2 \equiv g^2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

Unstable two-stream system

$$n(\mathbf{p}) = (2\pi)^3 \rho \left[\delta^{(3)}(\mathbf{p}-\mathbf{q}) + \delta^{(3)}(\mathbf{p}+\mathbf{q}) \right]$$

There is unstable longitudinal chromoelectric mode



- $$\varepsilon^{ij}(\omega,\mathbf{k}) = \varepsilon_L(\omega,\mathbf{k}) \frac{k^i k^j}{\mathbf{k}^2}$$
- $$(\Sigma^{-1})^{ij}(\omega,\mathbf{k}) = \frac{1}{\omega^2 \varepsilon_L(\omega,\mathbf{k})} \frac{k^i k^j}{\mathbf{k}^2}$$

longitudinal chromoelectric field only!

$$\mathbf{E}(\omega,\mathbf{k}) \parallel \mathbf{k}$$

$$\boxed{\frac{dE(t)}{dt} = ig^2 C_R \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-i(\omega-\overline{\omega})t}}{\omega^2 \varepsilon_L(\omega,\mathbf{k})} \frac{\overline{\omega}^2}{\mathbf{k}^2} \left[\frac{\omega}{\omega-\overline{\omega}} + \frac{\overline{\omega}}{\omega} \right]}$$

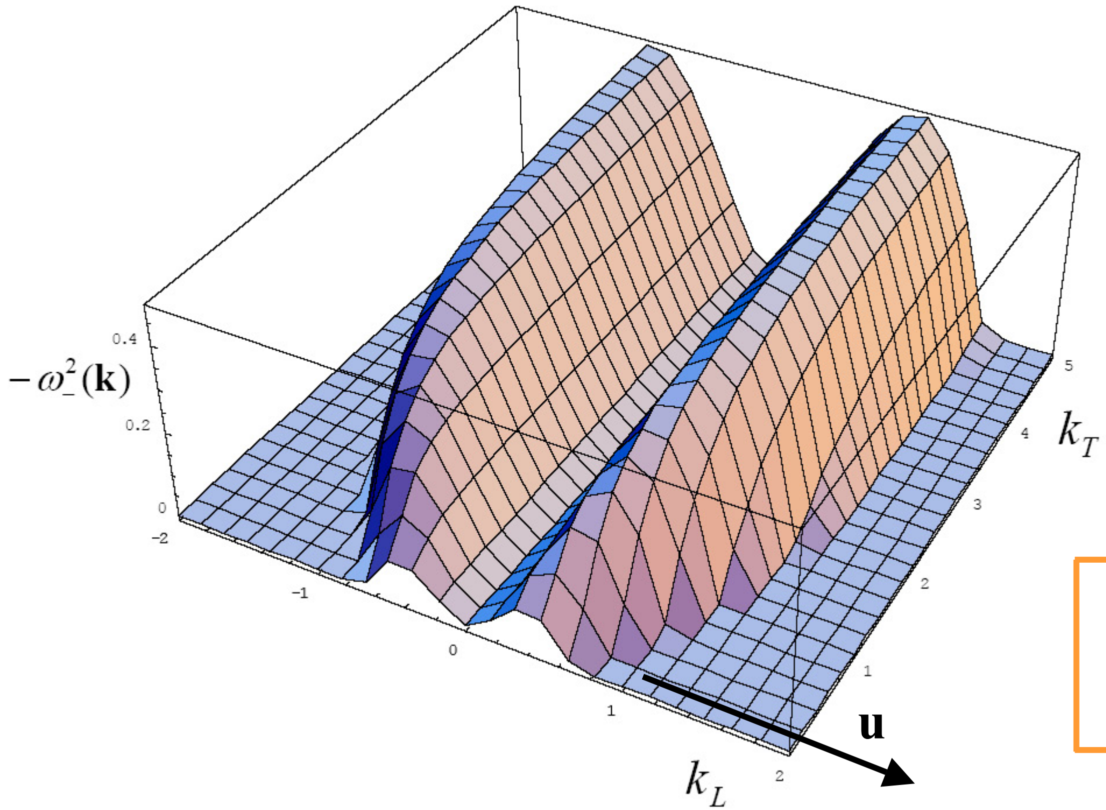
Collective modes in two-stream system

$$\varepsilon_L(\omega,\mathbf{k}) = \frac{(\omega-\omega_+(\mathbf{k}))(\omega+\omega_+(\mathbf{k}))(\omega-\omega_-(\mathbf{k}))(\omega+\omega_-(\mathbf{k}))}{(\omega^2-(\mathbf{k}\cdot\mathbf{u})^2)}$$

$\mathbf{u}$ - stream velocity

$\omega_+(\mathbf{k})$ - stable longitudinal mode

$\omega_-(\mathbf{k})$ - unstable longitudinal mode



$$\mathrm{Im}\,\omega_- > 0$$

$$\frac{dE(t)}{dt} \sim \int \frac{d^3k}{(2\pi)^3} e^{\mathrm{Im}\,\omega_- t} \ldots$$

Energy loss in two-stream system

$$\frac{dE(t)}{dt} = ig^2 C_R \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-i(\omega-\bar{\omega})t}}{\omega^2 \varepsilon_L(\omega,\mathbf{k})} \frac{\bar{\omega}^2}{\mathbf{k}^2} \left[\frac{\omega}{\omega-\bar{\omega}} + \frac{\bar{\omega}}{\omega} \right]$$

There are 6 contributions corresponding to $\omega = \pm\omega_+(\mathbf{k}), \pm\omega_-(\mathbf{k}), \bar{\omega} = \mathbf{k}\cdot\mathbf{v}, 0$

Only one dimensional parameter: $\mu^2 \equiv \dfrac{g^2 n}{2E_q}$

Remaining parameters: $g = 1, \quad |\mathbf{v}| = 1, \quad |\mathbf{u}| = 0.9, \quad C_R = 3$

The integral over **k** performed numerically for

$$-k_{\max} < k_L < k_{\max}, \quad 0 < k_T < k_{\max}$$

Vacuum contribution to energy loss

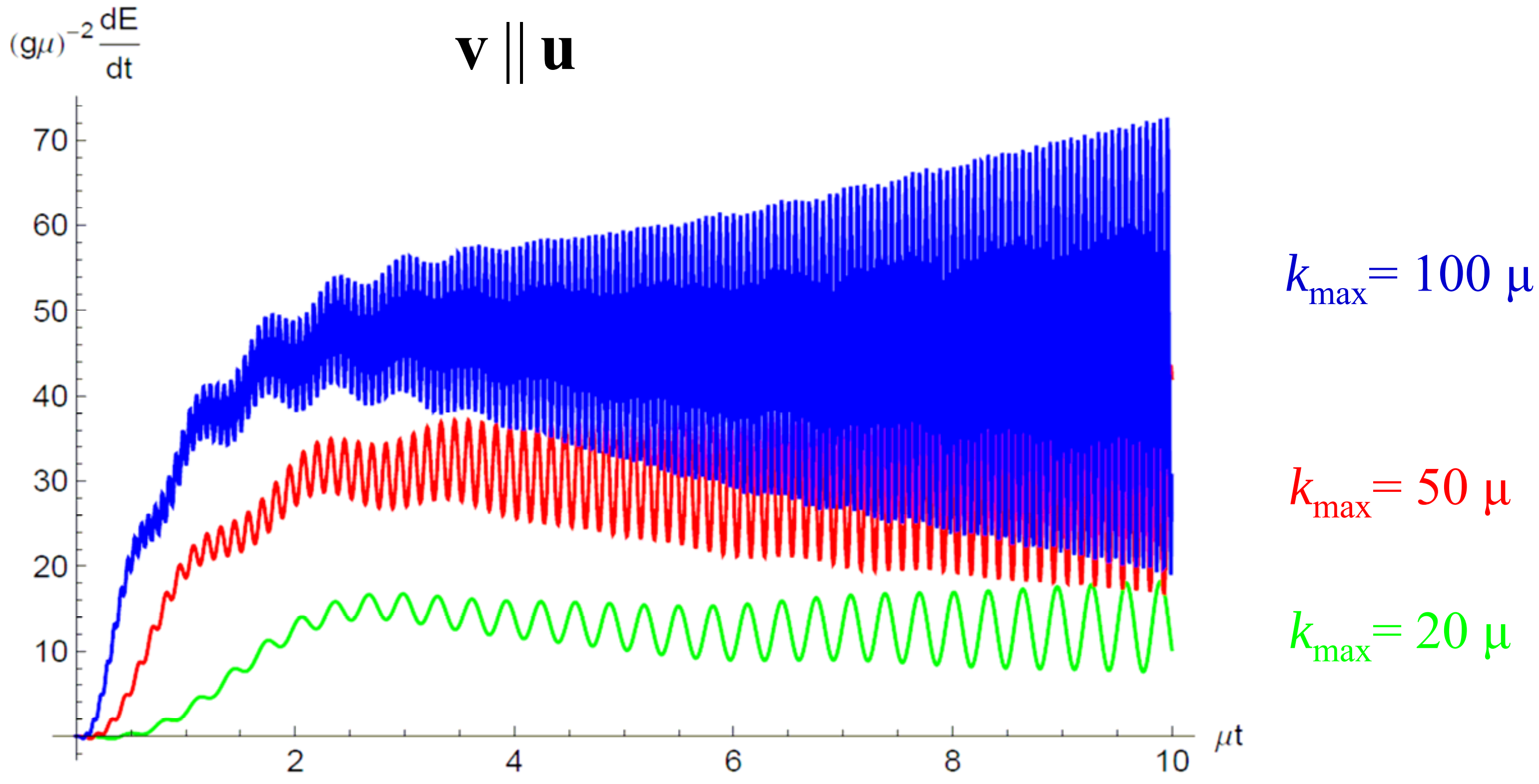
$$\frac{dE(t)}{dt} = ig^2 C_R \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-i(\omega-\varpi)t}}{\omega^2 \varepsilon_L(\omega,\mathbf{k})} \frac{\varpi^2}{\mathbf{k}^2} \left[\frac{\omega}{\omega-\varpi} + \frac{\varpi}{\omega} \right]$$

Vacuum contribution: $\varepsilon_L(\omega,\mathbf{k}) \to 1$

$$\boxed{\left.\frac{dE(t)}{dt}\right|_{\text{vacuum}} = ig^2 C_R \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-i(\omega-\varpi)t}}{\omega^2} \frac{\varpi^2}{\mathbf{k}^2} \left[\frac{\omega}{\omega-\varpi} + \frac{\varpi}{\omega} \right] = \frac{g^2 C_R}{8\pi} \frac{1}{t^2}}$$

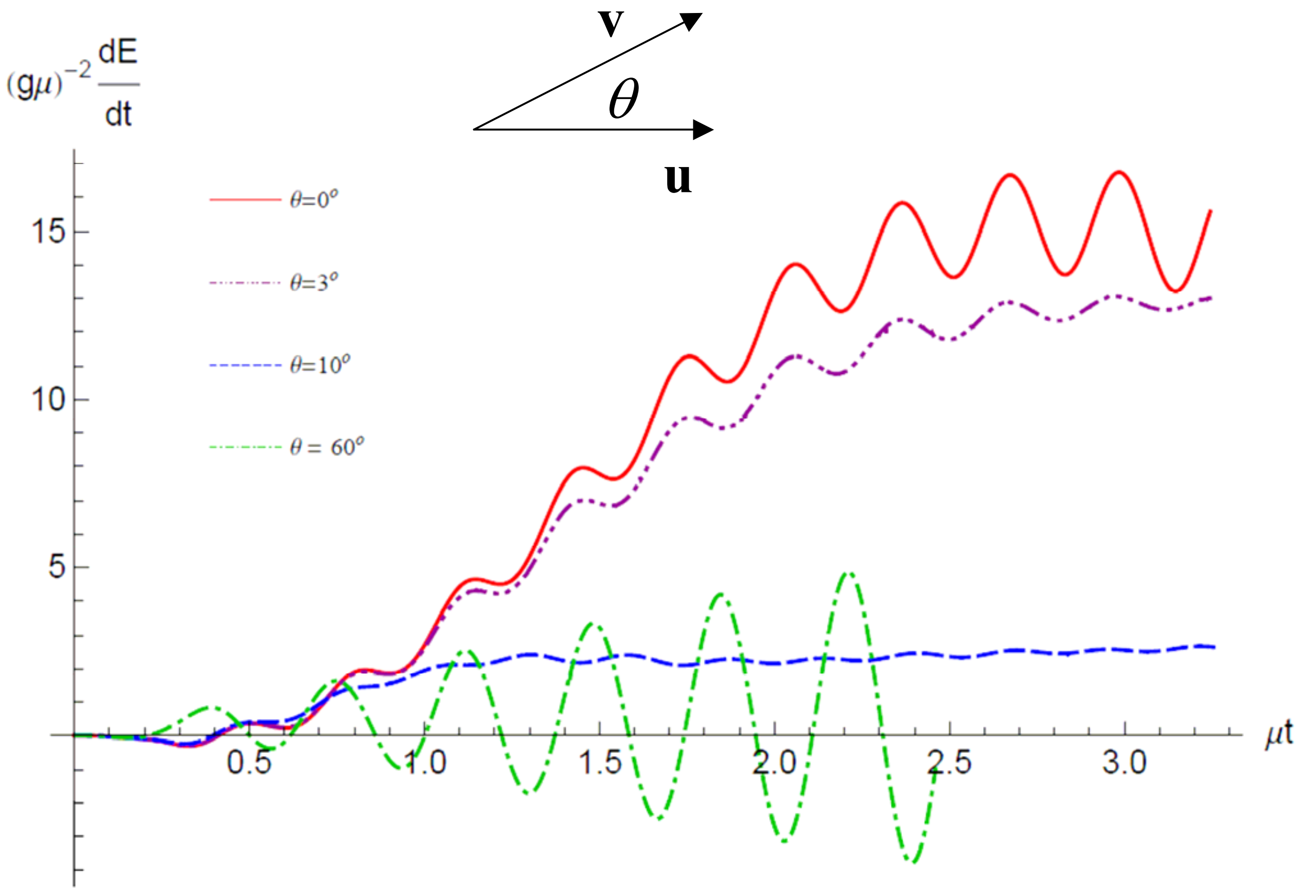
Vacuum contribution has to be subtracted

Energy loss in two-stream system



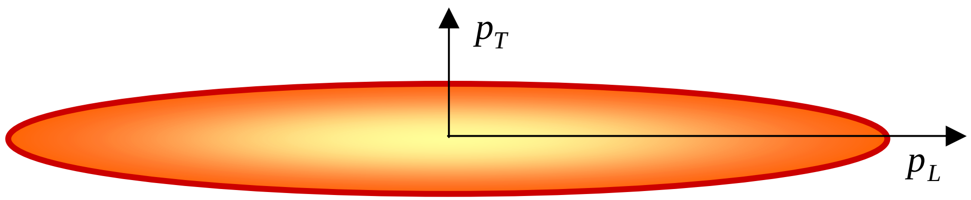
Strong ultraviolet dependence!

Energy loss in two-stream system



k_{max}= 50 μ

Extremely prolate system



$$f(\mathbf{p}) \sim \delta(p_x)\,\delta(p_y)$$

Collective modes

$$\det[\Sigma^{ij}(\omega,\mathbf{k})] = 0$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega,\mathbf{k})$$

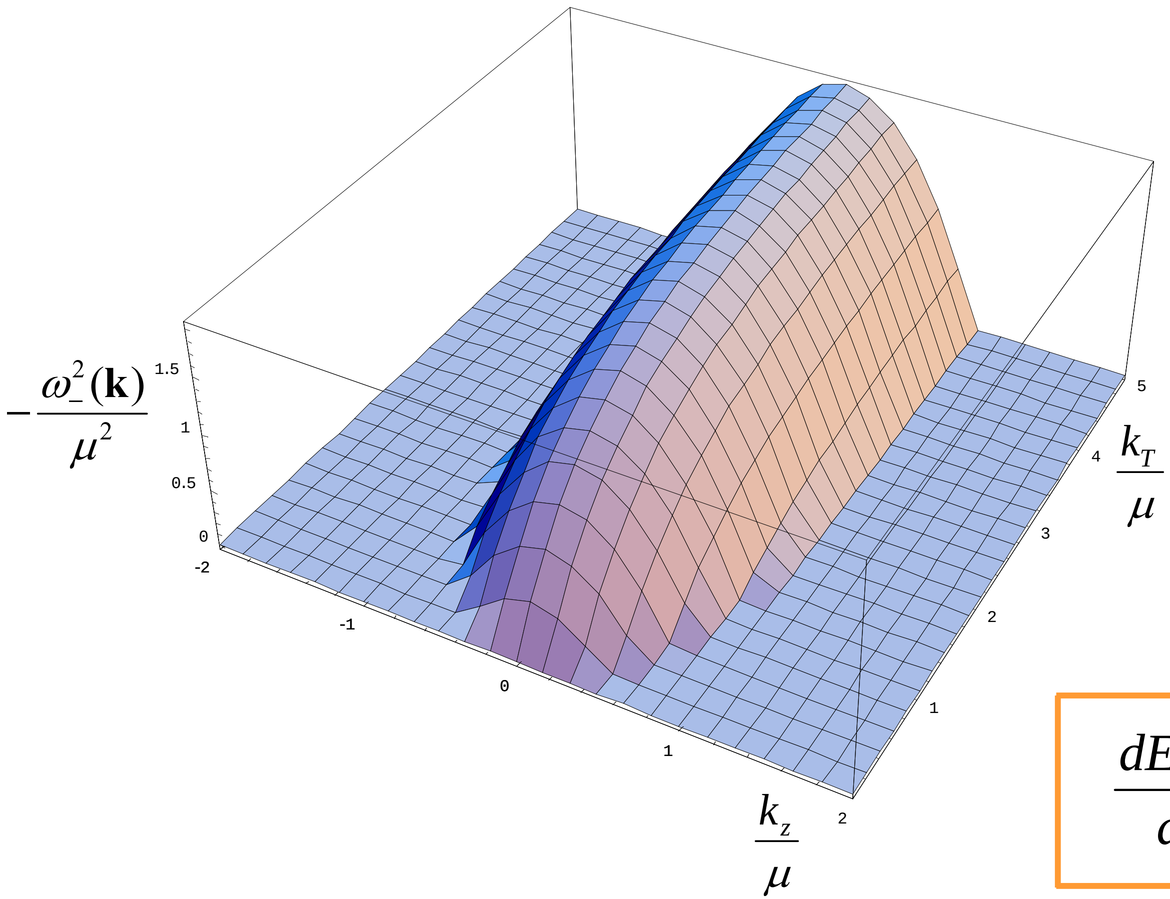
$$\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega}\int \frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega - \mathbf{kv} + i0^+}\frac{\partial f(\mathbf{p})}{\partial p^l}\Big[\Big(1-\frac{\mathbf{kv}}{\omega}\Big)\delta^{lj} + \frac{k^l v^j}{\omega}\Big]$$

Spectrum of collective modes

$$\begin{cases} \omega_1(\mathbf{k}) = \mu^2 + \mathbf{k}^2 \\ \omega_2(\mathbf{k}) = \mu^2 + (\mathbf{k}\cdot\mathbf{n})^2 \\ \omega_\pm(\mathbf{k}) = \dfrac{1}{2}\Big(\mathbf{k}^2 + (\mathbf{k}\cdot\mathbf{n})^2 \pm \sqrt{\mathbf{k}^4 + (\mathbf{k}\cdot\mathbf{n})^4 + 4\mu^2\mathbf{k}^2 - 4\mu^2(\mathbf{k}\cdot\mathbf{n})^2 - 2\mathbf{k}^2(\mathbf{k}\cdot\mathbf{n})^2}\Big) \end{cases}$$

$$\mathbf{n} \equiv (0,0,1)$$

Unstable chromomagnetic mode



$$\mathrm{Im}\,\omega > 0$$

$$\frac{dE(t)}{dt} \sim \int \frac{d^3k}{(2\pi)^3} e^{\mathrm{Im}\,\omega t} \dots$$

Energy loss in extremely prolate system

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t} (\Sigma^{-1})^{ij}(\omega,\mathbf{k})$$
$$\times \left[\frac{\omega\,\delta^{jl}}{\omega-\overline{\omega}} - (k^j k^k - \mathbf{k}^2 \delta^{jk})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k}) + \omega\overline{\omega}\varepsilon^{jk}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k}) \right]$$

- Inversion of matrix Σ depending on $\mathbf{k}$ and $\mathbf{n}$

$$\Sigma \;= aA + bB + cC + dD$$

$$\Sigma^{-1} = \alpha A + \beta B + \gamma C + \delta D$$

basis of matrices

$$\begin{cases} A^{ij} = \delta^{ij} - \dfrac{k^i k^j}{\mathbf{k}^2}, \quad B^{ij} = \dfrac{k^i k^j}{\mathbf{k}^2}, \\ C^{ij} = \dfrac{n_T^i n_T^j}{\mathbf{n}_T^2}, \quad D^{ij} = n_T^i k^j + k^i n_T^j \end{cases}$$

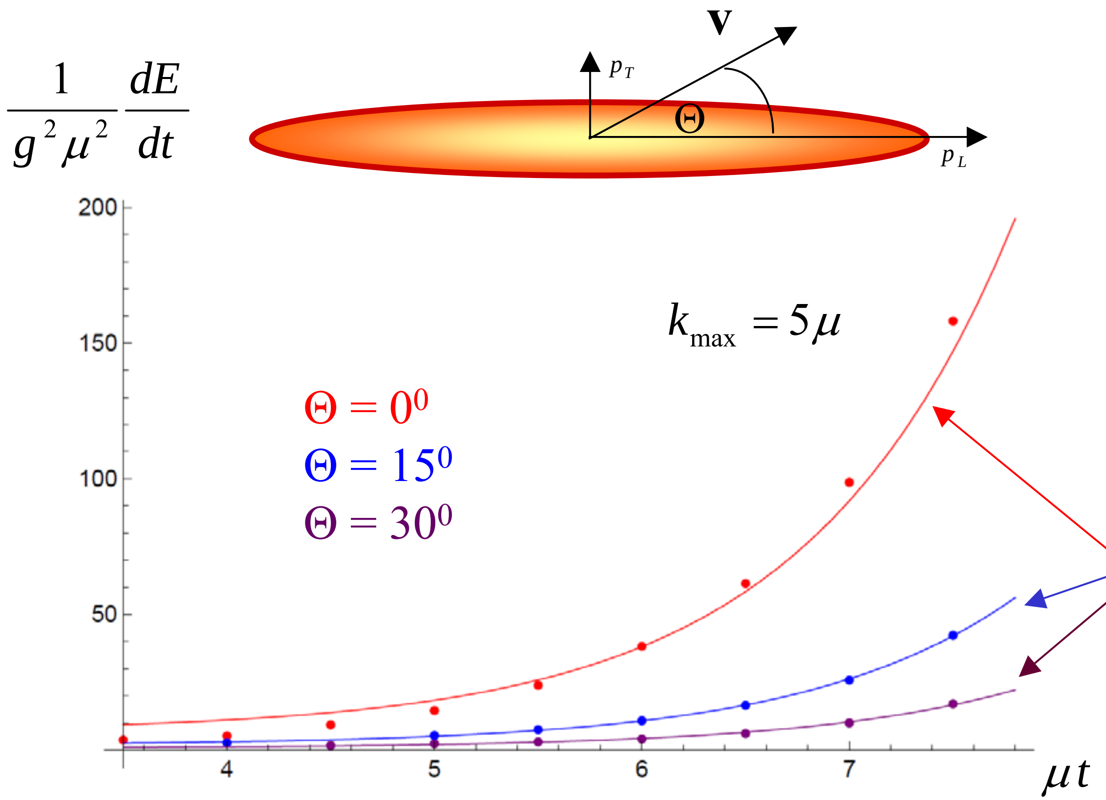
$$n_T^i \equiv \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) n^j$$

$$\mathbf{n}_T \perp \mathbf{k}$$

$$\Sigma\Sigma^{-1} = \mathbf{1} \quad \Rightarrow \quad \alpha, \beta, \gamma, \delta$$

P. Romatschke & M. Strickland, Physical Review D **68**, 036004 (2003)

Energy loss in extremely prolate system cont.



exponential fit

Debye mass

$$\mu^2 \equiv g^2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

Equilibrium value

$$\frac{1}{g^2\mu^2}\frac{dE}{dt} \approx 0.2$$

Conclusions

- Energy loss is found as a solution of initial value problem
- Two-stream & extremely prolate systems are discussed as examples
- Strong time and directional dependence of *dE/dx* is demonstrated
- *dE/dx* in unstable QGP is much bigger than in equilibrium one