From Regensburg to Universal Hard-Loop Action

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Happy Birthday!
Regensburg 1989 - 1990

Uli’s first postdoc
Towards Relativistic Transport Theory of Nuclear Matter

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Ph. D. project

- 1997 – Maldacena – AdS/CFT duality (gauge/gravity correspondence)

- To what extent $\mathcal{N}=4$ super YM is similar to QCD?

- To what extent $\mathcal{N}=4$ super YM plasma is similar to quark-gluon plasma?

- To what extent a supersymmetric plasma is similar to its non-supersymmetric counterpart in a weak coupling regime?


Two byproducts of the project:
- ghosts in the Keldysh-Schwinger formalism
- universality of the hard-loop action
Gauge theories under consideration

\[ \mathcal{L}_{\text{QED}} = -\frac{1}{4} F^\mu_\nu F^\nu_\mu + i \bar{\Psi} \gamma_\mu D^\mu \Psi \]

\[ F^\mu_\nu = \partial^\mu A^\nu - \partial^\nu A^\mu \]

\[ \mathcal{L}_{\text{ScQED}} = -\frac{1}{4} F^\mu_\nu F^\nu_\mu - (D_\mu \Phi)^* D^\mu \Phi \]

\[ \mathcal{L}_{\text{SUSY QED}} = -\frac{1}{4} F^\mu_\nu F^\nu_\mu + i \bar{\Psi} \gamma_\mu D^\mu \Psi + i \frac{1}{2} \Lambda \gamma_\mu \partial^\mu A + (D_\mu \phi_L)^* (D^\mu \phi_L) + (D_\mu \phi_R)^* (D^\mu \phi_R) \]

\[ + \sqrt{2} e (\bar{\Psi} P_R \Lambda \phi_L - \bar{\Psi} P_L \Lambda \phi_R^* + \phi_L^* \Lambda P_L \Psi - \phi_R^* \Lambda P_R \Psi) - \frac{e^2}{2} (\phi_L^* \phi_L - \phi_R^* \phi_R)^2 \]

\[ \mathcal{L}_{\text{YM}} = -\frac{1}{4} F^a_\mu F^a^\mu \]

\[ F^a_\mu = \partial^\mu A^a_\mu - \partial^\nu A^a_\nu + g f^{a bc} A^a_\mu A^b_\nu \]

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_\mu F^a^\mu + i \bar{\Psi}_i (\gamma_\mu D^\mu)_{ij} \Psi_j \]

\[ \mathcal{L}_{\text{SYM}} = -\frac{1}{4} F^a_\mu F^a^\mu + i \bar{\Psi}_i (\gamma_\mu D^\mu \Psi_i)^a + \frac{1}{2} (D_\mu \Phi_A)^a (D^\mu \Phi_A)_a \]

\[ - \frac{1}{4} g^2 f^{a bc} f_{cde} \Phi_A^a \Phi_B^b \Phi_C^c - \frac{1}{2} \frac{g}{6} \left( \bar{\Psi}_i \alpha_{ij} X_p^b \Psi_j + i \bar{\Psi}_i \beta_{ij} \gamma_5 Y_p^b \Psi_j \right) \]
Effective action

A system’s dynamics is encoded in an effective action.

How to find the effective action?

Self-energy constrains a form of the effective action

\[
\Pi^{\mu\nu}(x, y) = \frac{\delta^2 S[A]}{\delta A_\mu(x) \delta A_\nu(y)} \quad S = \int d^4 x \, \mathcal{L}(x)
\]

\[
\mathcal{L}^{(A)}_2(x) = \frac{1}{2} \int d^4 y \, A_\mu(x) \Pi^{\mu\nu}(x - y) A_\nu(y)
\]

Adopted strategy

self-energy \hspace{1cm} \rightarrow \hspace{1cm} \text{effective action}
Keldysh–Schwinger formalism

Description of non-equilibrium many-body systems

Contour Green function of gauge field

\[ i \mathcal{D}_{ab}^{\mu\nu}(x, y) \equiv \left\langle \tilde{T} A_{a}^{\mu}(x) A_{b}^{\nu}(y) \right\rangle \]

\[ \langle \ldots \rangle = \text{Tr}[\hat{\rho}(t_0)\ldots] \]

\( \tilde{T} \) - ordering along the contour

\[ \tilde{T}A(x)B(y) = \Theta(x_0, y_0)A(x)B(y) \pm \Theta(y_0, x_0)B(y)A(x) \]
Keldysh–Schwinger formalism

Contour Green’s function includes 4 Green’s functions with real time arguments:

$\left( D_{ab}^{\mu\nu} \right)^{>}(x, y) = \langle A_\alpha^\mu(x) A_\beta^\nu(y) \rangle$

$\left( D_{ab}^{\mu\nu} \right)^{<}(x, y) = \langle A_\beta^\nu(y) A_\alpha^\mu(x) \rangle$

$\left( D_{ab}^{\mu\nu} \right)^{c}(x, y) = \langle T^c A_\alpha^\mu(x) A_\beta^\nu(y) \rangle$

$\left( D_{ab}^{\mu\nu} \right)^{a}(x, y) = \langle T^a A_\alpha^\mu(x) A_\beta^\nu(y) \rangle$

Chronological time ordering

Anti-chronological time ordering
Retarded, advanced & symmetric Green’s functions

\[ \mathcal{D}^+(x, y) = \Theta(x_0 - y_0) \left( \mathcal{D}^>(x, y) - \mathcal{D}^<(x, y) \right) \]

\[ \mathcal{D}^-(x, y) = \Theta(y_0 - x_0) \left( \mathcal{D}^<(x, y) - \mathcal{D}^>(x, y) \right) \]

\[ \mathcal{D}^{\text{sym}}(x, y) = \mathcal{D}^>(x, y) + \mathcal{D}^<(x, y) \]
# Meaning of the functions

<table>
<thead>
<tr>
<th>$D^{&lt;,&gt;}(x, y)$</th>
<th>phase-space density</th>
<th>retarded &amp; advanced propagator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mass-shell constraint</td>
<td>no mass-shell constraint</td>
</tr>
<tr>
<td></td>
<td>real particles</td>
<td>virtual particles</td>
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<td>virtual particles</td>
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</tbody>
</table>

The Green functions $D(x, y)$ are gauge dependent.

**Physical results obtained from Green functions must be gauge independent**

For example

The poles of $D(x, y)$ - dispersion relations – are gauge independent
Green’s functions of free gluon field

\[ D(x, y) = D(x - y) \]

\[
\left( D^{ab}_{\mu\nu} \right)^{\times} (p) = \frac{i\pi}{E_p} g_{\mu\nu} \delta^{ab} \left[ \delta(E_p - p_0)[n_g(p) + 1] + \delta(E_p + p_0)n_g(-p) \right]
\]

\[
\left( D^{ab}_{\mu\nu} \right)^{-} (p) = \frac{i\pi}{E_p} g_{\mu\nu} \delta^{ab} \left[ \delta(E_p - p_0)n_g(p) + \delta(E_p + p_0)[n_g(-p) + 1] \right]
\]

\[
\left( D^{ab}_{\mu\nu} \right)^{\times} (p) = -g_{\mu\nu} \delta^{ab} \left[ \frac{1}{p^2 + i0^+} - \frac{i\pi}{E_p} \left( \delta(E_p - p_0)n_g(p) + \delta(E_p + p_0)n_g(-p) \right) \right]
\]

\[
\left( D^{ab}_{\mu\nu} \right)^{a} (p) = g_{\mu\nu} \delta^{ab} \left[ \frac{1}{p^2 - i0^+} + \frac{i\pi}{E_p} \left( \delta(E_p - p_0)n_g(p) + \delta(E_p + p_0)n_g(-p) \right) \right]
\]

\[ n_g(p) \text{ - gluon distribution function} \]
Green’s functions of free gluon field

\[ D(x, y) = D(x - y) \]

\[
\begin{align*}
\left( D_{\mu\nu}^{ab} \right)^>(p) &= \frac{i\pi}{E_p} g_{\mu\nu}\delta^{ab} \left[ \delta(E_p - p_0)[n_g(p) + 1] + \delta(E_p + p_0)n_g(-p) \right] \\
\left( D_{\mu\nu}^{ab} \right)^<(p) &= \frac{i\pi}{E_p} g_{\mu\nu}\delta^{ab} \left[ \delta(E_p - p_0)n_g(p) + \delta(E_p + p_0)[n_g(-p) + 1] \right] \\
\left( D_{\mu\nu}^{ab} \right)^+(p) &= \frac{-g_{\mu\nu}\delta^{ab}}{p^2 + i\text{sgn}(p_0)0^+} \\
\left( D_{\mu\nu}^{ab} \right)^-(p) &= \frac{g_{\mu\nu}\delta^{ab}}{p^2 - i\text{sgn}(p_0)0^+}
\end{align*}
\]

\[ n_g(p) \text{ - gluon distribution function} \]
Need for ghosts

- QCD computations in covariant gauges are usually much simpler than those in physical ones like the Coulomb gauge.

- Covariant gauges require ghosts to compensate unphysical degrees of freedom.

Gluon field: \( A^\mu (x) = (A^0(x), A(x)) \) - 4 degrees of freedom

Physical gluon: 2 polarizations

How to introduce ghosts in the Keldysh-Schwinger formalism?

What is the Green function of free ghosts?

\[
\begin{align*}
\Delta^>(p) \\
\Delta^<(p) \\
\Delta^c(p) \\
\Delta^a(p)
\end{align*}
\]
How to get Green’s function of free ghosts?

Ghost sector should be determined by the gauge symmetry of the theory!

\[ A^a_\mu \rightarrow (A^a_\mu)^U = A^a_\mu + f^{abc} \omega^b A^c_\mu - \frac{1}{g} \partial_\mu \omega^a \]

gauge symmetry of the theory

Slavnov-Taylor identities
Generating functional

\[ W_0[J, \chi, \chi^*] = N_0 \int_{BC} \mathcal{D}A \mathcal{D}c \mathcal{D}c^* e^{i \int_C d^4x \mathcal{L}_{\text{eff}}(x)} \]

boundary conditions:
the fields are fixed at \( t = -\infty \pm i0^+ \)

\[ \mathcal{L}_{\text{eff}}(x) = -\frac{1}{4} F^{\mu\nu}_a F^{a}_{\mu\nu} - \frac{1}{2} (\partial^{\mu} A^a_\mu)^2 - c^*_a (\partial^{\mu} \partial_\mu \delta_{ab} - g \partial^{\mu} f^{abc} A^c_\mu) c_b + J^a_\mu A^a_\mu + \chi^*_a c_a + \chi_a c^*_a \]

\[ W[J, \chi, \chi^*] = N \int DA' Dc' Dc'^* DA'' Dc'' Dc''^* \]
\[ \times \rho[A', c', c'^*| A'', c'', c''^*] W_0[J, \chi, \chi^*] \]

density matrix
The full Green’s function are generated as

\[ W[J, \chi, \chi^*] = N \int DA'Dc'Dc'* DA''Dc''Dc''' \]
\[ \times \rho[A', c', c*' A'', c'', c*'''] W_0[J, \chi, \chi^*] \]

The density matrix \( \rho[A', c', c*' A'', c'', c*'''] \) is not specified

The explicit forms of the functional and the Green’s function are not known

The functional provides various relations among Green’s functions
General Slavnov-Taylor identity

$W[J, \chi, \chi^*] = N \int_{BC} D\!A \Delta[A] e^{i\int_c d^4 x \mathcal{L}(x)}$

analog of the Fadeev-Popov determinant

$\Delta[A] \equiv \int_{BC} Dc Dc^* e^{-i\int_c d^4 x \left( -c^*_a (\partial^\mu \partial_\mu \delta_{ab} - g \partial^\mu f^{abc} A^c_\mu) c_b + \chi^*_a c_a + \chi_a c^*_a \right)}$

The invariance of $W[J, \chi, \chi^*]$ under the transformations

$A^a_\mu \rightarrow \left( A^a_\mu \right)^U = A^a_\mu + f^{abc} \omega^b A^c_\mu - \frac{1}{g} \partial^\mu \omega^a$

leads to

$$\left\{ i\hat{\partial}^\mu \frac{\delta}{\delta J^\mu_d(y)} - \int_c d^4 x J^\mu_a(x) \left( \partial^{(x)} \delta^{ab} + igf^{abc} \frac{\delta}{\delta J^\mu_c(x)} \right) M^{-1}_{bd} \left[ \frac{1}{i} \frac{\delta}{\delta J} \right] x, y \right\} W[J, \chi, \chi^*] = 0$$
Slavnov-Taylor identity for gluon Green’s function

\[ \frac{\delta}{\delta J^\nu(z)} \left\{ i \frac{\delta^{\mu}}{\delta J_a^\mu (y)} - \int_C d^4x J_a^\mu (x) \left( \frac{\partial^{(x)} \delta^{ab}}{\delta J_c^\mu (x)} + igf^{abc} \frac{\delta}{\delta J_c^\mu (x)} \right) M^{-1}_{bd} \left[ \frac{1}{i} \frac{\delta}{\delta J} \right]_{x, y} \right\} W[J, \chi, \chi^*] = 0 \]

\[ J = \chi = \chi^* = 0 \]

\[ - p^\mu D_{\mu \nu}^{ab} (p) = p_\nu \Delta_{ab} (-p) \]

free ghost Green’s function

The longitudinal component of the gluon Green’s function is free.

A. Czajka, St. Mrówczyński, PRD 89, 085035 (2014)
**Ghost functions**

\[- p_\mu D_{\mu \nu}^{ab}(p) = p_\nu \Delta_{ab}(-p)\]

\[
\Delta^>(p) = - \frac{i \pi}{E_p} \delta^{ab} \left[ \delta(E_p - p_0)[n_g(p) + 1] + \delta(E_p + p_0)n_g(-p) \right]
\]

\[
\Delta^<(p) = - \frac{i \pi}{E_p} \delta^{ab} \left[ \delta(E_p - p_0)n_g(p) + \delta(E_p + p_0)[n_g(-p) + 1] \right]
\]

\[
\Delta^c(p) = \delta^{ab} \left[ \frac{1}{p^2 + i0^+} - \frac{i \pi}{E_p} \left( \delta(E_p - p_0)n_g(p) + \delta(E_p + p_0)n_g(-p) \right) \right]
\]

\[
\Delta^a(p) = - \delta^{ab} \left[ \frac{1}{p^2 - i0^+} + \frac{i \pi}{E_p} \left( \delta(E_p - p_0)n_g(p) + \delta(E_p + p_0)n_g(-p) \right) \right]
\]

\[n_g(p) - \text{gluon distribution function}\]
<table>
<thead>
<tr>
<th>Polarization tensors</th>
</tr>
</thead>
<tbody>
<tr>
<td>photon in QED</td>
</tr>
<tr>
<td><img src="image1" alt="Diagram" /></td>
</tr>
<tr>
<td>photon in scalar QED</td>
</tr>
<tr>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>photon in SUSY QED</td>
</tr>
<tr>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>gluon in Yang-Mills</td>
</tr>
<tr>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td>gluon in QCD</td>
</tr>
<tr>
<td><img src="image5" alt="Diagram" /></td>
</tr>
<tr>
<td>gluon in $\mathcal{N} = 4$ super Yang-Mills</td>
</tr>
<tr>
<td><img src="image6" alt="Diagram" /></td>
</tr>
</tbody>
</table>
Different loop contributions

\[ \propto - \int \frac{d^3 p}{E_p} \frac{f(p)}{E_p} \left[ \frac{2p^\mu p^\nu + k^\mu p^\nu + p^\mu k^\nu - g^{\mu \nu} k \cdot p}{(p+k)^2 + i \text{sgn}((p+k)_0) 0^+} + \frac{2p^\mu p^\nu - k^\mu p^\nu + p^\mu k^\nu + g^{\mu \nu} k \cdot p}{(p-k)^2 - i \text{sgn}((p-k)_0) 0^+} \right] \]

\[ \propto - \int \frac{d^3 p}{E_p} \frac{f(p)}{E_p} \left[ \frac{(2p+k)^\mu (2p+k)^\nu}{(p+k)^2 + i \text{sgn}((p+k)_0) 0^+} + \frac{(2p-k)^\mu (2p-k)^\nu}{(p-k)^2 - i \text{sgn}((p-k)_0) 0^+} \right] \]

\[ \propto \int d^3 p \frac{f(p)}{E_p} \]

different structures \quad \longrightarrow \quad different behaviours
Wavelength of a quasi-particle is much bigger than inter-particle distance in the plasma: \[ \lambda \gg d \]

The only dimensional parameter in ultrarelativistic equilibrium plasma is temperature \( T \), so:

\[ \rho \sim \frac{1}{d^3} \sim T^3 \sim |p|^3 \]

\[ p \sim \frac{1}{d} \quad k \sim \frac{1}{\lambda} \]

momentum at which a plasma is probed \[ k^\mu \ll p^\mu \]

momentum of plasma constituent
Polarization tensor

\[ \Pi^{\mu\nu}(k) = C_\Pi \int \frac{d^3 p}{(2\pi)^3} \frac{f_\Pi(p)}{E_p} \frac{k^2 p^\mu p^\nu - [p^\mu k^\nu + k^\mu p^\nu - g^{\mu\nu}(k \cdot p)](k \cdot p)}{(k \cdot p + i0^+)^2} \]

After applying the HL approximation the polarization tensor gets the same structure for the \( \mathcal{N}=4 \) SYM, YM, QCD, SUSY QED and usual QED plasma.

- symmetric \( \Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k) \)
- transversal \( k_\mu \Pi^{\mu\nu}(k) = 0 \)

<table>
<thead>
<tr>
<th>Plasma system</th>
<th>( C_\Pi )</th>
<th>( f_\Pi(p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
<td>( e^2 )</td>
<td>( 2f_e(p) + 2\tilde{f}_e(p) )</td>
</tr>
<tr>
<td>Scalar QED</td>
<td>( e^2 )</td>
<td>( f_s(p) + \tilde{f}_s(p) )</td>
</tr>
<tr>
<td>( \mathcal{N} = 1 ) super QED</td>
<td>( e^2 )</td>
<td>( 2f_e(p) + 2\tilde{f}_e(p) + 2f_s(p) + 2\tilde{f}_s(p) )</td>
</tr>
<tr>
<td>Yang-Mills</td>
<td>( g^2 N_c \delta^{ab} )</td>
<td>( 2f_g(p) )</td>
</tr>
<tr>
<td>QCD</td>
<td>( g^2 N_c \delta^{ab} )</td>
<td>( 2f_g(p) + \frac{N_c}{N_c} (f_q(p) + \tilde{f}_q(p)) )</td>
</tr>
<tr>
<td>( \mathcal{N} = 4 ) super Yang-Mills</td>
<td>( g^2 N_c \delta^{ab} )</td>
<td>( 2f_g(p) + 8f_f(p) + 6f_s(p) )</td>
</tr>
<tr>
<td>Particle Type</td>
<td>Diagram</td>
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<td>-------------------------------</td>
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<tr>
<td>Electron in QED</td>
<td>![Diagram for electron in QED]</td>
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<tr>
<td>Electron in SUSY QED</td>
<td>![Diagram for electron in SUSY QED]</td>
<td></td>
</tr>
<tr>
<td>Photino in SUSY QED</td>
<td>![Diagram for photino in SUSY QED]</td>
<td></td>
</tr>
<tr>
<td>Quark in QCD</td>
<td>![Diagram for quark in QCD]</td>
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<tr>
<td>Fermion in $\mathcal{N}=4$ super Yang-Mills</td>
<td>![Diagram for fermion in $\mathcal{N}=4$ super Yang-Mills]</td>
<td></td>
</tr>
</tbody>
</table>
Fermion self-energy

\[ \Sigma(k) = C_\Sigma \int \frac{d^3 p}{(2\pi)^3} \frac{f_\Sigma(p)}{E_p} \frac{\hat{p}}{k \cdot p + i0^+} \]

The fermion self-energy in HL approximation has the same structure for all considered systems.

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<th>( f_\Sigma(p) )</th>
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<tr>
<td>QED</td>
<td>( \frac{e^2}{2} )</td>
<td>( 2f_\gamma(p) + f_e(p) + \bar{f}_e(p) )</td>
</tr>
<tr>
<td>Electron in ( \mathcal{N} = 1 ) super QED</td>
<td>( \frac{e^2}{2} )</td>
<td>( 2f_\gamma(p) + f_e(p) + \bar{f}<em>e(p) + 2f</em>\gamma(p) + f_s(p) + \bar{f}_s(p) )</td>
</tr>
<tr>
<td>Photino in ( \mathcal{N} = 1 ) super QED</td>
<td>( \frac{e^2}{2} )</td>
<td>( f_e(p) + \bar{f}_e(p) + f_s(p) + \bar{f}_s(p) )</td>
</tr>
<tr>
<td>QCD</td>
<td>( \frac{g^2}{2} \frac{N_c^2 - 1}{2N_c} \delta^{mn} \delta^{ij} )</td>
<td>( 2f_g(p) + N_f(f_q(p) + \bar{f}_q(p)) )</td>
</tr>
<tr>
<td>( \mathcal{N} = 4 ) super Yang-Mills</td>
<td>( \frac{g^2}{2} N_c \delta^{ab} \delta^{ij} )</td>
<td>( 2f_g(p) + 8f_f(p) + 6f_s(p) )</td>
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</tbody>
</table>
Scalar self-energies

<table>
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<th>$f_P(p)$</th>
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</thead>
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<tr>
<td>Scalar QED</td>
<td>$e^2$</td>
<td>$2f_\gamma(p) + f_s(p) + \bar{f}_s(p)$</td>
</tr>
<tr>
<td>$\mathcal{N} = 1$ super QED</td>
<td>$e^2$</td>
<td>$2f_\gamma(p) + f_e(p) + \bar{f}<em>e(p) + 2f</em>\bar{\gamma}(p) + f_s(p) + \bar{f}_s(p)$</td>
</tr>
<tr>
<td>$\mathcal{N} = 4$ super Yang-Mills</td>
<td>$g^2 N_c \delta^{ab} S^{AB}$</td>
<td>$2f_g(p) + 8f_f(p) + 6f_s(p)$</td>
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The scalar self-energy in HL approximation has the same structure for all considered systems.
From self-energies to effective action

The structure of self-energy of every field (vector, spinor, scalar) appears to be universal.

\[ \Pi^{\mu\nu}(x, y) = \frac{\delta^2 S[A]}{\delta A_\mu(x) \delta A_\nu(y)} \quad S = \int d^4x \, \mathcal{L}(x) \]

\[ \mathcal{L}^{(A)}_2(x) = C_\Pi \int \frac{d^3 p}{(2\pi)^3} \frac{f_{\Pi}(p)}{E_p} F^{\mu\nu}(x) \left( \frac{p^\nu p^\rho}{(p \cdot \partial)^2} \right) F_{\rho}^{\mu}(x) \]

\[ \mathcal{L}^{(\psi)}_2(x) = C_\Sigma \int \frac{d^3 p}{(2\pi)^3} \frac{f_\Sigma(p)}{E_p} \overline{\Psi}(x) \left( \frac{p \cdot \gamma}{p \cdot \partial} \right) \Psi(x) \]

\[ \mathcal{L}^{(\Phi)}_2(x) = -C_P \int \frac{d^3 p}{(2\pi)^3} \frac{f_P(p)}{E_p} \Phi^*(x) \Phi(x) \]

\[ \frac{1}{p \cdot \partial} \Psi(x) \equiv i \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik \cdot x}}{p \cdot k} \Psi(k) \]
The structure of each term of the effective action appears to be unique.
Limitations of universality

When is the universality valid?

Let us consider the limit \( k \to 0 \)

\[
\Sigma(k) \sim g^2 \int \frac{d^3 p}{(2\pi)^3} \frac{f_{\Sigma}(p)}{E_p} \frac{\hat{p}}{k \cdot p + i0^+}
\]

\[
k \sim g^2 T \quad \Rightarrow \quad \Sigma \sim O(g^0)
\]

(ultrasoft scale)

The wavevector \( k \) cannot be too small!


The universality works when

\[
k^\mu \ll p^\mu \quad \quad k^\mu \propto g P^\mu
\]

(HLA) (soft scale)
Physical consequences of universality

Microscopically different systems have the same long wavelength physical characteristics:

- response functions (dielectric function)
- screening lengths
- spectrum of collective modes (quasiparticles, instabilities)

Dispersion equations

gauge boson field: \( \det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0 \)

fermionic field: \( \det[\hat{k} - \Sigma(k)] = 0 \)

scalar field: \( k^2 + P(k) = 0 \)
**Why universality occurs?**

**Microscopic** dynamics of different systems is **different**

**Macroscopic** behaviour of different systems is very **similar**

Simple example:

**QED plasma** vs. **scalar QED plasma**

Why is there no effect of **quantum statistics** of plasma constituents?

Why is there no effect of **different interactions**?
Why universality occurs?

1. Hard loop condition: \[ k^\mu \ll p^\mu \]

\[ \frac{1}{k} \quad \text{length scale at which the system is probed} \quad \gg \quad \frac{1}{p} \quad \text{de Broglie wavelength of plasma constituents} \]

In classical limit fermions and bosons are not distinguishable!

2. Gauge symmetry determines the interaction
Conclusions

- The general Slavnov-Taylor identity allows one to express the ghost Green’s function through the gluon one.

- The hard-loop effective action is universal for a whole class of gauge theories.

Happy Birthday!