From Regensburg to Universal Hard-Loop Action

Stanisław Mrówczyński

Institute of Physics, Jan Kochanowski University, Kielce, Poland
and National Centre for Nuclear Research, Warsaw, Poland

Happy Birthday!

Regensburg 1989 - 1990

Uli's first postdoc

Keldysh-Schwinger formalism

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Towards Relativistic Transport Theory of Nuclear Matter

STANISŁAW MRÓWCZYŃSKI

High-Energy Department, Sołtan Institute for Nuclear Studies,
ul. Hoża 69, PL-00-681 Warsaw, Poland

AND

ULRICH HEINZ

Institut für Theoretische Physik, Universität Regensburg,
Postfach 101042, W-8400 Regensburg, Germany

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Ph. D. project

- 1997 – Maldacena – AdS/CFT duality (gauge/gravity correspondence)

- To what extent $\mathcal{N}$=4 super YM is similar to QCD?

- To what extent $\mathcal{N}$=4 super YM plasma is similar to quark-gluon plasma?

- To what extent a supersymmetric plasma is similar to its non-supersymmetric counterpart in a weak coupling reagime?

  - Ph. D. thesis of Alina Czajka: arXiv:1601.08215

Two byproducts of the project:

- ghosts in the Keldysh-Schwinger formalism
- universality of the hard-loop action

Gauge theories under consideration

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i\overline{\Psi}\gamma_\mu D^\mu \Psi$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\mathcal{L}_{\text{ScQED}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \left(D_\mu \Phi\right)^* D^\mu \Phi$$

$$\mathcal{L}_{\text{SUSY QED}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i\overline{\Psi}\gamma_\mu D^\mu \Psi + \frac{i}{2}\overline{\Lambda}\gamma_\mu \partial^\mu \Lambda + (D_\mu \phi_L)^* (D^\mu \phi_L) + (D^*_\mu \phi_R)(D^\mu \phi^*_R)$$
$$+ \sqrt{2} e (\overline{\Psi} P_R \Lambda \phi_L - \overline{\Psi} P_L \Lambda \phi^*_R + \phi^*_L \overline{\Lambda} P_L \Psi - \phi_R \overline{\Lambda} P_R \Psi) - \frac{e^2}{2} (\phi^*_L \phi_L - \phi^*_R \phi_R)^2$$

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f^{abc} A_b^\mu A_c^\nu$$

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + i\overline{\Psi}_i (\gamma_\mu D^\mu)_{ij} \Psi_j$$

$$\mathcal{L}_{\text{SYM}} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu} + \frac{i}{2} \overline{\Psi}^a_i (\gamma_\mu D^\mu \Psi_i)^a + \frac{1}{2} (D_\mu \Phi_A)_a (D^\mu \Phi_A)_a$$
$$-\frac{1}{4} g^2 f^{abe} f^{cde} \Phi^a_A \Phi^b_B \Phi^c_A \Phi^d_B - i\frac{g}{2} f^{abc} (\overline{\Psi}^a_i \alpha^p_{ij} X^b_p \Psi^c_j + i\overline{\Psi}^a_i \beta^p_{ij} \gamma_5 Y^b_p \Psi^c_j)$$

Effective action

A system's dynamics is encoded in an effective action.

How to find the effective action?

Self-energy constrains a form of the effective action

$$\Pi^{\mu\nu}(x,y) = \frac{\delta^2 S[A]}{\delta A_\mu(x)\delta A_\nu(y)} \qquad S = \int d^4x\, \mathcal{L}(x)$$

$$\mathcal{L}_2^{(A)}(x) = \frac{1}{2}\int d^4y\, A_\mu(x)\Pi^{\mu\nu}(x-y)A_\nu(y)$$

Adopted strategy

self-energy → effective action

Keldysh–Schwinger formalism

Description of non-equilibrium many-body systems

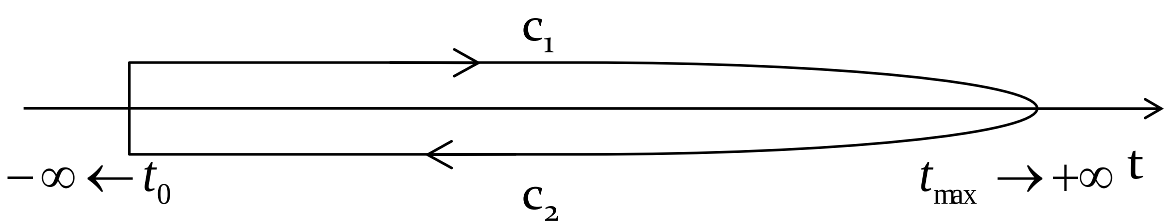
Contour Green function of gauge field

$$i\mathcal{D}^{\mu\nu}_{ab}(x,y) \overset{\text{def}}{=} \left\langle \tilde{T} A^{\mu}_{a}(x) A^{\nu}_{b}(y) \right\rangle$$

$$\langle \ldots \rangle = \mathrm{Tr}[\hat{\rho}(t_0)\ldots]$$

$\tilde{T}$ - ordering along the contour

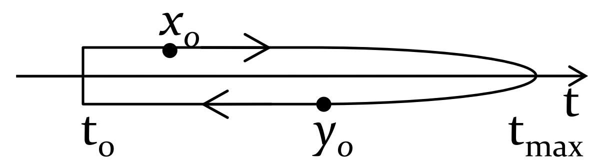
$$\tilde{T}A(x)B(y) = \Theta(x_0, y_0)A(x)B(y) \pm \Theta(y_0, x_0)B(y)A(x)$$



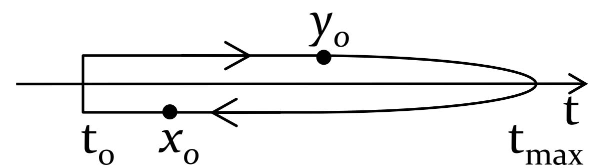
Keldysh–Schwinger formalism

Contour Green's function includes 4 Green's functions with real time arguments:

$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{>}(x,y) = \left\langle A^{\mu}_{a}(x) A^{\nu}_{b}(y) \right\rangle$$

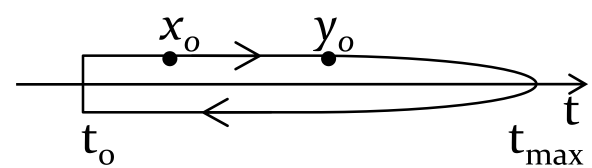


$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{<}(x,y) = \left\langle A^{\nu}_{b}(y) A^{\mu}_{a}(x) \right\rangle$$



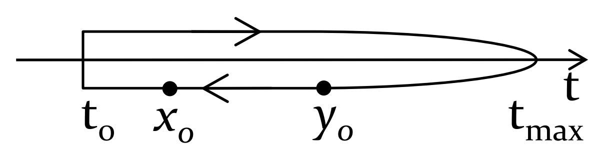
$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{c}(x,y) = \left\langle T^{c} A^{\mu}_{a}(x) A^{\nu}_{b}(y) \right\rangle$$

Chronological time ordering



$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{a}(x,y) = \left\langle T^{a} A^{\mu}_{a}(x) A^{\nu}_{b}(y) \right\rangle$$

Anti-chronological time ordering



Retarded, advanced & symmetric Green's functions

$$\boxed{\mathcal{D}^{+}(x,y)} = \Theta(x_0 - y_0)\left(\mathcal{D}^{>}(x,y) - \mathcal{D}^{<}(x,y)\right)$$

$$\boxed{\mathcal{D}^{-}(x,y)} = \Theta(y_0 - x_0)\left(\mathcal{D}^{<}(x,y) - \mathcal{D}^{>}(x,y)\right)$$

$$\boxed{\mathcal{D}^{sym}(x,y)} = \mathcal{D}^{>}(x,y) + \mathcal{D}^{<}(x,y)$$

Meaning of the functions

$$\mathcal{D}^{<,>}(x,y)$$

- phase-space density
- mass-shell constraint
- real particles

$$\mathcal{D}^{\pm}(x,y)$$

- retarded & advanced propagator
- no mass-shell constraint
- virtual particles

The Green functions $\mathcal{D}(x,y)$ are gauge dependent

Physical results obtained from Green functions must be gauge independent

For example

The poles of $\mathcal{D}(x,y)$ - dispersion relations – are gauge independent

Green's functions of free gluon field

$D(x,y) = D(x-y)$

Feynman gauge

$$\left(D^{ab}_{\mu\nu}\right)^{>}(p) = \frac{i\pi}{E_p} g_{\mu\nu}\delta^{ab}\left[\delta(E_p - p_0)[n_g(\mathbf{p})+1] + \delta(E_p + p_0)n_g(-\mathbf{p})\right]$$

$$\left(D^{ab}_{\mu\nu}\right)^{<}(p) = \frac{i\pi}{E_p} g_{\mu\nu}\delta^{ab}\left[\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)[n_g(-\mathbf{p})+1]\right]$$

$$\left(D^{ab}_{\mu\nu}\right)^{c}(p) = -g_{\mu\nu}\delta^{ab}\left[\frac{1}{p^2 + i0^+} - \frac{i\pi}{E_p}\left(\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)n_g(-\mathbf{p})\right)\right]$$

$$\left(D^{ab}_{\mu\nu}\right)^{a}(p) = g_{\mu\nu}\delta^{ab}\left[\frac{1}{p^2 - i0^+} + \frac{i\pi}{E_p}\left(\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)n_g(-\mathbf{p})\right)\right]$$

$n_g(\mathbf{p})$ - gluon distribution function

Green's functions of free gluon field

$D(x, y) = D(x - y)$

Feynman gauge

$$\left(D_{\mu\nu}^{ab}\right)^{>}(p) = \frac{i\pi}{E_p} g_{\mu\nu}\delta^{ab}\left[\delta(E_p - p_0)[n_g(\mathbf{p}) + 1] + \delta(E_p + p_0)n_g(-\mathbf{p})\right]$$

$$\left(D_{\mu\nu}^{ab}\right)^{<}(p) = \frac{i\pi}{E_p} g_{\mu\nu}\delta^{ab}\left[\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)[n_g(-\mathbf{p}) + 1]\right]$$

$$\left(D_{\mu\nu}^{ab}\right)^{+}(p) = -\frac{g_{\mu\nu}\delta^{ab}}{p^2 + i\,\mathrm{sgn}(p_0)0^+}$$

$$\left(D_{\mu\nu}^{ab}\right)^{-}(p) = \frac{g_{\mu\nu}\delta^{ab}}{p^2 - i\,\mathrm{sgn}(p_0)0^+}$$

$n_g(\mathbf{p})$ - gluon distribution function

Need for ghosts

- QCD computations in covariant gauges are usually much simpler than those in physical ones like the Coulomb gauge.

- Covariant gauges require ghosts to compenasate unphysical degrees of freedom.

Gluon field: $A^{\mu}(x) = (A^{0}(x), \mathbf{A}(x))$ - 4 degrees of freedom
Physical gluon: 2 polarizations

How to introduce ghosts in the Keldysh-Schwinger formalism?

What is the Green function of free ghosts?

$$\left.\begin{matrix} \Delta^{>}(p) \\ \Delta^{<}(p) \\ \Delta^{c}(p) \\ \Delta^{a}(p) \end{matrix}\right. = \ ?$$

How to get Green's function of free ghosts?

Ghost sector should be determined by the gauge symmetry of the theory!

$$A_\mu^a \to \left(A_\mu^a\right)^U = A_\mu^a + f^{abc}\omega^b A_\mu^c - \frac{1}{g}\partial_\mu \omega^a$$

↓ gauge symmetry of the theory

Slavnov-Taylor identities

Generating functional

$$W_0[J,\chi,\chi^*] = N_0 \int_{BC} \mathcal{D}A\,\mathcal{D}c\,\mathcal{D}c^*\, e^{i\int_C d^4x\, \mathcal{L}_{\text{eff}}(x)}$$

all fields are on the contour

boundary conditions:
the fields are fixed at $t = -\infty \pm i0^+$

$$\mathcal{L}_{\text{eff}}(x) = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a - \frac{1}{2}(\partial^\mu A_\mu^a)^2 - c_a^*(\partial^\mu \partial_\mu \delta_{ab} - g\partial^\mu f^{abc} A_\mu^c) c_b$$
$$+ J_\mu^a A_a^\mu + \chi_a^* c_a + \chi_a c_a^*$$

$$W[J,\chi,\chi^*] = N \int DA'\; Dc'\; Dc^{*\prime}\; DA''\; Dc''\; Dc^{*\prime\prime}$$
$$\times \rho[A', c', c^{*\prime} | A'', c'', c^{*\prime\prime}]\, W_0[J,\chi,\chi^*]$$

density matrix

Generating functional

$$W[J,\chi,\chi^*] = N\int DA' Dc' Dc^{*\prime} DA'' Dc'' Dc^{*\prime\prime} \times \rho[A',c',c^{*\prime}|A'',c'',c^{*\prime\prime}]\, W_0[J,\chi,\chi^*]$$

The full Green's function are generated as

$$i\mathcal{D}^{ab}_{\mu\nu}(x,y) = (-i)^2 \frac{\delta^2}{\delta J^a_\mu(x)\delta J^b_\nu(y)} W[J,\chi,\chi^*]\Big|_{J=\chi=\chi^*=0}$$

The density matrix $\rho[A',c',c^{*\prime}|A'',c'',c^{*\prime\prime}]$ is not specified

The explicit forms of the functional and the Green's function are not known

The functional provides various relations among Green's functions

General Slavnov-Taylor identity

$$W[J,\chi,\chi^*] = N\int_{BC}\mathcal{D}A\,\Delta[A]e^{i\int_C d^4x\,\mathcal{L}(x)}$$

analog of the Fadeev-Popov determinant

$$\Delta[A] \equiv \int_{BC}\mathcal{D}c\mathcal{D}c^*e^{-i\int_C d^4x\left(-c_a^*(\partial^\mu\partial_\mu\delta_{ab}-g\partial^\mu f^{abc}A_\mu^c)c_b+\chi_a^*c_a+\chi_a c_a^*\right)}$$

The invariance of $W[J,\chi,\chi^*]$ under the transformations

$$A_\mu^a \to \left(A_\mu^a\right)^U = A_\mu^a + f^{abc}\omega^b A_\mu^c - \frac{1}{g}\partial_\mu\omega^a$$

leads to

$$\left\{i\partial_{(y)}^\mu\frac{\delta}{\delta J_d^\mu(y)} - \int_C d^4x\,J_a^\mu(x)\left(\partial_\mu^{(x)}\delta^{ab} + igf^{abc}\frac{\delta}{\delta J_c^\mu(x)}\right)M_{bd}^{-1}\left[\frac{1}{i}\frac{\delta}{\delta J}\middle|x,y\right]\right\}W[J,\chi,\chi^*]=0$$

Slavnov-Taylor identity for gluon Green's function

$$\frac{\delta}{\delta J_e^{\nu}(z)} \left\{ i\partial_{(y)}^{\mu} \frac{\delta}{\delta J_d^{\mu}(y)} - \int_C d^4x\, J_a^{\mu}(x) \left(\partial_{\mu}^{(x)} \delta^{ab} + igf^{abc} \frac{\delta}{\delta J_c^{\mu}(x)} \right) M_{bd}^{-1} \left[\frac{1}{i} \frac{\delta}{\delta J} \bigg| x, y \right] \right\} W[J, \chi, \chi^*] = 0$$

$$J = \chi = \chi^* = 0$$

$$-p^{\mu} \mathcal{D}_{\mu\nu}^{ab}(p) = p_{\nu} \Delta_{ab}(-p)$$

free ghost Green's function

The longitudinal component of the gluon Green's function is free.

A. Czajka, St. Mrówczyński, PRD 89, 085035 (2014)

Ghost functions

$$-p^{\mu} D^{ab}_{\mu\nu}(p) = p_{\nu} \Delta_{ab}(-p)$$

$$\Delta^{>}(p) = -\frac{i\pi}{E_p}\delta^{ab}\left[\delta(E_p - p_0)[n_g(\mathbf{p})+1] + \delta(E_p + p_0) n_g(-\mathbf{p})\right]$$

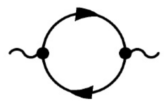
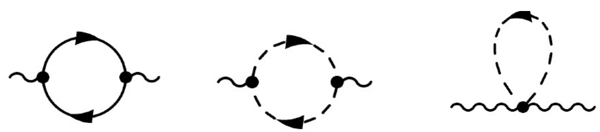
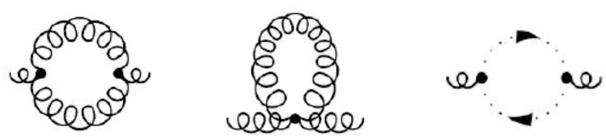
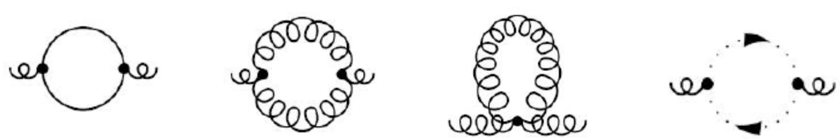
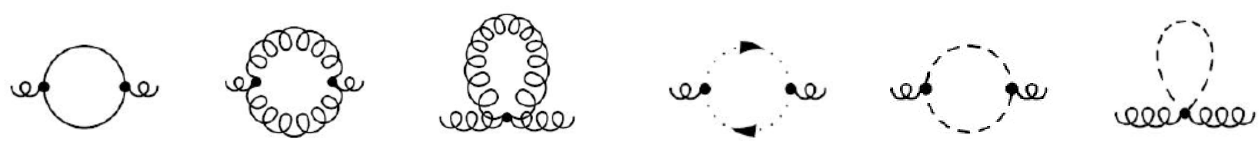
$$\Delta^{<}(p) = -\frac{i\pi}{E_p}\delta^{ab}\left[\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0)[n_g(-\mathbf{p})+1]\right]$$

$$\Delta^{c}(p) = \delta^{ab}\left[\frac{1}{p^2 + i0^+} - \frac{i\pi}{E_p}\left(\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0) n_g(-\mathbf{p})\right)\right]$$

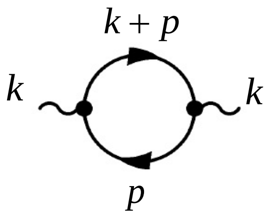
$$\Delta^{a}(p) = -\delta^{ab}\left[\frac{1}{p^2 - i0^+} + \frac{i\pi}{E_p}\left(\delta(E_p - p_0) n_g(\mathbf{p}) + \delta(E_p + p_0) n_g(-\mathbf{p})\right)\right]$$

$n_g(\mathbf{p})$ - gluon distribution function

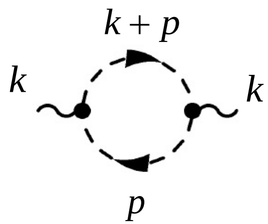
Polarization tensors

photon in QED	
photon in scalar QED	
photon in SUSY QED	
gluon in Yang-Mills	
gluon in QCD	
gluon in $\mathcal{N} = 4$ super Yang-Mills	

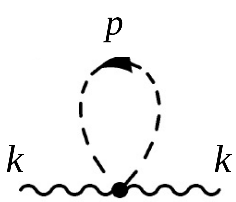
Different loop contributions



$$\propto -\int d^3p \frac{f(\mathbf{p})}{E_p}\left[\frac{2p^\mu p^\nu + k^\mu p^\nu + p^\mu k^\nu - g^{\mu\nu} k\cdot p}{(p+k)^2 + i\,\mathrm{sgn}((p+k)_0)0^+} + \frac{2p^\mu p^\nu - k^\mu p^\nu - p^\mu k^\nu + g^{\mu\nu} k\cdot p}{(p-k)^2 - i\,\mathrm{sgn}((p-k)_0)0^+}\right]$$



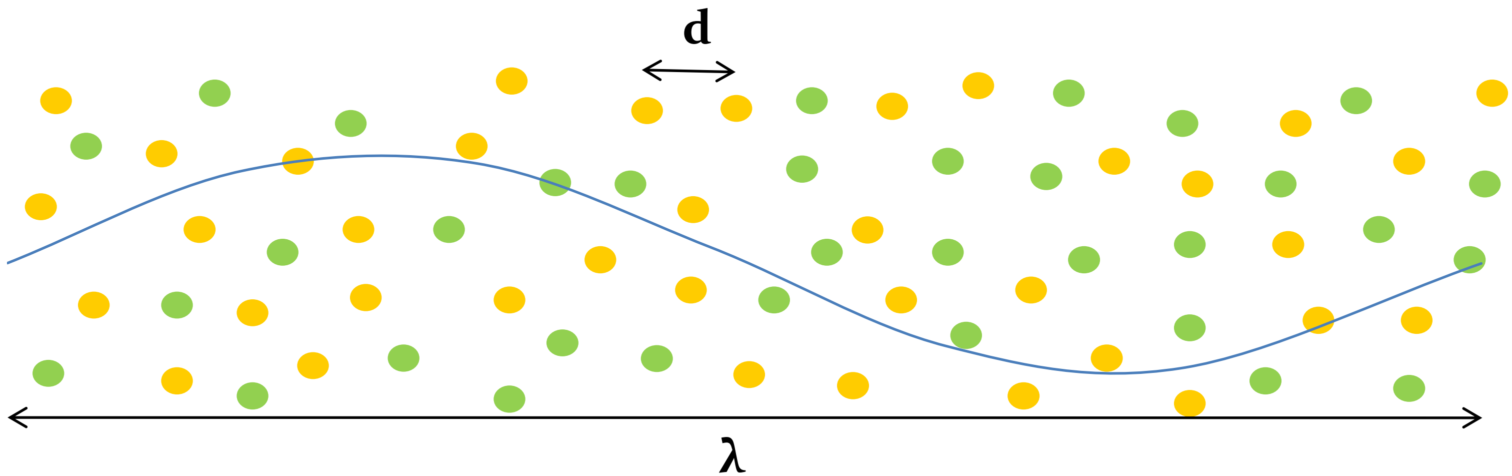
$$\propto -\int d^3p \frac{f(\mathbf{p})}{E_p}\left[\frac{(2p+k)^\mu (2p+k)^\nu}{(p+k)^2 + i\,\mathrm{sgn}((p+k)_0)0^+} + \frac{(2p-k)^\mu (2p-k)^\nu}{(p-k)^2 - i\,\mathrm{sgn}((p-k)_0)0^+}\right]$$



$$\propto \int d^3p \frac{f(\mathbf{p})}{E_p}$$

different structures → different behaviours

Hard Loop Approximation



Wavelength of a quasi-particle is much bigger than inter-particle distance in the plasma:

$$\lambda >> d$$

The only dimensional parameter in ultrarelativistic equilibrium plasma is temperature *T*, so:

$$\rho \sim \frac{1}{d^3} \sim T^3 \sim |\mathbf{p}|^3$$

$$p \sim \frac{1}{d} \qquad k \sim \frac{1}{\lambda}$$

momentum at which a plasma is probed

$$k^\mu << p^\mu$$

momentum of plasma constituent

Polarization tensor

$\text{HL}: \quad k^{\mu} << p^{\mu}$

$$\Pi^{\mu\nu}(k) = C_{\Pi} \int \frac{d^3 p}{(2\pi)^3} \frac{f_{\Pi}(\mathbf{p})}{E_p} \frac{k^2 p^{\mu} p^{\nu} - [p^{\mu} k^{\nu} + k^{\mu} p^{\nu} - g^{\mu\nu}(k \cdot p)](k \cdot p)}{(k \cdot p + i0^{+})^2}$$

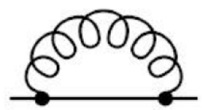
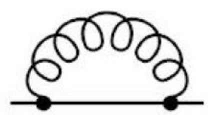
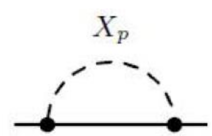
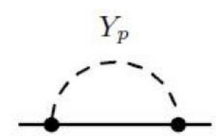
After applying the HL approximation the polarization tensor gets **the same structure** for the $\mathcal{N}$=4 SYM, YM, QCD, SUSY QED and usual QED plasma.

- **symmetric** $\quad \Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k)$
- **transversal** $\quad k_{\mu}\Pi^{\mu\nu}(k) = 0$

Gauge independence!

Plasma system	C_{Π}	$f_{\Pi}(\mathbf{p})$
QED	e^2	$2f_e(\mathbf{p}) + 2\bar{f}_e(\mathbf{p})$
Scalar QED	e^2	$f_s(\mathbf{p}) + \bar{f}_s(\mathbf{p})$
$\mathcal{N} = 1$ super QED	e^2	$2f_e(\mathbf{p}) + 2\bar{f}_e(\mathbf{p}) + 2f_s(\mathbf{p}) + 2\bar{f}_s(\mathbf{p})$
Yang-Mills	$g^2 N_c \delta^{ab}$	$2f_g(\mathbf{p})$
QCD	$g^2 N_c \delta^{ab}$	$2f_g(\mathbf{p}) + \frac{N_f}{N_c}(f_q(\mathbf{p}) + \bar{f}_q(\mathbf{p}))$
$\mathcal{N} = 4$ super Yang-Mills	$g^2 N_c \delta^{ab}$	$2f_g(\mathbf{p}) + 8f_f(\mathbf{p}) + 6f_s(\mathbf{p})$

Fermionic self-energies

electron in QED			
electron in SUSY QED			
photino in SUSY QED			
quark in QCD			
fermion in $\mathcal{N} = 4$ super Yang-Mills			

Fermion self-energy

$$\Sigma(k) = C_\Sigma \int \frac{d^3p}{(2\pi)^3} \frac{f_\Sigma(\mathbf{p})}{E_p} \frac{\hat{p}}{k\cdot p + i0^+}$$

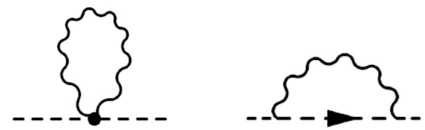
$$\text{HL}: \quad k^\mu << p^\mu$$

The fermion self-energy in HL approximation has **the same structure** for all considered systems.

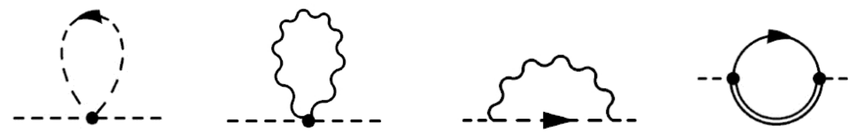
Plasma system	C_Σ	$f_\Sigma(\mathbf{p})$
QED	$\frac{e^2}{2}$	$2f_\gamma(\mathbf{p}) + f_e(\mathbf{p}) + \bar{f}_e(\mathbf{p})$
Electron in $\mathcal{N}=1$ super QED	$\frac{e^2}{2}$	$2f_\gamma(\mathbf{p}) + f_e(\mathbf{p}) + \bar{f}_e(\mathbf{p}) + 2f_{\tilde{\gamma}}(\mathbf{p}) + f_s(\mathbf{p}) + \bar{f}_s(\mathbf{p})$
Photino in $\mathcal{N}=1$ super QED	$\frac{e^2}{2}$	$f_e(\mathbf{p}) + \bar{f}_e(\mathbf{p}) + f_s(\mathbf{p}) + \bar{f}_s(\mathbf{p})$
QCD	$\frac{g^2}{2}\frac{N_c^2-1}{2N_c}\delta^{mn}\delta^{ij}$	$2f_g(\mathbf{p}) + N_f(f_q(\mathbf{p}) + \bar{f}_q(\mathbf{p}))$
$\mathcal{N}=4$ super Yang-Mills	$\frac{g^2}{2}N_c\delta^{ab}\delta^{ij}$	$2f_g(\mathbf{p}) + 8f_f(\mathbf{p}) + 6f_s(\mathbf{p})$

Scalar self-energies

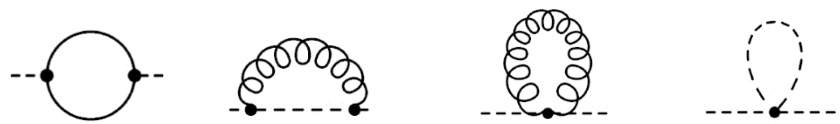
scalar QED



scalar in N=1 SUSY QED



scalar in N=4 Super Yang-Mills



$$P(k) = C_P \int \frac{d^3 p}{(2\pi)^3} \frac{f_P(\mathbf{p})}{E_p}$$

The scalar self-energy in HL approximation has **the same structure** for all considered systems.

Plasma system	C_P	$f_P(\mathbf{p})$
Scalar QED	e^2	$2f_\gamma(\mathbf{p}) + f_s(\mathbf{p}) + \bar{f}_s(\mathbf{p})$
$\mathcal{N} = 1$ super QED	e^2	$2f_\gamma(\mathbf{p}) + f_e(\mathbf{p}) + \bar{f}_e(\mathbf{p}) + 2f_{\tilde{\gamma}}(\mathbf{p}) + f_s(\mathbf{p}) + \bar{f}_s(\mathbf{p})$
$\mathcal{N} = 4$ super Yang-Mills	$g^2 N_c \delta^{ab} \delta^{AB}$	$2f_g(\mathbf{p}) + 8f_f(\mathbf{p}) + 6f_s(\mathbf{p})$

From self-energies to effective action

The structure of self-energy of every field (vector, spinor, scalar) appears to be universal.

$$\Pi^{\mu\nu}(x,y) = \frac{\delta^2 S[A]}{\delta A_\mu(x)\delta A_\nu(y)} \qquad S = \int d^4x\, \mathcal{L}(x)$$

$$\mathcal{L}_2^{(A)}(x) = C_\Pi \int \frac{d^3p}{(2\pi)^3} \frac{f_\Pi(\mathbf{p})}{E_p} F_{\mu\nu}(x) \left(\frac{p^\nu p^\rho}{(p\cdot\partial)^2} \right) F_\rho{}^\mu(x)$$

$$\mathcal{L}_2^{(\Psi)}(x) = C_\Sigma \int \frac{d^3p}{(2\pi)^3} \frac{f_\Sigma(\mathbf{p})}{E_p} \overline{\Psi}(x) \left(\frac{p\cdot\gamma}{p\cdot\partial} \right) \Psi(x)$$

$$\mathcal{L}_2^{(\Phi)}(x) = -C_P \int \frac{d^3p}{(2\pi)^3} \frac{f_P(\mathbf{p})}{E_p} \Phi^*(x)\Phi(x)$$

$$\frac{1}{p\cdot\partial}\Psi(x) \equiv i\int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik\cdot x}}{p\cdot k}\Psi(k)$$

Hard-loop effective action

$$\mathcal{L}_{\rm HL}^{(A)}(x) = C_\Pi \int \frac{d^3p}{(2\pi)^3} \frac{f_\Pi(\mathbf{p})}{E_p} F_{\mu\nu}(x) \left(\frac{p^\nu p^\rho}{(p\cdot D)^2} \right) F_\rho{}^\mu(x)$$

$$\mathcal{L}_{\rm HL}^{(\Psi)}(x) = C_\Sigma \int \frac{d^3p}{(2\pi)^3} \frac{f_\Sigma(\mathbf{p})}{E_p} \bar{\Psi}(x) \left(\frac{p\cdot\gamma}{p\cdot D} \right) \Psi(x)$$

$$\mathcal{L}_{\rm HL}^{(\Phi)}(x) = -C_P \int \frac{d^3p}{(2\pi)^3} \frac{f_P(\mathbf{p})}{E_p} \Phi^*(x)\Phi(x)$$

$$\frac{1}{p\cdot D}\Psi(x) \equiv \frac{1}{p\cdot\partial} \sum_{n=0}^{\infty} \left(igp\cdot A(x) \frac{1}{p\cdot\partial} \right)^n \Psi(x)$$

E. Braaten, R. D. Pisarski, PRD 45, 1827 (1992)

St. Mrówczyński, A. Rebhan, M. Strickland, PRD 70, 025004 (2004)

The structure of each term of the effective action appears to be unique.

A. Czajka, St. Mrówczyński, PRD 91, 025013 (2015)

Limitations of universality

When is the universality valid?

Let us consider the limit $k \to 0$

$$\Sigma(k) \sim g^2 \int \frac{d^3p}{(2\pi)^3} \frac{f_\Sigma(\mathbf{p})}{E_p} \frac{\hat{p}}{k \cdot p + i0^+}$$

$k \sim g^2 T$ (ultrasoft scale) ⇒ $\Sigma \sim O(g^0)$ the self-energy is not perturbatively small

The wavevector *k* cannot be too small!

A. D. Linde, Phys. Lett. B 96, 289 (1980)

The universality works when

$k^\mu << p^\mu$ (HLA)

$k^\mu \propto g P^\mu$ (soft scale)

Physical consequences of universality

Microscopically different systems have **the same long wavelength physical characteristics**:

- response functions (dielectric function)
- screening lengths
- spectrum of collective modes (quasiparticles, instabilities)

Dispersion equations

gauge boson field: $\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0$

fermionic field: $\det[\hat{k} - \Sigma(k)] = 0$

scalar field: $k^2 + P(k) = 0$

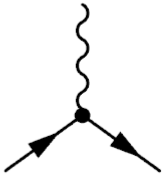
Why universality occurs?

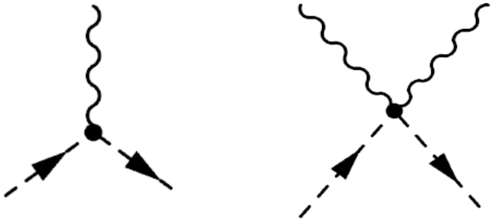
Microscopic dynamics of different systems is **different**

Macroscopic behaviour of different systems is very **similar**

Simple example:

QED plasma **vs.** **scalar QED plasma**





Why is there no effect of **quantum statistics** of plasma constituents?

Why is there no effect of **different interactions**?

Why universality occurs?

1. Hard loop condition: $$k^\mu << p^\mu$$

$\frac{1}{k}$ length scale at which the system is probed $\gg$ $\frac{1}{p}$ de Broglie wavelength of plasma constituents

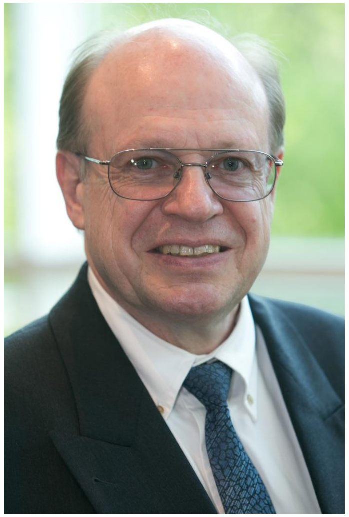
In classical limit fermions and bosons are not distinguishable!

2. Gauge symmetry determines the interaction

Conclusions

- The general Slavnov-Taylor identity allows one to express the ghost Green's function through the gluon one.

- The hard-loop effective action is universal for a whole class of gauge theories



Happy Birthday!