Equilibration of Quark-Gluon Plasma in Quasi-Linear Approach

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Motivation

- QGP for the early stage of RHIC is anisotropic
- Weakly coupled anisotropic QGP is unstable

How to describe equilibration of weakly coupled unstable QGP?

Equilibration in quasi-linear approach

The quasi-linear kinetic theory of weakly turbulent plasma

A.A. Vedenov, E. P. Velikhov and R.Z. Sagdeev,
Usp. Fiz. Nauk, **73**, 701 (1961) [in Russian];
Sov. Phys. Usp. **4**, 332 (1961).

A.A. Vedenov, Atomnaya Energiya **13**, 5 (1962) [in Russian];
J. Nucl. Energy C **5**, 169 (1963).

E.M. Lifshitz and L.P. Pitaevskii, *Physical Kinetics*

Application to QGP: St. Mrówczyński & B. Müller, Physical Review **D80**, 065021 (2010)

Collisionless transport equations

Weakly coupled QGP

fundamental:

$$p_\mu D^\mu Q - \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu Q\} = 0 \qquad \text{quarks}$$

$$p_\mu D^\mu \overline{Q} + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu \overline{Q}\} = 0 \qquad \text{antiquarks}$$

adjoint:

$$p_\mu \mathcal{D}^\mu G - \frac{g}{2} p^\mu \{\mathcal{F}_{\mu\nu}, (x) \partial_p^\nu G\} = 0 \qquad \text{gluons}$$

free streaming

mean-field force

mean-field generation

$$D_\mu F^{\mu\nu} = j^\nu[Q, \overline{Q}, G]$$

$$D^\mu \equiv \partial^\mu - ig[A^\mu, ...]$$

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

The dynamics is assumed to be dominated by strong mean fields!

Regular and fluctuating quantities

The distribution function of quarks

fluctuating part

$$Q(t,\mathbf{r},\mathbf{p}) = \langle Q(t,\mathbf{r},\mathbf{p})\rangle + \delta Q\,(t,\mathbf{r},\mathbf{p})$$

regular colorless part

$$\langle Q(t,\mathbf{r},\mathbf{p})\rangle = n(t,\mathbf{r},\mathbf{p})I$$

$$|n| \gg |\delta Q|, \quad |\nabla_p n| \gg |\nabla_p \delta Q|$$

$$\left|\frac{\partial n}{\partial t}\right| \ll \left|\frac{\partial \delta Q}{\partial t}\right|, \qquad |\nabla n| \ll |\nabla \delta Q|$$

$$\langle \mathbf{E}\rangle = 0, \quad \langle \mathbf{B}\rangle = 0, \quad \mathbf{E}, \mathbf{B}, A^0 \mathbf{A} \sim \delta Q$$

Quarks in fluctuating background

$$Q(t,\mathbf{r},\mathbf{p}) = n(t,\mathbf{r},\mathbf{p})I + \delta Q\,(t,\mathbf{r},\mathbf{p})$$

$$\left(D^0 + \mathbf{v}\cdot\mathbf{D}\right)Q - g\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right)\cdot\nabla_p Q = 0$$

$\mathrm{Tr}\langle\cdots\rangle$ ensemble averaging

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)n(t,\mathbf{r},\mathbf{p}) - \frac{g}{N_c}\mathrm{Tr}\left\langle\left(\mathbf{E}(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{r})\right)\cdot\nabla_p\delta Q(t,\mathbf{r},\mathbf{p})\right\rangle = 0$$

fluctuations provide a collision term

How to compute the collision terms?

$$C \equiv \frac{g}{N_c} \mathrm{Tr}\left\langle \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_p \delta Q \right\rangle = ?$$

$$Q(t, \mathbf{r}, \mathbf{p}) = n(t, \mathbf{r}, \mathbf{p}) I + \delta Q\ (t, \mathbf{r}, \mathbf{p})$$

$$\left(D^0 + \mathbf{v} \cdot \mathbf{D} \right) Q - g \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_p Q = 0$$

linearization

$$\left| \frac{\partial n}{\partial t} \right| << \left| \frac{\partial \delta Q}{\partial t} \right|$$

$$| \nabla n | << | \nabla \delta Q |$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \delta Q - g \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \nabla_p n = 0$$

Solution of the linearized transport equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\delta Q(t,\mathbf{r},\mathbf{p}) - g\left(\mathbf{E}(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{r})\right)\nabla_p n(\mathbf{p}) = 0$$

$$\delta Q(t,\mathbf{r},\mathbf{p}) = g\int_0^t dt'\left(\mathbf{E}(t',\mathbf{r}-\mathbf{v}(t-t')) + \mathbf{v}\times\mathbf{B}(t',\mathbf{r}-\mathbf{v}(t-t'))\right)\nabla_p n(\mathbf{p})$$

$$+\ \delta Q_0(\mathbf{r}-\mathbf{v}t,\mathbf{p})$$

initial value

$\mathrm{Tr}\left\langle\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)\cdot\nabla_p\delta Q\right\rangle$ expressed by $\left\langle E^i E^j\right\rangle, \left\langle B^i E^j\right\rangle, \left\langle B^i B^j\right\rangle$

Collision term is given by the field correlators

How to compute field correlators in unstable QGP?

- Equilibrium methods are not applicable
- We deal with the initial value problem

The kinetic theory method by Klimontovich & Silin, Rostoker, Tsytovich, see E.M. Lifshitz and L.P. Pitaevskii, *Physical Kinetics*

Developed and applied to QGP: St. Mrówczyński, Physical Review **D77**, 105022 (2008)

Linearized equations

Transport equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\delta Q(t,\mathbf{r},\mathbf{p}) - g\big(\mathbf{E}(t,\mathbf{r}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{r})\big)\nabla_p n(t,\mathbf{r},\mathbf{p}) = 0$$

Yang-Mills (Maxwell) equations

$$\nabla\cdot\mathbf{E}(t,\mathbf{r}) = \rho(t,\mathbf{r}), \qquad \nabla\cdot\mathbf{B}(t,\mathbf{r}) = 0,$$

$$\nabla\times\mathbf{E}(t,\mathbf{r}) = -\frac{\partial\mathbf{B}(t,\mathbf{r})}{\partial t}, \quad \nabla\times\mathbf{B}(t,\mathbf{r}) = \mathbf{j}(t,\mathbf{r}) + \frac{\partial\mathbf{E}(t,\mathbf{r})}{\partial t}$$

$$\begin{cases} \rho_a(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\,\mathrm{Tr}\left[\tau^a\,\delta Q(t,\mathbf{r},\mathbf{p})\right], \\ \mathbf{j}_a(t,\mathbf{r}) = -g\int\frac{d^3p}{(2\pi)^3}\,\mathbf{v}\,\mathrm{Tr}\left[\tau^a\,\delta Q(t,\mathbf{r},\mathbf{p})\right], \end{cases}$$

gauge dependence
discussed *a posteriori*

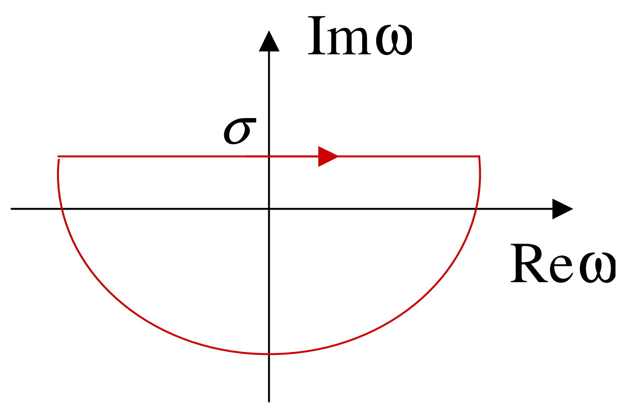
Initial value problem

$$\delta Q(t=0,\mathbf{r},\mathbf{p}) = \delta Q_0(\mathbf{r},\mathbf{p}),$$
$$\mathbf{E}(t=0,\mathbf{r},\mathbf{p}) = \mathbf{E}_0(\mathbf{r},\mathbf{p}), \quad \mathbf{B}(t=0,\mathbf{r},\mathbf{p}) = \mathbf{B}_0(\mathbf{r},\mathbf{p})$$

One-sided Fourier transformations

$$\begin{cases} f(\omega,\mathbf{k}) = \int_0^\infty dt \int d^3r\, e^{i(\omega t-\mathbf{kr})} f(t,\mathbf{r}) \\ f(t,\mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3}\, e^{-i(\omega t-\mathbf{kr})} f(\omega,\mathbf{k}) \end{cases}$$

$$0 < \sigma \in R$$



Transformed linear equations

Transport equation

$$-i(\omega-\mathbf{v}\cdot\mathbf{k})\delta Q(\omega,\mathbf{k},\mathbf{p})-g\big(\mathbf{E}(\omega,\mathbf{k})+\mathbf{v}\times\mathbf{B}(\omega,\mathbf{k})\big)\nabla_p n(\mathbf{p})=\delta Q_0(\mathbf{k},\mathbf{p})$$

Yang-Mills (Maxwell) equations

$$i\mathbf{k}\cdot\mathbf{E}(\omega,\mathbf{k})=\rho(\omega,\mathbf{k}),\qquad i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k})=0,$$

$$i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k})=i\omega\mathbf{B}(\omega,\mathbf{k})+\mathbf{B}_0(\mathbf{k}),$$

$$i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k})=\mathbf{j}(\omega,\mathbf{k})-i\omega\mathbf{E}(\omega,\mathbf{k})-\mathbf{E}_0(\mathbf{k})$$

$$\begin{cases}\rho_a(\omega,\mathbf{k})=-g\int\frac{d^3p}{(2\pi)^3}\mathrm{Tr}\left[\tau^a\delta Q(\omega,\mathbf{k},\mathbf{p})\right],\\ \mathbf{j}_a(\omega,\mathbf{k})=-g\int\frac{d^3p}{(2\pi)^3}\mathbf{v}\,\mathrm{Tr}\left[\tau^a\delta Q(\omega,\mathbf{k},\mathbf{p})\right],\end{cases}$$

Solution

$$\left[-\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})\right]E^j(\omega,\mathbf{k}) = -g\omega\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega - \mathbf{v}\cdot\mathbf{k}}\delta Q_0(\mathbf{k},\mathbf{p})$$

$$-i\frac{g^2}{2}\int\frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega - \mathbf{v}\cdot\mathbf{k}}\frac{\mathbf{v}\times\mathbf{B}_0(\mathbf{k})}{\omega}\cdot\nabla_p n(\mathbf{p}) + i\omega E_0^i(\mathbf{k}) - i\left(\mathbf{k}\times\mathbf{B}_0(\mathbf{k})\right)^i$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$$

chromodielectric tensor

Isotropic system

$$\varepsilon^{ij}(\omega,\mathbf{k}) \equiv \varepsilon_L(\omega,\mathbf{k})\frac{k^i k^j}{\mathbf{k}^2} + \varepsilon_T(\omega,\mathbf{k})\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

$$\left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k}) = \frac{1}{\omega^2\varepsilon_L(\omega,\mathbf{k})}\frac{k^i k^j}{\mathbf{k}^2} + \frac{1}{\omega^2\varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

Fluctuations of E field

The solution

$$E^i(\omega,\mathbf{k}) = \left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k})\left[\ldots\ldots\ \delta Q_0(\mathbf{k},\mathbf{p}) + \ldots\ldots\ \mathbf{E}_0(\mathbf{k}) + \ldots\ldots\ \mathbf{B}_0(\mathbf{k})\right]^j$$

initial values

The correlation function

$$\left\langle E^i(\omega,\mathbf{k})E^j(\omega',\mathbf{k}')\right\rangle = \left(\Sigma^{-1}\right)^{ik}(\omega,\mathbf{k})\left(\Sigma^{-1}\right)^{jl}(\omega',\mathbf{k}')\Big[\ldots\ldots\ \left\langle \delta Q_0(\mathbf{k},\mathbf{p})\delta Q_0(\mathbf{k}',\mathbf{p}')\right\rangle$$
$$+\ldots\ldots\ \left\langle \delta Q_0(\mathbf{k},\mathbf{p})E_0^m(\mathbf{k}')\right\rangle + \ldots\ldots\ \left\langle \delta Q_0(\mathbf{k},\mathbf{p})B_0^m(\mathbf{k}')\right\rangle$$
$$+\ldots\ldots\ \left\langle E_0^m(\mathbf{k})E_0^n(\mathbf{k}')\right\rangle + \ldots\ldots\ \left\langle E_0^m(\mathbf{k})B_0^n(\mathbf{k}')\right\rangle$$
$$+\ldots\ldots\ \left\langle B_0^m(\mathbf{k})B_0^n(\mathbf{k}')\right\rangle\Big]^{kl}$$

$\langle\cdots\rangle$ - statistical ensemble average

Initial values

Using Maxwell equations

$\mathbf{E}_0(\mathbf{k})$, $\mathbf{B}_0(\mathbf{k})$, $\rho_0(\mathbf{k})$, $\mathbf{j}_0(\mathbf{k})$ can be expressed through $\delta Q_0(\mathbf{k},\mathbf{p})$

Initial fluctuations

color indices $i, j, k, l = 1, 2, \dots, N_c$

$$\left\langle \delta Q_0^{ij}(\mathbf{r},\mathbf{p})\;\delta Q_0^{kl}(\mathbf{r}',\mathbf{p}')\right\rangle = ?$$

Assumption

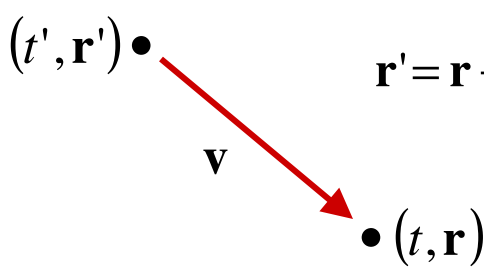
The initial fluctuations are given by $\left\langle \delta Q^{ij}(t=0,\mathbf{r},\mathbf{p})\;\delta Q^{kl}(t'=0,\mathbf{r}',\mathbf{p}')\right\rangle_{\text{free}}$

colorless state

$$\delta Q^{ij}(t,\mathbf{r},\mathbf{p}) \equiv Q^{ij}(t,\mathbf{r},\mathbf{p}) - \left\langle Q^{ij}(t,\mathbf{r},\mathbf{p})\right\rangle = Q^{ij}(t,\mathbf{r},\mathbf{p}) - \delta^{ij}\,n(\mathbf{p})$$

Classical limit

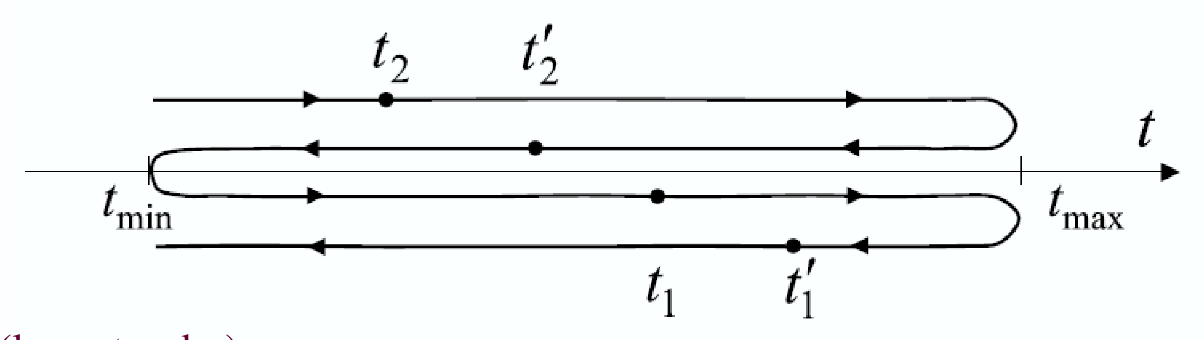
$$\left\langle \delta Q^{ij}(t,\mathbf{r},\mathbf{p})\;\delta Q^{kl}(t',\mathbf{r}',\mathbf{p}')\right\rangle_{\text{free}} = \delta^{il}\delta^{jk}(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')(2\pi)^3\delta^{(3)}\big(\mathbf{r}'-\mathbf{r}-\mathbf{v}(t'-t)\big)n(\mathbf{p})$$



$$\mathbf{r}' = \mathbf{r} + \mathbf{v}(t'-t)$$

Fluctuations of free distribution functions cont.

$$\left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle = \left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle$$



Wick theorem (lowest order)

$$\left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle = \left\langle T_c\left(\varphi_j^*(x'_1)\varphi_i(x_1)\right)\right\rangle\left\langle T_c\left(\varphi_l^*(x'_2)\varphi_k(x_2)\right)\right\rangle + \left\langle T_c\left(\varphi_j^*(x'_1)\varphi_k(x_2)\right)\right\rangle\left\langle T_c\left(\varphi_l^*(x'_2)\varphi_i(x_1)\right)\right\rangle$$

$$\left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\,\varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle = \left\langle \varphi_j^*(x'_1)\varphi_i(x_1)\right\rangle\left\langle \varphi_l^*(x'_2)\varphi_k(x_2)\right\rangle + \left\langle \varphi_j^*(x'_1)\varphi_k(x_2)\right\rangle\left\langle \varphi_i(x_1)\varphi_l^*(x'_2)\right\rangle$$

Fluctuations in isotropic (stable) system

$$\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle = \frac{g^2}{2}\delta^{ab}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\int\frac{d^3p}{(2\pi)^3}n(\mathbf{p})F(\omega,\mathbf{k},\omega',\mathbf{k}',\mathbf{p})$$

colorless background

translational invariance

$F(\omega,\mathbf{k},\omega',\mathbf{k}',\mathbf{p})$ has poles at:

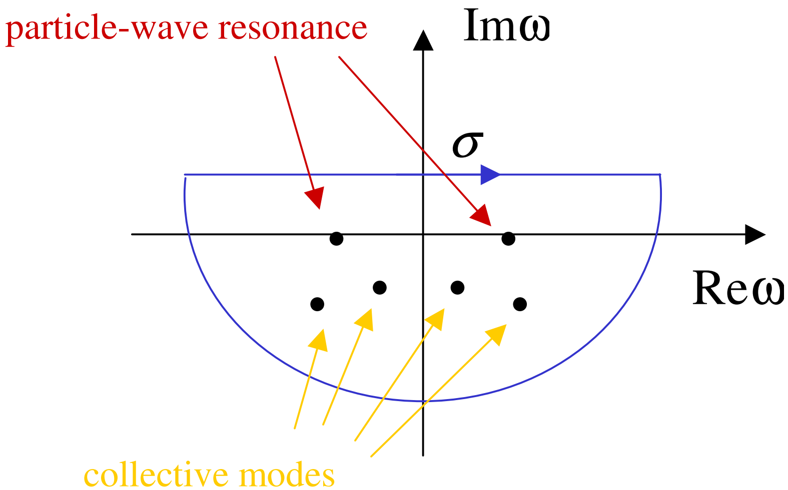
particle-wave resonance
$$\begin{cases}\omega-\mathbf{v}\cdot\mathbf{k}=0\\ \omega'-\mathbf{v}'\cdot\mathbf{k}'=0\end{cases}$$

collective longitudinal modes
$$\begin{cases}\varepsilon_L(\omega,\mathbf{k})=0\\ \varepsilon_L(\omega',\mathbf{k}')=0\end{cases}$$

collective transverse modes
$$\begin{cases}\omega^2\varepsilon_T(\omega,\mathbf{k})-\mathbf{k}^2=0\\ \omega'^2\varepsilon_T(\omega',\mathbf{k}')-\mathbf{k}'^2=0\end{cases}$$

Fluctuations in isotropic (stable) system

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle = \int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi}\int_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega'}{2\pi}\int\frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\,e^{-i(\omega t+\omega' t'-\mathbf{kr}-\mathbf{k}'\mathbf{r}')}\times\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle$$



$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle \sim f(\mathbf{r}-\mathbf{r}')$$

$$\left\langle E_a^i(\omega,\mathbf{k})E_b^j(\omega',\mathbf{k}')\right\rangle \sim \delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

$$\left\langle E_a^i(t,\mathbf{r})E_b^j(t',\mathbf{r}')\right\rangle = \begin{pmatrix}\text{collective}\\ \text{modes}\end{pmatrix}\left(e^{-\gamma t}\ \text{or}\ e^{-\gamma t'}\right) + \begin{pmatrix}\text{particle-wave}\\ \text{resonance}\end{pmatrix} f(t-t')$$

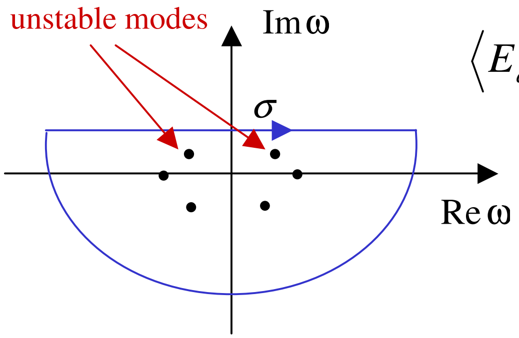
$$\gamma \equiv \mathrm{Im}\,\omega > 0$$

Fluctuations in unstable systems

Two-stream system

$$n(\mathbf{p}) = (2\pi)^3 n\left[\delta^{(3)}(\mathbf{p}-\mathbf{q}) + \delta^{(3)}(\mathbf{p}+\mathbf{q})\right]$$

Longitudinal electric field: $\omega_+(\mathbf{k})$ - stable mode, $\omega_-(\mathbf{k})$ - unstable mode



$$\left\langle E_a^i(\omega,\mathbf{k}) E_b^i(\omega',\mathbf{k}')\right\rangle = \frac{g^2}{2}\delta^{ab}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\frac{\mathbf{k}\cdot\mathbf{k}'}{\mathbf{k}^2\mathbf{k}'^2} \times \frac{1}{\varepsilon_L(\omega,\mathbf{k})}\frac{1}{\varepsilon_L(\omega',\mathbf{k}')}\int\frac{d^3p}{(2\pi)^3}\frac{n(\mathbf{p})}{(\omega-\mathbf{v}\cdot\mathbf{k})(\omega'-\mathbf{v}'\cdot\mathbf{k}')}$$

broken time translational invariance

$$\left\langle E_a^i(t,\mathbf{r}) E_b^i(t',\mathbf{r}')\right\rangle_{\text{unstable}} = \frac{g^2}{2}\delta^{ab} n\int\frac{d^3k}{(2\pi)^3}\frac{e^{-i\mathbf{k}(\mathbf{r}-\mathbf{r}')}}{\mathbf{k}^2}\frac{1}{\left(\omega_+^2-\omega_-^2\right)^2}\frac{\left(\gamma_\mathbf{k}^2+(\mathbf{k}\mathbf{u})^2\right)^2}{\gamma_\mathbf{k}^2} \times\left[\left(\gamma_\mathbf{k}^2+(\mathbf{k}\mathbf{u})^2\right)\cosh\left(\gamma_\mathbf{k}(t+t')\right)+\left(\gamma_\mathbf{k}^2-(\mathbf{k}\mathbf{u})^2\right)\cosh\left(\gamma_\mathbf{k}(t-t')\right)\right]$$

$\mathbf{u}\equiv\frac{\mathbf{q}}{E_\mathbf{q}}, \quad \gamma_\mathbf{k}\equiv \mathrm{Im}\,\omega_-(\mathbf{k})$

Gauge dependence

Generic correlation function: $L_{ab}(x,x') \equiv \langle H_a(x) K_b(x') \rangle$

Infinitesimal gauge transformation

$$H_a(x) \to H_a(x) + f_{abc}\lambda_b(x) H_c(x)$$

$$L_{ab}(x,x') \to L_{ab}(x,x') + f_{acd}\lambda_c(x) L_{db}(x,x') + f_{bcd}\lambda_c(x') L_{ad}(x,x')$$

colorless background

Actual correlation function: $L_{ab}(x,x') \equiv \delta^{ab} L(x,x')$

$$L_{ab}(x,x') \to \left(\delta^{ab} + f_{acb}\lambda_c(x) + f_{bca}\lambda_c(x')\right) L(x,x')$$

$L_{aa}(x,x') = \left(N_c^2 - 1\right) L(x,x')$ **- gauge invariant!**

Balescu-Lenard collision term for isotropic plasma

quite some work

$$\frac{g}{N_c}\mathrm{Tr}\left\langle \mathbf{E}(t,\mathbf{r})\cdot\nabla_p\delta Q(t,\mathbf{r},\mathbf{p})\right\rangle = \ldots\ldots\ldots = \nabla_p\mathbf{S}[n,\bar{n},n_g]$$

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)n(t,\mathbf{r},\mathbf{p}) = \nabla_p\mathbf{S}[n,\bar{n},n_g]$$

$$S^i[n,\bar{n},n_g] = \int\frac{d^3p'}{(2\pi)^3}B^{ij}(\mathbf{v},\mathbf{v}')[f(\mathbf{p}')\nabla_p^j n(\mathbf{p}) - n(\mathbf{p})\nabla_{p'}^j f(\mathbf{p}')]$$

$$f(\mathbf{p}) \equiv n(\mathbf{p}) + \bar{n}(\mathbf{p}) + 2N_c n_g(\mathbf{p})$$

$$B^{ij}(\mathbf{v},\mathbf{v}') = \frac{g^4}{8}\frac{N_c^2-1}{N_c}\int\frac{d^3k}{(2\pi)^3}\frac{k^ik^j}{\mathbf{k}^4}\frac{2\pi\delta(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}'))}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})|^2}$$

Landau collision term $(\varepsilon_L = 1)$

$$B^{ij}(\mathbf{v},\mathbf{v}') \approx \frac{g^4\ln(1/g)}{8}\frac{N_c^2-1}{N_c}\frac{1}{|\mathbf{v}-\mathbf{v}'|}\left(\delta^{ij} - \frac{(v^i-v'^i)(v^j-v'^j)}{(\mathbf{v}-\mathbf{v}')^2}\right)$$

Fokker-Planck collision term for isotropic plasma

$$\frac{g}{N_c}\mathrm{Tr}\left\langle \mathbf{E}(t,\mathbf{r})\cdot\nabla_p \delta Q(t,\mathbf{r},\mathbf{p})\right\rangle = [\nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j + \nabla_p^i Y^i(\mathbf{v})]n(\mathbf{p})$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla - \nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j - \nabla_p^i Y^i(\mathbf{v})\right)n(t,\mathbf{r},\mathbf{p}) = 0$$

$$X^{ij}(\mathbf{v}) \equiv \frac{g^4}{8}(N_c^2-1)\int\frac{d^3p'}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{k^i k^j}{\mathbf{k}^4}\frac{2\pi\delta(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}'))}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})|^2}f(\mathbf{p}')$$

$$Y^i(\mathbf{v}) \equiv \frac{g^4}{8}(N_c^2-1)\int\frac{d^3p'}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{k^i}{\mathbf{k}^4}\frac{2\pi\delta(\mathbf{k}\cdot(\mathbf{v}-\mathbf{v}'))}{|\varepsilon_L(\mathbf{k}\cdot\mathbf{v},\mathbf{k})|^2}\mathbf{k}\cdot\nabla_{p'}f(\mathbf{p}')$$

Fokker-Planck equation for two-stream system

Two-stream system $f(\mathbf{p}) = (2\pi)^3 \rho \left[\delta^{(3)}(\mathbf{p}-\mathbf{q}) + \delta^{(3)}(\mathbf{p}+\mathbf{q})\right]$

Longitudinal electric field: $\omega_+(\mathbf{k})$ - stable mode, $\omega_-(\mathbf{k}) = i\gamma_{\mathbf{k}}$ - unstable mode

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla - \nabla_p^i X^{ij}(\mathbf{v})\nabla_p^j - \nabla_p^i Y^i(\mathbf{v})\right) n(t,\mathbf{r},\mathbf{p}) = 0$$

$$X^{ij}(\mathbf{v}) \equiv \frac{g^4}{4}\frac{N_c^2-1}{N_c}\rho\int\frac{d^3k}{(2\pi)^3}\frac{k^ik^j}{\mathbf{k}^4}\frac{(\gamma_{\mathbf{k}}^2+(\mathbf{k}\cdot\mathbf{u})^2)^3}{(\omega_+^2+\gamma_{\mathbf{k}}^2)^2\gamma_{\mathbf{k}}(\gamma_{\mathbf{k}}^2+(\mathbf{k}\cdot\mathbf{v})^2)} \sinh(2\gamma_{\mathbf{k}}t)$$

$$Y^i(\mathbf{v}) = 0$$

$$\mathbf{u} \equiv \frac{\mathbf{q}}{E_{\mathbf{q}}}$$

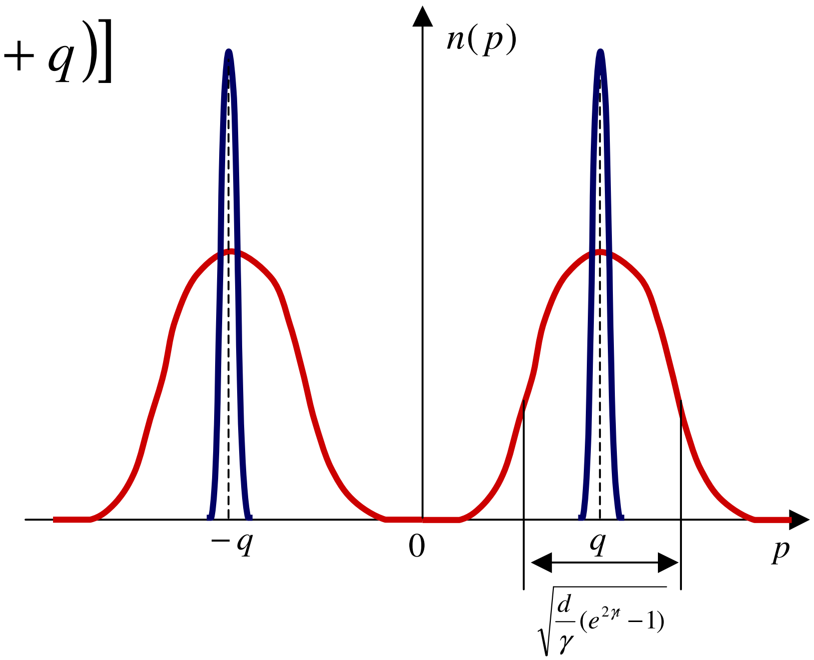
Evolution of the two-stream system

1D problem

$$n_0(p) = (2\pi)^3 \rho\left[\delta(p-q) + \delta(p+q)\right]$$

$$\frac{\partial n(t,p)}{\partial t} = D(t)\frac{\partial^2 n(t,p)}{\partial p^2}$$

$$D(t) = d\,e^{2\gamma t}$$



$$n(t,p) = \rho\sqrt{\frac{2\pi\gamma}{d(e^{2\gamma t}-1)}}\left\{\exp\left[-\frac{\gamma(p-q)^2}{2d(e^{2\gamma t}-1)}\right] + \exp\left[-\frac{\gamma(p+q)^2}{2d(e^{2\gamma t}-1)}\right]\right\}$$

Conclusions

- Aanalytic approach to equilibration of QGP is developed
- Two-stream plasma system is discussed as an example
- Important role of unstable modes is demonstrated