Quark-Gluon Plasma @ LHC

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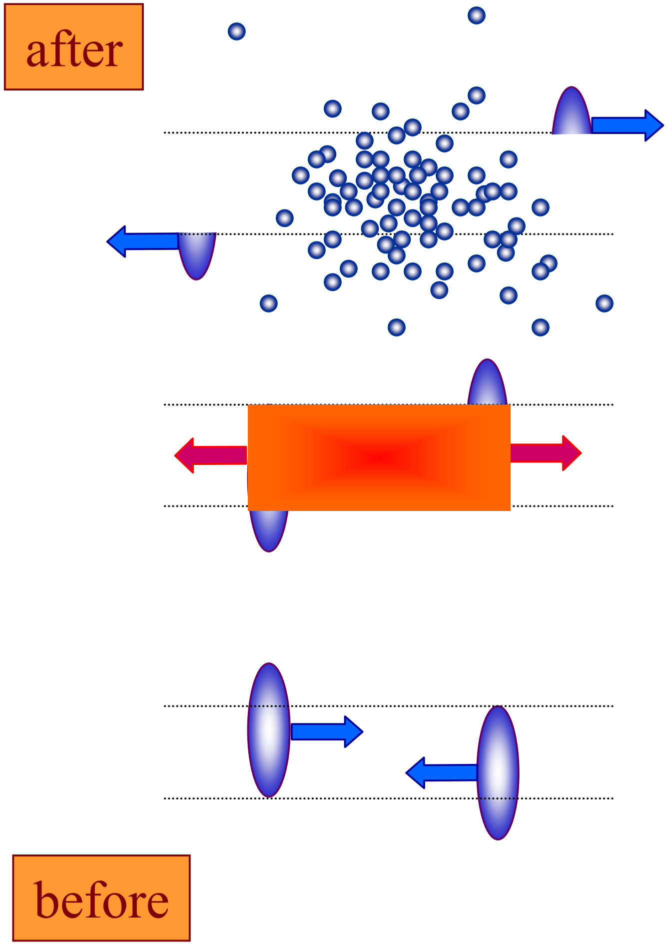
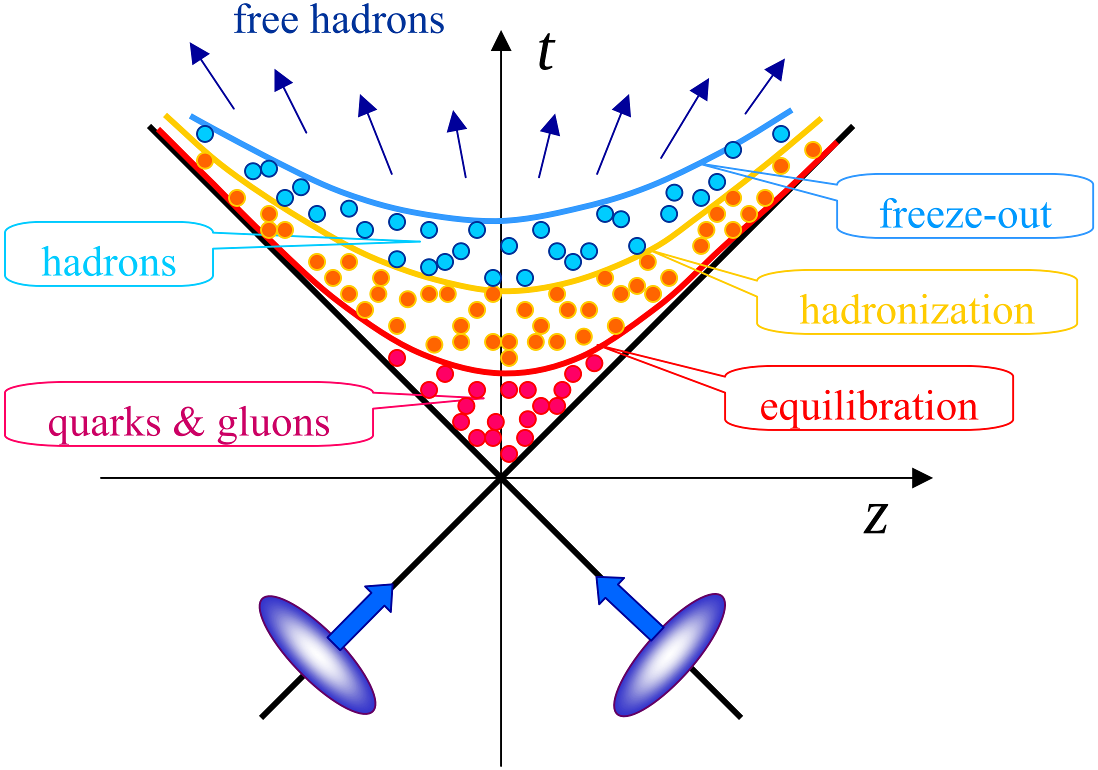
$$\text{QGP @ LHC} \left\{ \begin{array}{l} \bullet\ \text{weakly coupled} \\ \bullet\ \text{strongly collective} \\ \bullet\ \text{unstable} \\ \bullet\ \text{equilibrates fast} \end{array} \right.$$

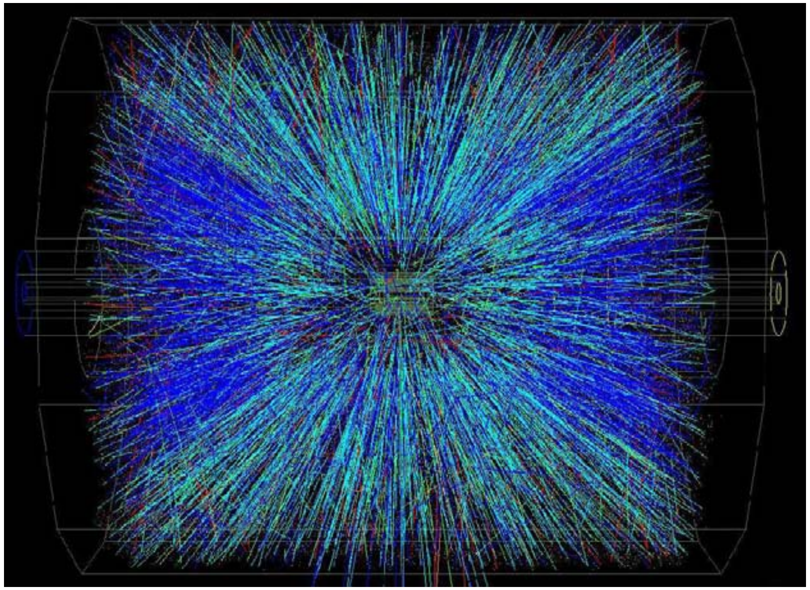
Relativistic heavy-ion collision – *Little Bang*

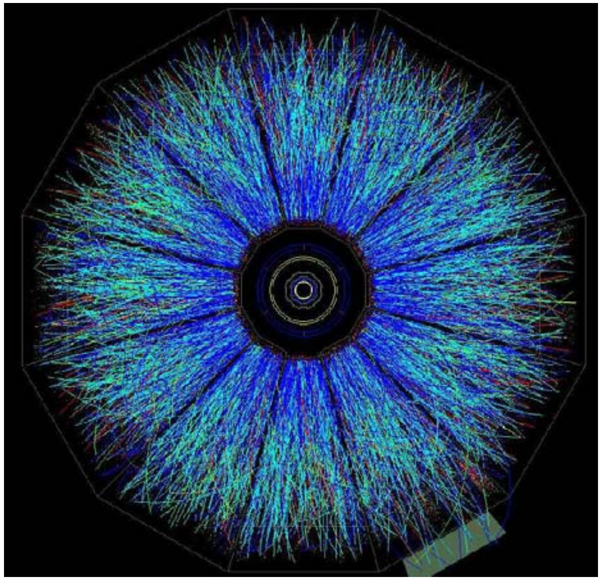




Au-Au collisions @ RHIC

$$\sqrt{s} = 100 + 100\ \text{GeV/NN}$$





STAR experiment

RHIC vs. LHC

$$\sqrt{s} = \begin{cases} 200\,\mathrm{GeV/NN} - \mathrm{RHIC} \\ 5500\,\mathrm{GeV/NN} - \mathrm{LHC} \end{cases}$$

$$\frac{\sqrt{s}_{\mathrm{LHC}}}{\sqrt{s}_{\mathrm{RHIC}}} \approx 30$$

Initial temperature

$$T_i \approx \begin{cases} 300\,\mathrm{MeV} - \mathrm{RHIC} \\ 700\,\mathrm{GeV} - \mathrm{LHC} \end{cases}$$

$$(30)^{1/4} \approx 2.3$$

RHIC vs. LHC cont.

Asymptotic freedom

$$\alpha_s = \frac{12\pi}{(33 - 2N_f)\ln\left(\frac{Q^2}{\Lambda^2_{\mathrm{QCD}}}\right)}$$

$$Q^2 \to \# T^2$$

$$\Lambda_{\mathrm{QCD}} \approx 200\ \mathrm{MeV}$$

RHIC

$$T \approx 300\ \mathrm{MeV}$$

$$\alpha_s \approx 0.3$$

LHC

$$T \approx 700\ \mathrm{MeV}$$

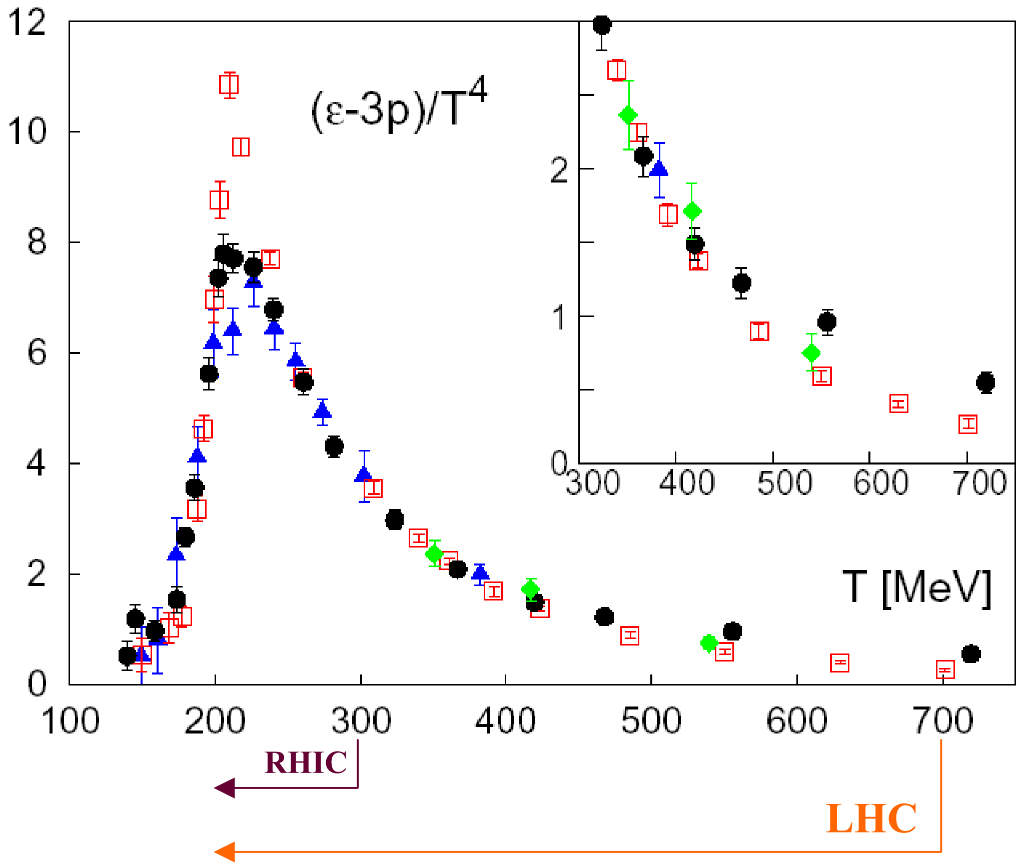
$$\alpha_s \approx 0.2$$

RHIC vs. LHC cont.

Lattice thermodynamics of Quark-Gluon Plasma

Ideal EoS

$$p = \frac{1}{3}\varepsilon$$



F. Karsch, arXiv:0711.0656 [hep-lat].

Mean fields vs. collisions

Transport equation

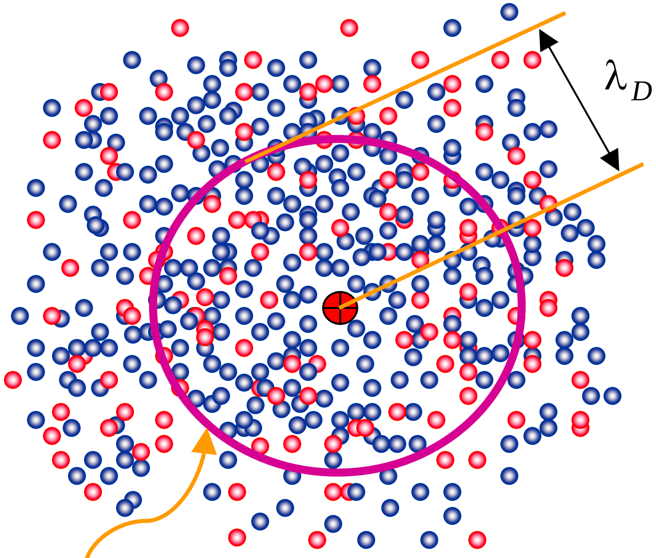
$$\underbrace{\left(\frac{\partial}{\partial t}+\mathbf{v}\nabla\right)f(t,\mathbf{r},\mathbf{p})}_{\text{free streaming}}+\underbrace{g\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)\nabla_p f(t,\mathbf{r},\mathbf{p})}_{\text{mean field }\sim\alpha_s}=\overbrace{C\left[f\right]}^{\text{collisions }\sim\alpha_s^2}$$

$$\alpha_s=\frac{g^2}{4\pi}$$

In weakly coupled plasmas the mean field dominates the dynamics

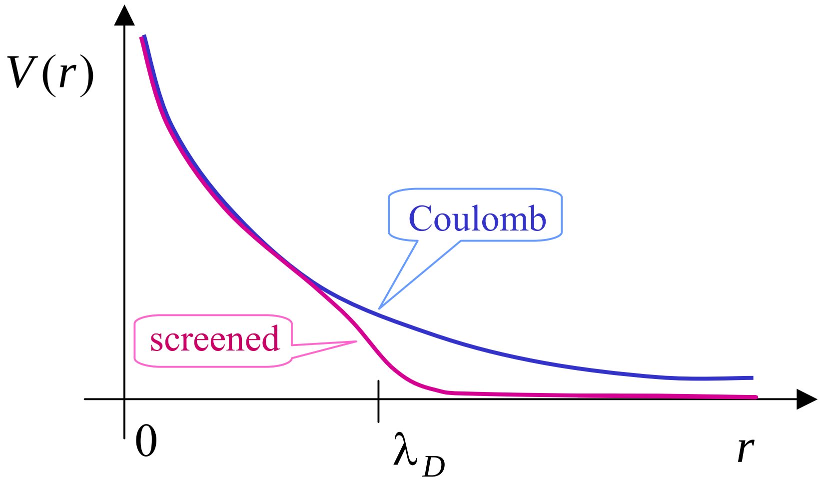
Weakly coupled plasmas are strongly collective

Plasma's collective behavior



Debye sphere

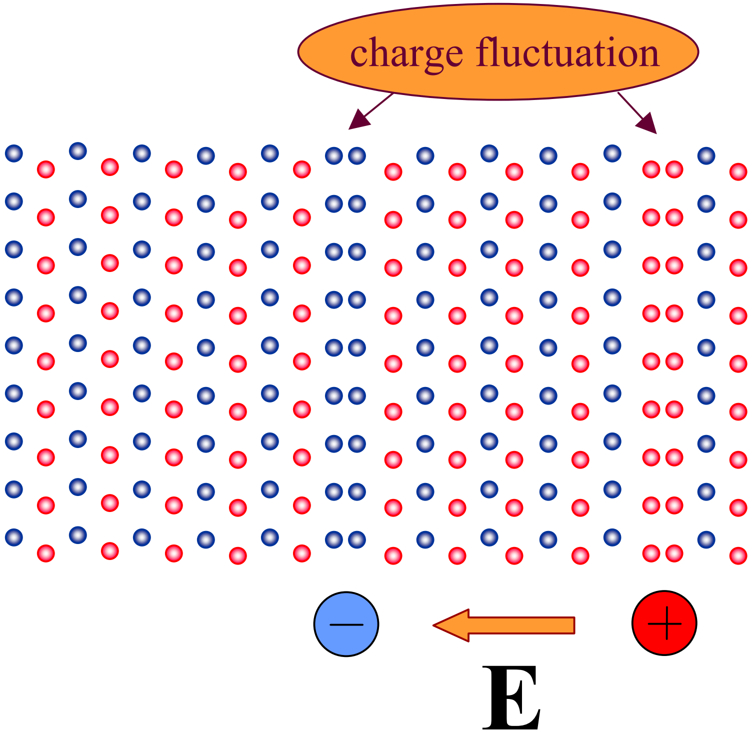
$$\lambda_D = \frac{1}{m_D} \sim \frac{1}{gT}$$ screening length $$V(r) \sim \frac{e^{-\frac{r}{\lambda_D}}}{r}$$



$$V_D = \frac{4}{3}\pi\lambda_D^3 \sim \frac{1}{g^3T^3}, \qquad n \sim T^3, \qquad \boxed{n\,V_D \sim \frac{1}{g^3} >> 1} \quad \text{if} \quad g << 1$$

In a weakly coupled plasma, there are many particles in a Debye sphere!

Plasma oscillations


charge fluctuation

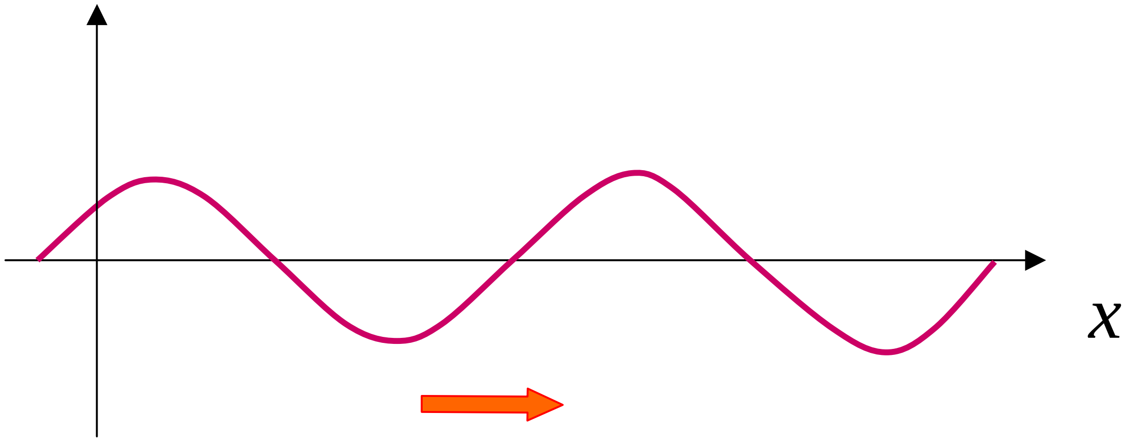
$$\mathbf{E}(t,\mathbf{r}) = \mathbf{E}_0 \cos(\omega(\mathbf{k})\,t - \mathbf{k}\cdot\mathbf{r} + \varphi)$$

$$\underset{\mathbf{k}\to 0}{\omega(\mathbf{k}) \approx} \omega_0 \sim gT$$

plasma frequency

Landau damping

$$E^x(t,x) = E_{\mathbf{0}} \cos(\omega_0 t - kx)$$



$$v_\varphi = \frac{\omega_0}{k}$$

Resonance energy transfer from electric field to particles with v = v_φ

Instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

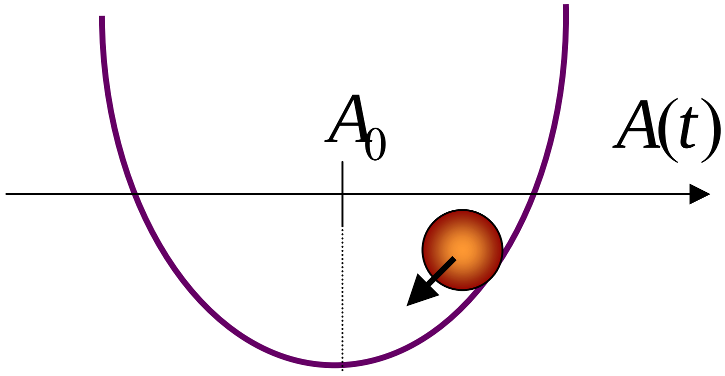
fluctuation

Instability

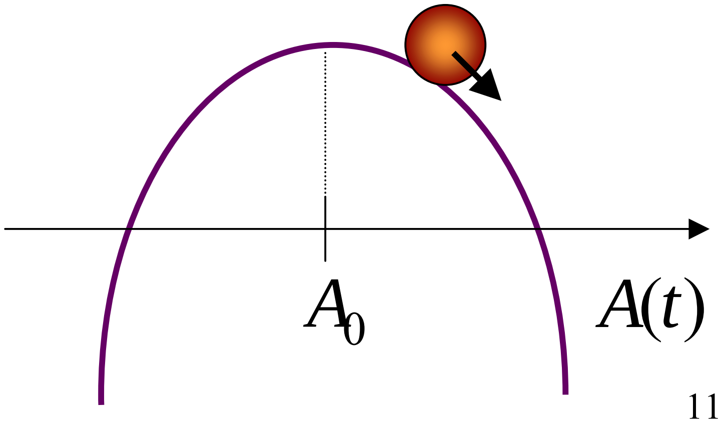
$$\delta A(t) \propto \mathrm{e}^{\gamma t}$$

$$\gamma > 0$$

stable configuration



unstable configuration



Terminology

Plasma instabilities – interplay of particles and classical fields

Quantum Field Theory – no particles, no classical fields

- particles – $\overbrace{\text{hard excitations, hard modes}}^{p_{\text{hard}} \sim T}$

- classical fields – $\underbrace{\text{highly populated}}_{\sim 1/g^2}\ \underbrace{\text{soft excitations, soft modes}}_{p_{\text{soft}} \sim gT}$

Plasma instabilities

- **instabilities in configuration space – hydrodynamic instabilities**

- **instabilities in momentum space – kinetic instabilities**

instabilities due to non-equilibrium momentum distribution

$$f(\mathbf{p}) \text{ is not } \sim \exp\left(-\frac{E}{T}\right)$$

Kinetic instabilities

- ► **longitudinal modes** – $\mathbf{k} \parallel \mathbf{E}\,,\ \ \delta\rho \sim \mathrm{e}^{-i(\omega t - \mathbf{kr})}$

- ► **transverse modes** – $\mathbf{k} \perp \mathbf{E}\,,\ \ \delta\mathbf{j} \sim \mathrm{e}^{-i(\omega t - \mathbf{kr})}$

E – electric field, **k** – wave vector, ρ – charge density, **j** - current

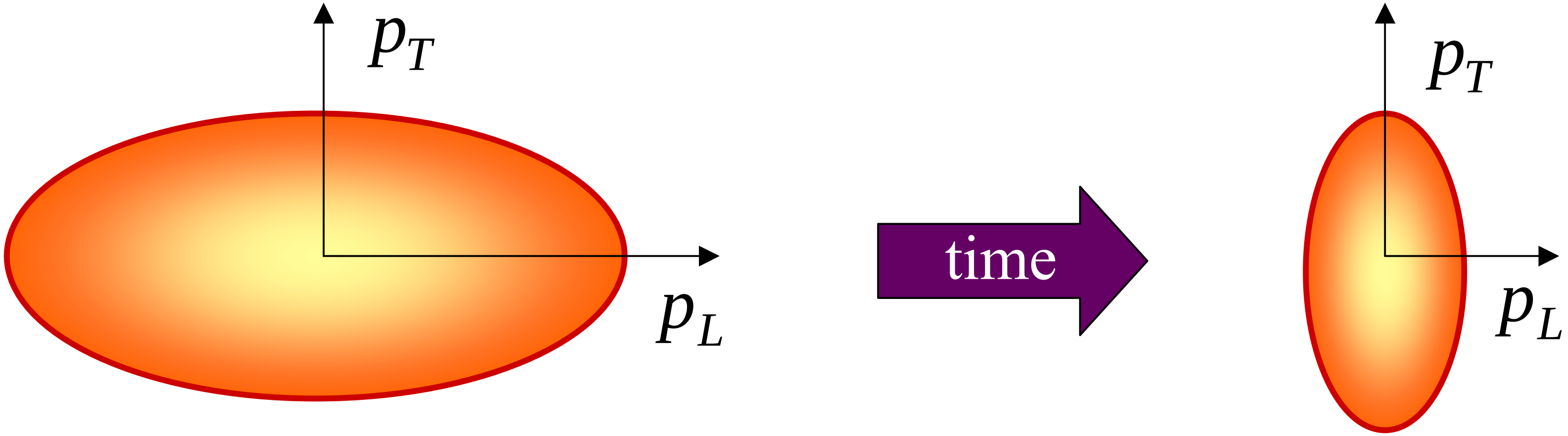
Transverse modes

Instabilities occur due to anisotropy of the momentum distribution

Transverse modes are relevant for relativistic nuclear collisions!

Momentum Space Anisotropy in Nuclear Collisions

Parton momentum distribution is initially strongly anisotropic



CM after 1-st collisions

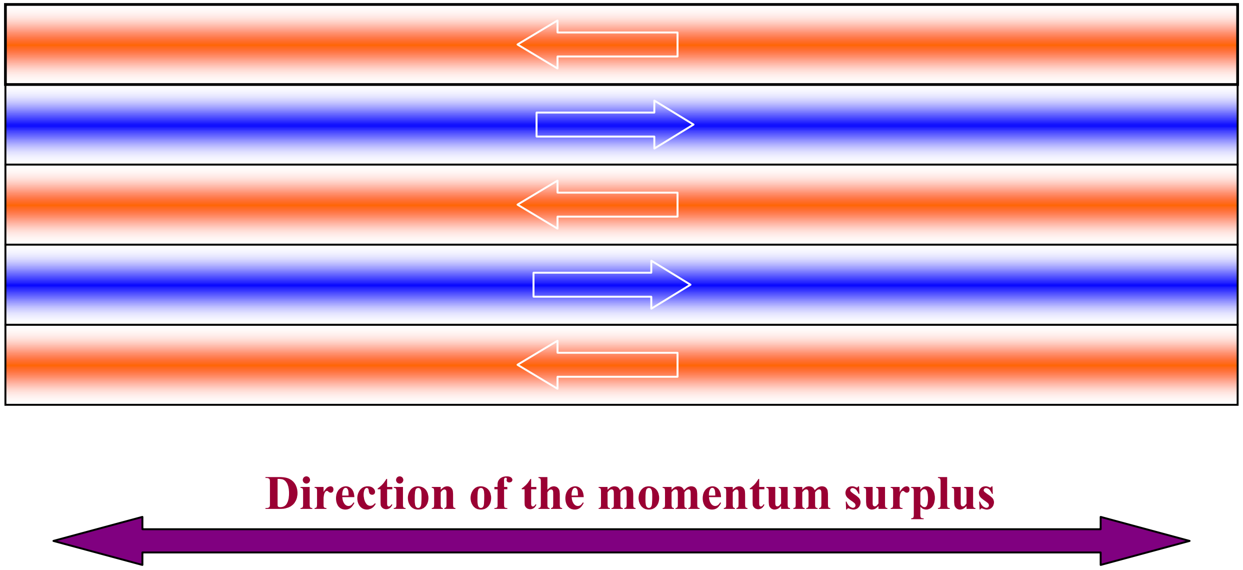
local rest frame

Seeds of instability

$\left\langle j_a^\mu(x) \right\rangle = 0$ **but current fluctuations are finite**

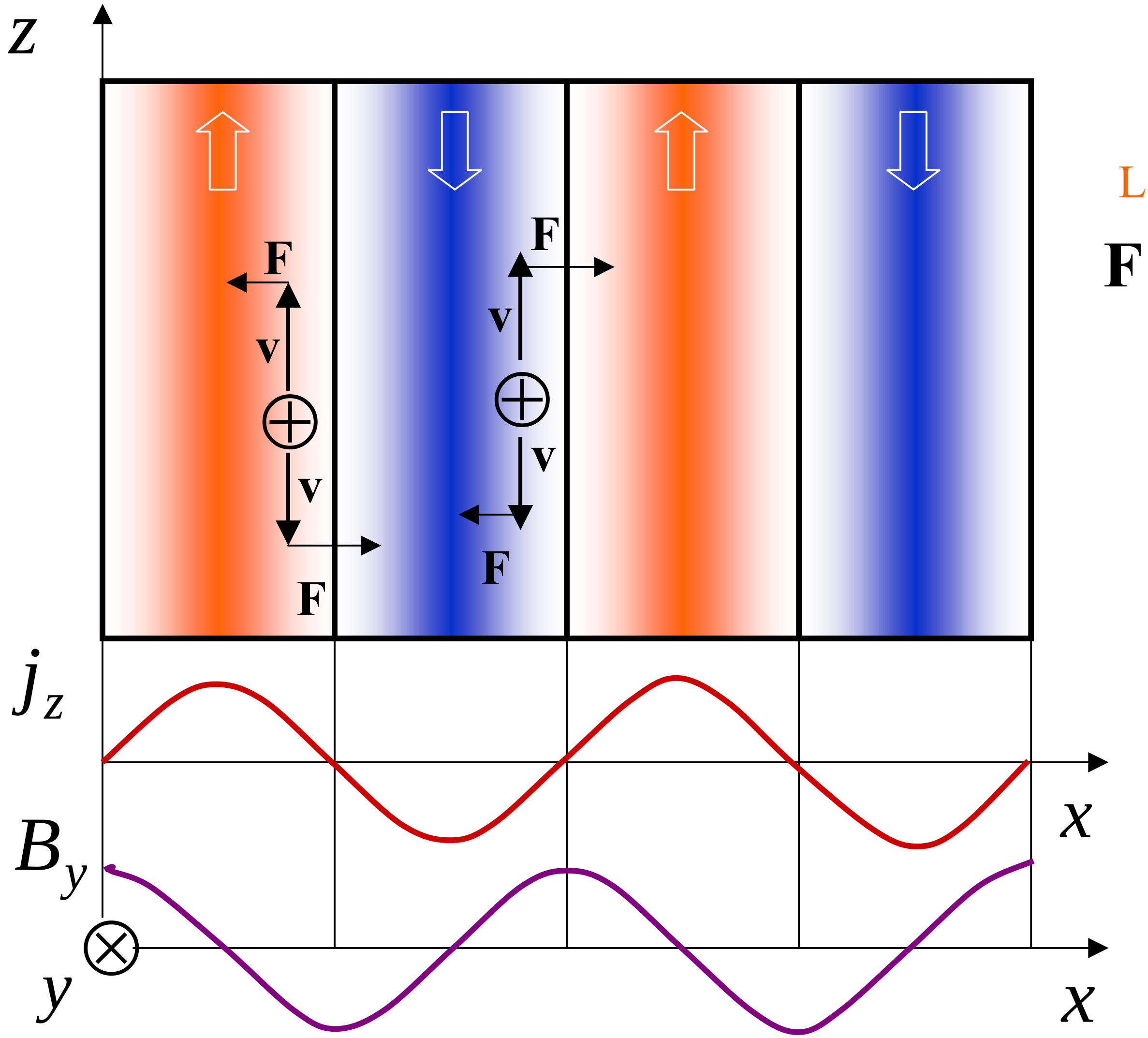
$$\boxed{\left\langle j_a^\mu(x_1)\, j_b^\nu(x_2) \right\rangle = \frac{1}{8}\delta^{ab} \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E_p^2} f(\mathbf{p}) \delta^{(3)}(\mathbf{x} - \mathbf{v}t) \neq 0}$$

$$x_1 = (t_1, \mathbf{x}_1)\,, \quad x_2 = (t_2, \mathbf{x}_2)\,, \quad x = (t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2)$$



Direction of the momentum surplus

Mechanism of filamentation



Lorentz force

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

Ampere's law

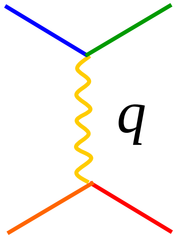
$$\nabla \times \mathbf{B} = \mathbf{j}$$

Instabilities vs. collisions

Time scale of collisional processes

$$t_{\text{hard}} \sim \frac{1}{g^4 \ln(1/g)\,T}$$

$$t_{\text{soft}} \sim \frac{1}{g^2 \ln(1/g)\,T}$$



hard scattering: $q \sim T$

soft scattering: $q \sim gT$

Time scale of collective phenomena

$$t_{\text{collec}} \sim \frac{1}{g\,T}$$

$$g^2 \ll 1 \quad \Rightarrow \quad t_{\text{hard}} \gg t_{\text{soft}} \gg t_{\text{collec}}$$

The instabilities are fast!

Dispersion equation

Equation of motion of chromodynamic field *A*$^\mu$ in momentum space

$$[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] A_\nu(k) = 0$$

gluon self-energy

Dispersion equation

$$\boxed{\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0}$$

$$k^\mu \equiv (\omega, \mathbf{k})$$

Instabilities – solutions with Im$\omega > 0$ $\Rightarrow$ $A^\mu(x) \sim \mathrm{e}^{\mathrm{Im}\,\omega t}$

Dynamical information is hidden in $\Pi^{\mu\nu}(k)$. **How to get it?**

Transport theory – transport equations

$$\text{fundamental}\ \left\{ \begin{array}{ll} p_\mu D^\mu Q - \frac{g}{2}\, p^\mu \{F_{\mu\nu}(x), \partial_p^\nu Q\} = C & \text{quarks} \\ p_\mu D^\mu \overline{Q} + \frac{g}{2}\, p^\mu \{F_{\mu\nu}(x), \partial_p^\nu \overline{Q}\} = \overline{C} & \text{antiquarks} \end{array} \right.$$

$$\text{adjoint}\quad p_\mu \mathcal{D}^\mu G - \frac{g}{2}\, p^\mu \{\mathcal{F}_{\mu\nu},(x)\partial_p^\nu G\} = C_g \quad \text{gluons}$$

free streaming

mean-field force

collisions

$$D^\mu \equiv \partial^\mu - ig[A^\mu, ...], \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$$

$$\boxed{D_\mu F^{\mu\nu} = j^\nu[Q, \overline{Q}, G]}$$

mean-field generation

collisionless limit: $C = \overline{C} = C_g = 0$

Transport theory - linearization

fluctuation

$$\boxed{Q(p,x) = Q_0(p) + \delta Q(p,x)}$$

stationary colorless state $Q_0^{ij}(p) = \delta^{ij} n(p)$

$$|Q_0(p)| >> |\delta Q(p,x)|, \quad |\partial_p^\mu Q_0(p)| >> |\partial_p^\mu \delta Q(p,x)|$$

Linearized transport equations

$$p_\mu D^\mu \delta Q(p,x) - g p^\mu F_{\mu\nu}(x) \partial_p^\nu Q_0(p) = 0$$

$$p_\mu D^\mu \delta \overline{Q}(p,x) + g p^\mu F_{\mu\nu}(x) \partial_p^\nu \overline{Q}_0(p) = 0$$

$$p_\mu \mathcal{D}^\mu \delta G(p,x) - g p^\mu \mathcal{F}_{\mu\nu}(x) \partial_p^\nu QG_0(p) = 0$$

Transport theory – polarization tensor

$$\delta Q(p,x) = g\int d^4x'\,\Delta_p(x-x')\,p^\mu F_{\mu\nu}(x)\partial_p^\nu Q_0(p)$$

$$p_\mu D^\mu \Delta_p(x) = \delta^{(4)}(x)$$

$$j^\mu[\delta Q, \delta\overline{Q}, \delta G]$$

$$j^\mu(k) = \Pi^{\mu\nu}(k)A_\nu(k)$$

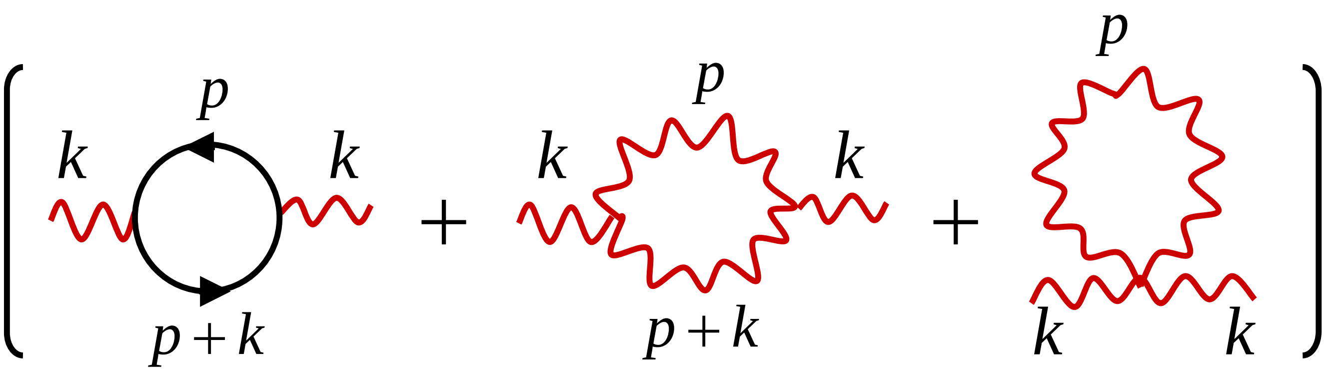
$$f(\mathbf{p}) \equiv n(\mathbf{p}) + \overline{n}(\mathbf{p}) + 2n_g(\mathbf{p})$$

$$\Pi^{\mu\nu}(k) = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E}\Big[g^{\mu\lambda} - \frac{p^\nu k^\lambda}{p^\sigma k_\sigma + i0^+}\Big]\frac{\partial f(\mathbf{p})}{\partial p^\lambda}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_\mu \Pi^{\mu\nu}(k) = 0$$

Diagrammatic Hard Loop approach

$$\Pi^{\mu\nu}(k) =$$



Hard loop approximation: $k^{\mu} << p^{\mu}$

$$\boxed{\Pi^{\mu\nu}(k) = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\,\frac{p^{\mu}}{E}\left[g^{\mu\lambda} - \frac{p^{\nu}k^{\lambda}}{p^{\sigma}k_{\sigma} + i0^{+}}\right]\frac{\partial f(\mathbf{p})}{\partial p^{\lambda}}}$$

$$\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k), \quad k_{\mu}\Pi^{\mu\nu}(k) = 0$$

St. M. & M. Thoma, Phys. Rev. C **62**, 036011 (2000)

Dispersion equation

Dispersion equation

$$\boxed{\det[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)] = 0}$$

$$k_\mu \Pi^{\mu\nu}(k) = 0$$

$$\varepsilon^{ij}(k) = \delta^{ij} - \frac{1}{\omega^2}\Pi^{ij}(k)$$

chromodielectric tensor

$$k^\mu \equiv (\omega, \mathbf{k})$$

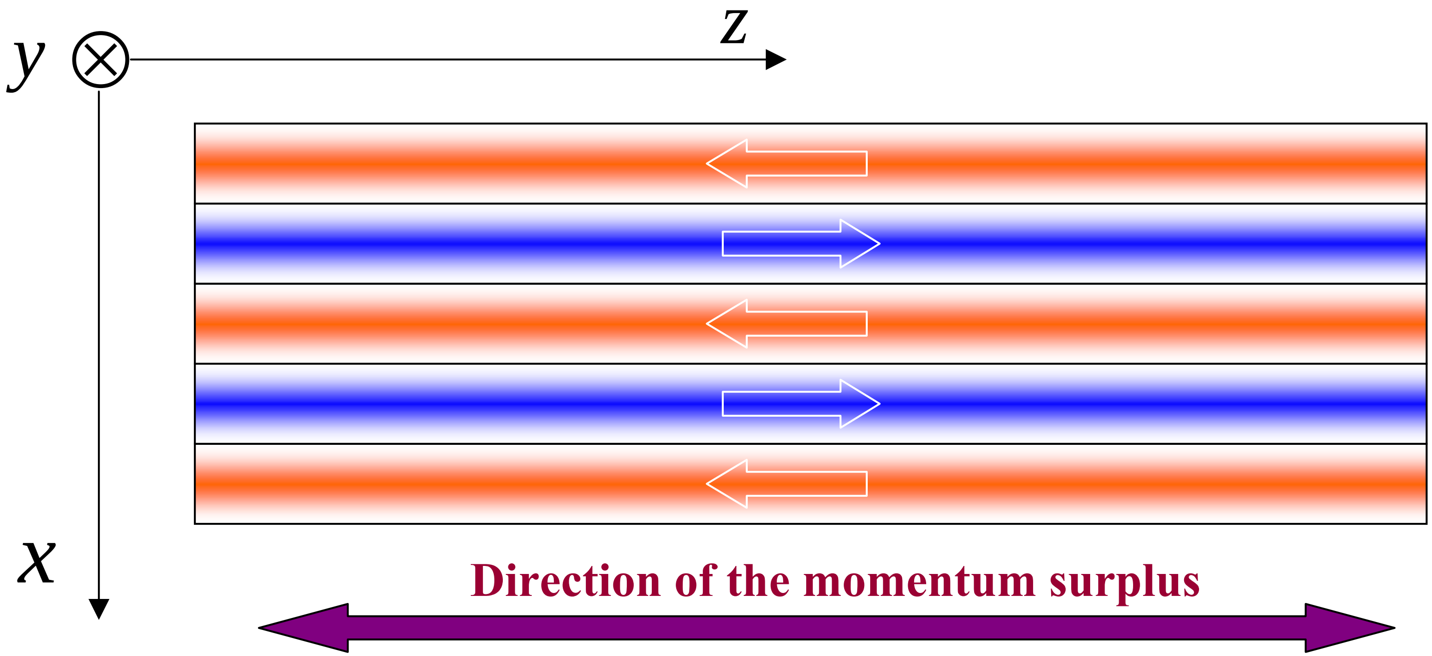
Dispersion equation

$$\boxed{\det[\mathbf{k}^2 \delta^{ij} - k^i k^j - \omega^2 \varepsilon^{ij}(k)] = 0}$$

$$\varepsilon^{ij}(k) = \delta^{ij} + \frac{g^2}{2\omega}\int \frac{d^3p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{kv} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^l}\left[\left(1 - \frac{\mathbf{kv}}{\omega}\right)\delta^{lj} + \frac{k^l v^j}{\omega}\right]$$

$$\mathbf{v} \equiv \mathbf{p}/E$$

Dispersion equation – configuration of interest



$$\mathbf{j} = (0,0,j), \quad \mathbf{E} = (0,0,E), \quad \mathbf{k} = (k,0,0)$$

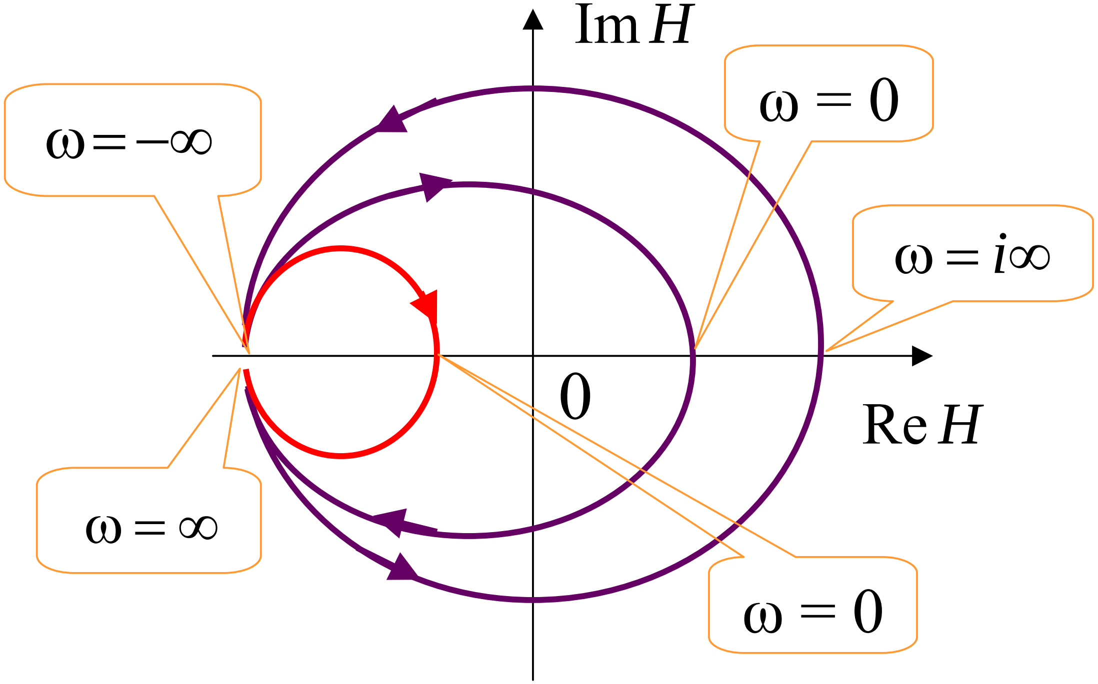
Dispersion equation

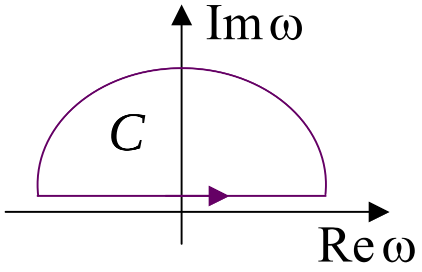
$$\boxed{k^2 - \omega^2 \varepsilon^{zz}(\omega, k) = 0}$$

Existence of unstable modes – Penrose criterion

$$\boxed{H(\omega) \equiv k^2 - \omega^2 \varepsilon^{zz}(\omega, k)}$$

$$\oint_C \frac{d\omega}{2\pi i}\frac{1}{H(\omega)}\frac{dH(\omega)}{d\omega} = \left\{ \begin{array}{l} \displaystyle\oint_C \frac{d\omega}{2\pi i}\frac{d\ln H(\omega)}{d\omega} = \ln H(\omega)\Big|_{\phi=\pi^+}^{\phi=\pi^-} \\ \text{number of zeros of } H(\omega) \text{ in } C \end{array} \right.$$





There are unstable modes if

$$\boxed{H(\omega = 0) < 0}$$

Anisotropy!

Unstable solutions

$$f(\mathbf{p}) = \frac{2^{1/2}}{\pi^{3/2}} \frac{\rho\sigma_\perp^4}{\sigma_\|} \frac{1}{(p_\perp^2+\sigma_\perp^2)^3} e^{-\frac{p_\|^2}{2\sigma_\|^2}}$$

$$\rho = 6 \text{ fm}^{-3}$$

$$\alpha_s = g^2/4\pi = 0.3$$

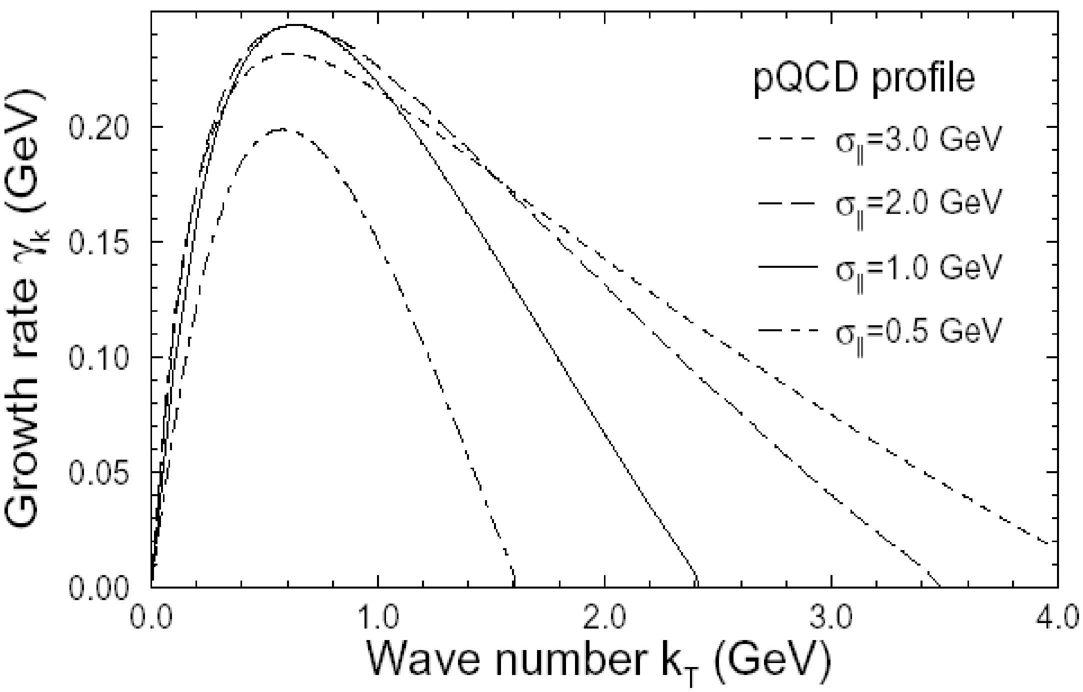
$$\sigma_\perp = 0.3 \text{ GeV}$$

$$k^2 - \omega^2\varepsilon^{zz}(\omega,k) = 0$$

solution

$$\omega(k) = \pm i\gamma_k$$

$$0 < \gamma_k \in \Re$$



J. Randrup & St. M., Phys. Rev. C **68**, 034909 (2003)

Hard-Loop dynamics

Soft fields in the passive background of hard particles

Braaten-Pisarski action generalized to anisotropic momentum distribution:

$$L_{\rm eff} = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\Big[f(\mathbf{p}) F^a_{\mu\nu}(x)\left(\frac{p^\nu p^\rho}{(p\cdot D)^2}\right)_{ab} F^{b\mu}_{\rho}(x)$$

$$+ i\,\frac{C_F}{3}\,\tilde{f}(\mathbf{p})\,\bar{\psi}(x)\,\frac{p\cdot\gamma}{p\cdot D}\,\psi(x)\Big]$$

$$k_\mu \Pi^{\mu\nu}(k) = 0, \qquad k_\mu \Lambda^\mu(p,q,k) = \Sigma(p) + \Sigma(q)$$

St. M., A. Rebhan & M. Strickland, Phys. Rev. D **74**, 025004 (2004)

Growth of instabilities – 1+1 numerical simulations

SU(2) Hard Loop Dynamics

1+1 dimensions

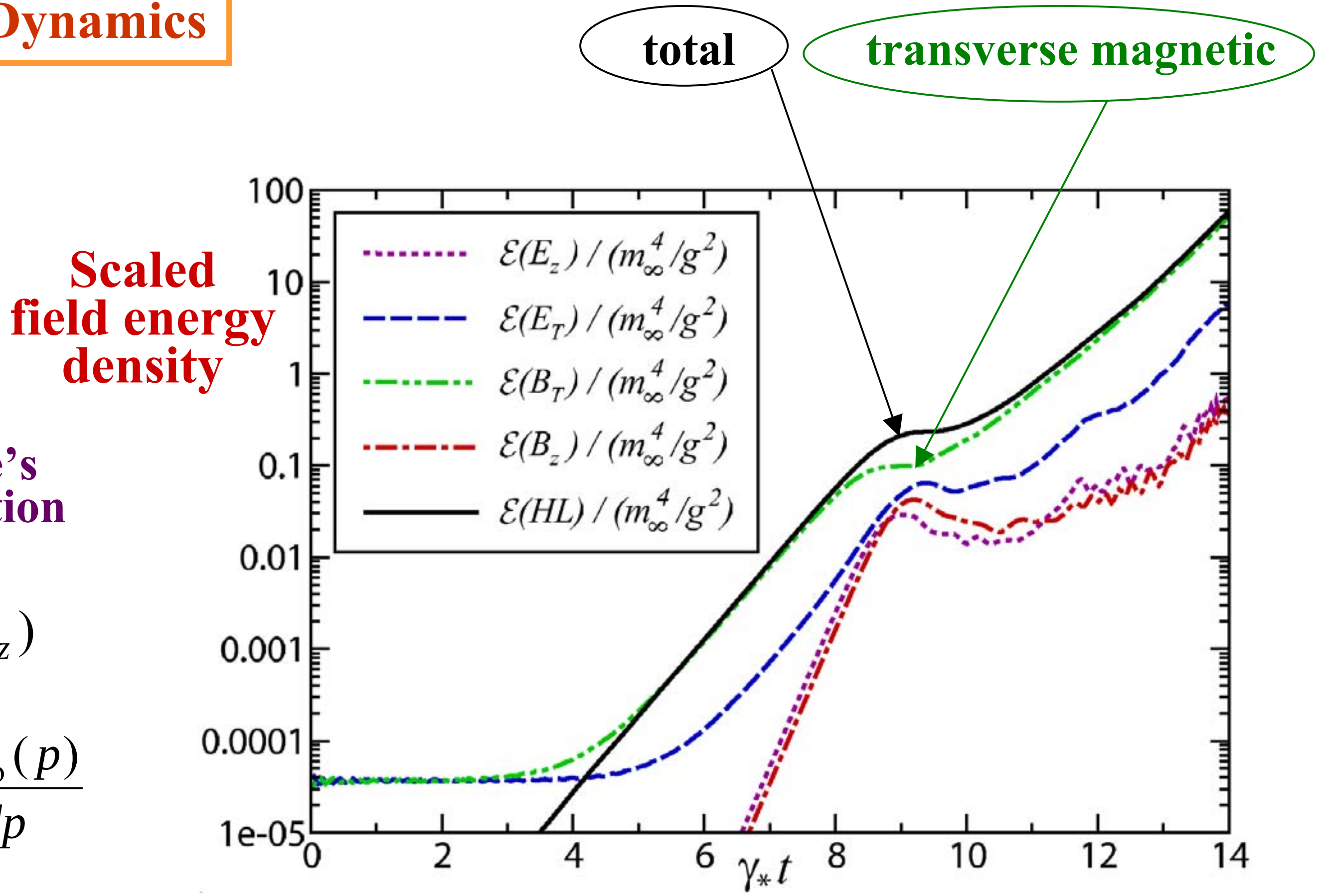
$$A_a^\mu = A_a^\mu(t,z)$$

Anisotropic particle's momentum distribution

$$f(\mathbf{p}) = f_{\text{iso}}(|\mathbf{p}| + \zeta p_z)$$

$$m_D^2 = -\frac{\alpha_s}{\pi}\int_0^\infty dp\,p^2 \frac{df_{\text{iso}}(p)}{dp}$$

$$(m_D, \zeta)$$

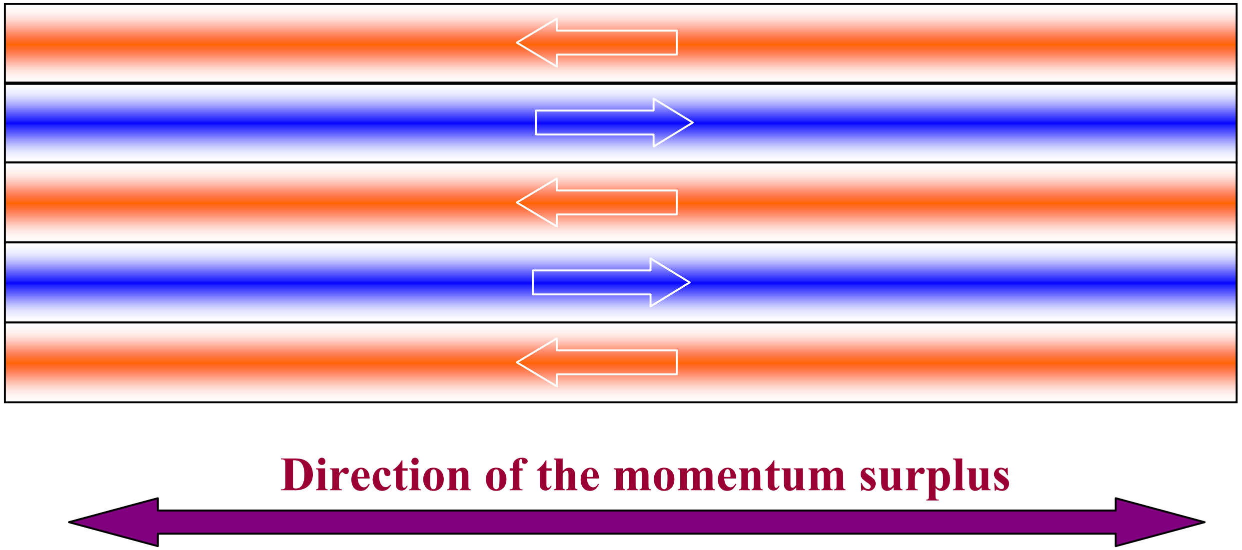


Strong anisotropy $\zeta = 10$

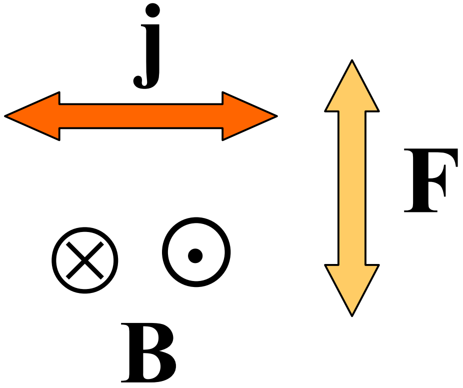
γ_* - maximal growth rate

A. Rebhan, P. Romatschke & M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005)

Isotropization - particles

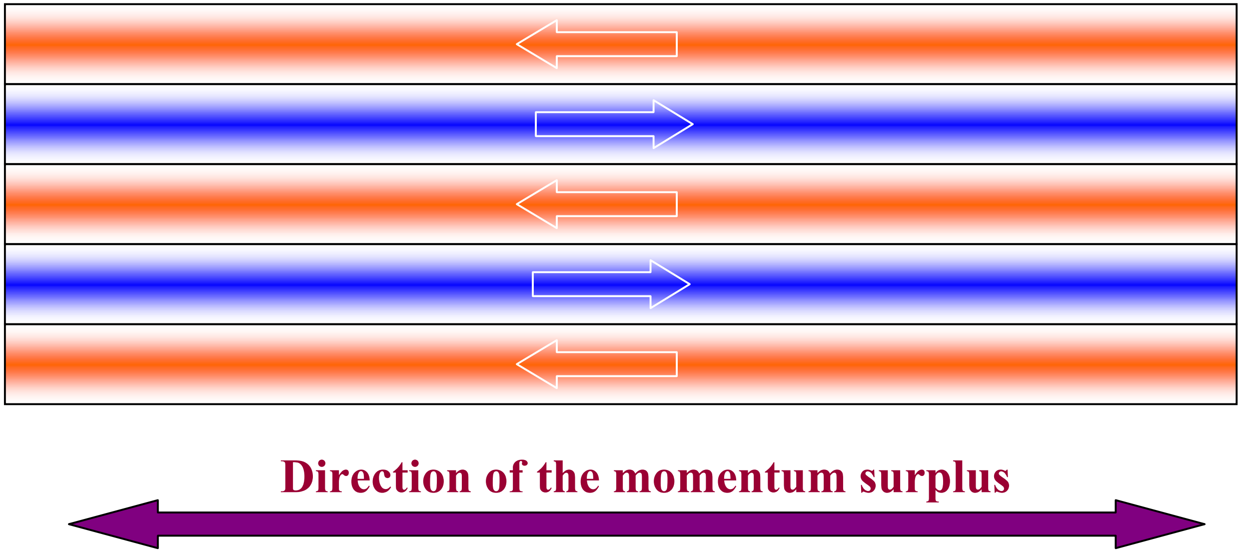


Direction of the momentum surplus

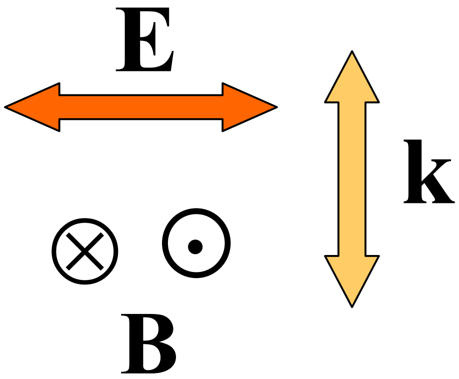


$$\Delta \mathbf{p} = \int dt\, \mathbf{F}$$

Isotropization - fields



Direction of the momentum surplus



$$\boxed{\mathbf{P}_{\text{fields}} \sim \mathbf{B}^{a} \times \mathbf{E}^{a} \sim \mathbf{k}}$$

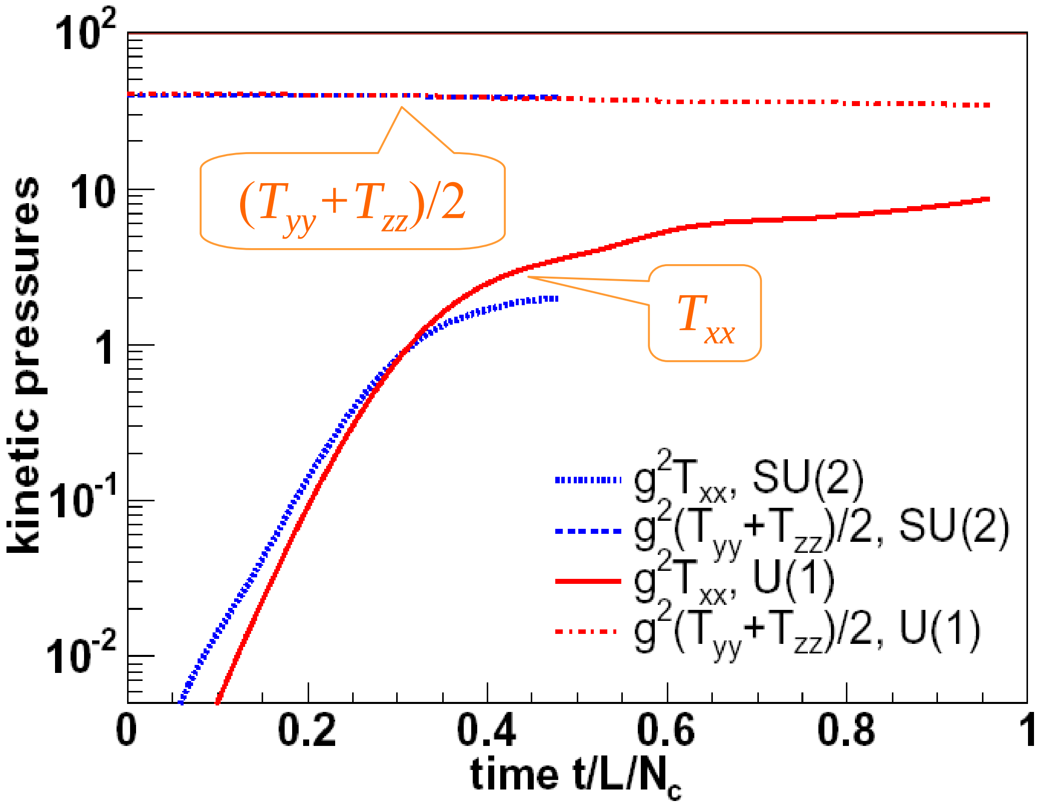
Isotropization – numerical simulation

Classical system of colored particles & fields

$$T_{ij} = \int \frac{d^3p}{(2\pi)^3} \frac{p_i p_j}{E} f(\mathbf{p})$$

Isotropy:

$$T_{xx} = (T_{yy} + T_{zz})/2$$



$(T_{yy}+T_{zz})/2$

T_{xx}

A. Dumitru & Y. Nara, Phys. Lett. **B621**, 89 (2005).

Conclusions

QGP @ LHC

- weakly coupled
- strongly collective
- unstable
- equilibrates fast

QGP is not a gas of partons