Production of light nuclei in relativistic heavy-ion collisions

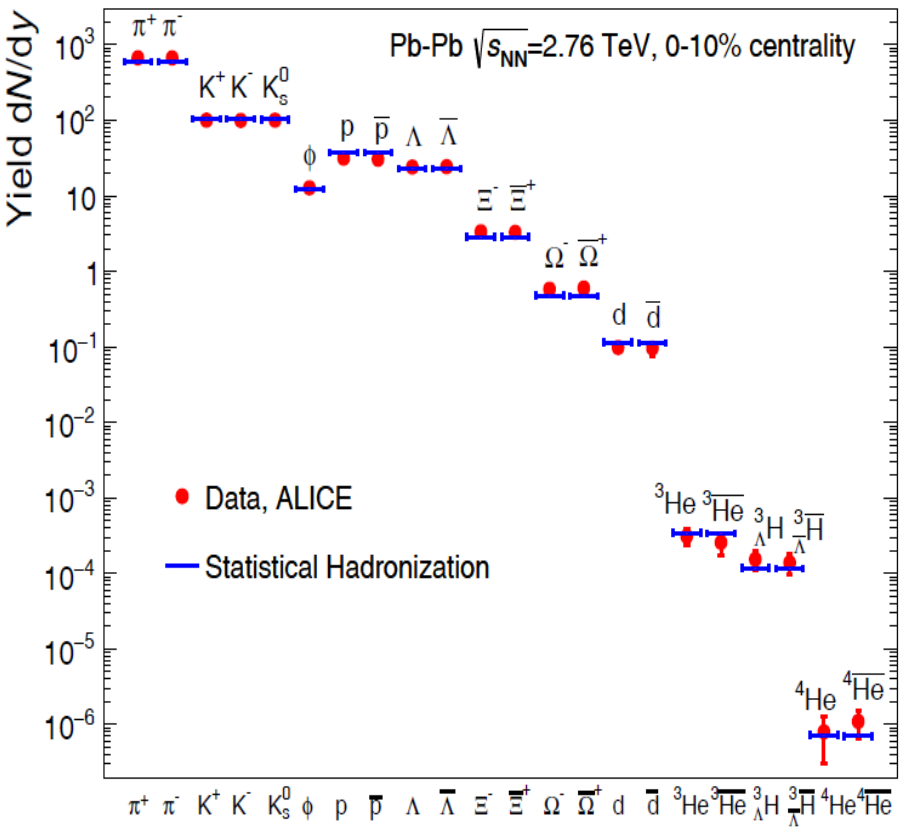
Stanisław Mrówczyński

Institute of Physics, Jan Kochanowski University, Kielce, Poland
and National Centre for Nuclear Research, Warsaw, Poland

Background

- Production of $^{2}\mathrm{H}$, $^{2}\overline{\mathrm{H}}$, $^{3}\mathrm{H}$, $^{3}\overline{\mathrm{H}}$, $^{3}\mathrm{He}$, $^{3}\overline{\mathrm{He}}$, $^{4}\mathrm{He}$, $^{4}\overline{\mathrm{He}}$, $^{3}_{\Lambda}\mathrm{H}$, $^{3}_{\overline{\Lambda}}\overline{\mathrm{H}}$ is observed in midrpidity at RHIC & LHC.

- Thermal model properly describes yields of light nuclei.



baryonless fireball

$$\text{Yield} \sim g\, e^{-\frac{m}{T}}$$

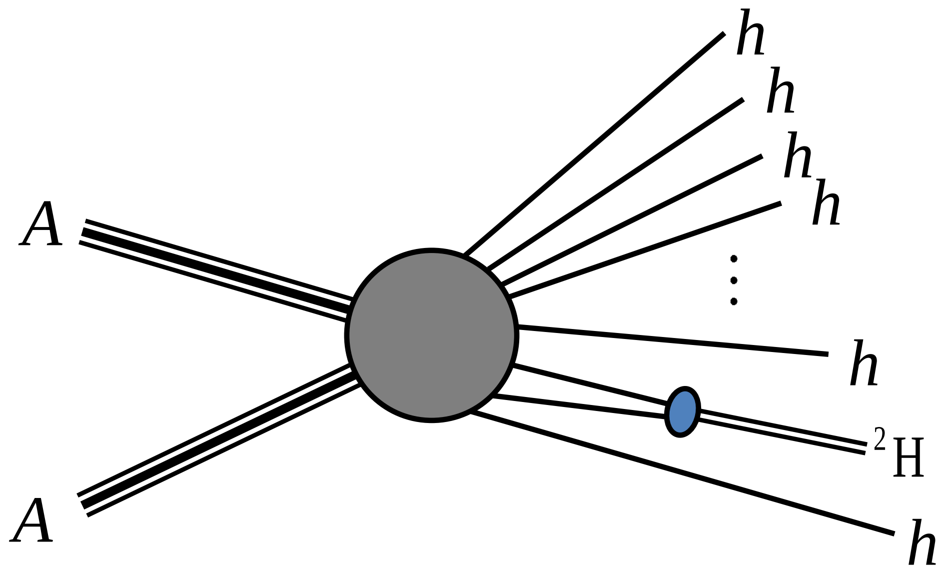
$$T = 156\ \text{MeV}$$

A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, arXiv:1710.09425 [nucl-th]

Can light nuclei exist in a fireball?

- Interparticle spacing in a hadron gas is about 1.5 fm at T = 156 MeV.
- Root mean square radius of a deuteron is 2.0 fm.
- Binding energy of a deuteron is 2.2 MeV.
- A hadron gas at T = 156 MeV is essentially a classical system.

Final state interaction – conventional approach to production of light nuclei



energy scale of particle production - particle's mass	>>	energy scale of formation of a nucleus - binding energy
fast process		slow process

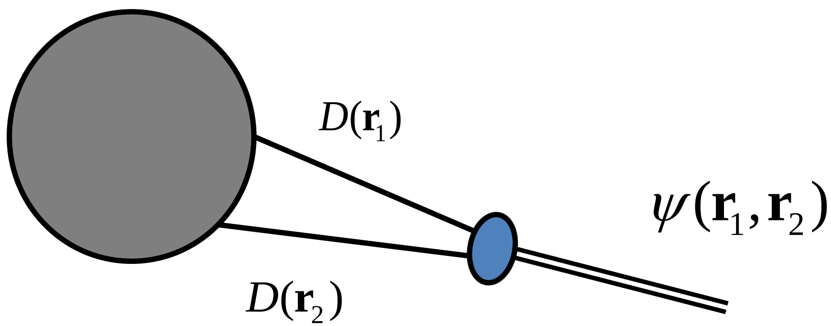
S.T. Butler & C.A. Pearson, Phys. Rev. **129**, 836 (1963)
A. Schwarzschild & C. Zupancic, Phys. Rev. **129**, 854 (1963)

Factorization of production of nucleons and formation of a nucleus

Deuteron production cross section

$$\frac{d\sigma^{D}}{d^{3}\mathbf{P}_{D}} = W\,\frac{d\sigma^{np}}{d^{3}\mathbf{p}_{n}d^{3}\mathbf{p}_{p}} \qquad \frac{1}{2}\mathbf{P}_{D} = \mathbf{p}_{n} = \mathbf{p}_{p}$$

source - fireball



Deuteron formation rate

spin factor

$$W = \frac{3}{4}(2\pi)^{3}\int d^{3}\mathbf{r}_{1}d^{3}\mathbf{r}_{2}\, D(\mathbf{r}_{1})D(\mathbf{r}_{2})\left|\psi(\mathbf{r}_{1},\mathbf{r}_{2})\right|^{2}$$

H. Sato and K. Yazaki, Phys. Lett. B **98** , 153 (1981)

Deuteron formation rate vs. n-p correlation

$$\psi(\mathbf{r}_1,\mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}}\varphi(\mathbf{r}) \qquad \mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2), \qquad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$\boxed{W = \frac{3}{4}(2\pi)^3 \int d^3\mathbf{r}\, D_r(\mathbf{r}) \left|\varphi(\mathbf{r})\right|^2}$$

$$D_r(\mathbf{r}) \equiv \int d^3\mathbf{R}\, D\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) D\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right)$$

n-p – correlation function

$$\boxed{C(\mathbf{q}) = \int d^3\mathbf{r}\, D_r(\mathbf{r}) \left|\varphi_{\mathbf{q}}(\mathbf{r})\right|^2}$$

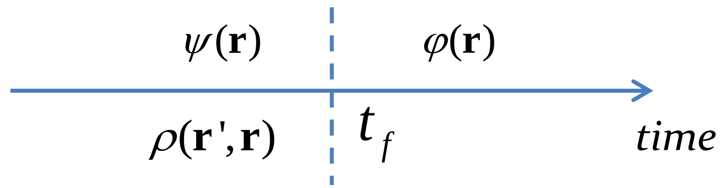
$\varphi(\mathbf{r})$ – wave function of a bound state

$\varphi_{\mathbf{q}}(\mathbf{r})$ – wave function of a scattering state

St. Mrówczyński, Phys. Lett. B **277**, 43 (1992)

Quantum-mechanical meaning of the formation rate formula

Sudden approximation



Transition matrix element

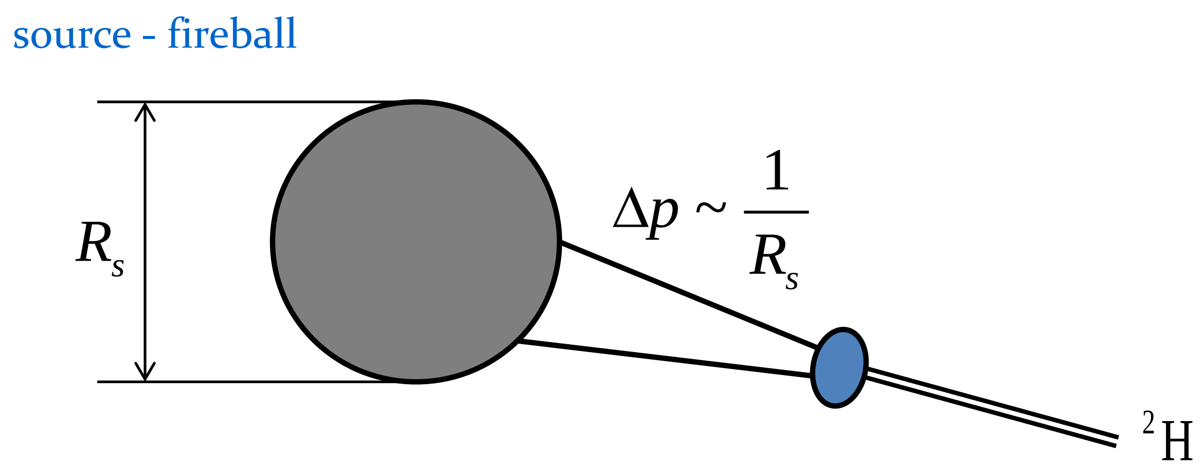
$$W = \left| \int d^3\mathbf{r}\, \psi^*(\mathbf{r})\, \varphi(\mathbf{r}) \right|^2 = \int d^3\mathbf{r}\, d^3\mathbf{r}'\, \varphi^*(\mathbf{r}')\, \underbrace{\psi(\mathbf{r}')\psi^*(\mathbf{r})}_{\rho(\mathbf{r}',\mathbf{r}) \ \text{density matrix}}\, \varphi(\mathbf{r})$$

$$W = \int d^3\mathbf{r}\, d^3\mathbf{r}'\, \varphi^*(\mathbf{r}')\, \rho(\mathbf{r}',\mathbf{r})\, \varphi(\mathbf{r})$$

If density matrix is diagonal (random phase approximation)

$$\rho(\mathbf{r}',\mathbf{r}) = D(\mathbf{r})\, \delta^{(3)}(\mathbf{r}'-\mathbf{r}) \qquad \Rightarrow \qquad \boxed{W = \int d^3\mathbf{r}\, D(\mathbf{r}) \left|\varphi(\mathbf{r})\right|^2}$$

Energy-momentum conservation



Nucleons are intermediate scattering states

$$E_i^2 - \mathbf{p}_i^2 \neq m_i^2 \quad i = n, p$$

Energy-momentum conservation

$$\begin{cases} \mathbf{p}_p + \mathbf{p}_n = \mathbf{p}_D \\ E_p + E_n = E_D \end{cases}$$

St. Mrówczyński, J. Phys. G **11**, 1087 (1987)

Yields of light nuclei

- Thermal model

$$\text{Yield} = g_A V \int \frac{d^3\mathbf{p}_A}{(2\pi)^3} \frac{1}{e^{\beta E_\mathbf{p}} \pm 1}$$

$$E_\mathbf{p} \equiv \sqrt{m_A^2 + \mathbf{p}_A^2}, \quad \beta \equiv \frac{1}{T}$$

- Coalescence model

$$\frac{dN^A}{d^3\mathbf{p}_A} = W_A \left(\frac{dN^N}{d^3\mathbf{p}_N} \right)^A \qquad \mathbf{p}_A = A\mathbf{p}_N$$

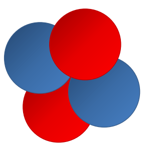
$$\text{Yield} = \int d^3\mathbf{p}_A \, \frac{dN^A}{d^3\mathbf{p}_A} = W_A \int d^3\mathbf{p}_A \left(\frac{dN^N}{d^3\mathbf{p}_N} \right)^A$$

- The models give rather similar yields of light nuclei.

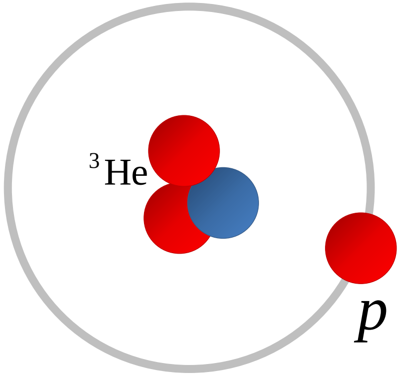
St. Mrówczyński, Acta Phys. Pol. B **48**, 707 (2017)

Thermal vs. Coalescence model

$^4\mathrm{He}$



$^4\mathrm{Li}$



$$r_{\mathrm{RMS}} = 1.68 \text{ fm}$$

$$\varepsilon_B = 28.3 \text{ MeV}$$

$$m = 3727.4 \text{ MeV}$$

$$s = 0$$

$$^4\mathrm{Li} \to {}^3\mathrm{He} + p$$

$$\Gamma = 6 \text{ MeV}$$

$$m = m_{^3\mathrm{He}} + m_p + 4.1 \text{ MeV}$$

$$m = 3749.7 \text{ MeV}$$

$$s = 2$$

St. Mrówczyński, Acta Phys. Pol. B **48**, 707 (2017)

Ratio of yields of ^{4}Li to ^{4}He

- Thermal model

$$\frac{\mathrm{Yield}(^4\mathrm{Li})}{\mathrm{Yield}(^4\mathrm{He})} = \frac{2S_{\mathrm{Li}}+1}{2S_{\mathrm{He}}+1} = 5$$

- Coalescence model

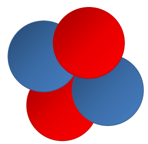
$$\frac{\mathrm{Yield}(^4\mathrm{Li})}{\mathrm{Yield}(^4\mathrm{He})} = \frac{W_{\mathrm{Li}}}{W_{\mathrm{He}}}$$

S. Bazak & St. Mrówczyński, arxiv: 1802.08212 [nucl-th]

Formation rates of ^{4}He & ^{4}Li

$$W = g_S g_I (2\pi)^9 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \int d^3\mathbf{r}_3 d^3\mathbf{r}_4 \, D(\mathbf{r}_1)D(\mathbf{r}_2)D(\mathbf{r}_3)D(\mathbf{r}_4)\left|\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2$$

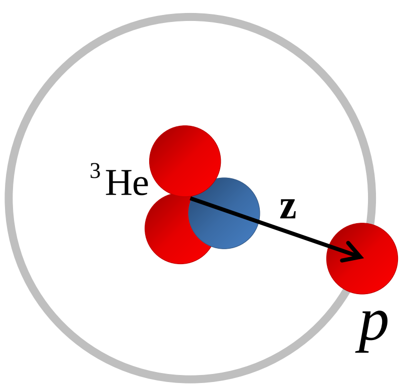
- ^{4}He



$$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

$$\left|\psi_{\mathrm{He}}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2 \sim \exp\left[-\alpha(\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2)\right]$$

- ^{4}Li



J. C. Bergstrom, Nucl. Phys. A **327**, 458 (1979)

$$\mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3)$$

$$\left|\psi_{\mathrm{Li}}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\right|^2 \sim \exp\left[-\beta(\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{23}^2)\right]\mathbf{z}^4 \exp\left(-\gamma\mathbf{z}^2\right)\left|Y_{lm}(\Omega_{\mathbf{z}})\right|^2$$

Formation rates of ^{4}He & ^{4}Li

Source function

$$D(\mathbf{r}_i) = \frac{1}{(2\pi R_s^2)^{3/2}} \exp\left(-\frac{\mathbf{r}_i^2}{2R_s^2}\right) \qquad i = 1, 2, 3, 4$$

If emission time included

$$R_s \to \sqrt{R_s^2 + v^2\tau^2}$$

Jacobi variables

$$\begin{cases} \mathbf{R} \equiv \frac{1}{4}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4) \\ \mathbf{x} \equiv \mathbf{r}_2 - \mathbf{r}_1 \\ \mathbf{y} \equiv \mathbf{r}_3 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2) \\ \mathbf{z} \equiv \mathbf{r}_4 - \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) \end{cases}$$

- $$\mathbf{r}_1^2 + \mathbf{r}_2^2 + \mathbf{r}_3^2 + \mathbf{r}_4^2 = 4\mathbf{R}^2 + \frac{1}{2}\mathbf{x}^2 + \frac{2}{3}\mathbf{y}^2 + \frac{3}{4}\mathbf{z}^2$$

- $$\mathbf{r}_{12}^2 + \mathbf{r}_{13}^2 + \mathbf{r}_{14}^2 + \mathbf{r}_{23}^2 + \mathbf{r}_{24}^2 + \mathbf{r}_{34}^2 = 2\mathbf{x}^2 + \frac{8}{3}\mathbf{y}^2 + 3\mathbf{z}^2$$

$$\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

Formation rates of ^{4}He & ^{4}Li

$$W_{\rm He} = \frac{\pi^{9/2}}{2^{9/2}} \frac{1}{\left(R_s^2 + R_\alpha^2\right)^{9/2}}$$

$$W_{\rm Li} = \frac{3\pi^{9/2}}{2^{11/2}} \begin{pmatrix} \frac{5}{2} \\ 1 \end{pmatrix} \frac{R_s^4}{\left(R_s^2 + \frac{1}{2} R_c^2\right)^3 \left(R_s^2 + \frac{4}{7} R_{\rm Li}^2 - \frac{3}{7} R_c^2\right)^{7/2}} \qquad \begin{pmatrix} l = 1 \\ l = 2 \end{pmatrix}$$

R_s – root mean square radius of the source

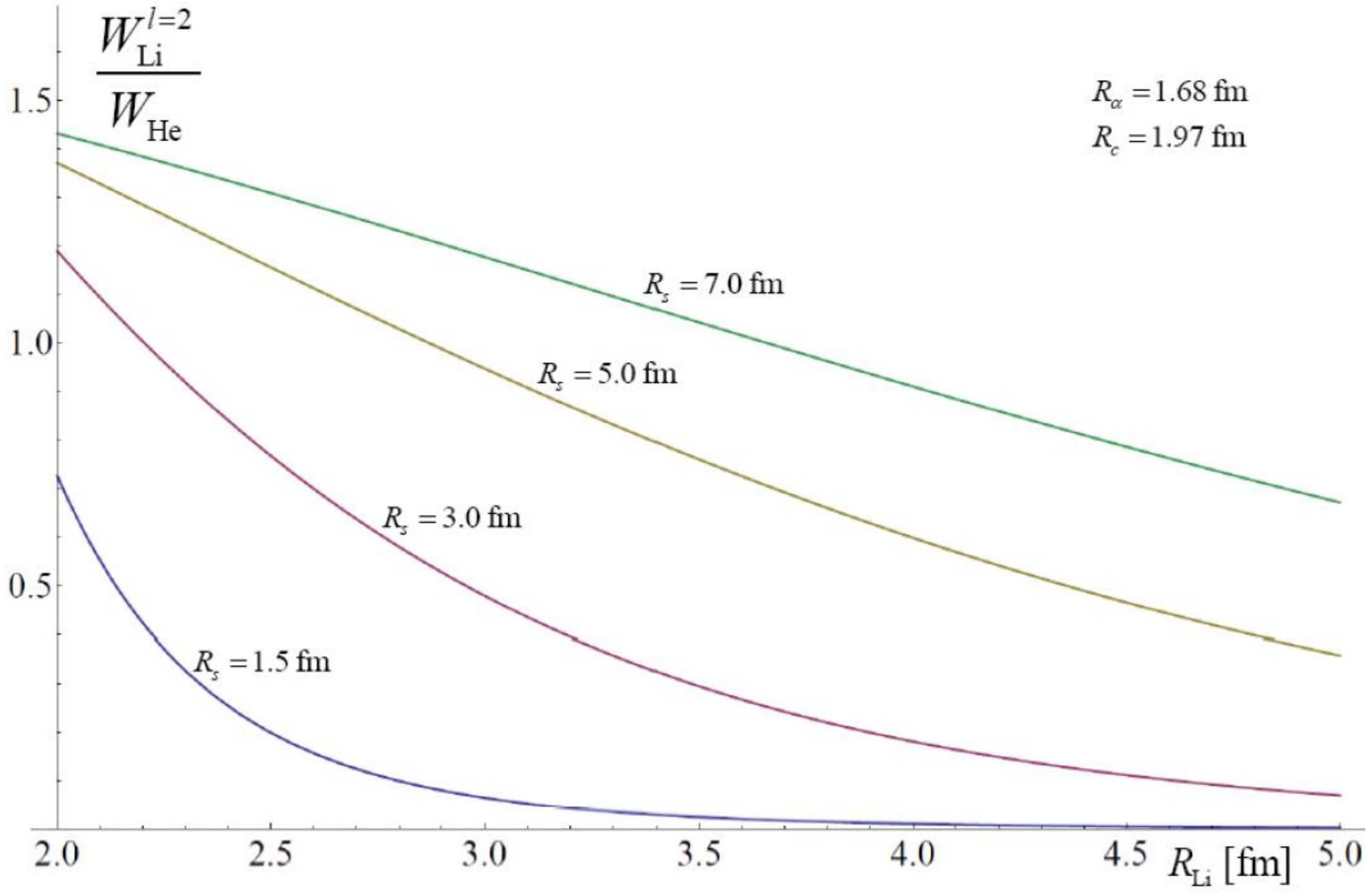
R_α – root mean square radius of ^{4}He

$R_{\rm Li}$ – root mean square radius of ^{4}Li

R_c – root mean square radius of ^{3}He cluster in ^{4}Li

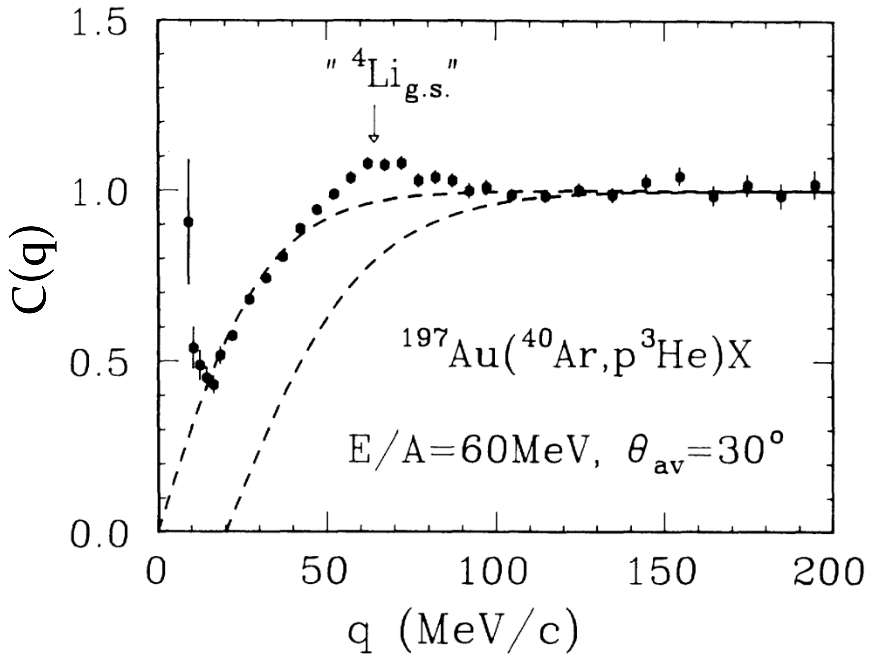
Ratio of yields of ^{4}Li to ^{4}He

In the thermal model the ratio equals 5.



How to observe ^{4}Li?

Measurement of the correlation function of ^{3}He-*p* is needed



J. Pochodzala et al. Phys. Rev. C **35**, 1695 (1987)

Conclusions & Outlook

- The thermal and coalescence models give different predictions on the ratio of yields of ^{4}Li to ^{4}He.

- In the thermal model the ratio of yields is rather independent of collision centrality.

- In the coalescence model the ratio is maximal for central collisions and rapidly decreases when one goes to peripheral collisions.

- Since ^{4}Li can be observed through the correlation function of ^{3}He-p, the correlation needs to be computed.