

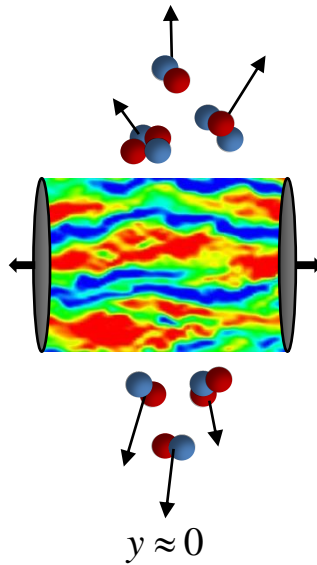
Coalescence model of production of light nuclei

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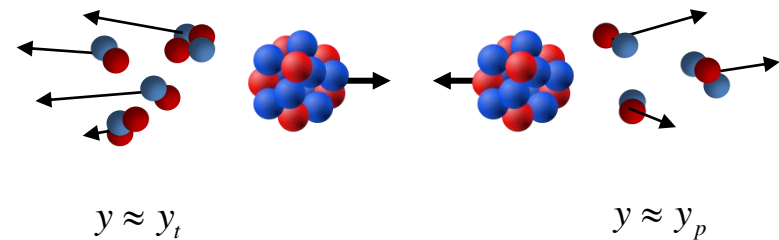
Two very different cases of producing light nuclei

Genuine production



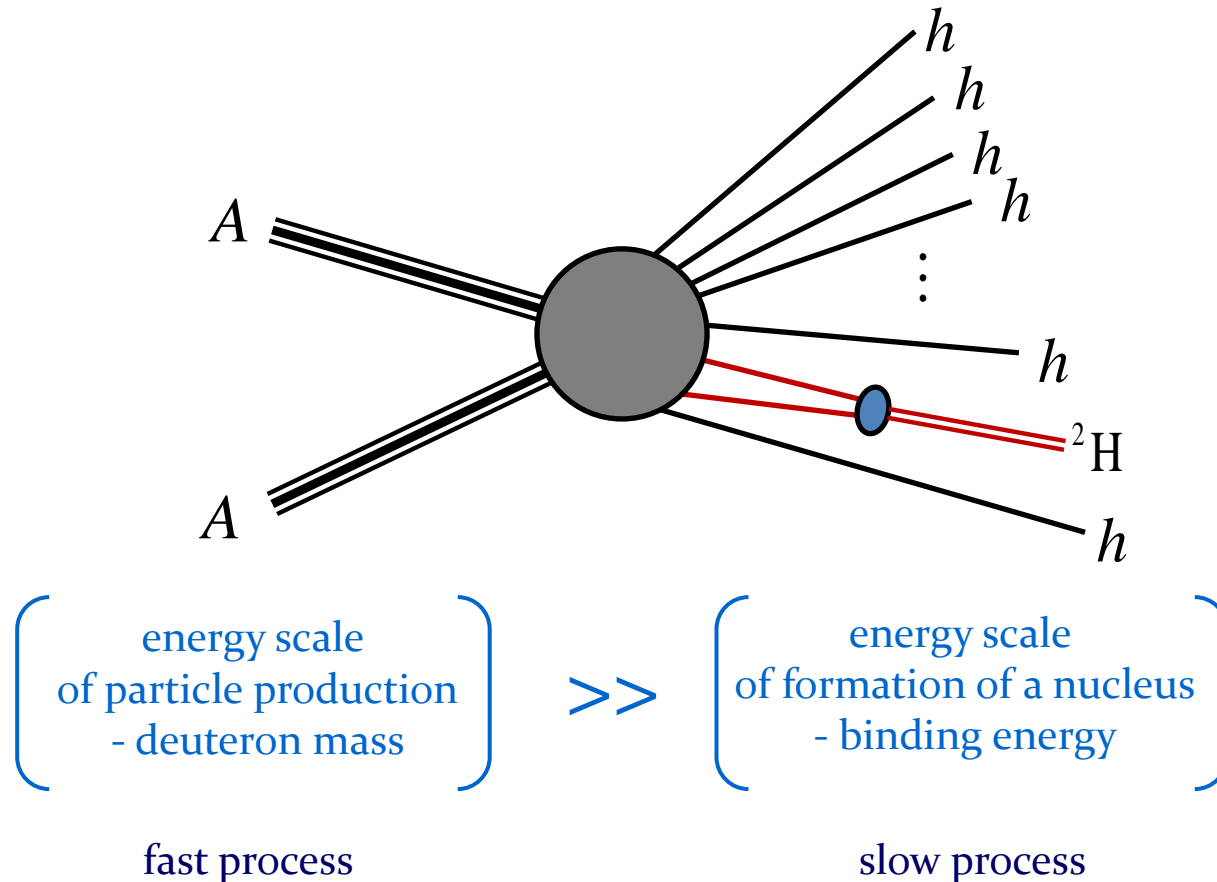
hard process

Shattering of incoming nuclei



soft process

Final state interaction



S.T. Butler & C.A. Pearson, Phys. Rev. **129**, 836 (1963)

A. Schwarzschild & C. Zupancic, Phys. Rev. **129**, 854 (1963)

Factorization of production of nucleons and formation of a deuteron

Deuteron yield

$$\frac{dN^D}{d^3\mathbf{P}_D} = \frac{1}{2} A_D \frac{dN^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p} \quad \frac{1}{2}\mathbf{P}_D = \mathbf{p}_n = \mathbf{p}_p$$

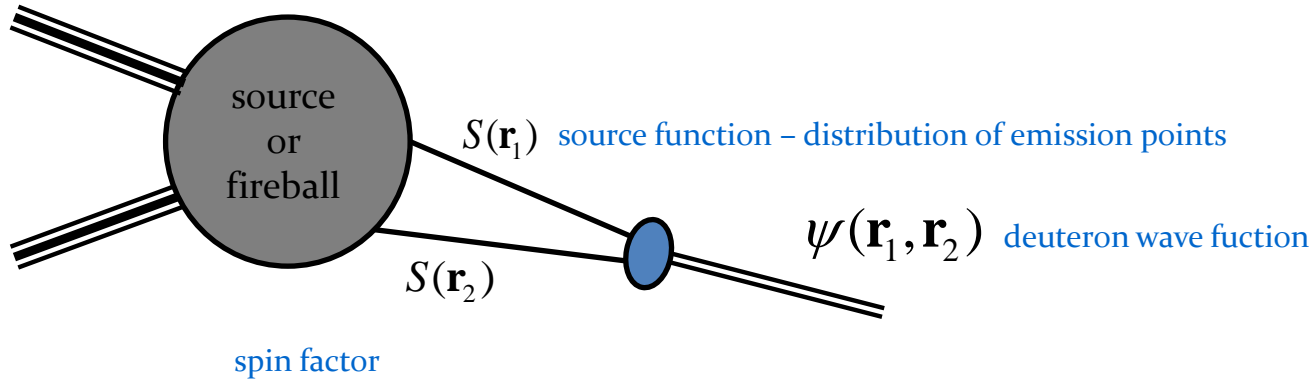
isospin factor
yield of np pairs

deuteron formation rate

$$\frac{1}{2} \frac{dN^{np}}{d^3\mathbf{p}_n d^3\mathbf{p}_p} \approx \frac{dN^{pp}}{d^3\mathbf{p}_p d^3\mathbf{p}_p} \approx \left(\frac{dN^p}{d^3\mathbf{p}_p} \right)^2$$

$$\frac{dN^D}{d^3\mathbf{P}_D} = A_D \left(\frac{dN^p}{d^3\mathbf{p}_p} \right)^2$$

Deuteron formation rate



$$A_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 S(\mathbf{r}_1) S(\mathbf{r}_2) |\psi(\mathbf{r}_1, \mathbf{r}_2)|^2$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{P}\cdot\mathbf{R}} \varphi_D(\mathbf{r})$$

$$A_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r} S_r(\mathbf{r}) |\varphi_D(\mathbf{r})|^2$$

$$S_r(\mathbf{r}) \equiv \int d^3\mathbf{R} S\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) S\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right) \quad \text{distribution of relative distance of } n \text{ and } p$$

Quantum-mechanical meaning of the formation rate formula

Sudden approximation

$$E\Delta t \ll 1$$

$\psi(\mathbf{r})$ $\varphi(\mathbf{r})$
 $\rho(\mathbf{r}', \mathbf{r})$ t_f *time*

Transition matrix element

$$M = \left| \int d^3\mathbf{r} \psi^*(\mathbf{r}) \varphi(\mathbf{r}) \right|^2 = \int d^3\mathbf{r} d^3\mathbf{r}' \varphi^*(\mathbf{r}') \underbrace{\psi(\mathbf{r}') \psi^*(\mathbf{r})}_{\rho(\mathbf{r}', \mathbf{r})} \varphi(\mathbf{r})$$

density matrix

$$M = \int d^3\mathbf{r} d^3\mathbf{r}' \varphi^*(\mathbf{r}') \rho(\mathbf{r}', \mathbf{r}) \varphi(\mathbf{r})$$

If density matrix is diagonal

$$\rho(\mathbf{r}', \mathbf{r}) = S(\mathbf{r}) \delta^{(3)}(\mathbf{r}' - \mathbf{r}) \quad \Rightarrow \quad M = \int d^3\mathbf{r} S(\mathbf{r}) |\varphi(\mathbf{r})|^2$$

Diagonal density matrix

$$\langle \psi | \hat{A} | \psi \rangle = \sum_{i,j} c_i^* c_j \langle \alpha_i | \hat{A} | \alpha_j \rangle = \sum_{i,j} \rho_{ji} A_{ij}$$

$$|\psi\rangle = \sum_i c_i |\alpha_i\rangle$$

$$\rho_{ji} \equiv c_i^* c_j \quad A_{ij} \equiv \langle \alpha_i | \hat{A} | \alpha_j \rangle$$

density matrix

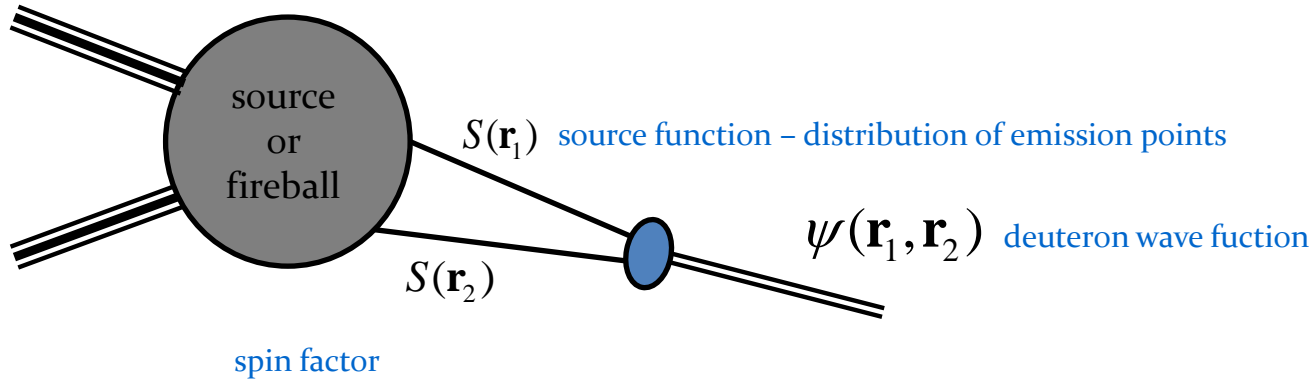
..... - averaging over time or events

$$\overline{\langle \psi | \hat{A} | \psi \rangle} = \sum_{i,j} \overline{c_i^* c_j} \langle \alpha_i | \hat{A} | \alpha_j \rangle = \sum_i |c_i|^2 A_{ii}$$

$$\overline{\rho_{ji}} = \overline{c_i^* c_j} = \delta^{ij} |c_i|^2 \quad \text{random phase approximation}$$

diagonal density matrix

Deuteron formation rate



$$A_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 S(\mathbf{r}_1) S(\mathbf{r}_2) |\psi(\mathbf{r}_1, \mathbf{r}_2)|^2$$

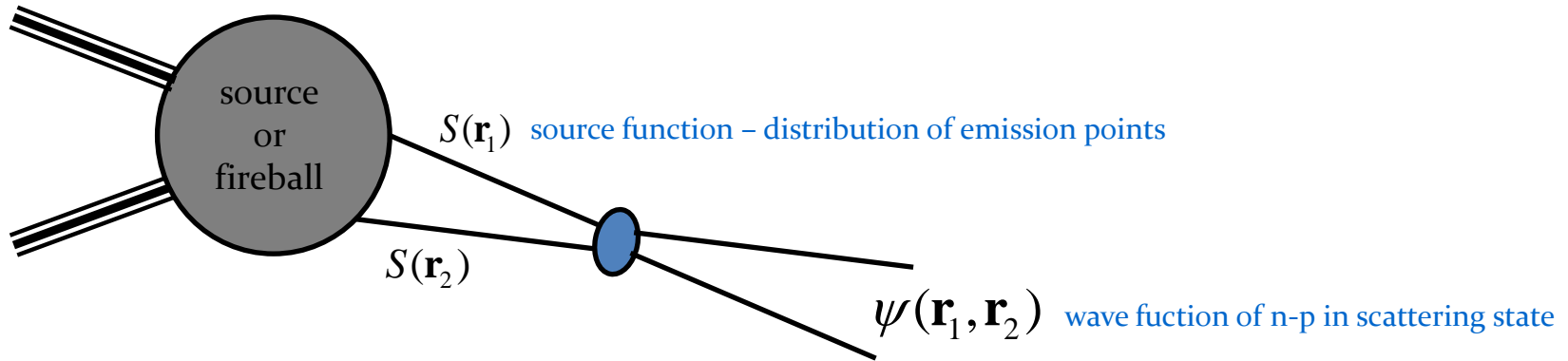
$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{P} \cdot \mathbf{R}} \varphi_D(\mathbf{r})$$

$$A_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r} S_r(\mathbf{r}) |\varphi_D(\mathbf{r})|^2$$

$$S_r(\mathbf{r}) \equiv \int d^3\mathbf{R} S\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) S\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right) \quad \text{distribution of relative distance of } n \text{ and } p$$

n-p correlation function



$$C(\mathbf{q}) = \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 S(\mathbf{r}_1) S(\mathbf{r}_2) \left| \psi_{\mathbf{q}}(\mathbf{r}_1, \mathbf{r}_2) \right|^2$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

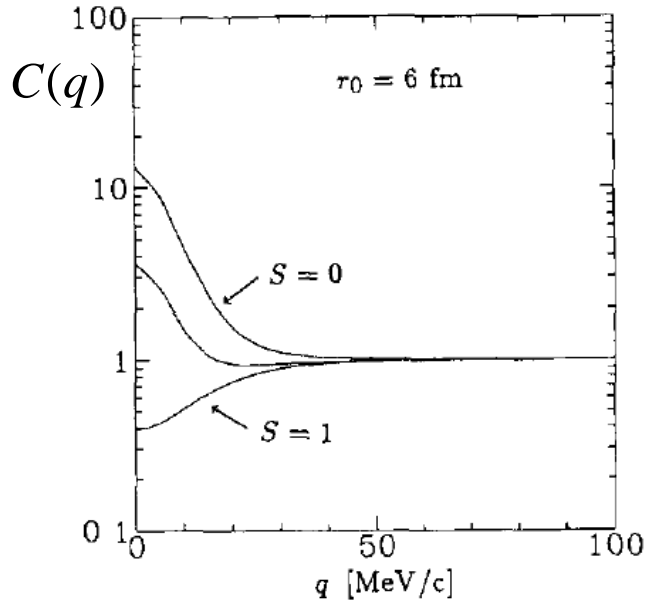
$$\psi_{\mathbf{q}}(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{P} \cdot \mathbf{R}} \varphi_{\mathbf{q}}(\mathbf{r})$$

$$C(\mathbf{q}) = \int d^3\mathbf{r} S_r(\mathbf{r}) \left| \varphi_{\mathbf{q}}(\mathbf{r}) \right|^2$$

$$S_r(\mathbf{r}) \equiv \int d^3\mathbf{R} S\left(\mathbf{R} - \frac{1}{2}\mathbf{r}\right) S\left(\mathbf{R} + \frac{1}{2}\mathbf{r}\right)$$

n-p correlation function

Sum rule due to completeness of quantum states

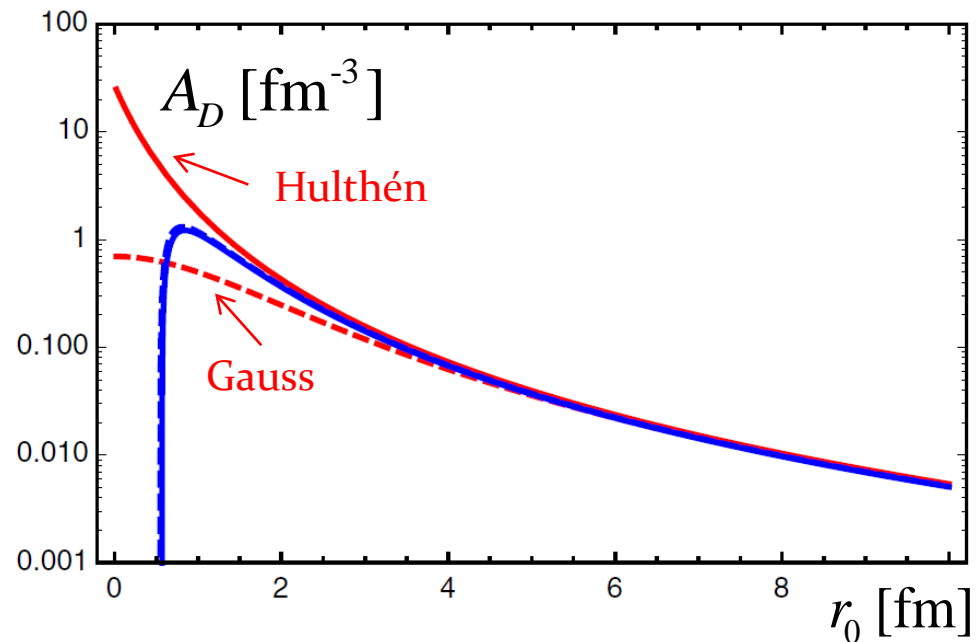


$$S(\mathbf{r}) = \left(\frac{1}{2\pi r_0^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2r_0^2} \right)$$

Lednický-Lyuboshitz formula

St. Mrówczyński, Phys. Lett. B **277**, 43 (1992)

$$\int d^3\mathbf{q} (C_1(\mathbf{q}) - C_0(\mathbf{q})) = -A_D$$



R. Maj & St. Mrówczyński, Phys. Rev. C **101**, 014901 (2020)

R. Maj & St. Mrówczyński, Phys. Rev. C **71**, 044905 (2005)

St. Mrówczyński, Phys. Lett. B **345**, 393 (1995)

Emission time

Instantaneous emission

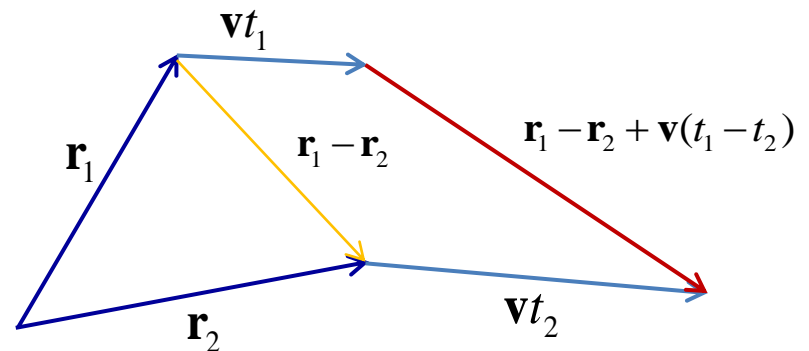
$$A_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 S(\mathbf{r}_1) S(\mathbf{r}_2) |\psi(\mathbf{r}_1, \mathbf{r}_2)|^2$$

Emission extended in time

$$A_D = \frac{3}{4} (2\pi)^3 \int dt_1 d^3\mathbf{r}_1 dt_2 d^3\mathbf{r}_2 S(t_1, \mathbf{r}_1) S(t_2, \mathbf{r}_2) |\psi(\mathbf{r}_1 + \mathbf{v}t_1, \mathbf{r}_2 + \mathbf{v}t_2)|^2$$

$$\int dt d^3\mathbf{r} S(t, \mathbf{r}) = 1$$

$$\mathbf{v} = \frac{\mathbf{P}_D}{E_D}$$



Emission time cont.

$$A_D = \frac{3}{4} (2\pi)^3 \int dt_1 d^3 \mathbf{r}_1 dt_2 d^3 \mathbf{r}_2 S(t_1, \mathbf{r}_1 - \mathbf{v}t_1) S(t_2, \mathbf{r}_2 - \mathbf{v}t_2) |\psi(\mathbf{r}_1, \mathbf{r}_2)|^2$$

$$\mathbf{R} \equiv \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$$

$$T \equiv \frac{1}{2}(t_1 + t_2), \quad t \equiv t_1 - t_2$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = e^{i\mathbf{P} \cdot \mathbf{R}} \varphi(\mathbf{r})$$

$$A_D = \frac{3}{4} (2\pi)^3 \int d^3 \mathbf{r} S_r(\mathbf{r}) |\varphi(\mathbf{r})|^2$$

$$S_r(\mathbf{r}) \equiv \int dt S_r(t, \mathbf{r} - \mathbf{v}t)$$

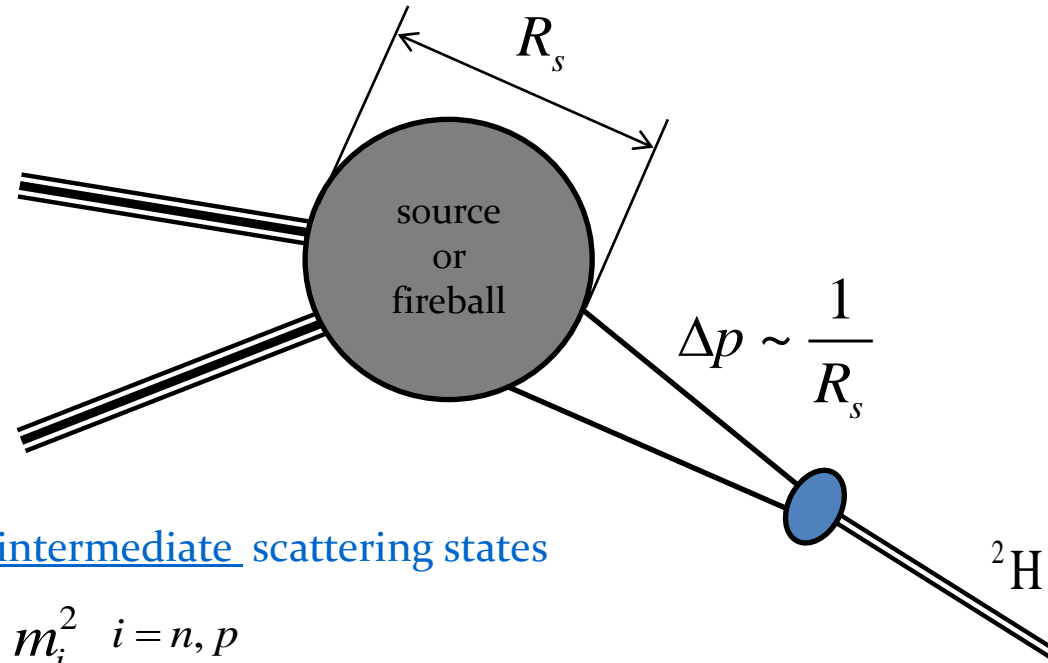
$$S_r(t, \mathbf{r}) \equiv \int dT d^3 \mathbf{R} S\left(T - \frac{1}{2}t, \mathbf{R} - \frac{1}{2}\mathbf{r}\right) S\left(T + \frac{1}{2}t, \mathbf{R} + \frac{1}{2}\mathbf{r}\right)$$

$$S(t, \mathbf{r}) = \left(\frac{1}{2\pi\tau^2}\right)^{1/2} \left(\frac{1}{2\pi R_s^2}\right)^{3/2} \exp\left(-\frac{t^2}{2\tau^2}\right) \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right)$$

$$S_r(\mathbf{r}) = \left(\frac{1}{2\pi(R_s^2 + v^2\tau^2)}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2(R_s^2 + v^2\tau^2)}\right)$$

$$R_s \rightarrow \sqrt{R_s^2 + v^2\tau^2}$$

Energy-momentum conservation



Nucleons are intermediate scattering states

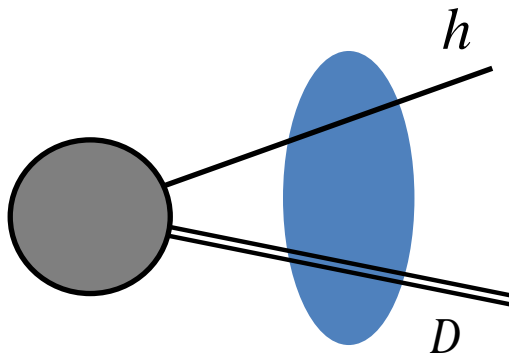
$$E_i^2 - \mathbf{p}_i^2 \neq m_i^2 \quad i = n, p$$

Energy-momentum conservation

$$\begin{cases} \mathbf{p}_p + \mathbf{p}_n = \mathbf{p}_D \\ E_p + E_n = E_D \end{cases}$$

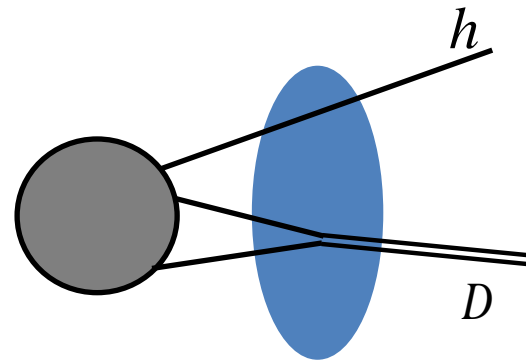
Hadron-deuteron correlations

Hadron-deuteron correlations carry information about a mechanism of deuteron production.



direct production

or



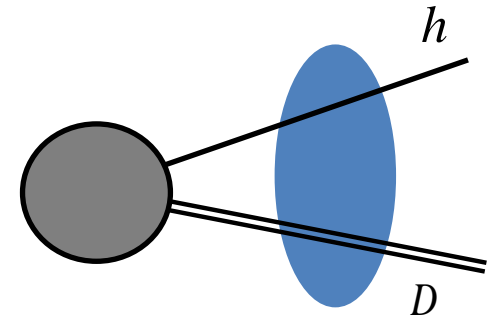
final state interaction

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = C(\mathbf{p}_h, \mathbf{p}_D) \frac{dN_h}{d\mathbf{p}_h} \frac{dN_D}{d\mathbf{p}_D}$$



Theoretical formula

$$C(\mathbf{p}_h, \mathbf{p}_D) = \int d^3r_h d^3r_D S(\mathbf{r}_h) S(\mathbf{r}_D) |\psi(\mathbf{r}_h, \mathbf{r}_D)|^2$$

distribution
of emission points

h - D wave function

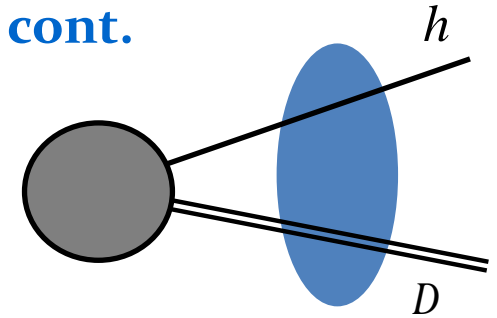
S.E. Koonin, Phys. Lett. B **70**, 43 (1977)

R. Lednicky and V.L. Lyuboshitz, Yad. Fiz. **35**, 1316 (1982)

Hadron-deuteron correlation function

1) Deuteron is treated as an elementary particle cont.

Separation of CM and relative motion



$$\left\{ \begin{array}{l} \mathbf{R} \equiv \frac{m_D \mathbf{r}_D + m_h \mathbf{r}_h}{m_D + m_h} \\ \mathbf{r} \equiv \mathbf{r}_D - \mathbf{r}_h \end{array} \right. \quad \psi(\mathbf{r}_h, \mathbf{r}_D) = e^{i\mathbf{P}\mathbf{R}} \phi_{\mathbf{q}}(\mathbf{r})$$

$$C(\mathbf{q}) = \int d^3 r S_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

„Relative” source function

$$S_r(\mathbf{r}) \equiv \int d^3 R S\left(\mathbf{R} - \frac{m_D}{m_D + m_h} \mathbf{r}\right) S\left(\mathbf{R} + \frac{m_h}{m_D + m_h} \mathbf{r}\right) = \left(\frac{1}{4\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R_s^2}\right)$$

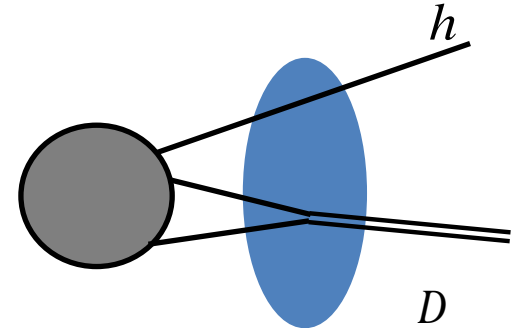
$$S(\mathbf{r}) = \left(\frac{1}{2\pi R_s^2}\right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R_s^2}\right)$$

Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton

Experimental definition

$$\frac{dN_{hD}}{d\mathbf{p}_h d\mathbf{p}_D} = C(\mathbf{p}_h, \mathbf{p}_D) A_D \frac{dN_h}{d\mathbf{p}_h} \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p}$$



Theoretical formula

$$C(\mathbf{p}_h, \mathbf{p}_D) A_D = \int d^3 r_h d^3 r_n d^3 r_p S(\mathbf{r}_h) S(\mathbf{r}_n) S(\mathbf{r}_p) |\psi_{hD}(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p)|^2$$

Deuteron formation rate

$$\frac{dN_D}{d\mathbf{p}_D} = A_D \frac{dN_n}{d\mathbf{p}_n} \frac{dN_p}{d\mathbf{p}_p} \quad \frac{1}{2} \mathbf{p}_D = \mathbf{p}_n = \mathbf{p}_p$$

$$A_D = \frac{3}{8} (2\pi)^3 \int d^3 \mathbf{r}_n d^3 \mathbf{r}_p S(\mathbf{r}_n) S(\mathbf{r}_p) |\psi_D(\mathbf{r}_n, \mathbf{r}_p)|^2 = \frac{3}{8} (2\pi)^3 \int d^3 r_{np} S_r(\mathbf{r}_{np}) |\phi_D(\mathbf{r}_{np})|^2$$

spin-isospin factor

$$\psi_D(\mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_D(\mathbf{r}_{np})$$

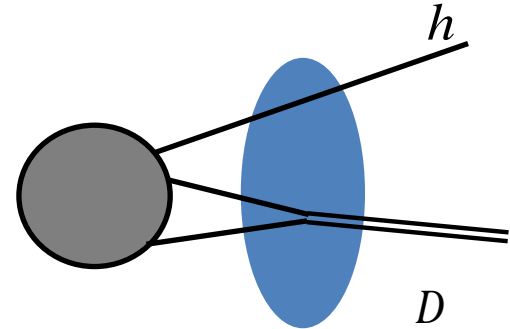
Hadron-deuteron correlation function

2) Deuteron is treated as a bound state of neutron and proton cont

Separation of CM and relative motion

$$\left\{ \begin{array}{l} \mathbf{R} \equiv \frac{m_p \mathbf{r}_p + m_n \mathbf{r}_n + m_h \mathbf{r}_h}{m_p + m_n + m_h} \\ \mathbf{r}_{np} \equiv \mathbf{r}_p - \mathbf{r}_n \\ \mathbf{r} \equiv \mathbf{r}_h - \frac{m_p \mathbf{r}_p + m_n \mathbf{r}_n}{m_p + m_n} \end{array} \right.$$

$$\psi(\mathbf{r}_h, \mathbf{r}_n, \mathbf{r}_p) = e^{i\mathbf{P}\mathbf{R}} \phi_{\mathbf{q}}(\mathbf{r}) \varphi_D(\mathbf{r}_{np})$$



$$C(\mathbf{q}) = \frac{1}{A_D} \int d^3 R d^3 r_{np} d^3 r S(\mathbf{r}_h) S(\mathbf{r}_n) S(\mathbf{r}_p) |\phi_{\mathbf{q}}(\mathbf{r})|^2 |\varphi_D(\mathbf{r}_{np})|^2$$

For Gaussian source

$$C(\mathbf{q}) = \int d^3 r S_{3r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

$$S_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2} \right)$$

For a non-Gaussian source, A_D remains in the correlation function!

Direct vs. final state interaction

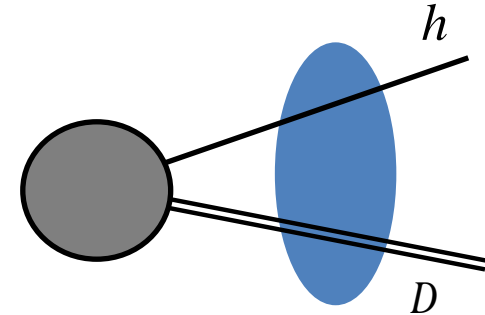
Direct production

$$C(\mathbf{q}) = \int d^3r S_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

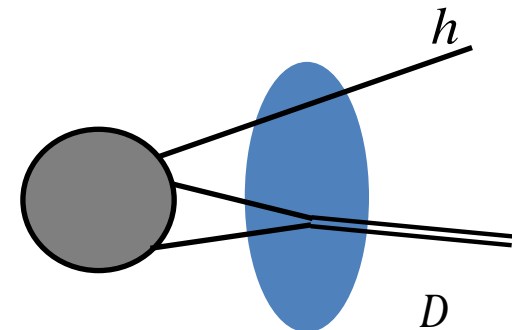


$$S_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2} \right)$$

$$S_{3r}(\mathbf{r}) = \left(\frac{1}{3\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{3R^2} \right)$$



$$\sqrt{\frac{4}{3}} \approx 1.15$$

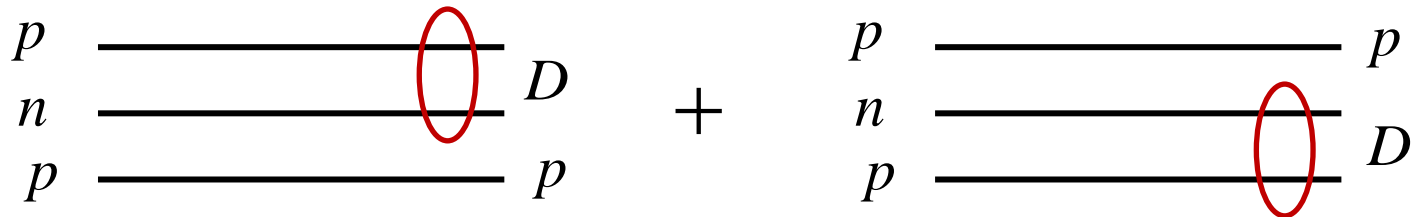


Final state interaction

$$C(\mathbf{q}) = \int d^3r S_{3r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$

p - D correlation function

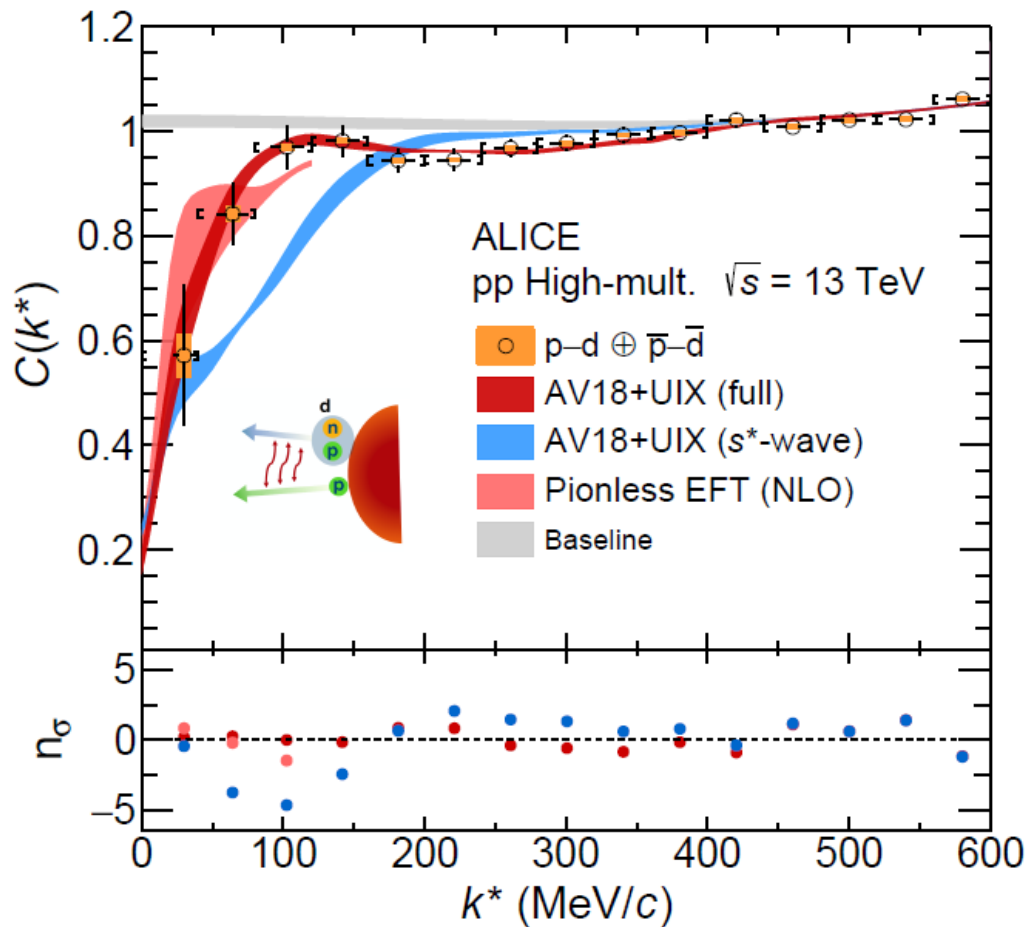
$$\psi_{pD}^{\mathbf{q}}(\mathbf{r}_n, \mathbf{r}_{p_1}, \mathbf{r}_{p_2})$$



Full three-body calculations

$$C(\mathbf{q}) = \frac{1}{A_D} \int d^3 r_n d^3 r_{p_1} d^3 r_{p_2} S(\mathbf{r}_n) S(\mathbf{r}_{p_1}) S(\mathbf{r}_{p_2}) \left| \psi_{pD}^{\mathbf{q}}(\mathbf{r}_n, \mathbf{r}_{p_1}, \mathbf{r}_{p_2}) \right|^2$$

p - D correlation function



$$R_s = 1.43 \pm 0.16 \text{ fm}$$

ALICE arXiv:2308.16120

M. Viviani et al, Phys. Rev. C **108**, 064002 (2023)

Deuteron-deuteron correlation function

Direct production

$$C(\mathbf{q}) = \int d^3r S_r(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$



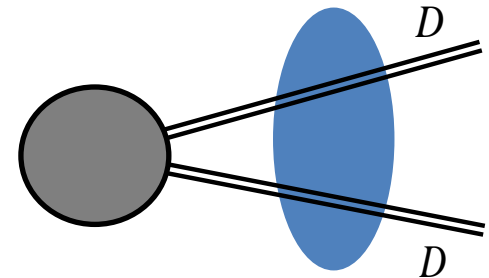
$$S_r(\mathbf{r}) = \left(\frac{1}{4\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{4R^2} \right)$$

$$S_{4r}(\mathbf{r}) = \left(\frac{1}{2\pi R^2} \right)^{3/2} \exp\left(-\frac{\mathbf{r}^2}{2R^2} \right)$$

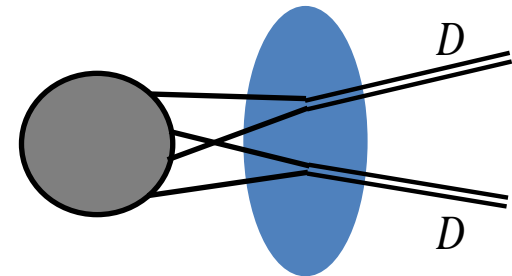
Final state interaction
& factorization



$$C(\mathbf{q}) = \int d^3r S_{4r}(\mathbf{r}) |\phi_{\mathbf{q}}(\mathbf{r})|^2$$



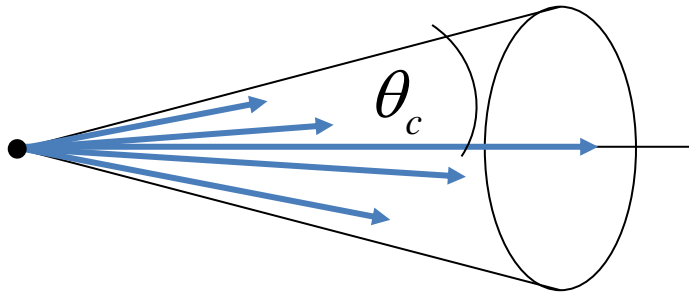
$$\sqrt{2} \approx 1.41$$



Jet-associated deuteron production

ALICE Collaboration, Phys. Lett. B 819, 136440 (2021)

ALICE Collaboration, Phys. Rev. Lett. 131, 042301 (2023)



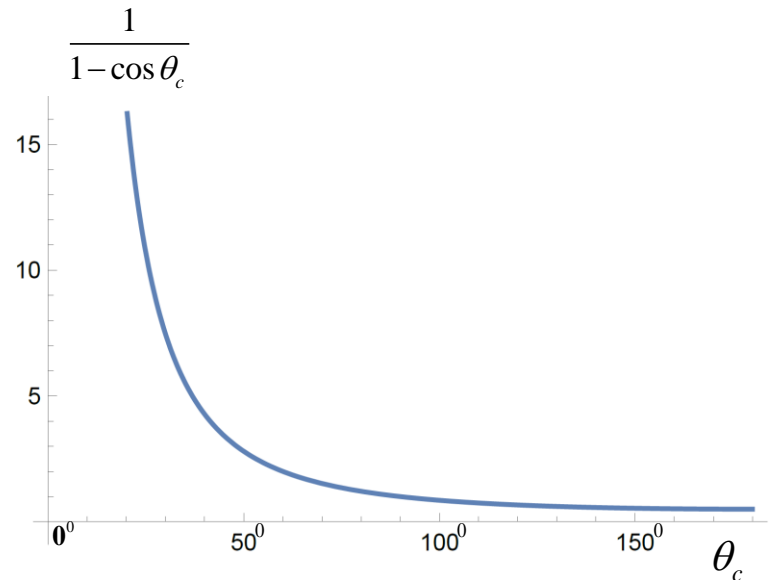
$$\frac{dN_D}{d^3\mathbf{P}_D} = A_D \left(\frac{dN^p}{d^3\mathbf{p}_p} \right)^2 \quad \mathbf{P}_D = 2\mathbf{p}_p$$

$$\frac{dN_p}{d^3\mathbf{p}} = N_p \frac{e^{-\alpha p}}{\pi\alpha^3} \frac{\Theta(\cos\theta - \cos\theta_c)}{1 - \cos\theta_c}$$

$$\int d^3\mathbf{p} \frac{dN_p}{d^3\mathbf{p}} = N_p$$

Deuteron yield

$$N_D = \int d^3\mathbf{p} \frac{dN_D}{d^3\mathbf{p}} = N_p^2 A_D \frac{2}{\pi\alpha^3} \frac{1}{1 - \cos\theta_c}$$



Jet-associated deuteron production

$$\frac{dN_D}{d^3\mathbf{P}_D} = A_D \left(\frac{dN^p}{d^3\mathbf{p}_p} \right)^2 \quad \mathbf{P}_D = 2\mathbf{p}_p$$

$$E_D \frac{dN_D}{d^3\mathbf{P}_D} = B_2 \left(E_p \frac{dN^p}{d^3\mathbf{p}_p} \right)^2 \quad E_D = 2E_p$$

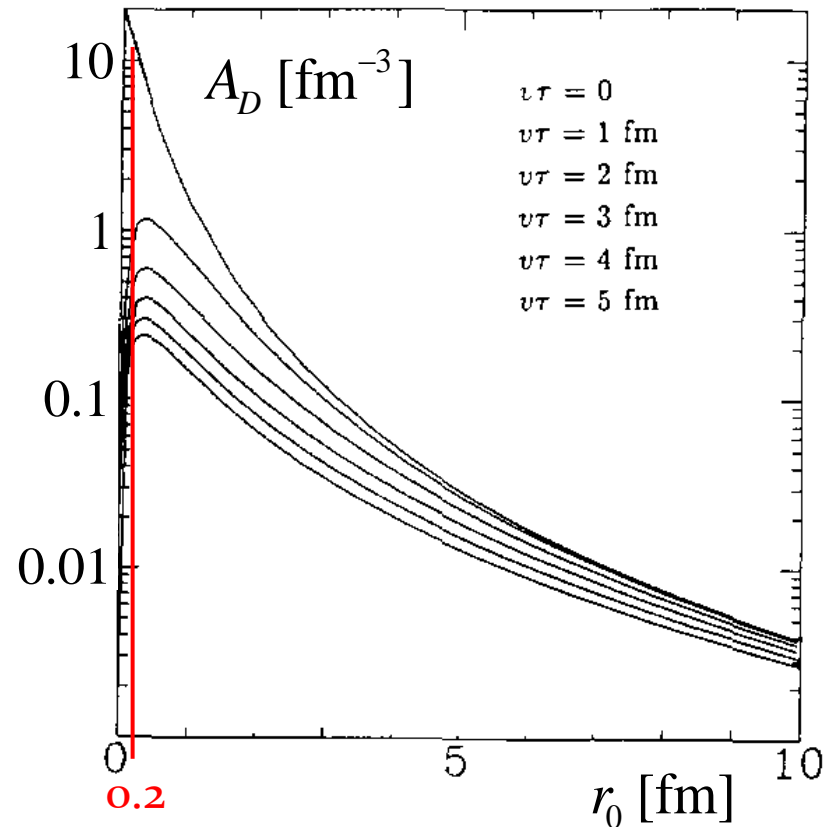
$$B_2 \approx 0.4 \pm 0.2 \text{ GeV}^2$$

$$A_D = \frac{1}{2} B_2 m$$

$$A_D \approx 24 \pm 12 \text{ fm}^{-3}$$

Hulthén wave function

$$\phi_D(r) = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}} \frac{\exp(-\alpha r) - \exp(-\beta r)}{r}$$



St. Mrówczyński, Phys. Lett. B 277, 43 (1992)

Jet-associated deuteron production

$$A_D = \frac{3}{4} (2\pi)^3 \int d^3\mathbf{r} S_r(\mathbf{r}) |\varphi(\mathbf{r})|^2 \approx \frac{3}{4} (2\pi)^3 |\varphi(r=0)|^2 \int d^3\mathbf{r} S_r(\mathbf{r})$$

$$r_0 \ll r_D$$

$$A_D = \frac{3}{4} (2\pi)^3 |\varphi(r=0)|^2 = 3\pi^2 \alpha\beta(\alpha + \beta) \approx 20.2 \text{ fm}^{-2}$$

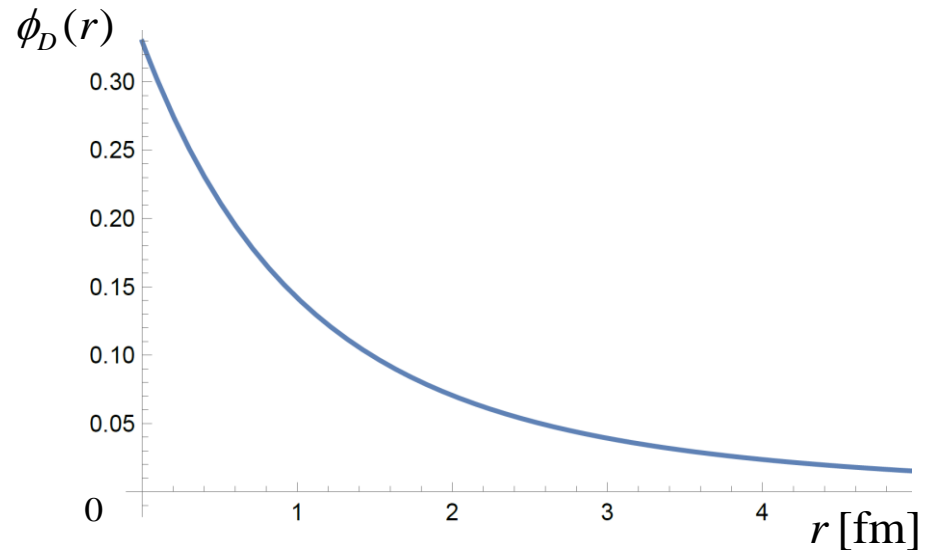
$$r_0 < 0.2 \text{ fm}$$

$$\text{Exp: } A_D \approx 24 \pm 12 \text{ fm}^{-3}$$

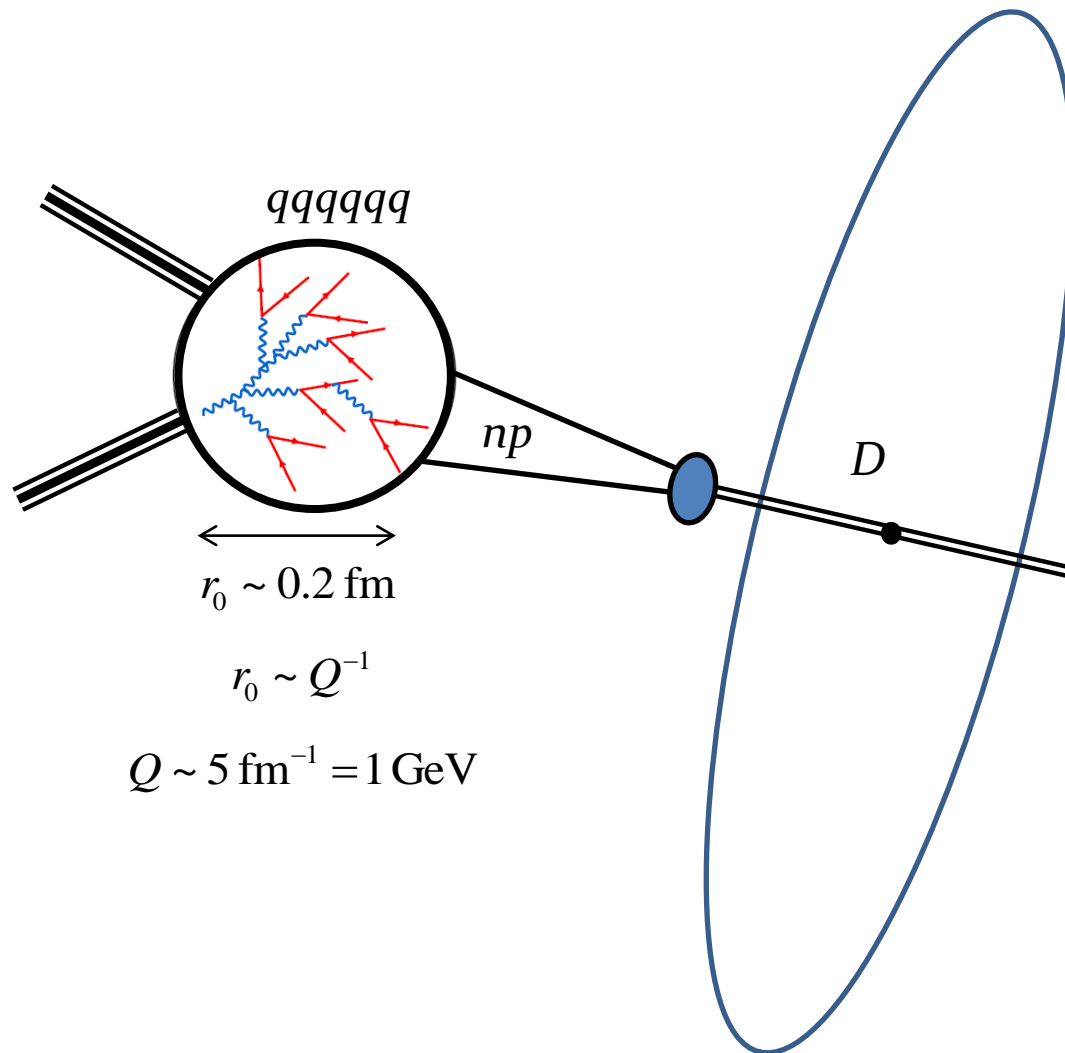
Hulthén wave function

$$\phi_D(r) = \sqrt{\frac{\alpha\beta(\alpha + \beta)}{2\pi(\alpha - \beta)^2}} \frac{\exp(-\alpha r) - \exp(-\beta r)}{r}$$

$$\alpha = 0.23 \text{ fm}^{-1}, \quad \beta = 1.61 \text{ fm}^{-1}$$



Jet-associated deuteron production



Big bound states from small sources

positronium

$$\pi^0 \rightarrow \gamma (e^+ e^-)$$

L. G. Afanasev et al., Phys. Lett. B **236**, 116 (1990)

pionium

$$p\text{Be} \rightarrow X (\pi^+ \pi^-)$$

DIRAC Collaboration, Phys. Rev. Lett. **122**, 082003 (2019)

$\pi \mu$ atom

$$K_L^0 \rightarrow (\pi^\pm \mu^\mp) \nu_\mu$$

$$\frac{r_B}{r_0} \sim 10^5$$

S. H. Aronson et al., Phys. Rev. D **33**, 3180 (1986)